Optimal Monetary Policy in HANK Economies

Sushant Acharya  Edouard Challe  Keshav Dogra

CEPR-BdF Heterogeneity Conference, Dec. 5-6 2019

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System.
How does imperfect insurance affect optimal monetary policy?
Challenge and framework

- **challenge:** infinite dimensional fixed point problem
  - social welfare function aggregates heterogeneous intertemporal utilities...
  - ...each of which is endogenous to policy

- **framework:** CARA-Normal HANK with closed-form expressions for
  - aggregate dynamics
  - distribution of agents & its law of motion
  - social welfare function
Challenge and framework

- **challenge:** infinite dimensional fixed point problem
  - social welfare function aggregates heterogenous intertemporal utilities...
  - ...each of which is endogenous to policy

- **framework:** CARA-Normal HANK with closed-form expressions for
  - aggregate dynamics
  - distribution of agents & its law of motion
  - social welfare function
Main results

- optimal policy governed by 2 forces:

1. price stability
Main results

- optimal policy governed by 2 forces:
  1. price stability
  2. consumption dispersion, as affected by
     - level and cyclicality of income risk
     - pass-trough to consumption risk (via time-varying MPC)

- CB may depart from price stability to reduce consumption dispersion
- breakdown of divine coincidence
Main results

▶ optimal policy governed by 2 forces:

1. **price stability**

2. **consumption dispersion**, as affected by
   - level and cyclicalilty of *income* risk
   - pass-trough to *consumption* risk (via time-varying MPC)

▶ CB may depart from price stability to reduce consumption dispersion

▶ breakdown of *divine coincidence*
Households

- Blanchard-Yaari demographics with population size 1 and survival rate $\theta$
Households

- Blanchard-Yaari demographics with population size 1 and survival rate $\theta$

- Newborn household $i$ at date $s$ maximizes:

\[
\max_{\{c^s_t(i), \ell^s_t(i), a^s_t(i)\}} \mathbb{E}_s \sum_{t=s}^{\infty} (\beta \theta)^{t-s} u[c^s_t(i), \xi^s_t(i) - \ell^s_t(i)]
\]

s.t.

\[
\begin{align*}
    c^s_t(i) + q^s_t a^s_{t+1}(i) &= w_t \ell^s_t(i) + a^s_t(i) + T_t \\
    a^s_s(i) &= 0 \\
    \xi^s_t(i) &\sim \mathcal{N}(\bar{\xi}, \sigma^2_t)
\end{align*}
\]
Households

- Blanchard-Yaari demographics with population size 1 and survival rate $\theta$
- Newborn household $i$ at date $s$ maximizes:

$$\max \{c_s^s(i), \ell_s^s(i), a_s^s(i)\}$$

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta \theta)^{t-s} u[c_t^s(i), \xi_t^s(i) - \ell_t^s(i)]$$

subject to

$$c_t^s(i) + q_t a_t^s(i+1) = w_t \ell_t^s(i) + a_t^s(i) + T_t$$

$$a_t^s(i) = 0$$

$$\xi_t^s(i) \sim \mathcal{N}(\bar{\xi}, \sigma_t^2)$$

- CARA utility:

$$u(c, \xi - \ell) = -\frac{1}{\gamma} e^{-\gamma c} - \rho e^{-\frac{1}{\rho}(\xi - \ell)}$$
Firms

- competitive final-goods firms + monopolistic competitive wholesale firms facing Rotemberg pricing frictions ⇒ NKPC:

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left[ 1 - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \frac{z_t}{(1 - \tau) w_t} \right] + \frac{1}{R_t} \left( \frac{y_{t+1} z_t w_t}{y_t z_{t+1} w_{t+1}} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]
Firms

- competitive final-goods firms + monopolistic competitive wholesale firms facing Rotemberg pricing frictions \( \Rightarrow \) NKPC:

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left[ 1 - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \frac{z_t}{(1 - \tau) w_t} \right] + \frac{1}{R_t} \left( \frac{y_{t+1} z_t w_t}{y_t z_{t+1} w_{t+1}} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]

- \( \ln z_t = \rho_z^t \ln z_0 \): one-off productivity change at \( t = 0 \) (no aggregate risk)
Firms

- competitive final-goods firms + monopolistic competitive wholesale firms facing Rotemberg pricing frictions ⇒ NKPC:

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left[ 1 - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \frac{z_t}{(1 - \tau) w_t} \right] + \frac{1}{R_t} \left( \frac{y_{t+1}z_tw_t}{y_tw_{t+1}} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]

- \( \ln z_t = \rho_t^z \ln z_0 \): one-off productivity change at \( t = 0 \) (no aggregate risk)

- \( \tau = \) (constant) payroll subsidy
Firms

- competitive final-goods firms + monopolistic competitive wholesale firms facing Rotemberg pricing frictions ⇒ NKPC:

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon}{\Phi} \left[ 1 - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \frac{z_t}{(1 - \tau) w_t} \right] + \frac{1}{R_t} \left( \frac{y_{t+1} z_t w_t}{y_t z_{t+1} w_{t+1}} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]

- \( \ln z_t = \rho^t_z \ln z_0 \): one-off productivity change at \( t = 0 \) (no aggregate risk)

- \( \tau = (\text{constant}) \) payroll subsidy

- \( y_t = \text{net output/income} \)

\[
y_t = z_t n_t - \frac{\Phi}{2} (\Pi_t - 1)^2
\]

\[
y_t = \frac{z_t n_t}{1 + \frac{\Phi}{2} (\Pi_t - 1)^2}
\]
Rest of model

- zero-profit life insurers
- gov't running a balanced budget
- market clearing

\begin{align*}
\textbf{goods: } (1 - \theta) \sum_{s=-\infty}^{t} \theta^{s-t} \int_{i} c_{t}^{s}(i) di &= y_{t} \\
\textbf{labor: } (1 - \theta) \sum_{s=-\infty}^{t} \theta^{s-t} \int_{i} \ell_{t}^{s}(i) di &= n_{t} \\
\textbf{bonds: } (1 - \theta) \sum_{s=-\infty}^{t} \theta^{s-t} \int_{i} a_{t}^{s}(i) di &= 0
\end{align*}
Equilibrium

- consumption decision rule, for any $HH \ i \in \text{cohort} \ s$:

$$c_t(x) = y_t + \mu_t x$$

where $x = w_t (\xi - \bar{\xi}) + a$ is (de-meaned) cash-on-hand

- $\mu_t$ is the MPC out of cash-on-hand:

$$\mu_t^{-1} = 1 + \gamma \rho w_t + \frac{\theta}{R_t} \mu_t^{-1}$$
Equilibrium

▶ consumption decision rule, for any HH $i \in$ cohort $s$:

$$c_t(x) = y_t + \mu_t x$$

where $x = w_t (\xi - \bar{\xi}) + a$ is (de-meaned) **cash-on-hand**

▶ $\mu_t$ is the MPC out of cash-on-hand:

$$\mu_t^{-1} = 1 + \gamma \rho w_t + \frac{\theta}{R_t} \mu_{t+1}^{-1}$$
Equilibrium

- consumption decision rule, for any HH $i \in$ cohort $s$:

$$c_t(x) = y_t + \mu_t x$$

where $x = w_t (\xi - \bar{\xi}) + a$ is (de-meaned) **cash-on-hand**

- $\mu_t$ is the MPC out of cash-on-hand:

$$\mu_t^{-1} = 1 + \gamma \rho w_t + \frac{\theta}{R_t} \mu_t^{-1}$$

- leisure

$$\xi - \ell_t(x) = \rho (\gamma y_t - \ln w_t) + (\rho \gamma \mu_t) x$$
Complete vs. incomplete markets

- With **complete mkts** consumption and leisure are unresponsive to $\xi$ shocks

\[
\frac{\partial c}{\partial \xi} = \mu_t \omega_t > 0 \quad \text{and} \quad \frac{\partial [\xi - \ell_t(x)]}{\partial \xi} = \gamma \rho \mu_t \omega_t > 0
\]

\[
\mu_t \text{ reflects ability to self-insure via labor and bond markets}
\]

\[
\mu_t - 1 = \infty \sum_{s=0}^{\infty} \theta_s \prod_{k=0}^{s-1} R_t + k (1 + \gamma \rho \omega_t + s)
\]

\[
\mu_t \text{ determines pass-through from earnings risk to consumption risk}
\]
Complete vs. incomplete markets

- With **complete mkts** consumption and leisure are unresponsive to $\xi$ shocks.

- With **incomplete mkts** both adjust (insurance through labor supply):

$$\frac{\partial c_t(x)}{\partial \xi} = \mu_t w_t > 0 \quad \text{and} \quad \frac{\partial [\xi - \ell_t(x)]}{\partial \xi} = \gamma \mu_t w_t > 0$$
Complete vs. incomplete markets

- With **complete mkts** consumption and leisure are unresponsive to $\xi$ shocks

- With **incomplete mkts** both adjust (insurance through labor supply):

  $$\frac{\partial c_t(x)}{\partial \xi} = \mu_t w_t > 0 \quad \text{and} \quad \frac{\partial [\xi - \ell_t(x)]}{\partial \xi} = \gamma \rho \mu_t w_t > 0$$

- $\mu_t$ reflects ability to self-insure via labor and bond markets

  $$\mu_t^{-1} = \sum_{s=0}^{\infty} \frac{\theta^s}{\prod_{k=0}^{s-1} R_{t+k}} (1 + \gamma \rho w_{t+s})$$
Complete vs. incomplete markets

▶ With **complete mkts** consumption and leisure are unresponsive to $\xi$ shocks

▶ With **incomplete mkts** both adjust (insurance through labor supply):

$$\frac{\partial c_t(x)}{\partial \xi} = \mu_tw_t > 0 \quad \text{and} \quad \frac{\partial [\xi - \ell_t(x)]}{\partial \xi} = \gamma \rho \mu_tw_t > 0$$

▶ $\mu_t$ reflects ability to self-insure via labor and bond markets

$$\mu_t^{-1} = \sum_{s=0}^{\infty} \frac{\theta^s}{\prod_{k=0}^{s-1} R_{t+k}} (1 + \gamma \rho w_{t+s})$$

▶ $\mu_t$ determines pass-through from earnings risk to consumption risk
Aggregate Euler equation

- individual Euler equation

\[ e^{-\gamma c^s(i)} = \beta \theta \frac{R_t}{\theta} \mathbb{E}_t e^{-\gamma c^s_{i+1}(i)} \]
Aggregate Euler equation

- Individual Euler equation

\[-\gamma c_t^s(i) = \ln[\beta R_t] - \gamma E_t c_{t+1}^s(i) + \frac{\gamma^2}{2} V_t c_{t+1}^s(i)\]
Aggregate Euler equation

- **individual Euler equation**

\[-\gamma c_t^s(i) = \ln [\beta R_t] - \gamma \mathbb{E}_t c_{t+1}^s(i) + \frac{\gamma^2}{2} \nabla_t c_{t+1}^s(i)\]

- **Using** $c_t^s(i) = y_t + \mu_t x_t^s(i)$ **and aggregating across all households:**

\[y_t = -\frac{1}{\gamma} \ln [\beta R_t] + y_{t+1} - \frac{\gamma \mu_{t+1}^2 w_{t+1}^2 \sigma_{t+1}^2}{2} \]  

precautionary savings motive
Equilibrium

\[
y_t = -\frac{1}{\gamma} \ln [\beta R_t] + y_{t+1} - \frac{\gamma \mu_{t+1}^2 w_{t+1}^2 \sigma_{t+1}^2}{2}
\]

\[
\mu_{t}^{-1} = 1 + \gamma \rho w_t + \frac{\theta}{R_t} \mu_{t+1}^{-1}
\]

\[
y_t = \frac{\rho \ln w_t + \xi}{z_t^{-1} + \gamma \rho + z_t^{-1} \Phi \frac{1}{2} (\Pi_t - 1)^2}
\]

\[
(\Pi_t - 1) \Pi_t = \frac{\varepsilon_t}{\Phi} \left[ 1 - \left( \frac{\varepsilon_t - 1}{\varepsilon_t} \right) \frac{z_t}{1 - \tau} w_t \right] + \frac{1}{R_t} \left( \frac{y_{t+1} z_t w_t}{y_t z_{t+1} w_{t+1}} \right) (\Pi_{t+1} - 1) \Pi_{t+1}
\]

\[
R_t = \frac{1 + i_t}{\Pi_{t+1}}
\]
Cyclicality of risk

- **cyclicality of risk** determines AD in incomplete-market economies
  (Acharya and Dogra, 2019; Bilbiie, 2019; Werning, 2015)
Cyclicality of risk

- **cyclicality of risk** determines AD in incomplete-market economies (Acharya and Dogra, 2019; Bilbiie, 2019; Werning, 2015)

- We allow for pro- or countercyclical risk. Assume variance of cash-on-hand $x$:

\[
\mathbb{V}_t(x) = \sigma^2_t w_t^2 = \sigma^2 w^2 e^{2\phi(y_t-y)}
\]

Linearized aggregate euler equation:

\[
\tilde{y}_t = \left(1 - \frac{\phi \Lambda}{\gamma}\right) \tilde{y}_{t+1} - \frac{1}{\gamma} (i_t - \pi_{t+1}) - \frac{1}{2} \frac{\Lambda}{\gamma} \mu_{t+1}
\]
Cyclicality of risk

- **cyclicality of risk** determines AD in incomplete-market economies (Acharya and Dogra, 2019; Bilbiie, 2019; Werning, 2015)

- We allow for pro- or countercyclical risk. Assume variance of cash-on-hand $x$:

$$V_t(x) = \sigma^2 w^2 = \sigma^2 w^2 e^{2\phi(y_t - y)}$$

Linearized aggregate euler equation:

$$\hat{y}_t = \left(1 - \frac{\phi \Lambda}{\gamma}\right)\hat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \frac{1}{2}\frac{\Lambda}{\gamma}\hat{\mu}_{t+1}$$

- **procyclical risk**: $\phi > 0$ or $\Theta < 1$: “discounted euler equation”
Cyclicality of risk

- **cyclicality of risk** determines AD in incomplete-market economies (Acharya and Dogra, 2019; Bilbiie, 2019; Werning, 2015)

- We allow for pro- or countercyclical risk. Assume variance of cash-on-hand $x$:

  $$\nabla_t(x) = \sigma_t^2 w_t^2 = \sigma^2 w^2 e^{2\phi(y_t-y)}$$

Linearized aggregate euler equation:

$$\hat{y}_t = \left(1 - \frac{\phi \Lambda}{\gamma}\right) \hat{y}_{t+1} - \frac{1}{\gamma} (i_t - \pi_{t+1}) - \frac{1}{2} \frac{\Lambda}{\gamma} \hat{\mu}_{t+1}$$

- **acyclical risk**: $\phi = 0$ or $\Theta = 1$ “undiscounted euler equation”
Cyclicality of risk

- **cyclicality of risk** determines AD in incomplete-market economies (Acharya and Dogra, 2019; Bilbiie, 2019; Werning, 2015)

- We allow for pro- or countercyclical risk. Assume variance of cash-on-hand $x$:

  \[ \nabla_t(x) = \sigma_t^2 w_t^2 = \sigma^2 w^2 e^{2\phi(y_t - y)} \]

  Linearized aggregate euler equation:

  \[ \hat{y}_t = \left(1 - \frac{\phi \Lambda}{\gamma}\right) \hat{y}_{t+1} - \frac{1}{\gamma} (i_t - \pi_{t+1}) - \frac{1}{2} \frac{\Lambda}{\gamma} \hat{\mu}_{t+1} \]

  - **countercyclical risk**: $\phi < 0$ or $\Theta > 1$ “compounded euler equation”
Optimal Monetary Policy
Optimal policy problem

Planner maximizes:

\[
W_0 = \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \theta) \sum_{s=-\infty}^{0} \theta^{-s} \int u[c^s_t(i), \xi^s_t(i) - \ell^s_t(i)] \, di \right\}
\]

Pareto weights:

- equal weights on all HH alive at any date \( t \), \( \beta^{s-t} \) on cohorts born at \( s \geq t \)
- ensures no time-inconsistency due to sequence of welfare weights (Calvo and Obstfeld, 1988)
Optimal policy problem

Lemma

\[ W_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, \bar{\xi} - \ell_t) \times \left\{ \begin{array}{c} \text{felicity of notional rep. agent} \\ \text{cost of inequality} \end{array} \right\} \]

where

\[ \Sigma_t = (1 - \theta) \sum_{s=-\infty}^{0} \theta^{-s} e^{\frac{1}{2} \sigma^2_c(t,s)} \geq 1 \]

\( \sigma^2_c(t,s) \): date t cross-sectional consumption dispersion amongst cohort s survivors
Optimal policy problem

Lemma

$$W_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, \bar{\xi} - \ell_t) \times \sum_t$$

where

$$\sum_t = (1 - \theta) \sum_{s=-\infty}^{0} \theta^{-s} e^{\frac{1}{2}\sigma_c^2(t,s)} \geq 1$$

$$\sigma_c^2(t,s):$$ date $t$ cross-sectional consumption dispersion amongst cohort $s$ survivors

RANK, $\sum_t = 1$
Optimal policy problem

Lemma

\[ W_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, \xi_t - \ell_t) \times \sum_t \frac{1}{2} \sigma_c^2(t,s) \geq 1 \]

where

\[ \sum_t = (1 - \theta) \sum_{s=-\infty}^{0} \theta^{-s} e^{\frac{1}{2} \sigma_c^2(t,s)} \geq 1 \]

\( \sigma_c^2(t,s) \): date t cross-sectional consumption dispersion amongst cohort s survivors

RANK, \( \sum_t = 1 \)  \hspace{1cm} HANK, \( \sum_t > 1 \)
Evolution of $\Sigma_t$

$$\ln \Sigma_t = \frac{1}{2} \gamma^2 \mu_t^2 w_t^2 \sigma_t^2 + \ln [1 - \theta + \theta \Sigma_{t-1}]$$

- Inequality is slow moving
Evolution of $\Sigma_t$

\[
\ln \Sigma_t = \frac{1}{2} \gamma^2 \mu_t^2 w_t^2 \sigma_t^2 + \ln [1 - \theta + \theta \Sigma_{t-1}] 
\]

- Inequality is slow moving
- Innovation to inequality depends on consumption risk
Evolution of $\Sigma_t$

$$\ln \Sigma_t = \frac{1}{2} \gamma^2 \mu_t^2 w_t^2 \sigma_t^2 + \ln [1 - \theta + \theta \Sigma_{t-1}]$$

- Inequality is slow moving

- Innovation to inequality depends on consumption risk which in turn depends on:
  - earnings risk $w_t^2 \sigma_t^2$
  - passthrough $\mu_t^2$
Evolution of $\Sigma_t$

\[
\ln \Sigma_t = \frac{1}{2} \gamma^2 \mu^2_t w_t^2 \sigma^2_t + \ln [1 - \theta + \theta \Sigma_{t-1}]
\]

- Inequality is slow moving
- Innovation to inequality depends on consumption risk which in turn depends on:
  - earnings risk $w_t^2 \sigma^2_t$
  - passthrough $\mu^2_t$
Evolution of $\Sigma_t$

- assume economy (incl. the wealth distribution) is initially in steady state

- $\Sigma_0$ has an additional component:

$$
\ln \Sigma_0 = \frac{1}{2} \gamma^2 \mu_0^2 w_0^2 \sigma_0^2 + \ln [1 - \theta + \theta \Sigma] + \ln \left( \frac{1 - \theta e^{\frac{\Lambda}{2}}}{1 - \theta e^{\frac{\Lambda}{2}} \left( \frac{\mu_0}{\mu} \right)^2} \right)
$$

- setting $\mu_0 < \mu$ reduces consumption inequality
Evolution of $\Sigma_t$

- assume economy (incl. the wealth distribution) is initially in steady state

- $\Sigma_0$ has an additional component:

$$\ln \Sigma_0 = \frac{1}{2} \gamma^2 \mu_0^2 w_0^2 \sigma_0^2 + \ln [1 - \theta + \theta \Sigma] + \ln \left( \frac{1 - \theta e^{\frac{A}{2}}}{1 - \theta e^{\frac{A}{2}} \left( \frac{\mu_0}{\mu} \right)^2} \right)$$

- setting $\mu_0 < \mu$ reduces **consumption inequality**

- unanticipated rate cut at date 0, given date 0 wealth distribution:
  - wealthy households get lower return on wealth $\Rightarrow \downarrow c$
  - indebted households pay lower interest on debt $\Rightarrow \uparrow c$
  - **not** revaluation effect
  - anticipated $\downarrow \mu$ does not have this additional effect since wealth distribution adjusts
Optimal policy problem

\[ \max_{\{i_t\}} W_0 \text{ s.t. law of motion for } \Sigma_t \text{ and equilibrium conditions} \]
Optimal policy problem

- \( \max_{\{i_t\}} \mathbb{W}_0 \) s.t. law of motion for \( \Sigma_t \) and equilibrium conditions

- optimality conditions given by f.o.c. of Lagrangian function
Optimal policy problem

- \[ \max_{\{i_t\}} W_0 \text{ s.t. law of motion for } \Sigma_t \text{ and equilibrium conditions} \]

- Optimality conditions given by f.o.c. of Lagrangian function

- Incomplete markets \( \Rightarrow \) incentive to lower \( \Sigma_t \)

1. Reduce earnings risk
2. Improve self insurance \( \Rightarrow \) reduce pass-through to consumption
3. Compress initial cons. distribution, by redistributing expected payoffs from assets
Optimal policy problem

- \( \max_{\{i_t\}} W_0 \) s.t. law of motion for \( \Sigma_t \) and equilibrium conditions

- optimality conditions given by f.o.c. of Lagrangian function

- incomplete markets \( \Rightarrow \) incentive to lower \( \Sigma_t \)

- 3 ways of achieving this purpose

1. reduce earnings risk
2. improve self insurance \( \Rightarrow \) reduce pass-through to consumption
3. compress initial cons. distribution, by redistributing expected payoffs from assets
Optimal policy problem

- $\max_{\{i_t\}} \mathbb{W}_0$ s.t. law of motion for $\Sigma_t$ and equilibrium conditions

- optimality conditions given by f.o.c. of Lagrangian function

- incomplete markets $\Rightarrow$ incentive to lower $\Sigma_t$

- 3 ways of achieving this purpose
  1. reduce earnings risk
Optimal policy problem

- \( \max \{ i_t \} \mathbb{W}_0 \) s.t. law of motion for \( \Sigma_t \) and equilibrium conditions

- Optimality conditions given by f.o.c. of Lagrangian function

- Incomplete markets \( \Rightarrow \) incentive to lower \( \Sigma_t \)

- 3 ways of achieving this purpose
  1. Reduce earnings risk
  2. Improve self insurance \( \Rightarrow \) reduce pass-through to consumption
Optimal policy problem

- $\max_{\{i_t\}} W_0$ s.t. law of motion for $\Sigma_t$ and equilibrium conditions

- optimality conditions given by f.o.c. of Lagrangian function

- incomplete markets $\Rightarrow$ incentive to lower $\Sigma_t$

- 3 ways of achieving this purpose
  1. reduce earnings risk
  2. improve self insurance $\Rightarrow$ reduce pass-through to consumption
  3. compress initial cons. distribution, by redistributing expected payoffs from assets
Refresher: Optimal monetary policy in RANK

- Target criterion after date 1:

$$\left( \hat{y}_t - \hat{y}_t^* \right) - \left( \hat{y}_{t-1} - \hat{y}_{t-1}^* \right) + \varepsilon \pi_t = 0$$

and date 0:

$$\left( \hat{y}_0 - \hat{y}_0^* \right) + \varepsilon \pi_0 = 0$$

where \( \hat{y}_t^* = \frac{1+\rho}{1+\gamma \rho} \hat{z}_t \) is the flexible-price level of output.
Refresher: Optimal monetary policy in RANK

- Target criterion after date 1:

\[
(\hat{y}_t - \hat{y}_t^*) - (\hat{y}_{t-1} - \hat{y}_{t-1}^*) + \varepsilon \pi_t = 0
\]

and date 0:

\[
(\hat{y}_0 - \hat{y}_0^*) + \varepsilon \pi_0 = 0
\]

where \( \hat{y}_t^* = \frac{1+\rho}{1+\gamma \rho} \hat{z}_t \) is the flexible-price level of output.

- Phillips curve:

\[
\pi_t = \beta \pi_{t+1} + \frac{\varepsilon}{\Phi} (\hat{y}_t - \hat{y}_t^*)
\]
Refresher: Optimal monetary policy in RANK

▶ Target criterion after date 1:

\[(\hat{y}_t - \hat{y}^*_t) - (\hat{y}_{t-1} - \hat{y}^*_{t-1}) + \varepsilon\pi_t = 0\]

and date 0:

\[(\hat{y}_0 - \hat{y}^*_0) + \varepsilon\pi_0 = 0\]

where \(\hat{y}^*_t = \frac{1+\rho}{1+\gamma \rho} \hat{z}_t\) is the flexible-price level of output.

▶ Phillips curve:

\[\pi_t = \beta \pi_{t+1} + \frac{\varepsilon}{\Phi} (\hat{y}_t - \hat{y}^*_t)\]

▶ divine coincidence: mp can set \(\hat{y}_t = \hat{y}^*_t\) and \(\pi_t = 0\) at all dates and states.
Optimal monetary policy in HANK

- can a version of divine coincidence hold in HANK?

\[ \hat{y}_t - \hat{y}_t^* - 1 = 0 \]

Of course, if \( \hat{y}_0 \neq \hat{y}_0^* \) then \( \hat{y}_t \geq 1 \neq \hat{y}_t^* \) and/or \( \pi_t \geq 1 \neq 0 \)
Optimal monetary policy in HANK

- can a version of divine coincidence hold in HANK?

- set $\phi$ so that the income-risk and self-insurance channels exactly offset each other
Optimal monetary policy in HANK

- can a version of divine coincidence hold in HANK?
- set $\phi$ so that the income-risk and self-insurance channels exactly offset each other
- this occurs under mildly procyclical income risk ($\phi > 0$)
Optimal monetary policy in HANK

- can a version of divine coincidence hold in HANK?

- set $\phi$ so that the income-risk and self-insurance channels exactly offset each other

- this occurs under mildly procyclical income risk ($\phi > 0$)

- target criterion for $t \geq 1$

$$
(\hat{y}_t - \hat{y}_t^*) - \frac{1}{\beta R} (\hat{y}_{t-1} - \hat{y}_{t-1}^*) + \varepsilon \pi_t = 0
$$

Of course, if $\hat{y}_0 \neq \hat{y}_0^*$ then $\hat{y}_t \geq 1 \neq \hat{y}_t^* \geq 1$ and/or $\pi_t \geq 1 \neq 0$
Optimal monetary policy in HANK

- Can a version of divine coincidence hold in HANK?

- Set $\phi$ so that the income-risk and self-insurance channels exactly offset each other.

- This occurs under mildly procyclical income risk ($\phi > 0$).

- Target criterion for $t \geq 1$

$$
(\hat{y}_t - \hat{y}_t^*) - \frac{1}{\beta R} (\hat{y}_{t-1} - \hat{y}_{t-1}^*) + \varepsilon \pi_t = 0
$$

- Of course, if $\hat{y}_0 \neq \hat{y}_0^*$ then $\hat{y}_{t \geq 1} \neq \hat{y}_{t \geq 1}^*$ and/or $\pi_{t \geq 1} \neq 0$
Optimal monetary policy in HANK

what about date 0? Recall nonrecursivity of law of motion for $\Sigma_t$

$\hat{y}_0 - \delta \hat{y}^*_{0} + \epsilon \alpha \pi_0 = \chi$

where $\alpha > 1$, $\delta \in (0, 1)$, $\chi > 0$

Initial wealth dispersion generates a form of time-inconsistency unrelated to Fisher channel or Barro-Gordon inflation bias. Disappears when initial bond holdings are equalized ($a_s(i) = 0 \forall i$ and $s \leq 0$).
Optimal monetary policy in HANK

- what about date 0? Recall nonrecursivity of law of motion for $\Sigma_t$

- optimal to implement a expansionary cut in real interest rates (relative to s.s.)
Optimal monetary policy in HANK

- what about date 0? Recall nonrecursivity of law of motion for $\Sigma_t$

- optimal to implement a expansionary cut in real interest rates (relative to s.s.)

- target criterion becomes:

\[
(\hat{y}_0 - \delta \hat{y}_0^*) + \frac{\varepsilon}{\alpha} \pi_0 = \chi
\]

where $\alpha > 1$, $\delta \in (0, 1)$, $\chi > 0$
Optimal monetary policy in HANK

- what about date $0$? Recall nonrecursivity of law of motion for $\Sigma_t$

- optimal to implement a expansionary cut in real interest rates (relative to s.s.)

- target criterion becomes:

$$
(\hat{y}_0 - \delta \hat{y}_0^*) + \frac{\varepsilon}{\alpha} \pi_0 = \chi
$$

where $\alpha > 1$, $\delta \in (0, 1)$, $\chi > 0$

- initial wealth dispersion generates a form of time-inconsistency
Optimal monetary policy in HANK

▸ what about date 0? Recall nonrecursivity of law of motion for $\Sigma_t$

▸ optimal to implement a expansionary cut in real interest rates (relative to s.s.)

▸ target criterion becomes:

$$(\hat{y}_0 - \delta \hat{y}_0^*) + \frac{\varepsilon}{\alpha} \pi_0 = \chi$$

where $\alpha > 1 \quad \delta \in (0, 1) \quad \chi > 0$

▸ initial wealth dispersion generates a form of time-inconsistency

▸ unrelated to Fisher channel or Barro-Gordon inflation bias
Optimal monetary policy in HANK

- what about date 0? Recall nonrecursivity of law of motion for $\Sigma_t$

- optimal to implement a expansionary cut in real interest rates (relative to s.s.)

- target criterion becomes:

$$\left(\hat{y}_0 - \delta \hat{y}_0^*\right) + \frac{\varepsilon}{\alpha} \pi_0 = \chi$$

where $\alpha > 1$, $\delta \in (0, 1)$, $\chi > 0$

- initial wealth dispersion generates a form of time-inconsistency

- unrelated to Fisher channel or Barro-Gordon inflation bias

- disappears when initial bond holdings are equalized ($a_0^s(i) = 0 \forall i$ and $s \leq 0$)
Optimal monetary policy in HANK

- now consider the (realistic) case of countercyclical earnings risk ($\phi < 0$)
Optimal monetary policy in HANK

- now consider the (realistic) case of countercyclical earnings risk ($\phi < 0$)

- earnings-risk and self-insurance channels pull monetary pol. in the same direction
Optimal monetary policy in HANK

- now consider the (realistic) case of countercyclical earnings risk ($\phi < 0$)

- earnings-risk and self-insurance channels pull monetary pol. in the same direction

- target criterion from time $t = 1$ onwards takes the form:

$$
(\hat{y}_t - \tilde{\delta}\hat{y}_t^*) - (\hat{y}_{t-1} - \tilde{\delta}\hat{y}_{t-1}^*) + \frac{\varepsilon y}{\tilde{\alpha}}\pi_t = 0
$$

where $\tilde{\delta} \in (0, 1)$
Optimal monetary policy in HANK

- now consider the (realistic) case of countercyclical earnings risk ($\phi < 0$)

- earnings-risk and self-insurance channels pull monetary policy in the same direction

- target criterion from time $t = 1$ onwards takes the form:

$$ (\hat{y}_t - \delta \hat{y}_t) - (\hat{y}_{t-1} - \delta \hat{y}_{t-1}) + \frac{\varepsilon y}{\bar{\alpha}} \pi_t = 0 $$

where $\tilde{\delta} \in (0, 1)$

- survives initial wealth equalization
Calibration

- Normalize \( y = 1 \) in RANK steady state
- set payroll subsidy so that zero steady-state inflation is always optimal
- \( \gamma = 5, \rho = 3 \)
- \( \beta = 0.95 \)
- \( 1 - \theta = 0.15 \) (Nistico 2016)
- NKPC: \( \kappa = 0.25, \Phi = 1.5/\kappa \)
Dynamics under optimal policy after contractionary $\varkappa$ shock
Conclusion

- with uninsurable risk, monetary policy affects consumption inequality by affecting:
  - level of income risk faced by households (cyclicality)
  - passthrough from income to consumption risk and from wealth inequality to consumption inequality

- optimal policy deviates from RANK
  - wealth inequality creates time inconsistency: desire to create surprise boom
  - deviate from divine coincidence, $y$ tracks $y^*$ less than one for one
  - cyclicality of risk matters

- time-inconsistency operative even absent nominal debt