Optimal Monetary Policy in HANK Economies

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Question

How does imperfect insurance affect optimal monetary policy?

Challenge and framework

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  - social welfare function aggregates heterogenous intertemporal utilities...
  - ...each of which is endogenous to policy

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  - ...each of which is endogenous to policy
- **framework:** CARA-Normal HANK with closed-form expressions for
  - aggregate dynamics
  - distribution of agents & its law of motion
  - social welfare function

Main results

- optimal policy governed by 2 forces:

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   - level and cyclicalilty of **income** risk
   - pass-trough to **consumption** risk (via time-varying MPC)

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   - level and cyclicalilty of **income** risk
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- CB may depart from price stability to reduce consumption dispersion
- breakdown of divine coincidence

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s.t.

$$\begin{aligned} c_t^s(i) + q_t a_{t+1}^s(i) &= w_t \ell_t^s(i) + a_t^s(i) + T_t \\ a_s^s(i) &= 0 \\ \xi_t^s(i) &\sim \mathcal{N}\left(\bar{\xi}, \sigma_t^2\right) \end{aligned}$$

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- CARA utility:

$$u(c, \xi - \ell) = -\frac{1}{\gamma} e^{-\gamma c} - \rho e^{-\frac{1}{\rho}(\xi - \ell)}$$

Firms

- competitive final-goods firms + monopolistic competitive wholesale firms facing Rotemberg pricing frictions ⇒ NKPC:

$$\left(\Pi_t - 1\right)\Pi_t = \frac{\varepsilon}{\Phi}\left[1 - \left(\frac{\varepsilon - 1}{\varepsilon}\right)\frac{z_t}{\left(1-\tau\right)w_t}\right] + \frac{1}{R_t}\left(\frac{y_{t+1}z_t w_t}{y_t z_{t+1} w_{t+1}}\right)\left(\Pi_{t+1} - 1\right)\Pi_{t+1}$$

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- $\tau =$ (constant) payroll subsidy
- $y_t =$ net output/income

$$y_t = z_t n_t - \frac{\Phi}{2}(\Pi_t - 1)^2 y_t = \frac{z_t n_t}{1 + \frac{\Phi}{2}(\Pi_t - 1)^2}$$

Rest of model

- zero-profit life insurers
- gov't running a balanced budget
- market clearing

$$\textbf{goods:}\ (1-\theta)\sum_{s=-\infty}^{t}\theta^{s-t}\int_i c_t^s(i)di = y_t$$

$$\textbf{labor:}\ (1-\theta)\sum_{s=-\infty}^{t}\theta^{s-t}\int_i \ell_t^s(i)di = n_t$$

$$\textbf{bonds:}\ (1-\theta)\sum_{s=-\infty}^{t}\theta^{s-t}\int_i a_t^s(i)di = 0$$

Equilibrium

- consumption decision rule, for any HH $i \in$ cohort s:

$$c_t(x) = y_t + \mu_t x$$

  where $x = w_t \left(\xi - \overline{\xi}\right) + a$ is (de-meaned) **cash-on-hand**

- μ_t is the MPC out of cash-on-hand:

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- leisure

$$\xi - \ell_t(x) = \rho(\gamma y_t - \ln w_t) + (\rho\gamma\mu_t)x$$

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- μ_t reflects ability to self-insure via labor and bond markets

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- μ_t determines pass-through from earnings risk to consumption risk

Aggregate Euler equation

- individual Euler equation

$$e^{-\gamma c_t^s(i)} = \beta\theta \frac{R_t}{\theta} \mathbb{E}_t e^{-\gamma c_{t+1}^s(i)}$$

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- individual Euler equation

$$-\gamma c_t^s(i) = \ln\left[\beta R_t\right] - \gamma \mathbb{E}_t c_{t+1}^s(i) + \frac{\gamma^2}{2} \mathbb{V}_t c_{t+1}^s(i)$$

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- Using $c_t^s(i) = y_t + \mu_t x_t^s(i)$ and aggregating across all households:

$$y_t = -\frac{1}{\gamma}\ln\left[\beta R_t\right] + y_{t+1} - \underbrace{\frac{\gamma \mu_{t+1}^2 w_{t+1}^2 \sigma_{t+1}^2}{2}}_{\text{precautionary savings motive}}$$

Equilibrium

$$
\begin{aligned}
y_t &= -\frac{1}{\gamma}\ln\left[\beta R_t\right] + y_{t+1} - \frac{\gamma \mu_{t+1}^2 w_{t+1}^2 \sigma_{t+1}^2}{2} \\
\mu_t^{-1} &= 1 + \gamma \rho w_t + \frac{\theta}{R_t}\mu_{t+1}^{-1} \\
y_t &= \frac{\rho \ln w_t + \bar{\xi}}{z_t^{-1} + \gamma\rho + z_t^{-1}\frac{\Phi}{2}\left(\Pi_t - 1\right)^2} \\
\left(\Pi_t - 1\right)\Pi_t &= \frac{\varepsilon_t}{\Phi}\left[1 - \left(\frac{\varepsilon_t - 1}{\varepsilon_t}\right)\frac{z_t}{\left(1-\tau\right)w_t}\right] + \frac{1}{R_t}\left(\frac{y_{t+1} z_t w_t}{y_t z_{t+1} w_{t+1}}\right)\left(\Pi_{t+1} - 1\right)\Pi_{t+1} \\
R_t &= \frac{1 + i_t}{\Pi_{t+1}}
\end{aligned}
$$

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$$\mathbb{V}_t(x) = \sigma_t^2 w_t^2 = \sigma^2 w^2 e^{2\phi(y_t - y)}$$

Linearized aggregate euler equation:

$$\widehat{y}_t = \underbrace{\left(1 - \frac{\phi\Lambda}{\gamma}\right)}_{\Theta} \widehat{y}_{t+1} - \frac{1}{\gamma}(i_t - \pi_{t+1}) - \frac{1}{2}\frac{\Lambda}{\gamma}\widehat{\mu}_{t+1}$$

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- **procyclical risk**: $\phi > 0$ or $\Theta < 1$: "*discounted euler equation*"

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- **acyclical risk**: $\phi = 0$ or $\Theta = 1$ "*undiscounted euler equation*"

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- **countercyclical risk**: $\phi < 0$ or $\Theta > 1$ "*compounded euler equation*"

Optimal Monetary Policy

Optimal policy problem

Planner maximizes:

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \left\{ (1-\theta) \sum_{s=-\infty}^{0} \theta^{-s} \int u[c_t^s(i), \xi_t^s(i) - \ell_t^s(i)] di \right\}$$

Pareto weights:

- equal weights on all HH alive at any date t, β^{s-t} on cohorts born at $s \geq t$
- ensures no time-inconsistency due to sequence of welfare weights (Calvo and Obstfeld, 1988)

Optimal policy problem

Lemma

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t u \underbrace{\left(c_t, \bar{\xi} - \ell_t;\right)}_{\substack{\text{felicity} \\ \text{of notional} \\ \text{rep. agent}}} \times \underbrace{\Sigma_t}_{\substack{\text{cost of} \\ \text{inequality}}}$$

where

$$\Sigma_t = (1-\theta) \sum_{s=-\infty}^{0} \theta^{-s} e^{\frac{1}{2}\sigma_c^2(t,s)} \geq 1$$

$\sigma_c^2(t,s)$: date t cross-sectional consumption dispersion amongst cohort s survivors

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RANK, $\Sigma_t = 1$ **HANK**, $\Sigma_t > 1$

Evolution of Σ_t

$$\ln \Sigma_t = \frac{1}{2}\gamma^2 \mu_t^2 w_t^2 \sigma_t^2 + \ln\left[1 - \theta + \theta \Sigma_{t-1}\right]$$

- Inequality is slow moving

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- assume economy (incl. the wealth distribution) is initially in steady state
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  - setting $\mu_0 < \mu$ reduces **consumption inequality**
  - unanticipated rate cut at date 0, given date 0 wealth distribution :
    - wealthy households get lower return on wealth $\Rightarrow \downarrow c$
    - indebted households pay lower interest on debt $\Rightarrow \uparrow c$
    - **not** revaluation effect
    - anticipated $\downarrow \mu$ does not have this additional effect since wealth distribution adjusts

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  1. reduce earnings risk
  2. improve self insurance $\Rightarrow$ reduce pass-through to consumption
  3. compress initial cons. distribution, by redistributing expected payoffs from assets

Refresher: Optimal monetary policy in RANK

- Target criterion after date 1:

$$\left(\widehat{y}_t - \widehat{y}_t^*\right) - \left(\widehat{y}_{t-1} - \widehat{y}_{t-1}^*\right) + \varepsilon \pi_t = 0$$

and date 0:

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- **divine coincidence**: mp can set $\widehat{y}_t = \widehat{y}_t^*$ and $\pi_t = 0$ at all dates and states.

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- of course, if $\widehat{y}_0 \neq \widehat{y}_0^*$ then $\widehat{y}_{t\geq 1} \neq \widehat{y}_{t\geq 1}^*$ and/or $\pi_{t\geq 1} \neq 0$

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- initial wealth dispersion generates a form of time-inconsistency
- unrelated to Fisher channel or Barro-Gordon inflation bias

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- target criterion becomes:

$$(\widehat{y}_0 - \delta\, \widehat{y}_0^*) + \frac{\varepsilon}{\alpha}\pi_0 = \chi$$

  where $\alpha > 1$ $\qquad \delta \in (0,1)$ $\qquad \chi > 0$

- initial wealth dispersion generates a form of time-inconsistency
- unrelated to Fisher channel or Barro-Gordon inflation bias
- disappears when initial bond holdings are equalized ($a_0^s(i) = 0\, \forall i$ and $s \leq 0$)

Optimal monetary policy in HANK

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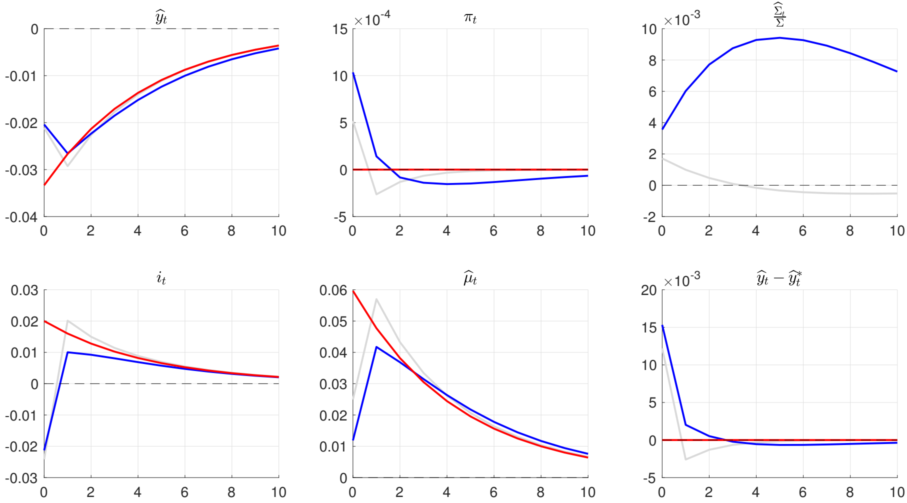
- survives initial wealth equalization

Calibration

- Normalize $y = 1$ in RANK steady state
- set payroll subsidy so that zero steady-state inflation is always optimal
- $\gamma = 5$, $\rho = 3$
- $\beta = 0.95$
- $1 - \theta = 0.15$ (Nistico 2016)
- NKPC: $\kappa = 0.25$, $\Phi = 1.5/\kappa$

Dynamics under optimal policy after contractionary z shock



Conclusion

- with uninsurable risk, monetary policy affects consumption inequality by affecting:
  - level of income risk faced by households (cyclicality)
  - passthrough from income to consumption risk and from wealth inequality to consumption inequality
- optimal policy deviates from RANK
  - wealth inequality creates time inconsistency: desire to create surprise boom
  - deviate from divine coincidence, y tracks y^* less than one for one
  - cyclicality of risk matters
- time-inconsistency operative even absent nominal debt