Financing constraints, micro adjustment of factor use and aggregate implications

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Abstract
Following a positive shock, financing constraints will prolong or impede economic expansion that would have been optimal in an unconstrained environment. The study of dynamic adjustment therefore offers a direct way of verifying the presence of financing constraints and assessing their consequences for economic allocation.

This paper compares the speed of adjustment of constrained and unconstrained firms using categorical information from survey data on the restrictions under which adjustment takes place. A set of moment conditions for the use in GMM estimation is developed, to cope with the problem of time varying speed of adjustment when the target level is partially unobserved.

After estimating the micro-dynamics of capital and labour demand, some of the aggregate consequences are worked out. The changing composition of the population makes for a time-varying sensitivity of the aggregate with respect to macroeconomic shocks. The asymmetry of microeconomic adjustment dynamics may induce a shortfall of the mean relative to the average of targets on the aggregate level, even in dynamic equilibrium. Whereas the first effect is sizeable though not overwhelming, the second looks surprisingly small.

Key Words: Dynamic panel data models, adjustment, investment and financing constraints

JEL Classification: C23, D21, D24

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Financing constraints, micro adjustment of factor use and aggregate implications

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1. Introduction

With informational frictions, indivisibilities and irreversibilities, the reaction of aggregate factor demand to aggregate shocks will not be time-invariant. Instead, it will depend on the distribution of individuals in the relevant state-space. In a prototypical (S,S) model of irreversible investment, a region of inaction is bounded by an upper and a lower threshold that triggers investment and hiring, or disinvestment and firing. The individual reaction to measures of fundamentals will be nonlinear: investment bursts (spikes) will be followed by relative inaction. On the aggregate level, the reaction to a positive shock will be sharp if a large mass of individual agents is situated near the upper threshold, as opposed to a situation where a series of negative shocks has driven agents to the edge of disinvestment. Models of informational asymmetry describe a similar range of inactivity for individuals. If the market value of their assets is insufficient to sustain externally financed expansion, firms will not be able to make use of profitable investment opportunities, or have to take the slow lane of accumulation by internal finance. Again, the sensitivity of aggregate factor demand to productivity or demand shocks and user cost changes induced by tax reforms or monetary policy will depend on the distribution of equity or liquidity ratios among firms, as induced by recent history. Therefore the action of the financial accelerator is quite asymmetric in booms and busts. This "asymmetry", as we may call the state dependence of correlations between macroeconomic aggregates and prices, is important for policy makers as well as for market participants, forecasters and analysts.

In such a situation, there is a high value attached to direct information on the position of individuals in state space that would allow us to infer on the aggregate sensitivity. This paper argues that survey data can be a timely and informative source of information about the relevant constraints for individuals. More specifically, it deals with financing constraints on the firm level and their effect on the aggregate. We argue first that the most important real effect of financing constraints is to slow down the adjustment of capital and labour input to positive, expansionary shocks (Section 2). In explicitly relating financing constraints to the

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2 Caballero, Engel and Haltiwanger (1995, 1999),
3 See Bernanke and Gertler (1989, 1995) and Bernanke, Gertler and Gilchrist (1996, 1999) on the credit channel and the financial accelerator.
adjustment dynamics, our paper is related to Basu and Guariglia (2002), von Kalckreuth (2006) and Bayer (2006). We then develop a GMM based technique to estimate micro level adjustment equations when the target is unobserved and the adjustment speed varies over time, and categorical information on adjustment regimes is available (Section 3). This estimation technique is then taken to the data (Section 4). Using micro data from a survey on the investment behaviour of German industrial companies (the Ifo Investitionstest), we estimate capital and labour adjustment functions. The behaviour of financially constrained firms is contrasted with the adjustment of unconstrained expanders on the one hand, and stationary or contracting firms on the other. These estimates, though preliminary, yield meaningful results and show that regime specific dynamic behaviour can successfully be disentangled. The estimation method proposed has a wide range of application for other important adjustment processes, such as sales price adjustment by firms, interest pass through by banks or the adjustment of equity ratios. It needs qualitative information on the adjustment regime.

In a last step, some of the aggregate implications are worked out (Section 5). First, the aggregate sensitivity changes with the composition of the aggregate. With estimated adjustment functions in hand, it is easy to trace the time-varying aggregate sensitivity. This offers an important new way in which survey data can be used by analysts and forecasters. Second, the asymmetry of adjustment may entail long run level effects. Individual companies are subject to idiosyncratic and aggregate shocks. If companies below their target – aiming at growth – systematically adjust slower than those for which contraction is indicated, then, on average, the aggregate factor use will be below the sum of individual targets even in a stationary stochastic equilibrium. Depending on the ranking of adjustment speed, a positive or negative equilibrium shortfall will result. Using the estimated adjustment equations, the time variation of targets and the correlation structure, we can estimate the aggregate shortfall. According to our preliminary estimates, the average shortfall is of moderate size and would not persist in dynamic equilibrium.

2. Financing constraints and investment dynamics

Empirical work on financing constraints has traditionally been based on an approach pioneered by Fazzari, Hubbard and Peterson (1988). If the investment of supposedly financially constrained firms shows a higher sensitivity to internal finance than the investment of their supposedly unconstrained counterparts, this is seen as evidence for the existence of binding financial constraints. In recent years, this approach has been forcefully criticised. Kaplan and Zingales (1997) state that there is no theoretical reason why – in a comparison between firms – a larger cost differential between internal and external finance might lead to a higher cash-flow sensitivity, as opposed to just comparing the extreme cases of a constrained firm and an absolutely unconstrained one. A non-monotonic relationship between the cost
differential and excess sensitivity is perfectly conceivable.\(^4\) On the other hand, it has been shown theoretically that, under certain conditions, cash flow terms can be significant even in the absence of financing constraints.\(^5\) Ultimately, there is a pervasive missing variable problem. Cash flow is a close relative to profit, a summary measure of all that is important for a firm, and it is useful in predicting future values of variables relevant to the current investment decision.\(^6\)

Here, we use a direct approach by relying on explicit statements by the firms themselves. We are able to explore the micro data base of the Ifo institute’s *Investitionstest* (Investment Test, IT) for the manufacturing sector in West Germany during the years 1988 to 1998. During these eleven years, the autumn wave yields 25,643 observations on a total of 4,443 firms, with 2,331 firms per year on average. Apart from its size and coverage, the data set has two important characteristics that are relevant to our problem. First, it contains many small firms, on which very little information is available from micro data sets based on quoted companies. Although large firms are clearly over-sampled, almost 50% of the IT observations refer to firms with fewer than 200 employees, and 19.5% of the firms have fewer than 50 employees. Second, the data set contains information on financing constraints that firms face in their investment decisions. Notably, a number of firms (around 26.2% of respondents) explicitly state that their investment demand is limited by the cost and/or the unavailability of finance. Although part of this may be due to the workings of the classical interest rate channel, these aggregate effects can be eliminated by the use of time dummies, focussing on differential changes in time.

In von Kalckreuth (2004) argued that a specific pattern with respect to the distribution of investment over time should be expected to hold for financially constrained firms. Given a shock, an unconstrained investor can adapt rapidly or even instantaneously if other types of adjustment costs are unimportant. The bulk of investment spending will take place in the first few periods, and there may be a spike in the first period. If the investor is financially constrained, however, marginal costs of finance will increase with the amount of spending, possibly to infinity. In such a setting, the investor has to equalise marginal costs of finance and the marginal value of new investment in each period. After an initial debt-financed increase in the capital stock that leads to a worsening of the financial position, the firm needs internal finance to continue the expansion and to repair balance sheets gradually. Thus, the adjustment will be spread over time. This crucial difference in the adjustment dynamics can be used to identify financially constrained firms, or better, to test whether a subset of supposedly constrained firms really is, without having to take recourse to cash flow sensitivities.

\(^4\) The discussion was continued in Fazzari, Hubbard and Peterson (2000) and Kaplan and Zingales (2000).
\(^5\) See the models by Abel and Eberly (2003), Cooper and Ejarque (2001), and Gomes (2001).
\(^6\) This argument is developed formally in Appendix B of Chirinko and von Kalckreuth (2002).
It is possible to condense the dynamics of the model into a single diagram. In Figure 1, the decreasing schedule represents marginal return on investment, whereas the increasing schedule with the flat portion depicts the costs of finance. Given a profitability shock, indicated by arrow (1), the financially constrained firm will immediately invest up to the point where the costs of external finance are equal to marginal profitability of investment. The difference between marginal profitability and the costs of finance in the steady state winds up a “clockwork”. The firm retains profits, indicated by arrow (2), expanding equity base and physical capital at the same time. The “clock” winds down, along the falling schedule that depicts marginal returns on investment, until dynamic equilibrium is reached again.

Figure 1: The clockwork

The hypothesis is examined empirically by von Kalckreuth (2006), using an entirely qualitative set of survey data on UK firms. The duration of capacity restrictions is compared between firms that characterised themselves as being financially constrained, and others that did not.

The Ifo Investment Test has a different and more specific informational content, in that many key variables are continuously scaled, so that the adjustment dynamics can be much more closely observed. Von Kalckreuth (2004) proceeds to sort firms into two groups, according to whether they are predominantly financially constrained or not. This approach has a serious drawback: it does not make use of the time variation in the financing constraints variable for a given firm. This is the most valuable sort of information in micro-econometrics, when many important aspect of the process in question are unobserved, but can be trusted to be relatively stable in time. In this case it is the variation in the left hand variable following a variation of the explanatory variable that helps to identify structural relationships. However, a time

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7 I thank John van Reenen for making this point in the course of a conference discussion. Doing so, he set me on the track that led to this paper.
varying adjustment coefficient poses special problems if the target is unobserved. These will be discussed in the next section.

The interrelation of financing constraints and investment behaviour is studied also by Basu and Guariglia (2002), looking at the dynamics of capital returns. Our approach is closest in spirit to Bayer (2006). In this paper, a gap model of adjustment is estimated, where capital imbalances are measured as an imputed difference between capital stock and imputed target, whereas financing constraints are proxied simply using the equity ratio. For both of these fundamental magnitudes, a treatment of unobserved differences between firms is extremely difficult, and we believe that the empirical approach presented next may greatly alleviate some of these measurement problems.

3. State dependent adjustment dynamics with unobserved target

This section draws on von Kalckreuth (2007), a companion paper that studies moment conditions that can be used in the estimation of an adjustment model with unobserved target and time-varying speed of adjustment in a context of panel data. The econometric method is tailor-made for the problem at hand, but has considerably more general use.

In a rather general form, economic adjustment can be framed by a "gap equation", as formalised by Caballero, Engel and Haltiwanger (1995):

$$\Delta y_{t,i} = \Lambda(x_{t,i}, z_{t,i}) \cdot x_{t,i}, \quad \text{where} \quad x_{t,i} = y_{t,i-1}^* - y_{t,i}^*.$$  

Here, subscripts refer to individual i at time t, and $x_{t,i}$ is the gap between the state $y_{t,i-1}^*$ inherited from the last period and the target $y_{t,i}^*$ that would be realised if adjustment costs were zero for one period of time. The speed of adjustment, written as a function $\Lambda$ of the gap itself and additional state variables $z_{t,i}$, determines the fraction of the gap that is removed within one period of time. The adjustment function will reflect convex or non-convex adjustment costs, irreversibility and indivisibilities, financing constraints or other restrictions, and the uncertainty of expectation formation. With quadratic adjustment costs or Calvo-type probabilistic adjustment, $\Lambda$ will be a constant.

Estimating the function $\Lambda$ is inherently difficult. In general, both $y_{t,i}^*$ and $x_{t,i}$ will be not observable. But some measure of the gap is needed for any estimation, and if $\Lambda$ explicitly depends on $x_{t,i}$, the measure will actually move to the centre stage.

In linear dynamic panel estimation, this problem can successfully been addressed by positing an error component structure for the measurement error and eliminating the individual fixed effect by a suitable transformation, such as first differencing. See Bond et al. (2003) and Bond and Lombardi (2007) for an error correction model of capital stock adjustment. The GMM
estimator developed by Arellano and Bond (1991) accounts for the presence of lagged endogenous variables, the endogeneity of other explanatory variables, and unobserved individual specific effects. Individual effects (including a possible measurement error in the target) are differenced out. Endogenous explanatory variables can be instrumented using lagged dependent variables if serial correlation of the error process is limited. Time fixed effects can also be accommodated; the remaining idiosyncratic component of the measurement error needs to be uncorrelated with the instruments.

In the unrestricted, non-linear case, this approach is not feasible, as a host of incidental parameters will threaten identification. But there may be direct qualitative information on the level of \( \Lambda(\cdot) \), e.g. from survey data, ratings or market information services. If one is willing to treat the adjustment process as piecewise linear, distinguishing regimes of adjustment, then, as will be shown, this information can be harnessed to eliminate the incidental parameters from the problem completely.

3.1. The estimation problem

We want to examine a situation where a variable \( y_{it} \), e.g. the capital stock of a firm, or the number of employees, or the equity ratio, reverts to some target level \( y_{it}^* \), characteristic of individual \( i \). The speed of adjustment is variable and depends on the value of \( r_{it} \). This is an \( L \)-dimensional column vector of regime indicator variables, with one element taking a value of 1, and all others being zero. The equation is

\[
\Delta y_{it} = -(1 - \alpha_{it}) (y_{it-1} - y_{it}^*) + \varepsilon_{it},
\]

with \( \alpha_{it} = \alpha' r_{it} \).

The target level \( y_{it}^* \) is unobservable. It follows an equation that contains an individual-specific latent term:

\[
y_{it}^* = \beta' x_{it} + \mu_i.
\]

The vector \( x_{it} \) may encompass deterministic time trends or time dummies. The vector \( \alpha \) holds the adjustment coefficients. The \( \alpha_{it} \) thus vary over time and individuals, and \((1 - \alpha_{it-1})\) is the adjustment speed at date \( t \). If it is stable, the process would eventually settle in the target in the absence of shocks. The expectation of \( \varepsilon_{it} \) is assumed to be zero. Generally, there will be a non-zero covariance between the error term and the regime indicators, \( \text{cov}(\varepsilon_{it}, r_{it}) \neq 0 \).

If, for example, \( \varepsilon_{it} \) is the error term in a capital accumulation equation and \( r_{it} \) is the regime indicating the degree of financing constraints, there should be a contemporaneous correlation between those two.
As we do not observe the target, we have no direct information on the position of the individual relative to the target. But if the idiosyncratic component of the target which may contain the measurement error is constant over time, the panel dimension can help us to identify the adjustment process nonetheless. Given the lack of knowledge on the process that generates the regimes, a GMM approach suggests itself. Unlike the standard model in dynamic panel data analysis, the fixed effect interacts with a time varying and endogenous variable. Simple differencing will therefore be not enough to eliminate it. In order to simplify notation, the problem of finding appropriate moment conditions will be discussed on the basis of a version of the process with a target that is constant over time and heterogeneous among individuals:

\[
\Delta y_{i,t} = -\left(1 - \alpha_{i,t-1}\right)(y_{i,t-1} - \mu_i) + \epsilon_{i,t},
\]

or

\[
y_{i,t} = \alpha_{i,t-1}y_{i,t-1} + \left(1 - \alpha_{i,t-1}\right)\mu_i + \epsilon_{i,t}.
\]

Anderson and Hsiao (1982) have devised the classic strategy for estimating linear dynamic panel equations with fixed effects. Consider a first-order autoregressive equation with fixed effects:

\[
y_{i,t} = \gamma y_{i,t-1} + \mu_i + \epsilon_{i,t}.
\]

Obviously, the latent fixed effect \( \mu_i \) is correlated with the explanatory variable. Transforming the equation by taking first differences eliminates the fixed effect:

\[
\Delta y_{i,t} = \gamma \Delta y_{i,t-1} + \Delta \epsilon_{i,t}.
\]

Now, however, the transformed error term \( \Delta \epsilon_{i,t} \) is correlated with the transformed regressor, \( \Delta y_{i,t-1} \). This can be accommodated using an instrument variable procedure. Anderson and Hsiao propose using either lagged first differences or lagged levels as instruments. Employing second and further lags of the level as instruments for the differenced equation makes use of the following moment restrictions:

\[
E\left(y_{i,t-\lambda} \cdot \Delta \epsilon_{i,t}\right) = 0, \quad \lambda = 2, 3, \ldots
\]

If these moment restrictions hold, then the lagged levels will be valid instruments, because they are correlated with the regressor variable. The suggestion of Anderson and Hsiao was refined by Holtz-Eakin, Newey and Rosen (1988) and Arellano and Bond (1991), who propose to use an efficient GMM estimator that uses all available moment restrictions optimally. Formally, the moment equations are written as a system, in order to be able to use a varying number of instruments according to availability. The instruments are weighted
optimally using the Hansen (1982) two-stage procedure. In the following, we will try to modify this approach in such a way that it can be put to use in our non-linear setting.

3.2. A transformation made to measure

We start by looking more closely at the first difference of \( y_{i,t} \):

\[
\Delta y_{i,t} = \begin{cases} 
\alpha' r_{i,t-1} \Delta y_{i,t-1} + \Delta \epsilon_{i,t} & \text{for } r_{i,t-1} = r_{i,t-2} \\
\alpha' r_{i,t-1} \Delta y_{i,t-1} + \alpha' \Delta r_{i,t-1} y_{i,t-2} + (1 - \alpha') \Delta r_{i,t-1} \mu_{i} + \Delta \epsilon_{i,t} & \text{for } r_{i,t-1} \neq r_{i,t-2} 
\end{cases}
\]

For \( r_{i,t-1} = r_{i,t-2} \), this expression looks very much like the first difference in the linear case, although there is more than one coefficient to estimate. Taking first differences of observations that belong to different regimes has two consequences. First, the equation will contain the product of a lagged level term and a difference of regime indicators. Second, there is a latent term \((1 - \alpha') \Delta r_{i,t-1} \mu_{i}\) that will be correlated with the lagged dependent variable under a variety of circumstances.

As it is this term that makes the use of the standard technique difficult, the following strategy comes to mind: Differences are only formed for observations with \( r_{i,t-2} = r_{i,t-1} \). On the basis of cases where two consecutive observations belong to the first regime, we could estimate \( \alpha(1) \), and using differences of observations that both belong to the second regime, we could infer on \( \alpha(2) \), etc. In this straight fashion, however, the idea will not work. The transformed residual \( \Delta \epsilon_{i,t} \) has an expectation different from zero in the two groups of observations. This is because \( r_{i,t-1} \) and \( \epsilon_{i,t-1} \) are correlated by assumption. The expectation \( \mathbb{E}(\epsilon_{i,t-1} | r(1)_{i,t-1} = 1) \) is not equal to zero, and neither is \( \mathbb{E}(\epsilon_{i,t-1} | r(2)_{i,t-1} = 1) \). Selecting residuals according to regimes will lead to biased estimators.

If \( \epsilon_{i,t} \) is uncorrelated with past regime indicators, \( r_{i,t-1}, r_{i,t-2}, \ldots \), we are able to use a modified differencing approach. Autocorrelation of \( \epsilon_{i,t} \) is permitted if the usual requirement of limited memory is satisfied. The following two principles will generate moment conditions involving the use of lagged endogenous variables as instruments:

1. Let \( q \) be the maximum \( \tau \) for which there is a correlation between \( r_{i,t} \) and \( \epsilon_{i,t-\tau} \), eg. as a consequence of an MA structure of the state driving the regime indicator. Then the observation is to be transformed subtracting past observations of the same regime with a lag of at least \( \lambda = 2 + q \).

2. If an observation is not matched by a \( 2 + q \)-lag in the same regime, it may be transformed using any other lag \( \lambda > q + 2 \).
The second principle avoids the loss of many observations in cases where regimes in \( t \) and \( t+q \) do not match because of regime switches. What we propose here is a dynamic filter, which varies according to regimes.

Similar to (2) we obtain for the \( \lambda \)’th difference:

\[
(y_{ij} - y_{ij-\lambda}) = \alpha'(r_{ij-1}y_{ij-1} - r_{ij-\lambda-1}y_{ij-1}) + (1 - \alpha')(r_{ij-1} - r_{ij-\lambda-1}) \mu_i + (e_{ij} - e_{ij-\lambda}),
\]

which simplifies to

\[
(y_{ij} - y_{ij-\lambda}) = \alpha' r_{ij-1} (y_{ij-1} - y_{ij-\lambda-1}) + (e_{ij} - e_{ij-\lambda})
\]

if the two observations are characterised by the same regime, such that \( r_{ij-1} = r_{ij-\lambda-1} \). When does the conditional expectation of the residual term, \( (e_{ij} - e_{ij-\lambda}) \), become zero? It is sufficient that \( e_{ij} \) and \( e_{ij-\lambda} \) are both uncorrelated with the conditioning variables, which are \( r_{ij-1} \) and \( r_{ij-\lambda-1} \). Now assume \( e_{ij} \) to be uncorrelated with \( r_{ij-1} \) and \( r_{ij-\lambda-1} \). Then the same is true with respect to \( e_{ij-\lambda} \) and \( r_{ij-\lambda-1} \). Therefore, by choosing \( \lambda \), we have only to make sure that \( e_{ij-\lambda} \) and \( r_{ij-1} \) are uncorrelated. This will never happen with \( \lambda = 1 \), as we have seen before. However, if \( r_{ij} \) is uncorrelated with all lags of \( e_{ij} \), then \( \lambda = 2 \) will ensure that

\[
E(e_{ij} - e_{ij-\lambda} \mid r_{ij-1} = r_{ij-\lambda-1}) = 0,
\]

regardless of whatever value \( r_{ij-1} \) and \( r_{ij-\lambda-1} \) take. More generally, if there is correlation between \( r_{ij} \) and \( e_{ij-\tau} \) up to lag \( \tau = q \), the difference that guarantees the above equation to hold will be at least of order \( 2 + q \). GMM estimation on the basis of this transformation may be called forward difference estimation. But we are not restricted to using only differences of the order that is "just right", i.e. \( 2 + q \). Any other difference of order \( \lambda \geq 2 + q \) will fulfil eq. (3) just as well. Therefore we construct the difference using the most proximate observation of the same regime with lag \( \lambda \geq 2 + q \). With respect to admissibility and validity of instruments, the rules of the classic approach apply: the instruments need to be uncorrelated with the earlier of the two observations that make up the difference. In the following, this procedure will be called the generalised difference estimator. The viability of this procedure and the conditions for its use are worked out in the companion paper, von Kalckreuth (2007).

To sum up, there are two identifying assumptions to be made on the regime. The regime indicator \( r_{ij-1} \) must be uncorrelated with the current error term \( e_{ij} \), or otherwise a selective use of data according to regime will not lead to a consistent estimator. This needs to be considered when choosing suitable regime indicators. Second, the process that drives the regime indicator needs to have finite memory with respect to innovations \( e_{ij} \). If \( r_{ij} \) were correlated with all past values of \( e_{ij} \), the conditional expectation of the transformed error
term resulting from a difference of two observations from the same regime would not disappear. For the estimation problem at hand, the situation is this: A shock in the capital accumulation equation will spill over to any equation describing financing conditions, as financing conditions naturally depend on the size of investment to be financed. However, a stint of financing constraints triggered by a shock in investment demand will end in finite time, therefore warranting the assumption of a limited memory of \( r_{t,i} \) with respect to \( \varepsilon_{t,i} \).

When implementing the generalised difference estimator, we use the moment conditions in a way that greatly facilitates the calculation of moments. Equation (4) requires us to calculate the \( \lambda \)’th difference of every observation, with \( \lambda \geq q + 2 \) and differences being taken using only observations in the same regime. Then we may use levels lagged \( \lambda + 1, \lambda + 2, \ldots \) as instruments. It seems that this requires us to make the set of instruments for a specific observation dependent on whether or not there are two observations in the same regime within a specific time distance. By taking the earlier of the two observations as a point of reference \( y_{t,i} \) and assigning to it the nearest lead \( y_{t+i} \) of the same regime with \( \lambda \geq 2 + q \), the definition of suitable instruments is straightforward. We can uniformly use lags \( y_{t,i-1}, y_{t,i-2} \) and earlier as instruments.

4. Estimating the micro-dynamics of adjustment

We want to build our econometric model in a rather simple way on the error correction factor demand equation invented by Charles Bean (1981), and introduced to the micro literature by Bond, Elston, Mairesse and Mulkay (2003).\(^9\) We will estimate adjustment equations both for capital and labour. Whereas the value of real capital must be imputed using data on investment and depreciations, the number of employees is observed directly. The regime information refers to purchases of fixed capital. But the number of employees and the capacity provided by fixed capital are complements, and an expansion will entail a concurrent increase in the use of both factors. The estimations reported in this section are preliminary and need to be worked out further, checking for robustness and testing a larger range of specifications.

The point of departure is the static neoclassical equation for factor demand. Using a constant returns CES production function, the following linear equation for capital results from the first-order conditions of profit maximisation:

\[
\log K_i^* = \log S_i^* - \sigma \log UC_i^* + \log h_i^*,
\]

where \( K_i \) is capital, \( S_i \) is real output, \( UC_i \) is the user costs of capital, \( \sigma \) is the elasticity of substitution, and \( h_i \) is a magnitude that depends on technology parameters and the time

\(^9\) See von Kalckreuth (2004), Appendix B for a detailed discussion of the capital demand ECM.
varying total factor productivity. A star denotes a long-run, equilibrium value. We may model adjustment by replacing the starred values by distributed lags of observed quantities. If the DL coefficients add up to one, the long-run effects will be preserved. Using a lag length of 1 for the distributed lags on both sides of the equation and reordering leads to

$$\Delta \log K_t = \beta \Delta \log S_t + \gamma \Delta \log UC_t + \nu \Delta h_t - \phi (\log K_{t-1} - \log S_{t-1} - \sigma \log UC_{t-1} - h_{t-1})$$

For a given firm, we may represent the influence of technology parameters, and the effects of user cost changes and technology shocks by an individual specific error component, pure time effects and an idiosyncratic error term. Making the remaining adjustment coefficients depend on the regime \( r_{it} \) and making observations specific to individual \( i \), one obtains:

$$\Delta \log K_{it} = \beta \left( r_{it} \right) \Delta \log S_{it} - \phi \left( r_{it} \right) (\log K_{i,t-1} - \log S_{i,t-1} - \mu_t - \psi_t) + \lambda_t + \epsilon_{it}.$$  

The term in brackets stands for the long-run equilibrium relationship the firm is aiming at. Adjustment dynamics thus have two components: a short run reaction \( \beta \left( r_{i,t-1} \right) \) to contemporaneous changes in sales conditions and error correcting behaviour with regard to accumulated deviations from target represented by \( \phi \left( r_{i,t-1} \right) \). Both parameters are functions of the predetermined regime indicator. Ignoring the transitory dynamics (restricting \( \beta \left( r_{i,t-1} \right) \) to zero) yields a partial adjustment equation with time-varying adjustment speed, where the target is given by a firm specific capital-output ratio determined by user cost changes and technology. Note that in this case the time variation of the target is represented by \( S_{i,t-1} \), not by \( S_{it} \). The pure time effect is split into a regime independent component \( \lambda_t \) and a component that enters the target relationship and thus has interaction effects with regimes. Of course, these two components are not separately identified.

For labour, an exactly analogous equation can be derived, featuring wage rates in the place of user costs, and different technology terms. The adjustment dynamics, of course, may also be quite different.

The Ifo data set and its information on investment and financing conditions have been discussed in detail by von Kalckreuth (2004). With respect to financing constraints, we may use Question 5 of the autumn survey, which asks for the factors stimulating or limiting investment in the current year and in the coming year. We will use the answers on financing constraints expected for the next year. The financing constraints indicator receives a 1 if financing conditions are regarded as "limiting" or "very limiting", and zero otherwise. The current change of the capital stock on the left-hand side uses the fixed capital investment in the year after the autumn survey question was asked. With respect to current investment, the financing constraints information is predetermined.
Similarly, we need information on whether or not the firm is planning to expand, as it is with respect to expansion that financing constraints are predicted to have the retarding dynamic consequences sketched above. We use the information on the investment purposes from Question 4.1 of the Spring Survey. Again, this information has to be predetermined with respect to the investment of the current period. Therefore we use the purpose of planned investment one year before the spring survey that gives us the information on the current change of the capital stock. Financially constrained firms are below their target by definition. For unconstrained firm-years, we use three different ways of grouping firms as 'expansive' and as 'stationary/contractive'. Partition I is specifically geared to capital demand. We
consider a firm as expanding its capital stock if either "expansion" is given as the main purpose of investment, or "rationalisation with respect to labour costs", as the latter will involve a capital deepening. We consider a firm as a firm as 'stationary or contracting', if "replacement" is given as the main purpose of investment or "rationalisation directed at other costs" and if the firm does not claim to be financially constrained. Partition II allocates the entire group of rationalising firms into the expansive regime, leaving for 'stationary of contracting' only those firms that have ticked "replacement" as their main purpose of investment. Partition III, finally, classifies as expanding only those firm-years where "expansion" was indicated as principal purpose. The number of those firms is rather small, and capital intensity has increased markedly during the nineties, partly by substituting labour. The data show that the average firm in the sample has increased its capital stock while shedding labour at the same time. Therefore, concerning capital demand, Partition III is probably too narrow.

A number of firms that report not to plan any investment do not answer Question 4.1 at all. In those cases a firm is also classified as not expanding or contracting, if it is not financially constrained. The preparation of the capital stock and sales data is as described by von Kalckreuth (2004). The results for 'stationary or contracting' firms have to be interpreted with care as far as investment demand is concerned: when imputing the capital stocks from investment information, an eternal inventory method had to be used, with sector specific and time-varying depreciation rates. Sales of capital goods are not observed. Therefore the adjustment rate of rapidly downsizing firms may be understated. We do not have this problem with the labour demand equation. Labour input is measured as number of full-time employees. Variation in hours worked, however, is not observed.

We start by estimating the partial adjustment version of the equation for the capital stock:

\[ \Delta \log K_{i,t} = -\phi \left( r_{i,t} \right) \left( \log K_{i,t-1} - \log S_{i,t-1} - \mu_i - \psi_i \right) + \lambda_i + \epsilon_{i,t}. \]

We predict that adjustment to a higher target capital stock will be slower when the firm has to overcome financing constraints, compared to a situation where the same gap has to be closed without financing constraints. What we want to test is whether the adjustment dynamics is different depending on the financial regime a firm is in. Coefficients in the adjustment equation may take three different values. It is important that the regime classification be made on the basis of past information, i.e. with survey data that arrives prior to the information on the investment in a given year.

For the generalised difference transformation, we use a minimum lead of 2. The second step results with Windmeijer-corrected standard errors are reported. Starting with Partition I, we observe error correction behaviour for all three regimes, for both types of time-effect modelling, although the adjustment coefficient for constrained firms is too small to be
significant. The adjustment of contracting firms is fastest – which is a strong result, given that it is probably understated. The p-values of the Sargan-Hansen test are in the neighbourhood of 10%. The regression with simple time effects in column (1) passes the regime coefficient test described by von Kalckreuth (2007): a significant regime dummy would have indicated a lag length of differentiation that is too low given the memory of the regime indicator. For the regression with interacted time effects in column (2) this test is not feasible, as a pure regime effect cannot be separately estimated. For the regression in column (1), the difference in adjustment speed between 'expanding' and 'constrained' firm years is significant on a 10% level, and testing the hypothesis of equal coefficients in all regimes leads to a p-value of 5.9%. Interacting time effects with regime effects makes the difference insignificant.

With Partition II, the Sargan-Hansen test statistics turn out less favourable, with a p-value of around 2-3%. The measured adjustment speeds related to 'expansive' and 'stationary/contractive' firm years do not differ much in this specification, nor do the results for 'expansive' and 'constrained' firm-years.

Concerning labour adjustment, it does not make much sense to group firms as 'expanding' that report to install new capital with the principal aim of saving labour costs. Therefore we use Regime Partition II as a point of departure, and contrast these results with Partition III, limiting the first regime to pure expanders. In the first two columns, only the regression with interacted time dummies shows clear differences between regime-specific adjustment speeds, with the adjustment speed of 'constrained' and 'stationary/contractive' firms insignificant from zero and those of 'expanders' at around 24%. The hypothesis of equal adjustment coefficients is rejected at a 5% level, the equality of adjustment among unconstrained expanders and constrained firms is rejected at a 10% level. However, with a p-value of 0.033, the Sargan-Hansen statistic does not fully support this specification. The specification in Column (1), featuring simple time effects and not showing any differences, is rejected even more strongly.

The regressions for Regime Partition III, finally, are both accepted by the Sargan-Hansen-test, and the regression in column (3) featuring simple time effects also passes the regime dummy test. Estimated speeds of adjustment are quite different between expanders and constrained firms, with between 20% and 25% for the former, and below 10% for the latter. Adjustment speeds for stationary/contracting firms are in between, as in column (3), or about the same size as the constrained firms, as in column (4). However, the hypothesis of equal adjustment coefficients among the first two regimes or among all three is rejected only for the regression with simple time dummies.

In Table 3 we show results for the full constant returns error correction model of labour demand:

$$\Delta \log L_{t,t} = \beta \left( r_{t,t} \right) \Delta \log S_{t} - \phi \left( r_{t,t} \right) \left( \log K_{t,t-1} - \log S_{t,t-1} - \mu_{t} - \psi_{t} \right) + \lambda_{t} + \epsilon_{t,t}$$
In this specification, we can also compare the short-run reactions between regimes, in addition to the error correction term. The regime partitions are identical to the ones used for the regressions in Table 2. The regressions for Partition I show a strong positive differential between the adjustment speeds of 'expanding' and 'constrained' firms only for the specification featuring interacted time effects, also the 'stationary/contracting' firms adjust very slowly. With simple time dummies, the estimated difference between these groups is almost zero. The short run reaction is rather sizeable in all three regimes, invalidating the partial adjustment results somewhat. The Sargan-Hansen statistics indicates some problems with the moment conditions, whereas the regime dummy test for the simple time effects regression is innocent. In columns (3) and (4) we report the results for Partition III. This time, there are sizeable differences between adjustment speeds only for the regression featuring simple time dummies. With 35%, measured adjustment speed for expanders is very high, compared to financially constrained firms and stationary/contractive firm, both at around 15%. The short run adjustment coefficients do not differ sizeably, in all three cases they are at around 0.3. The differences practically disappear when time dummies are interacted with regimes (column (4). However, this worsens also the Sargan-Hansen test. The error correction model has also been estimated for capital demand. The results were similar in nature to the ones obtained in Table II, although none of the differences was significant.

From our preliminary results, we can take away the following results. First, adjustment seems to be much faster for labour input than for capital demand. Although this has nothing to do with the problem of regime-dependent adjustment, it came as a surprise to the author, who was raised to believe in Eurosclerosis. Second, the differences in adjustment dynamics are more robust for labour demand, as compared to fixed investment. This may be due to the fact that the real capital stock is an imputed variable. Whereas the absolute changes of the capital stock can be observed relative precisely, the levels are much less reliable. Furthermore, the problems of the eternal inventory method become more acute when considering downsizing firms, as their downward adjustment may be severely mismeasured. This problem could be mended if direct observations on the capital stock were available. Third, the estimator seems in fact to be able to discern regime specific differences in dynamic behaviour in a real world setting: if differences are large enough, they become statistically significant.

Ultimately, there is an interesting qualitative pattern in the measured adjustment speed for constrained firms that merits further attention. It is Partition I for capital demand and Partition III for labour demand that seem most adequate on a priori grounds to measure expansionary behaviour. In both cases, it is the simple time dummy specification that shows significant differences between 'unconstrained expanding' and 'financially constrained' firm years in the direction that was theoretically expected, whereas the differences become insignificant when the time dummies are interacted with regimes. This indicates that a large part of the dynamic consequences of financing constraints is explained by adjustments of the target. Following a
positive shock, the financing costs of a financially constrained firm will be higher than for an unconstrained firm, affecting the current time user costs of capital. In the process of adjustment, the financial structure is improved after an initial deterioration, leading to lower user costs and a rising target. With uniform time dummies, only the pure macroeconomic effect of changing tax and interest rates will enter the regression. The expanding target as a consequence of financing constraints will then show up as a reduced speed of adjustment. If we additionally condition the time effects on an indicator of financing constraints, this user cost effect will be partialled out, and what remains is the adjustment with respect to given firm specific user costs, where firm specific financing constraints have been taken into account. There is no obvious reason why this should be lower for constrained firms.

5. Aggregate implications of adjustment heterogeneity

In this section, we want to trace two roads on which microeconomic heterogeneity of adjustment behaviour can translate into the aggregate. First, if the speed of adjustment is regime dependent, then the aggregate reaction to an overall shock depends on the composition, and changes in this composition are equivalent to changes in aggregate sensitivity. This is well known, but our method of relating the dynamic behaviour of individuals to survey information makes it particularly easy to trace the aggregate sensitivity and give up-to-date estimates about the current stance. This will be demonstrated here. Second, the nonlinearity of adjustment entails a difference between the average realisation and the average of targets that may persist even in dynamic equilibrium. Potentially, this gap may have important long-run consequences on employment and productivity.

5.1. Time-varying sensitivity

In Graph 1 and 2, we display the changing composition of the estimation panel with respect to adjustment regimes and the sensitivity to an aggregate shock that results in an equiproportionate increase in the target level of all firms. For this example we have chosen the partial adjustment model, because in this simple case, the dynamic behaviour is a function of just one variable. The same type of approach is possible for other, more complex adjustment equations, and various dynamic multipliers, eg first period, second, third etc. period effects. We have chosen the estimates with simple time effects, as they do not partial out possible effects on the cost of finance for the constrained firms, and we used Partition I for investment demand and Partition III for labour demand, as these are the most appropriate partitions for the respective factor demand.

In the two graphs, the first row displays the time variance in composition, and we have to distinguish between the unweighted composition (number of firms) and the composition of the aggregate. As weights, the levels of the respective factor input are used. It can easily be seen that the variation in composition is considerable and closely follows the business cycle in
Germany. In the current setting, with just one parameter, we simply need to multiply the shares with the associated adjustment coefficient to get the one-period sensitivity of labour or capital demand with respect to an aggregate shock in the target in the last period. The time variation in aggregate sensitivity is considerable, though not overwhelming: Aggregate sensitivity of capital demand fluctuates between 9.6% in 1993 and 12.5% in 1990 and 1991, the reunification boom years. For labour demand in Table 2, the respective figures are 14.2% in 1993 and 17.6% in 1999.

5.2. Aggregate shortfall in stationary equilibrium

The adjustment process as given in equation (1) is characterised by a nonlinear stochastic difference equation where the speed of adjustment is endogenous and may be related to the position of the variable in question relative to its target. If the speed of adjustment varies with the sign of the shortfall relative to the target, the microeconomic adjustment dynamics can have important aggregate implications. This is more general than the problem of factor demand – similar effects may be given for the adjustment of capital equity ratios, sales prices or commercial banks interest rates.

Suppose firms with a large factor demand have a slow speed of adjustment because a large gap confronts them with financing constraints. The speed of adjustment for contracting firms, on the other hand, might be relatively high. The opposite situation may be given in the dynamics of prices or wages, if firms try to avoid price or wage decreases. When the distance to target is relevant for the speed of adjustment, then the average realisation will divert from the target in a systematic way. If upward adjustment is slow relative to downward adjustment, the average realisation will be below the target, and vice versa. The downward rigidity of prices will lead to an average price level that is "too high", whereas the capital stock of firms will be below target on average if strong expansion is slowed down in an asymmetric way by financing constraints.

Micro dynamics thus can matter for the aggregate. In this section, the stochastic properties of asymmetric adjustment are described. On this basis, we work out a way to estimate the average excess of shortfall that is due to the nonlinearity of adjustment.

In order to gain an intuitive understanding of the significance of regime dependent adjustment, it helps to consider first the dynamics of the process with the random shock $\varepsilon_{i,t}$ "turned off". Depending on the regime the individual is in, adjustment is faster or slower, but in any case the stationary equilibrium is given by $\mu_i$, the target. This value of the process is mapped onto itself. Figure 2 illustrates the so-called "skeleton" of the process, the regression $E\left(y_{i,t} \mid y_{i,t-1}\right)$, separately for the case of two regimes, with $\alpha_2 > \alpha_1$. 
With a random disturbance $\varepsilon_{i,t}$ present, the process will be moved away from this stationary equilibrium, even if it ever happens to reach it. The consequences of nonlinear adjustment for the mean level of the process crucially depend on whether the regime indicator $r_{i,t}$ is correlated with the position of $y_{i,t}$ relative to $\mu_i$. If $y_{i,t} - \mu_i$ is correlated with the position of the regime in such a way that adjustment is – on average – faster for $y_{i,t} > \mu_i$ than for $y_{i,t} < \mu_i$, then the process will on average spend more time in a low state than in a high state. On average, $y_{i,t}$ will be below $\mu_i$. Things would be different if a low value of $y_{i,t}$ relative to $\mu_i$ enhances the probability that the individual is in a regime with a high speed of adjustment.

**Figure 2: Conditional expectations according to regimes: the regressions**

We will now further investigate the process $\{y_{i,t}\}$ in (1) and its expected value. Looking at $\{y_{i,t}\}$ alone, we see that formally it is an autoregressive equation with a stochastic coefficient $\alpha_{i,t-1}$ and a shift term $(1-\alpha_{i,t-1})\mu_i$. For a given starting value $y_{i,0}$, the general solution can therefore be found recursively by substitution. For $t \geq 1$, the backward solution to the adjustment equation is given by:

$$y_{i,t} = \left[y_{i,0} - \mu_i\right] \prod_{k=0}^{t-1} \alpha_{i,k} + \mu_i + A_{i,t},$$

(4)
with \[ A_{i,t} = \sum_{l=1}^{t-1} \varepsilon_{i,l} \prod_{k=l}^{t-1} \alpha_{i,k} + \varepsilon_{i,t}. \] (5)

Whereas the terms \( [y_{i,0} - \mu_i] \) and \( \mu_i \) capture the influence of the initial condition and the deterministic equilibrium, respectively, the term \( A_{i,t} \) gives us the effect of shocks, past and present. This effect is characterised by an interplay between the process \( \{\varepsilon_{i,t}\} \) and \( \{\alpha_{i,t}\} \), the adjustment coefficient. The term \( A_{i,t} \) follows a homogeneous version of the stochastic adjustment equation for \( y_{i,t} \) without a shift term:

\[ A_{i,t} = \alpha_{i,t-1} A_{i,t-1} + \varepsilon_{i,t}. \] (6)

If one disregards the (vanishing) influence of the initial condition, \( A_{i,t} \) is the distance between \( y_{i,t} \) and the deterministic equilibrium, \( \mu_i \). A closer inspection of (6) shows that the covariance between \( \alpha_{i,t-1} \), the speed of adjustment in \( t \), and \( A_{i,t-1} \), the deviation in \( t-1 \), is of crucial importance for the question of whether or not a persistent deviation between \( y_{i,t} \) and the deterministic equilibrium will exist.

We now need to explore the asymptotic properties of \( A_{i,t} \). Specifically, we want to know whether the expected value converges and how it can be statistically identified. Given the structure of \( A_{i,t} \), as exposed in (5), convergence implies some stationarity on the joint process \( \{y_{i,t}, \alpha_{i,t}\} \). We make the following two assumptions:

**Assumption 1:** The moments \( E\alpha_{i,t} \) and \( E\left(\varepsilon_{i,t} \prod_{\tau=k}^{t} \alpha_{i,\tau}\right) \) exist and are time invariant, i.e. they depend only on \( k \);

**Assumption 2:** The conditional expectation of \( |\alpha_{i,t}| \), given \( \varepsilon_{i,t} \) and the entire history of \( \alpha_{i,t} \), is bounded by a value \( \theta \) less than 1: \( E\left(|\alpha_{i,t}| \varepsilon_{i,t}, \alpha_{i,1}, \ldots, \alpha_{i,t-1}\right) \leq \theta < 1 \).

**Proposition:** Under Assumptions 1 and 2, \( E A_{i,t} \) converges to a constant for \( t \to \infty \).

**Proof:** Given the structure of \( A_{i,t} \) as depicted in (5), the difference between the expected deviation of two adjacent periods is given by:

\[ E\left(A_{i,t+1}\right) - E\left(A_{i,t}\right) = E\left(\varepsilon_{i,t} \prod_{\tau=t}^{t} \alpha_{i,\tau}\right). \]
A series \( \sum_{n=1}^{\infty} a_n \) is said to converge absolutely if the series \( \sum_{n=1}^{\infty} |a_n| \) converges. We now show that under Assumption 2, the series \( \sum_{i=1}^{\infty} E \left( e_{i,1} \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \right) \) is majorised by a convergent geometric series:

\[
E \left( e_{i,1} \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \right) \leq E \left( \left| e_{i,1} \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \right| \right) = E \left( \left| e_{i,1} \right| \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \cdot E \left( \left| \alpha_{i,1} \right| \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \right) \right) \leq E \left( \left| e_{i,1} \right| \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \cdot \theta \right) = \theta \cdot E \left( \left| e_{i,1} \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \right| \right).
\]

The third step uses the fact that \( \left\{ e_{i,1}, \alpha_{i,1}, \ldots, \alpha_{i,i-1} \right\} \) is a coarser information set than \( \left\{ e_{i,1} \prod_{\tau=1}^{i-1} \alpha_{i,\tau} \right\} \). The upper bound for the conditional expectation \( \left| \alpha_{i,\tau} \right| \) will therefore hold a fortiori. Repeating this argument \( t-1 \) times gives us

\[
E(A_{i,t}) - E(A_{i,i}) \leq \theta^t E(\left| e_{i,i} \right|).
\]

The convergence proof does not directly yield an analytical expression for the asymptotic mean of \( A_{i,t} \). Using the recursive structure of \( A_{i,t} \) gives us:

\[
E A_{i,t} = E(\alpha_{i,t-1} A_{i,t-1}) = E \alpha_{i,t-1} E A_{i,t-1} + \text{cov}(\alpha_{i,t-1}, A_{i,t-1}).
\]

This is a deterministic difference equation for the expectation of \( A_{i,t} \). Its coefficients will not necessarily be constant. But as \( E A_{i,t} \) converges, the stationary value is characterised by:

\[
\lim_{t \to \infty} E A_{i,t} = \lim_{t \to \infty} \frac{\text{cov}(A_{i,t-1}, \alpha_{i,t-1})}{1 - E(\alpha_{i,t-1})}.
\]

With Assumption 1, it follows as a corollary that \( \text{cov}(A_{i,t-1}, \alpha_{i,t-1}) \) also converges.

### 5.3. Three estimators for the aggregate shortfall

In order to assess the economic significance of the asymmetry, we need an estimator for the excess or shortfall in dynamic equilibrium. We will investigate two different approaches.

#### 5.3.1. An estimator based on the covariance between the level of \( y_{i,t} \) and the regime

The deviation \( A_{i,t} \) is unobservable. However, we do observe \( y_{i,t} \), and we know that it is additively composed as:
\[ y_{i,t} = \left[ y_{i,0} - \mu_{i} \right] \prod_{k=0}^{t-1} \alpha_{i,k} + \mu_{i} + A_{i,t}. \]

If \( \mu_{i} \) and \( r_{i,j} \) are uncorrelated, as we assume, then \( \text{cov}(y_{i,t}, \alpha_{i,t}) \) and \( \text{cov}(A_{i,t}, \alpha_{i,t}) \) are asymptotically equal, as for large \( t \) the influence of initial condition wanes. As we have seen, in Section 3, there is an N-consistent estimator for \( \alpha_{i,t} = \alpha \cdot r_{i,t} \). Therefore the statistic

\[ S_{i} = \frac{\text{cov}(y_{i,t}, \hat{\alpha} \cdot r_{i,t})}{1 - \frac{1}{N} \sum \hat{\alpha} \cdot r_{i,t}} \]

will be consistent for \( \lim E A_{i,t} \), the shortfall that is due to the non-linearity of adjustment and will persist in dynamic equilibrium.

It is interesting to consider an alternative derivation of this estimator. Departing directly from equation (1), and taking expectations,

\[ E \Delta y_{i,t} = -E(1 - \alpha_{i,t-1})(y_{i,t-1} - \mu_{i}) = -E(1 - \alpha_{i,t-1}) \cdot E(y_{i,t-1} - \mu_{i}) - \text{cov}[1 - \alpha_{i,t-1}, y_{i,t-1} - \mu_{i}], \]

we can solve for \( E(y_{i,t-1} - \mu_{i}) \). As \( \text{cov}[1 - \alpha_{i,t-1}, y_{i,t-1} - \mu_{i}] = -\text{cov}[\alpha \cdot r_{i,t-1}, y_{i,t-1}] \) if \( \mu_{i} \) and \( r_{i,t} \) are uncorrelated, we have

\[ E\left(y_{i,t-1} - \mu_{i}\right) = \frac{\text{cov}[\alpha \cdot r_{i,t-1}, y_{i,t-1}] - E \Delta y_{i,t}}{E(1 - \alpha \cdot r_{i,t-1})}. \]

This expectation still refers to individual \( i \) at time \( t \), given target \( \mu_{i} \) and initial conditions \( y_{i,0} \).

In dynamic equilibrium, \( E \Delta y_{i,t} = 0 \), and the right hand side converges to \( E \lim A_{i,t} \) for all individuals. Replacing the theoretical moments by empirical moments, thereby replacing \( \alpha \) by \( \hat{\alpha} \), results in expression \( S_{i} \) above.

### 5.3.2. Two alternatives based on the covariance of \( \Delta y_{i,t} \) and reciprocal adjustment speeds

Substituting the deviation \( A_{i,t} \) by the level of \( y_{i,t} \) in the estimation of the covariance \( \text{cov}(A_{i,t}, \alpha_{i,t}) \) can add a lot of noise in cases where the variation of \( \mu_{i} \) between individuals is high relative to the variation of \( A_{i,t} \). Fortunately, there is an alternative that circumvents this problem. From equation (1) we obtain:

\[ -\frac{1}{1 - \alpha_{i,t-1}} \Delta y_{i,t} = y_{i,t-1} - \mu_{i} + \frac{\epsilon_{i,t}}{1 - \alpha_{i,t-1}}, \text{ or} \]

\[ \gamma \cdot r_{i,t-1} \Delta y_{i,t} = y_{i,t-1} - \mu_{i} + \gamma \cdot r_{i,t-1} \epsilon_{i,t}, \quad \text{where} \]

(7)
\[ \gamma' = -\left( \frac{1}{1-\alpha_1}, \ldots, \frac{1}{1-\alpha_L} \right). \]

If we assume that \( r_{it-1} \) is uncorrelated with the error term \( \varepsilon_{it} \), then averaging the left hand side of (7) over all individuals will constitute a random variable that converges to the expected value of \( y_{it} - \mu_i \), in the population of individuals. Therefore with

\[ S_2 = \frac{1}{N} \sum_{i} \hat{\gamma}' r_{it-1} \Delta y_{it}, \]

we have an unbiased and N-consistent estimator of the mean excess or shortfall in the population at time \( t-1 \). Furthermore, when the process has been running long enough, the expected shortfall converges to the same value \( \lim E A_{it} \) for all firms, regardless of initial conditions. For this limit shortfall, we can also use a slightly modified estimator:

\[ S_3 = \text{cov}\left( \hat{\gamma}' r_{it-1}, \Delta y_{it} \right). \]

This statistic derives from \( S_2 \), additionally making use of the information that \( E \Delta y_{it} = 0 \) in the stationary state. Simulations have shown that \( E S_3 \) converges much faster to \( \lim E A_{it} \) than \( E S_2 \), as the former is distorted by the non-zero expected value of \( \Delta y_{it} \) outside the stationary state.

In principle, standard deviations for these estimators can be generated by the delta method. This is not as straightforward as it looks. We will discuss this for \( S_2 \). Defining

\[ \hat{\kappa} = \frac{1}{N} \sum_{i} r_{it-1} \Delta y_{it}, \]

we can write the estimator \( S_2 \):

\[ S_2 = \hat{\gamma}' \cdot \hat{\kappa}. \]

This estimator for the expected aggregate shortfall is thus the vector product of two rather simple structures. The vector \( \hat{\gamma} \) holds the reciprocal values of the estimated speeds of adjustment. The expression \( \hat{\kappa} \) is the sample average of an observable vector, holding the value \( \Delta y_{it} \) in the position \( k \) with \( r(k)_{it} = 1 \) and 0 elsewhere. In order to use the delta method, we need to estimate the two structures jointly, in order to obtain the covariance matrix for the elements of \( \hat{\gamma} \) and \( \hat{\kappa} \). A joint estimation is not easy, given that equation (1) with its time varying, stochastic coefficient constitutes a nonlinear dynamic panel equation that needs to be estimated consistently in order to gather \( \hat{\gamma} \). Alternatively, standard errors can be bootstrapped. We may resample entire firm histories, making use of the assumption of independence between firms, and generating repeated estimations for \( \hat{\gamma} \) and \( \hat{\kappa} \).
5.4. Estimates of the aggregate shortfall

Estimates of the aggregate shortfall using the three procedures developed in the last subsection are easily done, once there is a consistent estimate of the alpha-coefficients. Again, we use the Partition I estimates for capital demand and Partition III estimates for labour demand. Table 4 presents the results, both for simple and for interacted time effects. Standard errors were bootstrapped by repeated sampling the observations of entire firms, with 100 iterations each. The bootstrapped standard errors proved to be unbiased in simulation. For the current estimation, the bootstraps did not indicate any bias in gap estimation.

The result is a rather simple pattern. The estimates yielded by method S1 and S3 that intend to estimate the gap in dynamic equilibrium where \( \Delta y_{it} \) has died down, yield a gap estimate of almost nil. The estimated pattern of adjustment speeds in the three regimes is therefore not such that, taking into account the correlation of endogenous variables with the regimes, a sizeable gap would persist in equilibrium. The estimator S2, however, gives a direct estimate of the average gap that was realised in the sample period. It is a meaningful estimator of the asymptotic shortfall only when the average growth of the endogenous variable has subsided. This gap is -1.4% for capital according to both estimates of the alpha-coefficients and between 1.3% and 1.4% according to the two estimates for labour. This reflects the fact that, over the sample period, firms have been shedding labour at an average rate of 1.4%, whereas they have been accumulating real capital at a rate of 2.3% on average. Presumably following a trend in the relative price of labour and capital, the firms have increased capital intensity considerably, shedding labour while still increasing output (4.0% in the over the average of all firms). On average, given delayed adjustment, firms were lagging behind this trend to a certain extent. The size of the average gap is equivalent to about one year of growth (for labour) or even less (for capital). In a hypothetical stationary distribution, the gap would not persist.

This result is surprisingly clear, and it makes a strong statement on allocational efficiency. A possible aggregate divergence between "mandated" capital or labour and the actual factor use will result from incomplete adjustment to macro shocks, not from the asymmetry of the adjustment process as such.

6. Outlook

At this stage, our estimation results are not so much intended to give a definite and reliable quantitative description of the adjustment dynamics of firms as to outline a productive way of dealing with the non-linearities involved in dynamic adjustment. Survey data have two decisive advantages: they are timely, and they give a direct access to expectations and plans of economic agents, readily yielding crucial distributional information. If it is possible to relate the dynamics of adjustment – be it households, non-financial firms or financial
institutions – to the categorical information of survey data, then the aggregate consequences of economic adjustment can be assessed in a way that is relevant both for scientific analysis and for economic policy. As there are many potential adjustment mechanisms, there remains a lot of work to be done. But as we have seen in the last section, important aggregate results may be quite invariant to the details of the specification chosen. And this is where new knowledge on the workings of the economy is to be expected!
References


Bond, Stephen and Domenico Lombardi, To Buy or Not to Buy? Uncertainty, Irreversibility and Heterogeneous Investment Dynamics in Italian Company Data. *IMF Staff Papers* Vol. 53, 375-400


Table 1
State dependent partial adjustment model for log capital stock

<table>
<thead>
<tr>
<th></th>
<th>Regime Partition I</th>
<th>Regime Partition II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. var.</td>
<td>Δ log K_{it}</td>
<td></td>
</tr>
<tr>
<td>R1: unconstr.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>expansion</td>
<td>log(K_{it-1}/S_{it-1})</td>
<td>-0.1204*** (0.0394)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime constant</td>
<td>implied</td>
<td>implied</td>
</tr>
<tr>
<td># obs. in regime</td>
<td>2156</td>
<td>2156</td>
</tr>
<tr>
<td>R2: financing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constraints</td>
<td>log(K_{it-1}/S_{it-1})</td>
<td>-0.0489 (0.0379)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime constant</td>
<td>implied</td>
<td>implied</td>
</tr>
<tr>
<td># obs. in regime</td>
<td>933</td>
<td>933</td>
</tr>
<tr>
<td>R3: stationary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>or contracting</td>
<td>log(K_{it-1}/S_{it-1})</td>
<td>-0.1564*** (0.0401)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime constant</td>
<td>implied</td>
<td>implied</td>
</tr>
<tr>
<td># obs. in regime</td>
<td>1196</td>
<td>970</td>
</tr>
<tr>
<td>Additional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>regressors</td>
<td>8 year dummies</td>
<td>24 regime/ year</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dummies</td>
</tr>
<tr>
<td># firms</td>
<td>1406</td>
<td>1406</td>
</tr>
<tr>
<td># obs. total</td>
<td>4285</td>
<td>4285</td>
</tr>
<tr>
<td>Tests</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sargan-Hansen</td>
<td>χ²(123)=144.1 p=0.0809</td>
<td>χ²(123)=143.2 p=0.102</td>
</tr>
<tr>
<td>R1 and R2 same</td>
<td>χ² (1) = 2.8917 p=0.089</td>
<td>χ² (1) = 0.1127 p=0.7260</td>
</tr>
<tr>
<td>adjustment speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All adjustment speeds homogeneous</td>
<td>χ² (2) = 5.6778 p=0.0385</td>
<td>χ² (2) = 2.3177 p=0.3138</td>
</tr>
</tbody>
</table>

Regimes are defined on the basis of principal investment purposes and factors influencing investment activity, as stated by respondents. Column (1) and (2): R1 – financially unconstrained and expansion or rationalisation directed at labour costs as principal purpose; R2 – financially constrained; R3 – financially unconstrained and replacement investment or rationalisation directed at other costs as principal purpose. Column (3) and (4): R1 – financially unconstrained and expansion or rationalisation as principal purpose; R2 – financially constrained; R3 – financially unconstrained and replacement investment as principal purpose. Estimates in cols. (1) and (3) are with uniform year dummies, estimates in cols. (2) and (4) are with year dummies interacted with adjustment regime dummies. Estimation: GMM Generalised Difference estimator for regime specific adjustment as described in von Kalckreuth (2007), using a lead of at least 2. Time dummies are transformed alongside the other variables. Instruments: regime dummies, lag 0 and 1 of log K_{it-1} and log S_{it-1} interacted with regime dummies, plus transformed time dummies. The Sargan-Hansen statistic is a test of overidentifying restrictions proposed by Sargan (1958) and Hansen (1982). The tests on equality of adjustment coefficients are standard $\chi^2$ tests. The robust standard errors from the second step estimation with a small sample correction based on Windmeijer (2005) are in parentheses: *** significant at the 1% level; ** significant at the 5% level, * significant at the 10% level. Estimation was executed using DPD package version 1.2 on Ox version 3.30 and additional Ox routines.
Table 2
State dependent partial adjustment model for log employment

<table>
<thead>
<tr>
<th>Regime Partition II</th>
<th>Regime Partition III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep. var.: Δlog$L_{ij}$</td>
<td>(1)</td>
</tr>
<tr>
<td>(year dummies)</td>
<td>(interacted year dummies)</td>
</tr>
<tr>
<td>R1: unconstr. expansion</td>
<td>log($L_{ij} / S_{ij}$)</td>
</tr>
<tr>
<td></td>
<td>(0.0497)</td>
</tr>
<tr>
<td>Regime constant</td>
<td>implied</td>
</tr>
<tr>
<td># obs. in regime</td>
<td>2694</td>
</tr>
<tr>
<td>R2: financing constraints</td>
<td>log($L_{ij} / S_{ij}$)</td>
</tr>
<tr>
<td></td>
<td>(0.0602)</td>
</tr>
<tr>
<td>Regime constant</td>
<td>-0.0012</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
</tr>
<tr>
<td># obs. in regime</td>
<td>959</td>
</tr>
<tr>
<td>R3: stationary or contracting</td>
<td>log($L_{ij} / S_{ij}$)</td>
</tr>
<tr>
<td></td>
<td>(0.0524)</td>
</tr>
<tr>
<td>Regime constant</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>(0.0030)</td>
</tr>
<tr>
<td># obs. in regime</td>
<td>1011</td>
</tr>
<tr>
<td>Additional regressors</td>
<td>8 year dummies</td>
</tr>
<tr>
<td># firms</td>
<td>1472</td>
</tr>
<tr>
<td># obs. total</td>
<td>4664</td>
</tr>
<tr>
<td>Tests</td>
<td>Sargan-Hansen</td>
</tr>
<tr>
<td></td>
<td>$p=0.03$</td>
</tr>
<tr>
<td>R1 and R2 same adjustment speed</td>
<td>$\chi^2$(1)=1.577</td>
</tr>
<tr>
<td></td>
<td>$p=0.6913$</td>
</tr>
<tr>
<td>All adjustment speeds homogeneous</td>
<td>$\chi^2$(2)=0.6241</td>
</tr>
<tr>
<td></td>
<td>$p=0.7319$</td>
</tr>
</tbody>
</table>

Notes: See Table 1. Regimes are defined on the basis of principal investment purposes and factors influencing investment activity, as stated by respondents. Column (1) and (2): R1 – financially unconstrained and expansion or rationalisation as principal purpose; R2 – financially constrained; R3 – financially unconstrained and replacement investment as principal purpose. Column (3) and (4): R1 – financially unconstrained and expansion as principal purpose; R2 – financially constrained; R3 – financially unconstrained and rationalisation or replacement investment as principal purpose. Instruments: regime dummies, lag 0 and 1 of log$L_{ij-1}$ and log$S_{ij-1}$ interacted with regime dummies, plus transformed time dummies.
<table>
<thead>
<tr>
<th>Table 3</th>
<th>State dependent Constant Returns ECM for log employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regime Partition II</td>
</tr>
<tr>
<td>Dep. var.: $\Delta \log L_{t,i}$</td>
<td>(1) (year dummies)</td>
</tr>
<tr>
<td>R1: unconstrained expansion</td>
<td>$\Delta \log S_{t,i}$</td>
</tr>
<tr>
<td></td>
<td>$\log (L_{t,i}/S_{t,i})$</td>
</tr>
<tr>
<td></td>
<td>Regime constant</td>
</tr>
<tr>
<td></td>
<td># obs. in regime</td>
</tr>
<tr>
<td>R2: financing constraints</td>
<td>$\Delta \log S_{t,i}$</td>
</tr>
<tr>
<td></td>
<td>$\log (L_{t,i}/S_{t,i})$</td>
</tr>
<tr>
<td></td>
<td>Regime constant</td>
</tr>
<tr>
<td></td>
<td># obs. in regime</td>
</tr>
<tr>
<td>R3: stationary or contracting</td>
<td>$\Delta \log S_{t,i}$</td>
</tr>
<tr>
<td></td>
<td>$\log (L_{t,i}/S_{t,i})$</td>
</tr>
<tr>
<td></td>
<td>Regime constant</td>
</tr>
<tr>
<td></td>
<td># obs. in regime</td>
</tr>
<tr>
<td>Additional regressors</td>
<td>8 year dummies</td>
</tr>
<tr>
<td># firms</td>
<td>1472</td>
</tr>
<tr>
<td># obs. total</td>
<td>4664</td>
</tr>
<tr>
<td>Tests</td>
<td>Sargan-Hansen</td>
</tr>
<tr>
<td></td>
<td>R1 and R2 same short run reaction</td>
</tr>
<tr>
<td></td>
<td>R1 and R2 same adjustment speed</td>
</tr>
<tr>
<td></td>
<td>Both coeff. homogen. in R1 and R2</td>
</tr>
<tr>
<td></td>
<td>All short run reactions homogen.</td>
</tr>
<tr>
<td></td>
<td>All adjustment speeds homogen.</td>
</tr>
<tr>
<td></td>
<td>All coeff. homogen. between regimes</td>
</tr>
</tbody>
</table>

Notes: See Table 1. Regimes are defined on the basis of principal investment purposes and factors influencing investment activity, as stated by respondents. Column (1) and (2): R1 – financially unconstrained and expansion or rationalisation as principal purpose; R2 – financially constrained; R3 – financially unconstrained and replacement investment as principal purpose. Column (3) and (4): R1 – financially unconstrained and expansion as principal purpose; R2 – financially constrained; R3 – financially unconstrained and rationalisation or replacement investment as principal purpose. Instruments: regime dummies, lag 0 and 1 of $\log L_{t,i-1}$ and $\log S_{t,i-1}$ interacted with regime dummies, plus transformed time dummies.
Table 4
Estimates of aggregate gap between state and target

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>Capital demand (Regime Partition I)</th>
<th>Labour demand (Regime Partition III)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (year dummies)</td>
<td>(2) (interacted year dummies)</td>
</tr>
<tr>
<td>S1</td>
<td>0.0001 (0.0051)</td>
<td>0.0070 (0.0066)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.0046 (0.0025)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.0084 (0.0057)</td>
</tr>
<tr>
<td>S2</td>
<td>-0.0135 (0.0014)</td>
<td>-0.0138 (0.0014)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0132 (0.0012)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0139 (0.0014)</td>
</tr>
<tr>
<td>S3</td>
<td>0.0001 (0.0003)</td>
<td>-0.0003 (0.0004)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0005 (0.0002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0007 (0.0004)</td>
</tr>
</tbody>
</table>

Note: Estimation methods S1, S2 and S3 as described in Section 5.3. Column (1) and (2) use the adjustment speed estimates of columns (1) and (2) of Table 1, respectively. Column (3) and (4), for labour demand, draw on the coefficient estimates of column (3) and (4) of Table 2. For the definition of Partitions see Section 4 and the notes to Table 1. Bootstrapped standard errors (100 replications) in brackets, calculated by resampling entire firm histories and reestimating both the adjustment coefficients and the relevant sample covariances.
Graph 1: Time varying sensitivity of capital demand

(for Regime Partition I, simple time effects, Table 1, Column I)
Graph 2: Time varying sensitivity of labour demand

(for Regime Partition II, simple time effects, Table 2, Column 3)