The New Area-Wide Model II:
An Updated Version of the ECB’s Micro-Founded Model for Forecasting and Policy Analysis with a Financial Sector

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* The views expressed should not be interpreted as reflecting the views of the European Central Bank.
Motivation

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- There is a pressing need to facilitate the analysis of the quantitative effects of NSMs by developing coherent *structural macroeconomic modelling frameworks*. 
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- Asset price reactions suggest that these NSMs had expansionary effects but the quantitative impact on other macroeconomic variables remains uncertain.

- There is a pressing need to facilitate the analysis of the quantitative effects of NSMs by developing coherent structural macroeconomic modelling frameworks.

- Standard DSGE models are silent on the transmission channels of NSMs or, more generally, on the role of financial frictions and the propagation of financial disturbances.
What we do

- Depart from the estimated baseline version of the ECB’s New Area-Wide Model (NAWM):
  - stylised financial frictions in the form of risk premium shocks
  - no *endogenous* financial intermediation frictions
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  - stylised financial frictions in the form of risk premium shocks
  - no *endogenous* financial intermediation frictions
- Extend the baseline NAWM with a rich financial intermediary sector:
  - study how shocks originating in the financial sector transmit to the real economy and contribute to business-cycle fluctuations
  - account for the prominent role of bank lending rates in the transmission of monetary policy operations
  - allow NSMs to have a meaningful role in affecting the economy
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  - account for the prominent role of bank lending rates in the transmission of monetary policy operations
  - allow NSMs to have a meaningful role in affecting the economy

- Conduct some policy counterfactuals and, especially, simulate the macroeconomic impact of large-scale asset purchases.
Outline

1. The model
2. Bayesian estimation
3. Inspecting the model
   - Transmission mechanisms
   - Historical decompositions
4. Effects of asset purchases
5. Conclusion
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The baseline version of the NAWM

- The baseline NAWM is a small-open-economy extension of the Smets-Wouters (2003, 2007) model, designed for forecasting and policy analysis at the ECB:
  - agents: households, (intermediate and final-good) firms, central bank and fiscal authority
  - real and nominal frictions: habit formation, adjustment costs, sticky prices and wages, limited exchange-rate pass-through, ...
  - financial frictions: (exogenous) domestic and external risk premia
  - Rest-of-the-World block (SVAR)

- Estimated on 18 macro time series employing Bayesian inference methods.

The financial extension of the NAWM: A bird’s eye view

- Financial intermediaries (“banks”) engage in maturity transformation:
  - banks offer long-term loans to the private sector to finance investment projects and hold long-term government bonds (Carlstrom-Fuerst-Paustian, 2014)
  - banks fund these assets with short-term household deposits and with their equity/net worth (accumulated through retained earnings)

- Imperfect financial markets:
  - the option to abscond (“agency problem”) limits the leverage of banks (Gertler-Karadi, 2011 and 2013)
  - banks’ capital position influences the transmission of shocks (“financial accelerator” mechanism)
  - foreign trade is intermediated by banks
The financial extension of the NAWM: A bird’s eye view

- Central bank can purchase long-term private sector loans and/or government bonds:
  - relief of banks’ balance sheets/leverage constraints (“stealth recapitalisation”) and improvement of lending conditions
  - banks’ holdings of foreign currency-denominated bonds accounts for exchange-rate channel of asset purchases

- Other key elements:
  - bank-based intermediation with delayed pass-through to lending rates (Gerali-et-al., 2011)
  - presence of nominal debt gives rise to a debt-deflation channel
Other extensions of the NAWM

- Non-zero import content of exports
- Kimball aggregator in deriving price Phillips curves (real rigidity)
- Partial indexation of wages to trend productivity in deriving wage Phillips curve
- Uncertainty about persistent shifts in trend productivity (signal extraction problem) linked to external measure of potential output
- Time-varying equilibrium real interest rate in monetary policy rule linked to perceived trend productivity growth
- Time-varying misperceptions on part of private agents about central bank’s inflation objective
The financial extension in some detail: Market for loans

- Demand for loans: loans are taken out by households

- Supply of loans: loans are originated by wholesale banks and distributed by retail banks

- Loan characteristics (at the retail level): nominal consoles with geometrically decaying coupons (as in Woodford, 2001):
  - loan value: \( Q_{I,t} B_{I,t+1} \)
  - loan rate (“yield to maturity”):
    \[
    R_{I,t} = \frac{1}{Q_{I,t}} + q_I,
    \]
    where \( Q_{I,t} \) is the “discount price” of the loan \( B_{I,t+1} \) and \( q_I \) denotes the decay factor of the coupons
The financial extension in some detail: Demand for loans

- Investments by household $h$ require new bank loans (“loan-in-advance (LIA) constraint”, Carlstrom-Fuerst-Paustian, 2014):

$$ P_{C,t} \tilde{p}_{I,t} I_{h,t} \leq Q_{I,t} (B_{I,t+1} - \rho_{I} B_{I,t}) , $$

where $\tilde{p}_{I,t}$ is the relative price of the investment good obtained from the capital good producer.

- Value of the loan for household $h$:

$$ \beta \mathbb{E}_t \left[ \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} \cdot \frac{P_{C,t}}{P_{C,t+1}} \cdot \frac{1 + \rho_{I} Q_{I,t+1} (1 + \varsigma_{h,t+1})}{Q_{I,t}(1 + \varsigma_{h,t})} \right] = 1 $$

$$ \tilde{p}_{I,t} (1 + \varsigma_{h,t}) = Q_{h,t} \epsilon_{t}^{I}, $$

where $\varsigma_{h,t} \Lambda_{h,t}/P_{C,t}$ is the Lagrange multiplier associated with the household’s LIA constraint and $Q_{h,t}$ represents Tobin’s $Q$. 


The financial extension in some detail: Supply of loans

- Funding-constrained wholesale banks (Gertler-Karadi, 2013):
  - originate long-term loans and hold domestic and foreign long-term government bonds, with expected excess returns s.t. no-arbitrage conditions
  - face a leverage constraint $\Phi_t$ reflecting the assumption that they can abscond with a portion $\Psi$ of loans, and a portion $\omega_L \Psi (\omega_{L,t}^*, \Psi)$ of domestic (foreign) government bonds
  - cannot raise equity, but only rebuild net worth $NW_t$ through retained earnings
    - die with probability $1 - \theta_t$
    - new banks receive startup funds $\Theta$

- Monopolistically competitive retail banks (Gerali-et-al., 2011): distribute loans and adjust loan rates sluggishly
The financial extension in some detail: Asset purchases

- In the full model, also households are allowed to hold domestic government bonds s.t. portfolio adjustment costs:

\[
\Gamma^h_L(B^p_{L,h,t}) = \frac{1}{2} \gamma^h_L \left( B^p_{L,h,t} - \overline{B}^p_{L,h} \right)^2 / \overline{B}^p_{L,h}
\]

- The central bank can purchase loans and government bonds from wholesale banks s.t. aggregate supply:

\[
\begin{align*}
B^p_{I,t} + B^g_{I,t} &= B_{I,t} \\
B^p_{L,t} + B^g_{L,t} &= B_{L,t} \\
B^*_{L,t} + B^*_{L,t} &= B^*_{L,t},
\end{align*}
\]

where superscript \( p \) (\( g \)) denotes assets held privately (publicly)

- Asset purchases (against issuance of short-term excess reserves) ease wholesale banks’ leverage constraint: they can extend more loans
Outline

1. The model

2. Bayesian estimation

3. Inspecting the model
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   - Historical decompositions

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The data

- Macro data:
  - 18 macro time series of baseline NAWM
  - Source: Area-Wide Model database

- Financial data:
  - 10-year government bond yields for the euro area (AAA) and for the United States, and a composite euro area long-term lending rate
  - Source: ECB’s Statistical Data Warehouse, Bundesbank, FRED

- Survey data:
  - long-term inflation expectations, and long-term growth expectations
  - Source: ECB’s Survey of Professional Forecasters (SPF)

- Measure of the euro area output gap; source: AMECO database

- Estimation sample extending from 1985 to 2014
The financial, survey and output gap data

Note: The euro area 10-year government bond yield series (AAA) is available from 2004Q3 onwards, while the earlier observations concern the German 10-year government bond yield. The euro area long-term lending rate is available from 2003Q1 onwards and covers (new business) lending with an original maturity of over 1 year to households for house purchases and to non-financial corporations.
Selection of calibrated financial sector parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Wholesale banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Leverage ratio</td>
<td>6.00</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival rate</td>
<td>0.95</td>
</tr>
<tr>
<td>B. Retail banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi^I$</td>
<td>Mark-down parameter</td>
<td>0.987</td>
</tr>
<tr>
<td>C. Duration of assets</td>
<td>$= 1/(4(1 - \frac{\varphi}{R}))$</td>
<td></td>
</tr>
<tr>
<td>$\varphi_I$</td>
<td>Decay parameter: loans</td>
<td>0.974</td>
</tr>
<tr>
<td>$\varphi_L$</td>
<td>Decay parameter: dom. gov. bonds</td>
<td>0.976</td>
</tr>
<tr>
<td>$\varphi^*_L$</td>
<td>Decay parameter: foreign gov. bonds</td>
<td>0.976</td>
</tr>
<tr>
<td>D. Spreads (in annualised percentage points)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_I - R$</td>
<td>Retail loan rate over deposit rate</td>
<td>2.17</td>
</tr>
</tbody>
</table>
Selection of estimated financial sector parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mode</td>
</tr>
<tr>
<td>A. Relative absconding rates for wholesale banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_L$</td>
<td>gamma(0.72,0.05)</td>
<td>0.805</td>
</tr>
<tr>
<td>$\tilde{\omega}_L^*$</td>
<td>gamma(1.00,0.05)</td>
<td>0.977</td>
</tr>
<tr>
<td>B. Loan price re-set parameter for retail banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_I$</td>
<td>beta(0.75,0.0375)</td>
<td>0.742</td>
</tr>
<tr>
<td>C. Households’ portfolio adjustment parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^h_L$</td>
<td>gamma(0.01,0.0025)</td>
<td>0.009</td>
</tr>
<tr>
<td>D. Autoregressive parameters of financial shock processes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>beta(0.75,0.05)</td>
<td>0.824</td>
</tr>
<tr>
<td>$\rho_{\varphi^I}$</td>
<td>beta(0.75,0.05)</td>
<td>0.718</td>
</tr>
</tbody>
</table>
Checking the posterior mode estimates

Note: This slide shows plots of the conditional log posterior densities (blue solid lines) around the estimated posterior mode of the model’s (transformed) parameters, along with the (scaled) conditional log-likelihood functions (red dashed lines). Since some of the model parameters have bounded support, the original parameters have been transformed for estimation purposes such that the domain of the transformed parameters is the real line. To avoid scaling problems, the value of the log prior at the mode has been added to the log-likelihood.
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Transmission of an interest rate shock

Note: This slide depicts the impulse responses of selected domestic variables to an interest rate shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the model's non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.
Transmission of an interest rate shock (cont’d)

Note: See above.
Transmission of a shock to the survival rate $\theta$

Note: This slide depicts the impulse responses of selected domestic variables to a shock to the wholesale banks' survival rate equal to one standard deviation. All impulse responses are reported as percentage deviations from the model's non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.
Transmission of a shock to $\theta$ (cont’d)

Note: See above.
Transmission of a shock to the mark-down parameter $\varphi^I$

Note: This slide depicts the impulse responses of selected domestic variables to the retail banks’ mark-down parameter equal to one standard deviation. All impulse responses are reported as percentage deviations from the model’s non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.
Transmission of a shock to $\varphi^I$ (cont’d)

Note: See above.
Historical decomposition of real GDP growth

Note: The upper panel in this slide depicts the decomposition of annual real GDP growth into the contributions of the model’s shocks which are classified into 7 groups: technology, demand, markup, foreign, financial and perception shocks, plus a monetary policy shock. The decomposition is shown in deviation from the model-implied mean real GDP growth rate using the posterior mode estimates of the model parameters. The lower panel shows the contributions of the individual financial sector shocks in the real GDP growth decomposition.
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Simulating the effects of asset purchases

- Asset Purchase Programme (APP) as announced in January 2015:
  - 11% of GDP, 8-year maturity (9% in “ten-year equivalents”)
  - 20% AAA government bonds, 80% “risky” assets
  - hump-shaped, peaks in 2 years’ time, exits as bonds mature

- Alternative calibration:
  - higher fraction of AAA government bonds: 70% (vs. 20%)
Simulating the effects of asset purchases (cont’d)

Note: This slide depicts the impulse responses of selected domestic variables to an asset purchase shock for the baseline and for the alternative calibration. All impulse responses are reported as percentage deviations from the model’s non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.
Effects with unchanged interest rate

Note: This slide depicts the impulse responses of selected domestic variables to an asset purchase shock for the baseline calibration with interest rates unchanged for 4 quarters. All impulse responses are reported as percentage deviations from the model’s non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.
Impact of the exchange rate channel

Note: This slide depicts the impulse responses of selected domestic variables to an asset purchase shock for the baseline calibration and in a model version without the exchange rate channel but with a standard UIP condition. All impulse responses are reported as percentage deviations from the model’s non-stochastic balanced growth path, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations.
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Conclusion

- There exist a variety of frameworks for analysing the effects of the ECB’s NSMs – reflecting their specific purpose and nature.

- The extension of the NAWM with a financial sector goes a long way towards providing a rich and detailed quantitative framework:
  - central bank asset purchases alleviate the funding constraints of banks and operate through domestic and external channels
  - central bank refinancing operations, which offer long-term funding to banks, have comparable effects
  - emphasis on realistic bank-based financing with gradual lending-rate pass-through

- Preliminary findings regarding the macroeconomic impact of the ECB’s asset purchases are broadly in line with alternative estimates.
Background slides
<table>
<thead>
<tr>
<th>Financial frictions</th>
<th>(E)APP</th>
<th>(T)LTRO</th>
<th>OMT</th>
<th>SMP</th>
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<tbody>
<tr>
<td>Imperfect financial markets, funding constraint</td>
<td>X</td>
<td>(X)</td>
<td>X</td>
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<tr>
<td>Maturity transformation, nominal debt</td>
<td>X</td>
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<td>X</td>
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<tr>
<td>Interbank frictions</td>
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<td>X</td>
<td></td>
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<tr>
<td>Multiple equilibria in sovereign financing</td>
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<td></td>
<td>X</td>
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<tr>
<td>Bank run equilibria</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Inability to commit to future interest rates</td>
<td>X</td>
<td>X</td>
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</tbody>
</table>
Baseline NAWM: Agents

- **Households**: consume, accumulate physical capital, supply differentiated labour services, set wages in monopolistically competitive markets, trade in domestic and foreign bonds.

- **Firms**: produce tradable intermediate and non-tradable final goods
  - **domestic intermediate-good firms**: use labour and capital services as inputs, produce tradable differentiated goods, set prices in *producer* currency in monopolistically competitive markets at home and abroad
  - **foreign intermediate-good firms**: sell differentiated goods in domestic markets, set prices in *local* currency in monopolistically competitive markets
  - **final-good firms**: combine domestic and foreign intermediate goods into three non-tradable goods: a private consumption good, a private investment good, a public consumption good
Baseline NAWM: Agents

- **Central bank**: sets the short-term nominal interest rate by following a Taylor-type interest-rate rule.

- **Fiscal authority**: purchases public consumption goods, issues bonds, levies distortionary as well as lump-sum taxes.
Baseline NAWM: Frictions and shocks

- The NAWM features a relatively large number of frictions:
  - external habit formation in consumption
  - generalised adjustment cost in investment, imports and exports
  - fixed cost in intermediate-good production
  - monopolistic competition in intermediate-good and labour markets
  - sticky prices and wages à la Calvo, with dynamic indexation
  - domestic and external financial intermediation costs
  - non-state-contingent bonds

- The model contains a relatively large number of shocks (classified as demand, technology, markup and foreign shocks, plus a monetary policy shock).
Baseline NAWM: Data

- The NAWM uses data on 18 key macroeconomic variables:
  - real GDP
  - private consumption
  - government consumption
  - total investment
  - extra-euro area exports
  - extra-euro area imports
  - GDP deflator
  - consumption deflator
  - import deflator
  - employment
  - nominal wages
  - nominal interest rate (EURIBOR)
  - nominal effective exchange rate
  - competitors’ export prices\(^\dagger\)
  - foreign demand\(^\dagger\)
  - foreign prices\(^\dagger\)
  - foreign nominal interest rate\(^\dagger\)
  - oil price\(^\dagger\)

- The variables with a dagger (‘\(^\dagger\)’) are modelled using a structural VAR, while government consumption is modelled as an AR process.
Wholesale banks

- **Wholesale banks’ balance sheet:**

\[
\tilde{Q}_{I,t} B^p_{I,b,t+1} + Q_{L,t} B^p_{L,b,t+1} + S_t Q^*_L t B^*_p_{L,b,t+1} = NW_{b,t} + D^h_{b,t+1}
\]

- **Evolution of net worth:**

\[
NW_{b,t+1} = \tilde{R}_{I,t+1} \tilde{Q}_{I,t} B^p_{I,b,t+1} + R_{L,t+1} Q_{L,t} B^p_{L,b,t+1} + S_{t+1} / S_t R^*_L t+1 Q^*_L t B^*_p_{L,b,t+1} - \epsilon_t^R R_t D^h_{b,t+1}
\]

- **Objective:**

\[
V_{b,t}(NW_{b,t}) = \max E_t \left[ \sum_{k=1}^{\infty} (1 - \theta_{t+k}) \Pi_{i=1}^{k-1} \theta_{t+i} \Lambda_{t+k} \Pi_{C,t+k}^{-1} NW_{b,t+k} \right]
\]
Agency problem: banker can divert
- the fraction $\Psi$ of investment loans,
- the fraction $\omega_{L,t} \Psi$ of domestic government bonds, and
- the fraction $\omega^*_{L,t} \Psi$ of foreign government bonds, with $0 \leq \omega_{L,t}$, $\omega^*_{L,t} \leq 1$

Lenders can recover the residual funds and shut the bank down

Incentive constraint:

$$V_{b,t}(NW_{b,t}) \geq \Psi \left( \tilde{Q}_{I,t} B_{I,b,t+1}^p + \omega_{L,t} Q_{L,t} B_{L,b,t+1}^p + \omega^*_{L,t} S_t Q_{L,t} B_{L,b,t+1}^{*,p} \right)$$
The wholesale banks’ balance sheets are s.t. a constraint on the “risk-weighted” leverage ratio $\Phi_t$ w.r.t. net worth $NW_t$:

$$\tilde{Q}_{I,t} B_{I,t+1}^p + \omega_L Q_{L,t} B_{L,t+1}^p + \omega_{L,t}^* S_t Q_{L,t}^* B_{L,t+1}^{*,p} = \Phi_t NW_t,$$

where $\tilde{Q}_{I,t}$, $Q_{L,t}$ and $Q_{L,t}^*$ are the “discount prices” of the privately (p) intermediated assets $B_{I,t+1}^p$, $B_{L,t+1}^p$ and $B_{L,t+1}^{*,p}$.

In equilibrium, the wholesale banks’ leverage is given by:

$$\Phi_t = \frac{E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi_{C,t+1}^{-1} \epsilon_t^{RP} R_t \right]}{\Psi - E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi_{C,t+1}^{-1} \left( \tilde{R}_{I,t+1} - \epsilon_t^{RP} R_t \right) \right]},$$

where $\Lambda_{t,t+1} = \Lambda_{t+1} / \Lambda_t$, $\Pi_{C,t+1} = P_{C,t+1} / P_{C,t}$ and $\Omega_{t+1}$ is a discount factor modifier.
Supply of loans: Wholesale banks

Low of motion for the wholesale banks’ aggregate net worth:

\[
NW_t = \theta_t \left( (\tilde{R}_{I,t} - \epsilon_{t-1} R_{t-1}) \frac{\tilde{Q}_{I,t-1} B_{I,t}^p}{NW_{t-1}} \right. \\
+ (R_{L,t} - \epsilon_{t-1} R_{t-1}) \frac{Q_{L,t-1} B_{L,t}^p}{NW_{t-1}} \\
+ \left( \frac{S_t \tilde{R}_{L,t}^*}{S_{t-1}} - \epsilon_{t-1} R_{t-1} \right) \frac{S_t Q_{L,t-1}^* B_{L,t}^*,p}{NW_{t-1}} \\
+ \epsilon_{t-1} R_{t-1} \right) NW_{t-1} + \Theta,
\]

where \(\tilde{R}_{I,t}\), \(R_{L,t}\) and \(R_{L,t}^*\) are the “expected returns” of the wholesale banks’ assets in excess of the interest on household deposits \(\epsilon_{t-1} R_{t-1}\).
Arbitrage relation between loans and domestic government bonds:

\[
\omega_L E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi^{-1}_{C,t+1} \left( \tilde{R}_{I,t+1} - \epsilon_t^{RP} R_t \right) \right] = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi^{-1}_{C,t+1} \left( R_{L,t+1} - \epsilon_t^{RP} R_t \right) \right]
\]

Arbitrage relation between domestic and foreign government bonds:

\[
\tilde{\omega}_{L,t}^* E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi^{-1}_{C,t+1} \left( \frac{S_{t+1} R_{L,t+1}^*}{S_t} - \epsilon_t^{RP} R_t \right) \right] = E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi^{-1}_{C,t+1} \left( R_{L,t+1} - \epsilon_t^{RP} R_t \right) \right],
\]

where \( \tilde{\omega}_{L,t}^* = \tilde{\omega}_L(1 - \Gamma_L^*(s_{L,t+1}; \epsilon_t^{RP^*})) \) and \( \tilde{\omega}_L = \omega_L / \omega_L^* \)
Supply of loans: Retail banks

- Retail banks obtain an amount $\tilde{Q}_{I,t} B_{I,t}$ of loans from the wholesale banks, differentiate them costlessly and distribute an amount $Q_{I,t} B_{I,t} \leq \tilde{Q}_{I,t} B_{I,t}$ of loans to the households.

- Retail banks face rigidities constraining their ability to re-set the loan price, like in Calvo (1983), resulting in sluggish loan rate adjustments:
  - loan price re-set (Calvo) parameter: $\xi_I$
  - “mark-down” shock: $\varphi^I_t$, with $\varphi^I_t \leq 1$

- Log-linearised equations for optimal loan price of retail bank $r$ and for aggregate loan price index:

\[
\hat{Q}^\circ_{I,r,t} = (1 - \beta \xi_I)(\hat{Q}_{I,t} + \hat{\varphi}^I_t) + \beta \xi_I \hat{Q}^\circ_{I,r,t+1} \\
\hat{Q}_{I,t} = \xi_I \hat{Q}_{I,t-1} + (1 - \xi_I) \hat{Q}^\circ_{I,r,t}
\]