Risk Incentives in an Interbank Network

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Abstract

I develop a model of the interbank market where financial institutions endogenously form a network of bilateral debt contracts as a response to idiosyncratic liquidity shocks. Counterparty risk and regulatory constraints interact with endowment heterogeneity to generate interest rate dispersion and differing roles in the trading process, such as intermediation. The interbank market allows for socially desirable liquidity transfers, but limited liability may generate perverse incentives that increase risk-taking. These interact with the network structure to generate bank herding and endogenously magnifying aggregate risk. The endogenous nature of the network allows for the analysis of passive (regulatory) and active (interventionary) policies. Numerical simulations suggest that regulatory policies have perverse effects that tend to amplify risk, while interventions are more effective at containing the emergence of systemic risk and the propagation of shocks. No commitment problems arise with banking sector bailouts: by committing to bailout, the authority endogenously contains bank herding, and the formation of risk.

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1 Introduction

I develop a model of endogenous formation of mutual exposures between financial institutions. These institutions are subject to a fundamental maturity mismatch problem: they hold (risky) long-term assets, and short-term fixed rate liabilities (such as demandable deposits). Idiosyncratic liquidity and rollover shocks generate heterogeneity in the endowments of short-term liquid assets, motivating the trade of bilateral debt contracts. The market for liquidity is segmented, in the sense that institutions may choose to lend to some counterparties but not to others. Optimal lending and borrowing behavior induces a network of mutual claims and exposures, an interbank network. Heterogeneous endowments, regulatory constraints, and counterparty risk induce interest rate dispersion and lead different institutions to play different roles in the market: some supply or absorb liquidity, while other intermediate. The endogenous formation of the network allows us to study the emergence of systemic risk, and how do private incentives affect the general properties of the financial system as a whole. More importantly, by accounting for the behavioral response to changes in the fundamental parameters of the system, this model allows us to study how do changes in policy influence private incentives and, therefore, the process of network formation.

In this model, banks have potentially heterogeneous balance sheets, consisting initially of long-term assets and short-term debt. In the first period, a rollover shock on deposits is realized, before long-term assets mature. This leaves some institutions with excess liquidity, while others face a deficit. Banks can choose to invest their liquidity in zero-return cash reserves, or can access an outside market for short-term funding from where they can borrow at increasing costs. This provides banks with a motive to trade their net liquidity endowments among themselves. Trade takes place in a networked market: each bank has access to a market from where it can borrow funds, and chooses which counterparties to lend to (by participating in their respective markets). This allows us to keep track of the identity of the participants in every transaction in a tractable manner, making the analysis of counterparty risk more meaningful.

In the final period, after banks have optimally chosen their portfolios, and the network of mutual exposures has been formed, the return on long-term assets is realized. This return may not be sufficient to cover all liabilities that have been accumulated by the bank: to other banks, to depositors and/or to outside investors. In such an event, the institution becomes insolvent and its residual capital is distributed amongst creditors according to a seniority rule. The risk of default influences lending and borrowing decisions in the first period, which take into account direct counterparty risk. It is well known that limited liability may induce distressed banks (with low equity) to gamble on the upside, at the expense of debtholders. This may induce a phenomenon known in the literature as *bank herding*: banks tend to correlate their investments, by exposing themselves to counterparties that default in the same states of the world. Due to the networked structure of the economy, this risk can propagate and a single default can propagate through balance sheet contagion. In the presence of such externalities, and since defaults are costly, a welfare-maxiziming regulator faces a trade-off between the risk-sharing and liquidity benefits of the interbank market, and the incentives to amplify risk.

I study conventional regulatory measures in the spirit of existing regulation of the banking sector (the Basel framework). When choosing their lending and borrowing strategies, banks face two regulatory constraints: a capital adequacy ratio (or leverage constraint) and a liquidity requirement. Both constraints are governed by a regulatory parameter each, which is set by an external regulator. I find that policies aimed at curbing the emergence of systemic risk may have unintended effects: constrains on leverage raise the cost of funds of borrowing banks, leading otherwise inactive banks to start lending. This results in a more connected network, where the potential for negative shocks to spread is greater. On the other hand, reserve requirements tend to affect disproportionately banks that are liquidity constrained. This has an asymmetric impact on interest rates, and creates incentives for lending banks to expose themselves more to those that need liquidity. Once again, this may result in a more connected network with greater overall exposures to risk. I also study active interventions: direct lending facilities by the Central Bank (similar to the discount window), and bailouts. By providing an upper bound on the cost of funds, the Central Bank is able to reduce risk both directly (by providing banks with funds at lower rates), and indirectly (since the Central Bank absorbs losses due to counterparty risk and prevents their propagation). Expected bailouts are welfare improving, and dominate unexpected ones. By committing to bailout, the authority reduces the amount of perceived risk in the economy. This lowers interest rates, and reduces risk-shifting incentives, thereby endogenously mitigating the amount of risk. I find that when the regulator commits to bailout the banking sector, it needs to do so in less states of the world (compared to the situation in which the regulator does not commit, and the bailout is unexpected).

1.1 Relation to the Literature

While the use of network theory in the analysis of financial systems has become increasingly popular, a significant part of the literature tends to focus on pure network analysis: the study of properties of financial systems, taking the networked structure as given. While useful to understand certain phenomena (such as stability and resilience), this approach has limited used from the point of view of policy analysis, since it ignores behavioral responses by agents to changes in the environment. Meaningful policy analysis must account for the fact that financial systems are not environment-invariant objects, and be robust to the Lucas Critique. While reviewing the literature on financial networks, I focus on work that has taken this aspect into consideration, and endogenized, to some extent, the network formation process.

Financial Networks The literature in financial networks has grown too vast to be reviewed in a comprehensive manner. An excellent literature review is provided in Allen and Babus (2009). In spite of a considerable amount of recent work, the literature on the analysis of endogenous network formation in finance is considerably more limited. The seminal work in this field is that by Allen and Gale (2000): the authors extend the standard Allen and Gale (2007) model to allow for an interbank market, where banks from different regions are allowed to trade liquidity contracts so as to hedge against regional shocks. They conclude that a more connected (or more densely connected) network of mutual exposures is the most stable, since it allows different banks to fully hedge negatively correlated liquidity shocks. Another early work on this topic is that by Freixas et al. (2000). Other, more recent, works in the same tradition of financial network formation theory include Allen et al. (2012), Babus (2007), Babus (2011), Gale and Kariv (2007). This approach usually relies on the decentralization of first-best allocations. It also offers limited role and scope for agent/bank heterogeneity, thereby being unable to explain, for example, the emergence of core-periphery structures as we observe in most financial networked markets. More recently, other works have studied the formation of financial networks without relying on decentralization arguments, and allowing for some heterogeneity. Fique and Page (2013), Kondor and Babus (2013), Malamud and Rostek (2012) and Cohen-Cole et al. (2011) are examples of this new strand.

Decentralized Formation of Interbank Networks The works that are most closely related to mine are those by Acemoglu et al. (2013), Farboodi (2014) and Bluhm et al. (2014) since they specifically study systemic risk and intermediation in an environment where banks endogenously form a network of debt contracts. I proceed to discuss how this work compares to each of them in greater detail.

Acemoglu et al. (2013) comprehensively generalizes the concept of repayment equilibrium developed by Eisenberg and Noe (2001), and incorporates it in a two-stage game that allows for potential network formation in the first stage. The authors focus on specific types of structures of debt contract networks and establish that while the Allen and Gale (2000) result that fully connected networks are more efficient under a small shock regime (in which shocks to banks' balance sheets are small), the result may not hold when shocks to balance sheets are large. This is consistent with the "robust-yet-fragile" hypothesis put forward by Haldane (2009), that for small shocks, connections serve as shock-absorbers, but tend to help propagate negative shocks if these are large. I build on the general environment developed by the authors, in the sense that I also adopt a multi-stage game that involves endogenous network formation in the first stage and a realization of a shock that generates a payment equilibrium in the second stage. I extend their work by considering a richer environment where banks that face regulatory constraints fully optimize over: a) which counterparties to lend to; and b) how much to lend to each of these counterparties. I also allow for general heterogeneity in initial balance sheets and shocks, and do not not impose any prior on the emerging network structure.

Another paper that closely relates to my work is that by Farboodi (2014), where banks are heterogeneous in their investment opportunities. This is a highly stylized model that incorporates three main stages in a game with *ex-ante* identical banks: 1) establishment of credit lines that must be honored, 2) realization of investment opportunities (some banks have access to a constant returns to scale investment, while others do not) and interbank trade, 3) realization of investment payoffs and repayments. The author achieves equilibrium intermediation through the *ex-ante* establishment of credit lines that must be honored by banks with limited endowments of liquidity: since banks do not know whether they will be able to invest or not, and they must honor any credit line they open, this may provide a rationale for establishing very few connections with counterparties, thereby generating intermediation. The author focuses in the welfare properties of intermediation in the interbank market, and concludes that equilibrium networks tend to be inefficient due to overconnection and overexposure to counterparty risk. While I retain several conceptual features from this model (namely the costs of intermediation), this work is more quantitative in nature and provides banks with a richer portfolio problem, while also allowing for greater *ex-ante* heterogeneity. I also give greater emphasis to the welfare impact of existing regulatory constraints.

Finally, Bluhm et al. (2014) develop a very similar model to the present one that also allows for a potentially large degree of heterogeneity across banks, as well as the analysis of the impact of regulatory constraints. My paper does improve on their framework in two substantial ways, however: first, the network formation process is not truly endogenous, in the sense that interbank trading takes place in a quasi-Walrasian market, and counterparties are matched according to a quantity-matched algorithm. I fully endogenize the network formation process in my model. Secondly, while individual banks correctly anticipate the probabilities of counterparty default, they do not account for the fact that they, themselves, may default, and that this may affect funding costs. In my model, individual banks fully account for the impact of their actions in their own default probabilities. Thirdly, default does not, in my model, produce an "all-or-nothing" situation, and residual value of bankrupt counterparties is distributed amongst creditors according to a seniority rule. Not taking into account the distribution of residual assets can greatly exacerbate measures of systemic risk.

Bank Herding and Risk-Shifting I study the emergence of bank herding and risk-shifting incentives in a systems context. Bank herding is the systemic risk-shifting incentive defined by Acharya (2009), where banks optimally choose to undertake correlated investments and increase aggregate risk. This effect is further studied, in different forms and due to different reasons by Acharya and Yorulmazer (2008) (who coin the term) and Zetlin-Jones (2014). While the authors study the phenomenon in different contexts, this externality is fundamentally caused by limited liability. This feature biases risk-shifting incentives of agents (even when they are risk averse), to the extent that they care about payoffs conditional on no default. Banks care about counterparty repayments when they, themselves, do not default. This provides incentives to correlate their own states of default with those of the counterparty, and thereby enjoying full (or almost full) repayments in the only payoff-relevant states (those in which they do not default).

Over-the-Counter Markets in Finance This paper more broadly relates to the literature on over-the-counter markets in finance. Seminal work in the field, such as Duffie et al. (2005) and Lagos and Rocheteau (2009) adapts search-and-matching theory to the dynamic formation of financial contracts. More recently, Afonso and Lagos (2012) provide a search-and-matching based description of the microstructure of the Federal

Funds Market, the short-term liquidity market for US banks. The search and matching paradigm is greatly complementary to network analysis: while the first approach focuses on the disaggregated-level interactions between the participants in financial markets, the second is useful to study the systems that emerge. One can envision the network formation mechanism in this model as being the outcome or reduced form of a deeper, search mechanism such as the one studied in Afonso and Lagos (2012).

1.2 Interbank Markets

"Interbank markets" is, by itself, a vague and loose term and can generally refer to any market where several types of financial institutions interact. Participants in these markets are of varied nature and range from commercial banks to government sponsored enterprises (GSEs). I focus on wholesale funding markets, where financial institutions trade and hedge liquidity needs through short-term debt contracts. These markets can be more or less decentralized, depending on the institutional context. While most transactions in these markets take place over-the-counter and not in decentralized exchanges, aggregate activity is usually summarized by short-term reference interest rates that attempt to measure the overall cost of funding in the economy, examples being the Federal Funds rate in the US, the LIBOR in the UK and the Euribor in the Eurosystem.

Afonso et al. (2013) provides a comprehensive review of some of the defining characteristics of the interbank market for liquidity in the US, the Federal Funds market, prior to the 2008 financial crisis. The authors find that banks trade mostly to hedge and satisfy liquidity needs, and tend to establish lending/borrowing contracts with counterparties that face negatively correlated liquidity shocks. Informational asymmetries and counterparty opacity do not seem to play a significant role in the functioning of this market. The market seems to operate in a competitive fashion, with lenders not seeming to take advantage of market power during aggregate liquidity shortages. These negative liquidity shocks tend to be related with rising interest rates and costs of funding, as well as with declines in traded volumes.

While my model is highly stylized, it intend to capture some of the key features of these interbank markets for liquidity. Banks interact in a competitive manner and trade is motivated by liquidity management. Due to the competitive nature of the market, there are pressures towards harmonization of costs of funding across banks, in spite of market segmentation and a variety of other constraints. Average costs of funding and market activity are negatively correlated with the aggregate stock of liquidity. While I focus primarily on balance-sheet contagion through the interbank lending market, the model is easily extended to allow for trade in other classes of assets that may make banks vulnerable to other forms of contagion, such as common exposures and fire sales of assets.

2 A Model of Network Formation in the Interbank Market

In this section I present a model of segmented markets where banks endogenously form a network of bilateral exposures. I start by describing the environment, and the particular market structure for interbank relationships. I describe the repayment equilibrium in the final period, when payoffs are realized, and the portfolio allocation problem faced by banks in the initial period. I conclude by describing the roles of intermediation, risk-sharing and risk-shifting in the model, as well as by discussing numerical examples.

2.1 Environment

Time is discrete and there are two periods, $t = 0, 1$. There is one good in each period, which I call liquidity, or cash. There are three types of agents in the economy: financial institutions, outside investors and depositors. There are N islands, and each of them is populated by a continuum of outside investors and a representative financial institution. This institution is risk-neutral, maximizes expected profits at $t = 1$ and operates under limited liability. I will call it bank for the remainder of the paper. There is a continuum of depositors that banks in all islands.

At the beginning of $t = 0$, banks receive a liquidity shock that affects their endowments of short-term liquid assets. After this shock is realized, banks can trade and solve portfolio allocation problems. At $t = 1$, all uncertainty is resolved and repayments are undertaken. The timeline of the model is summarized in figure 1.

Figure 1: Sequence of Events

$t = 0$

- Liquidity shocks are realized
- Markets open
- Banks allocate their portfolios
- Network is formed

$t = 1$

- Long-term asset shocks are realized
- Banks may default
- Repayments are made

2.1.1 Initial Conditions and Liquidity Shocks

In each island $i \in N$ there is a representative bank. This bank is indexed by an initial portfolio of long-term assets $a_i > 0$ and an initial stock of short-term debt, or demandable deposits $d_i > 0$. These liabilities are claims owned by the depositors on the bank. The initial balance sheet identity is given by

$$a_i = d_i + n_i$$

where $n_i > 0$ is the residual equity. Long-term assets yield a stochastic return $R_i^a \sim g_i(R)$ at $t = 1$, and deposits can be redeemed at any time. For simplicity, I normalize the interest rate on deposits to 1.

At the beginning of $t = 0$, depositors randomly reallocate their deposits across the different islands. I abstract from the foundations and motives behind this reallocation and take it as given and exogenous [1]. Depositors withdraw their balances d_i from bank i and deposit a new amount $d_i' \geq 0$. We can define the total liquidity surplus or shortfall faced by bank i at the beginning of $t = 0$ as

$$s_i \equiv d_i' - d_i$$

If $s_i > 0$, the bank received more new deposits than the amount that was withdrawn, and so it faces a liquidity surplus. Conversely, $s_i < 0$ corresponds to a situation in which the bank faces a liquidity deficit. This liquidity shock generates heterogeneity in initial endowments, constituting a motive to trade. I assume that banks are obliged to clear their liquidity positions at the end of $t = 0$, that is, banks have to be able to satisfy all depositor withdrawals by the end of the initial period.

2.1.2 Asset Structure

In order to clear their liquidity positions, banks have access to several instruments. Banks with a liquidity deficit can finance their needs by either borrowing from banks in other islands, or by borrowing from the outside investors in their own island. Banks with a liquidity surplus can lend to other banks, lend to outside investors or invest in risk-free cash reserves. All markets and investment in assets takes place at $t = 0$, with the returns and repayments on such investments being realized at $t = 1$.

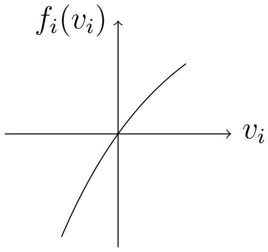
Interbank Lending and Borrowing Banks can establish bilateral links between themselves to trade liquidity. Let $\ell_{ij} \geq 0$ denote the amount of cash that is lent from bank i to bank j at $t = 0$. I focus on debt contracts that involve the contractual repayment of $r_{ij}\ell_{ij}$ at $t = 1$, where r_{ij} is an endogenous bilateral interest rate. The focus on bilateral debt contracts is in accordance with most of the literature on the endogenous formation of interbank networks (see Allen and Gale (2000), Acemoglu et al. (2013), Farboodi (2014) as examples) as well as with the institutional character of interbank interactions (see Afonso et al. (2011), for example, for a detailed description). Due to limited liability, this debt contract is potentially risky and the counterparty may be unable to fully repay its

[1] One could think of a typical shock à la Diamond and Dybvig (1983), coupled with the arrival of new depositors.

contractual value. If the counterparty defaults, it may only be able to repay a fraction $\theta_j \in [0, 1]$ of this loan. I defer a description of the repayment protocol in case of default to the next section. I also assume that a loan transaction between bank i and j entails a cost that may potentially depend on the value of the loan and the identities of the counterparties, $\kappa_{ij}(\ell_{ij}) \geq 0$ with $\kappa_{ij}(0) = 0$. Without loss of generality, I assume that this cost is borne by the lender and is to be repaid at $t = 1$. This cost is a reduced form for transaction and matching frictions that prevail in over-the-counter markets. It represents costly search for counterparties in the interbank market, and the structural cost of installation of platforms to access payment clearing systems and broker-dealer facilities. Most interbank payments and settlement systems, such as the Fedwire and CHIPS in the U.S., TARGET2 in the Eurosystem and CHAPS Sterling in the U.K. entail the payment of volume-dependent fees. See Afonso and Shin (2011) for a detailed discussion of the institutional features of Fedwire and CHIPS, and Adams et al. (2010) for a description of CHAPS

Outside Market Banks can also access an outside market in the island or location where they reside. Each bank can access this outside market to either deposit excess liquidity, or to request cash that the bank was unable (or unwilling) to obtain in the interbank market. This can be thought of as a market for short-term debt, such as commercial paper. Let $v_i \in \mathbb{R}$ denote bank i's net position in the outside market, where $v_i < 0$ if the bank is borrowing and $v_i > 0$ if the bank is investing. Bank i has access to a continuum of investors that are willing to provide/demand liquidity according to a function $f_i : \mathbb{R} \to \mathbb{R}$. This function is strictly increasing, strictly concave and satisfies $f_i(0) = 0$. Given a net outside position v_i, bank i acquires a debt (claim) with face value $f_i(v_i)$ that must be repaid (received) at $t = 1$. Figure 2 plots this function for reference. This investment is completely risk-free from the point of view of the bank, as I assume that outside investors can never default in their promises should the bank's net position be positive. If the net position in this market is zero, the bank acquires neither a claim nor a liability. Concavity of this function implies that the returns on investing in this market are decreasing, and costs of accessing it are increasing. This function can be seen as a reduced form of a situation in which the bank has market power vis-à-vis risk-neutral investors whose outside option is a concave storage technology. One can also think of it as representing borrowing from the market at a linear price, but with nonlinear broker-dealer fees. Kacperczyk and Schnabl (2010) provide a detailed account on the use of commercial paper as a source of funding by financial institutions prior to and during the 2007-2009 financial crisis. They mention, for example, that most commercial paper issued by financial institutions is unsecured.

Figure 2: Outside Investment Function



Cash Reserves The last type of asset that is available to banks are risk-free cash reserves. Banks can keep a non-negative amount of cash $c_i \geq 0$ stored at the normalized unity return.

Long-Term Assets For simplicity, I assume that long-term asset holdings a_i are completely illiquid and/or bank-specific, and so no investor is willing to purchase them. This means that banks cannot purchase or sell long-term assets at $t = 0$. Bank-specificity of assets is a common assumption in the financial intermediation literature (see, for example, Acharya et al. (2012a)). In the appendix, I relax this assumption and show that allowing for rebalancing of the long-term asset portfolio at $t = 0$ does not substantially alter the results.

2.1.3 Flow of Funds Constraint and Profits

To summarize, upon the realization of the liquidity shock s_i, bank i can raise funds at $t = 0$ by

1. Borrowing from banks $j \in N \setminus i$, an amount ℓ_{ji} that entails a promised repayment of $r_{ji}\ell_{ji}$ at $t = 1$
2. Borrowing from outside investors, an amount $v_i \leq 0$ that generates a cost $f(v_i) \leq 0$ at $t = 1$

or invest excess funds by

1. Lending to banks $j \in N \setminus i$ an amount ℓ_{ij} for a promised repayment $r_{ij}\ell_{ij}$ at $t = 1$
2. Lending to outside investors, an amount $v_i \geq 0$ that generates a claim $f(v_i) \geq 0$ at $t = 1$
3. Investing in cash reserves, an amount c_i that yields c_i at $t = 1$

As mentioned, I assume that each bank is forced to clear its liquidity position at $t = 0$, and satisfy all depositors' withdrawals by the end of the period. This allows us to write the $t = 0$ liquidity budget constraint, or flow of funds constraint, for the bank as

$$\sum_{j \neq i} \ell_{ij} + c_i + v_i = \sum_{j \neq i} \ell_{ji} + s_i \tag{1}$$

where the left-hand side represents total outflows: total interbank lending, cash reserves and outside investment, potentially negative; while the right-hand side represents total inflows: total interbank borrowing and liquidity endowment, potentially negative.

At $t = 1$, the returns on long-term assets R_i^a are realized, and repayments take place. Under limited liability, profits for bank i can be written as

$$\pi_i^+ = \left[R_i^a a_i + c_i + f_i(v_i) + \sum_{j \neq i} \theta_j r_{ij} \ell_{ij} - \sum_{j \neq i} r_{ji} \ell_{ji} - d_i' - \sum_{j \neq i} \kappa_{ij}(\ell_{ij}) \right]^+ \tag{2}$$

Banks earn revenues from maturing long term assets $R_i^a a_i$, cash reserves, and interbank repayments $\sum_{j \neq i} \theta_j r_{ij} \ell_{ij}$, where contractual repayments from each bank j are weighted by the fraction $\theta_j \in [0, 1]$ that is actually repaid. Their outflows correspond to the repayment of interbank borrowing $\sum_{j \neq i} r_{ji} \ell_{ji}$, repayment of deposits d_i' and payment of interbank lending costs $\sum_{j \neq i} \kappa_{ij}(\ell_{ij})$. Furthermore, the bank may either receive returns, or pay debts on outside investment $f_i(v_i)$, depending on its net position. Due to limited liability, actual profits are the maximum between zero and revenues net of costs, where $x^+ \equiv \max(0, x)$.

2.1.4 Regulatory Constraints

In the spirit of existing regulations, banks operate under two different regulatory constraints. The first is a capital adequacy ratio (CAR) that imposes that the bank's net worth divided by risk-weighted assets must exceed some level ϕ (the inverse of maximum regulatory leverage). The relevant net worth is computed after portfolio decisions have been made, at the end of $t = 0$. This intermediate net worth can be written as

$$n_i' = a_i + \sum_{j \neq i} \ell_{ij} + c_i + v_i - \sum_{j \neq i} \ell_{ji} - d_i'$$

It is an accounting measure that consists of the book-value of assets at the end of $t = 0$, minus the book-value of liabilities. Total assets are equal to long-term assets [2], total interbank claims, cash reserves, and outside investment (if positive). Total liabilities equal interbank debt $\sum_{j \neq i} \ell_{ji}$, and new deposits d_i'. The constraint is given by

$$\frac{a_i + \sum_{j \neq i} \ell_{ij} + c_i + v_i - \sum_{j \neq i} \ell_{ji} - d_i'}{\omega_a a_i + \omega_\ell \sum_{j \neq i} \ell_{ij}} \geq \phi \tag{3}$$

[2] Long-term assets are assigned a book-value of 1. If one prefers a mark-to-market interpretation, it can be argued that a_i already incorporates price-valuation effects, and prices do not change after the liquidity shock and portfolio decisions are made since no new information regarding the quality of these assets is revealed.

where $\omega_\iota > 0$ is the risk-weight on asset class ι. Note that cash reserves and outside investment (if positive) are attributed a risk-weight of zero due to their risk-free nature, consistent with current regulatory requirements. Only the risky components of the asset-side of the balance sheet, long-term assets and interbank claims, are assigned a positive risk-weight.

The second regulatory constraint is a reserve requirement, that bank i hold in risk-free cash reserves c_i a fraction τ of its total short-term demandable deposits d'_i

$$c_i \geq \tau d'_i \tag{4}$$

This constraint mirrors current regulatory reserve requirements by giving c_i the broader interpretation of risk-free reserves at the Central Bank. Unlike the leverage constraint, which also plays a technical role in the model, the reason why this constraint is introduced in this model may not be obvious at first. Historically, reserve requirements have served the dual function of a monetary policy tool (albeit less perfect than open market operations) and as a primitive liquidity ratio ("primitive" since this concept has only been formally introduced in the Basel III framework). While a liquidity ratio is not meaningful in the current model, since all decisions that are constrained by this ratio are taken after the liquidity shock has been realized, reserve requirements will play an important role in disciplining some of the results of the model, and their role as a monetary policy tool will be discussed.

2.1.5 Banks' Problem

Banks are risk-neutral and maximize the expected value of their profits after observing liquidity shocks. There are two main sources of uncertainty over which bank i takes expectations: the idiosyncratic return on their long-term assets R_i^a and the risk of default by their counterparties in the interbank market, summarized by the repayment fractions $\{\theta_j\}_{j\neq i}$. We can write the problem for bank i as

$$\max_{c_i, v_i, \{\ell_{ij}\}_{j\neq i}, \{\ell_{ji}\}_{i\neq j}} \mathbb{E}[\pi_i^+] \tag{5}$$

subject to

$$\sum_{j\neq i} \ell_{ij} + c_i + v_i = \sum_{j\neq i} \ell_{ji} + s_i$$

$$\frac{a_i + \sum_{j\neq i} \ell_{ij} + c_i + v_i - \sum_{j\neq i} \ell_{ji} - d'_i}{\omega_a a_i + \omega_\ell \sum_{j\neq i} \ell_{ij}} \geq \phi$$

$$c_i \geq \tau d'_i$$

where the objective function is the expectation of (2), and the three constraints are the flow of funds at $t = 0$, the capital adequacy ratio and the reserve requirement. Each bank takes the initial states (a_i, d_i, d'_i) as given. The bank solves a portfolio allocation problem that consists of choosing cash reserves c_i, (net) outside positions v_i, interbank lending $\{\ell_{ij}\}_{j \neq i}$ and interbank borrowing $\{\ell_{ji}\}_{i \neq j}$. This problem may seem daunting at first glance due to the large number of control variables and complicated objective function (due to limited liability and multiple sources of endogenous uncertainty). Before solving the banks' problem, I simplify these two aspects in the following subsections.

2.2 Networked Markets

As described in the previous sections, banks can trade loan contracts at $t = 0$ after observing the liquidity shock, where ℓ_{ij} denotes the total amount of cash transferred from bank i to bank j in exchange for a contractual repayment $r_{ij}\ell_{ij}$ at $t = 1$. Graph theory can help us conveniently summarize the information regarding trades in the interbank market. Formally, a graph $\mathbf{G} = (N, \mathbf{L})$ is an ordered pair that comprises a set of vertices (or nodes) N and a set of edges (or connections) $\mathbf{L}$. In our application, each representative bank (or island) i is an element of the set of nodes [3]. The edges or connections between these nodes are summarized in the $N \times N$ interbank matrix $\mathbf{L}$. This is an adjacency matrix, whose ij-th entry is ℓ_{ij}, the loan extended from bank i to bank j

$$\mathbf{L} = \begin{bmatrix} 0 & \ell_{12} & \dots & \ell_{1N} \\ \ell_{21} & 0 & \dots & \ell_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{N1} & \ell_{N2} & \dots & 0 \end{bmatrix}$$

The diagonal of this matrix is composed of zeros, since I do not allow banks to lend to themselves [4]. The only additional restriction on $\mathbf{L}$ is that its other entries be non-negative: this matrix records *gross* lending volumes only. In principle, it is possible for bank i to be lending and borrowing at the same time from bank j, acquiring a negative net position. This would correspond to $\ell_{ij} > 0$ and $\ell_{ji} > 0$ occurring at the same time. Row i of the interbank matrix corresponds to loans extended by bank i to the remaining $N - 1$ banks in the economy, whereas column j gives us the loans that are contracted by bank j from all other $N - 1$ banks.

The interbank graph, or network, $\mathbf{G} = (N, \mathbf{L})$ is a *directed* and *weighted* network. It is directed since $\mathbf{L}$ is not restricted to be a symmetric matrix: the direction of each connection matters, since bank i lending to bank j is economically different from bank j lending to bank i. It is a weighted network since the entries of $\mathbf{L}$ can take any non-negative real values: the intensity of each connection, interpreted as the volume of each loan, also has economic meaning.

[3] I am abusing notation by letting N denote both the number and the set of banks.

[4] This is without loss of generality, since it would never be optimal in the presence of lending costs. Even without explicit costs, this is an activity with zero return that entails the cost of tightening constraints.

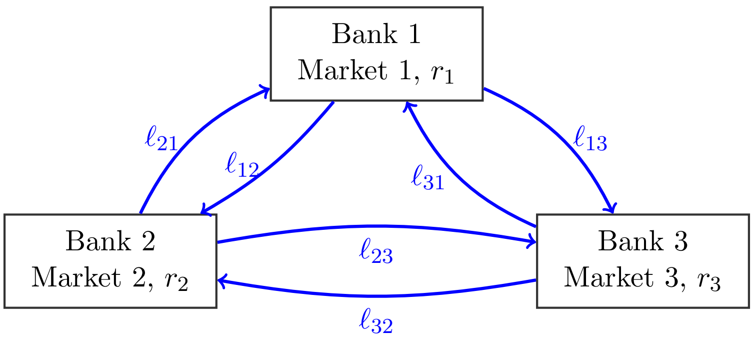
One of the main contributions of this paper is to generate $\mathbf{L}$ as the outcome of interactions between rational, optimizing agents. In particular, each bank i is allowed to optimize over the set of loans it extends $\{\ell_{ij}\}_{j\neq i}$ (the i-th row of $\mathbf{L}$) and the set of loans it contracts $\{\ell_{ji}\}_{j\neq i}$ (the i-th column of $\mathbf{L}$). To render the problem tractable, I assume a particular structure for the interbank market, by imposing that it consists of a set of segmented markets indexed by $i \in N$. Bank i "creates" a Walrasian market where it can borrow, and all other $N-1$ banks can choose to participate in this market by paying the costs of lending $\kappa_{ji}(\ell_{ji}), j \neq i$ and lending to bank i. This is a convenient but powerful simplification, as it allows us to model both explicit and implicit costs of trading (inherent to over-the-counter markets) while still retaining the tractability of a Walrasian framework.

In market $i \in N$, bank i borrows at a single interest rate r_i. Since this market is competitive (due to the assumption of a representative bank in each island), this interest rate does not depend on the identity of the lender: bank i has the option of borrowing from the outside market, so it must be able to borrow at the same interest rate from other banks (with a potential risk-adjustment). For the borrower, the identity of the lender is irrelevant (since all lenders offer the same rate), but the converse is not true due to counterparty risk. From now onwards, I define total interbank debt contracted by bank i as

$$B_i \equiv \sum_{j\neq i} \ell_{ji}$$

Heterogeneity in endowments, counterparty risk and lending costs all interact to generate interest rate dispersion across these segmented markets. Figure 3 illustrates the networked market structure of the economy.

Figure 3: Networked Market Structure with 3 Banks



2.3 Equilibrium at $t = 1$

Before looking at each banks' optimal decisions, let us look at what happens in period $t = 1$, when payoffs from long-term assets are realized and repayments are undertaken. At this stage, banks have already made their portfolio allocation decisions and the interbank network $\mathbf{L}$ has been formed.

2.3.1 Limited liability and seniority

For low enough realizations of the idiosyncratic shock on long-term assets R_i^a, bank i may find itself earning negative profits, $\pi_i < 0$. Due to limited liability, equity holders are not liable for excess costs and dividends are zero in this case, $\pi^+ = 0$.This means, however, that losses fall upon debt holders and the bank may be unable to honor part or the totality of the contractual claims that are owned by its creditors. I abstract away from incentive and agency problems on the part of the banker, and assume that bank equity holders receive nothing in case of default.

I assume *absolute priority of debt over equity*. This means that if revenues are not sufficient to repay creditors in full, all value is paid to creditors. Even if a bank makes negative profits, it still generates some revenues from long-term assets, cash reserves, outside claims and interbank claims, however small they might be. If profits are negative, all of this value is to be distributed amongst the bank's creditors according to a seniority rule. Recall that banks at $t = 1$ may have four types of costs: demand deposits d_i', outside debt $f_i(v_i) \leq 0$, interbank debt $r_i B_i$ and total costs of lending $\sum_{j \neq i} \kappa_{ij}(\ell_{ij})$. I assume that demand deposits are senior, and all other forms of debt are equally junior, *pari passu*.

$$\begin{aligned}
&\text{senior debt} : d_i' \\
&\text{junior debt} : Q_i \equiv r_i B_i + [-f_i(v_i)]^+ + \sum_{j \neq i} \kappa_{ij}(\ell_{ij})
\end{aligned}$$

So, in case of bankruptcy, the bank uses its salvage value to repay depositors first, and then proceeds to repay other banks, outside investors and lending fees in a proportional manner. The fraction of contractual value that creditors of bank i are able to recover depends on how much residual value is left, after bank i repays its depositors. Since profits of creditor banks depend on this fraction, so does their bankruptcy status. It is possible for a bank with a high realization of the idiosyncratic shock R_i^a to default due to the failure of its counterparties to repay their contractual obligations. This is be the main source of contagion and systemic risk in the model.

Seniority of depositors is a natural assumption that is consistent with existing regulations in several countries. In the United States, Australia and Switzerland, depositor preference is established by law, and the European Council has recently approved legislation in this direction (see Hardy (2013) for a complete discussion).

2.3.2 Costs of default

In the absence of explicit costs, defaults merely redistribute the bank's assets amongst different creditors (depositors, outsiders, receivers of lending fees, and other banks). In order to make defaults socially costly, I assume that a fraction δ of the value of non-interbank revenues is lost upon default. Let total non-interbank revenues of bank i be denoted as

$$e_i = R_i^a a_i + c_i + [f_i(v_i)]^+ \tag{6}$$

that is, the sum of proceedings from long-term assets, cash reserves, and otuside investments. in case bank i defaults, only $(1-\delta)e_i$ becomes available to repay creditors. Bankruptcy costs of financial institutions tend to be large and to extend well beyond direct creditor losses. A large literature has emerged to study the deadweight cost losses of bankruptcy. James (1991) estimates average direct expenses of bankruptcy proceedings for banks as 10% of the institutions' pre-default assets. Hardy (2013) provides a review of the literature on this topic. Several references in the literature on contagion in financial systems emphasize the role of bankruptcy and default costs as sources of amplification: Cifuentes et al. (2005) and Elliott et al. (2014) are two examples.

2.3.3 Repayment equilibrium

Due to limited liability, seniority rules and the interconnected structure of the economy, it becomes a non-trivial task to compute profits given a realization of the long-term asset payoffs $\mathbf{R} = \{R_i^a\}_{i=1}^N$. To proceed, I adopt the concept of repayment equilibrium introduced by Eisenberg and Noe (2001) and extended by Acemoglu et al. (2013), Glasserman and Young (2013), and Rogers and Veraart (2013). This consists of formulating a fixed point problem that solves for the vector of repayment fractions $\boldsymbol{\theta} = \{\theta_i\}_{i=1}^N$ as a function of the realization of shocks $\mathbf{R}$ and the banks' portfolio decisions at $t = 0$ (which include the interbank network $\mathbf{L}$).

We define salvage value for a bank as its total revenues, given by

$$[1 - \delta \mathbb{1}(\pi_i < 0)]e_i + \sum_{j \neq i} \theta_j r_j \ell_{ij}$$

That is, salvage value equals non-interbank revenues as defined in (6) (net of default costs) plus total interbank revenues, which comprise the sum of the face value of all extended interbank credit weighted by the fraction of the face value that is actually repaid by each bank. Since depositors are senior, they have a priority claim over these revenues in case of default. The amount of residual value that is left to repay junior creditors is then given by

$$\rho_i = \left\{ [1 - \delta \mathbb{1}(\pi_i < 0)]e_i + \sum_{j \neq i} \theta_j r_j \ell_{ij} - d_i' \right\}^+$$

That is, either depositors are repaid in full and some residual value is left to repay junior creditors, or total salvage value is smaller than deposits, in which case depositors bear some losses and junior creditors receive nothing.

Let $x_i \in [0, Q_i]$ denote the total amount of junior debt that bank i is able to repay. If residual value ρ_i exceeds total junior liabilities Q_i, the bank generates positive value for the equity holders. Otherwise, if $\rho_i < Q_i$, the bank is unable to fulfill its contractual obligations and defaults. Since equity holders are not liable for negative profits, we assume that the bank repays all residual value that it can repay to junior creditors, and we call this amount x_i. Note that our previously defined repayment fractions can be conveniently defined as $\theta_i = \frac{x_i}{Q_i}$. We can then write realized profits as

$$\pi_i^+ = \rho_i - x_i$$

where $\pi_i^+ = 0$ if $x_i < Q_i$ [5]. Let Π denote the matrix of relative junior liabilities, whose ij-th element is defined as

$$\Pi_{ij} = \begin{cases} \frac{r_i \ell_{ji}}{Q_i} & \text{if } Q_i > 0 \\ 0 & \text{if } Q_i = 0 \end{cases}$$

That is, Π_{ij} tells us the share of junior liabilities of bank i that are owned by bank j. Due to the presence of outside liabilities, Π will be substochastic, since its rows sum to ≤ 1 (i.e., $Q_i \geq r_i B_i$). The total contractual liabilities from bank j to bank i can be written as $\Pi_{ji} Q_j = r_j \ell_{ij}$. This means that we can write contractual interbank revenues for bank i as

$$\sum_{j \neq i} r_j \ell_{ji} = \sum_{j \neq i} \Pi_{ji} Q_j$$

Since banks may default and repay only a part of their junior liabilities, $x_i \leq Q_i$, we can rewrite actual interbank revenues as

$$\sum_{j \neq i} \theta_j r_j \ell_{ij} = \sum_{j \neq i} \Pi_{ji} x_j$$

Repayments by bank i can be written as

$$x_i = \begin{cases} Q_i & \text{if } Q_i \leq e_i + \sum_{j \neq i} \Pi_{ji} x_j - d_i' \\ \max\left\{(1-\delta)e_i + \sum_{j \neq i} \Pi_{ji} x_j - d_i', 0\right\} & \text{if } Q_i > e_i + \sum_{j \neq i} \Pi_{ji} x_j - d_i' \end{cases} \tag{7}$$

That is, either the bank honors all of its junior liabilities when $\rho_i > Q_i$ and $x_i = Q_i$, or the bank repays its residual value (net of default costs and senior repayments) otherwise.

Note, however, that the amount of residual value that is repaid depends itself on junior debt repayments by banks to whom bank i has extended a loan, which may in turn depend on junior repayments of banks with whom bank i is not connected. This means that, in principle, residual value repayments will depend on the entire series of debt repayments by the other $N-1$ banks, $\{x_j\}_{j \neq i}$. This requires solving for $\mathbf{x} = [x_i]_{i=1}^N$ simultaneously. We can do this by solving a fixed point problem. To achieve this, define the operator $\mathbf{\Phi}(\mathbf{x}) : \times_{i=1}^N [0, Q_i] \to \times_{i=1}^N [0, Q_i]$ as

$$[\mathbf{\Phi}(\mathbf{x})]_i = \begin{cases} Q_i & \text{if } Q_i \leq e_i + \sum_{j \neq i} \Pi_{ji} x_j - d_i' \\ \max\left\{(1-\delta)e_i + \sum_{j \neq i} \Pi_{ji} x_j - d_i', 0\right\} & \text{if } Q_i > e_i + \sum_{j \neq i} \Pi_{ji} x_j - d_i' \end{cases} \tag{8}$$

Following Rogers and Veraart (2013), it is possible to show that $\mathbf{\Phi}$ is bounded above by $\mathbf{Q} = (Q_1, \ldots, Q_N)^T$, and that Φ is monotone. Even if, in general, it is not possible to

[5] This is not an if and only if statement due to the possibility of the bank generating just enough profits to cover all of its liabilities.

show that a unique fixed point for this operator exists, it is enough for my purposes to show that a *greatest* fixed point exists. This is a natural selection mechanism in case of multiplicity; to understand why, assume that defaults take place in several rounds, in fictitious time. In the first round, all banks receive their idiosyncratic shock to long-term asset returns. Given this shock, some banks become fundamentally insolvent and default. In the second round, banks update their profits (given defaults and repayments by banks that defaulted), and may default or not. In the third round, banks reupdate their profits once more, and so on. Since there are N banks in the economy, and default is an absorbing state (due to monotonicity of the operator), there are at most N rounds of defaults. This sequential default mechanism stops, by construction, at the greatest fixed point of $\mathbf{\Phi}$. See Eisenberg and Noe (2001) and Glasserman and Young (2013) for detailed discussions.

The following proposition summarizes the results

Proposition 2.1. *Given a portfolio allocation and a joint realization of long-term asset payoffs* $\mathbf{R}$*, a greatest equilibrium repayment vector* $\mathbf{x}$ *satisfying*

$$x_i = \begin{cases} Q_i & \text{if } Q_i \leq e_i + \sum_{j\neq i} \Pi_{ji} x_j - d'_i \\ \max\left\{(1-\delta)e_i + \sum_{j\neq i} \Pi_{ji} x_j - d'_i, 0\right\} & \text{if } Q_i > e_i + \sum_{j\neq i} \Pi_{ji} x_j - d'_i \end{cases}, \qquad \forall i \in N$$

always exists and is unique.

Proof. See appendix. □

Once x_i is determined, we can extract repayment fractions as

$$\theta_i = \frac{x_i}{Q_i}$$

2.4 Equilibrium at $t = 0$

We have seen that, given portfolio allocations, a realization of shocks $\mathbf{R}$ maps into a vector of repayment fractions $\boldsymbol{\theta}$ through the repayment equilibrium at $t = 1$. The equilibrium selection procedure in case of non-uniqueness (selecting the greatest repayment vector) allows us to define the map $\boldsymbol{\theta}(\mathbf{R})$. When solving their portfolio allocation problem, and deciding which counterparties to lend to, banks will rationally take expectations over the distribution of these repayments, $\boldsymbol{\theta}(\mathbf{R})$.

The convenient market structure that we assume for interbank lending and borrowing allows us to rewrite the problem for bank i in a much more tractable way.

$$\max_{c_i, v_i, \{\ell_{ij}\}_{j \neq i}, B_i} \mathbb{E}_{\mathbf{R}} \left\{ \left[R_i^a a_i + c_i + f_i(v_i) + \sum_{j \neq i} \theta_j(\mathbf{R}) r_j \ell_{ij} - r_i B_i - d_i' - \sum_{j \neq i} \kappa_{ij}(\ell_{ij}) \right]^+ \right\} \tag{9}$$

subject to

$$\sum_{j \neq i} \ell_{ij} + c_i + v_i = B_i + s_i$$
$$\frac{a_i + \sum_{j \neq i} \ell_{ij} + c_i + v_i - B_i - d_i'}{\omega_a a_i + \omega_\ell \sum_{j \neq i} \ell_{ij}} \geq \phi$$
$$c_i \geq \tau d_i'$$

where the expectation is taken with respect to the joint distribution of returns $\mathbf{R} \sim G(\mathbf{R})$. Since bank i is a representative bank in island i, it takes all prices and the structure of the network as given, while computing rational expectations of repayments $\boldsymbol{\theta}(\mathbf{R})$ over the joint distribution of returns G. Crucially, however, bank i accounts for the impact of its portfolio decisions in its own probability of default. The solution to this problem is summarized in proposition 2.2.

Proposition 2.2. *Let*

$$\bar{R}_i^a(R_{-i}^a) = \frac{r_i B_i + d_i' + \sum_{j \neq i} \kappa_{ij}(\ell_{ij}) - f_i(v_i) - c_i - \sum_{j \neq i} \theta_j(R_{-i}^a) r_j \ell_{ij}}{a_i} \tag{10}$$

denote the minimum realization of R_i^a for which bank i does not default. Then, optimal policies are given by the following first-order conditions

$$(B_i): P_i(f_i' - r_i) \leq 0$$
$$(c_i): P_i(1 - f_i' + \lambda_i) \leq 0$$
$$(\ell_{ij}): \int_{R_{-i}^a} \int_{\bar{R}_i^a(R_{-i}^a)}^{\infty} \theta_j(\mathbf{R}) r_j \mathrm{d}G(\mathbf{R}) - P_i[f_i' + \phi\omega_\ell \mu_i + \kappa_{ij}'(\ell_{ij})] \leq 0, \forall j \neq i$$

where $f_i' = f_i'(v_i)$, μ_i and λ_i are the (normalized) Lagrange multipliers on the capital adequacy ratio and reserve requirement constraints, respectively.

Proof. See appendix. □

I focus on analyzing the first-order conditions for a bank such that $P_i > 0$ (the problem is not very interesting otherwise). The first-order conditions for borrowing yields

$$f_i' \leq r_i \tag{11}$$

The bank only decides to borrow from the interbank market if the available interest rate in market i, given by r_i, is at most equal to the cost of funds in the outside market, given by f_i'. This is a simple non-arbitrage condition: since the states of default are payoff-irrelevant due to limited liability, and all sources of financing are in the form of debt contracts, the bank must be indifferent between them in equilibrium. This allows us, without loss of generality, to conveniently define

$$r_i \equiv f_i'$$

the price of debt in market i as the outside cost of funding for bank i. This allows us to discuss prices even in the absence of any trade in this market, and to use the terms "interbank rate of borrowing" and "cost of funds" interchangeably [6].

The first-order condition for cash holdings is

$$1 + \lambda_i = f_i' \tag{12}$$

where we assume that $\tau d_i' > 0$, and so cash holdings must be strictly positive, making the first-order condition bind. Once again, since cash is a risk-free asset in all payoff-relevant states, we are left with another simple non-arbitrage condition. The left-hand side represents the marginal benefit of investing one unit of cash: it yields a return of 1 and loosens the reserve requirement by λ_i. The right-hand side is the marginal cost of cash, captured by the outside cost of funds. The monetary policy role of reserve requirements is evident here, as they effectively impose a lower bound on the bank's cost of funds and, correspondingly, interbank rate of borrowing. Since the bank can invest at cash reserves that yields return 1, and returns to the outside market are strictly concave, it will never invest beyond the point in which the outside market rate of return falls below one. This effectively floors interest rates at $r_i \geq 1$.

Finally, the first-order condition for lending to bank j can be more elegantly written by dividing through by P_i,

$$r_j \mathbb{E}[\theta_j | \pi_i \geq 0] \leq f_i' + \phi \omega_\ell \mu_i + \kappa_{ij}'(\ell_{ij}) \tag{13}$$

The left-hand side of the FOC is the marginal benefit from lending: an interest rate weighted by the expected repayment, conditional on bank i not defaulting. Note that

[6]This is relevant for computing reference rates even in the absence of any trade, as it will be discussed later. From the LIBOR website: "On every working day at around 11 a.m. (London time) the panel banks inform Thomson Reuters for each maturity at what interest rate they would expect to be able to raise a substantial loan in the interbank money market at that moment.". The reference rate is computed based on a hypothetical cost of funds that may not correspond to any realized market price in case there is no trade.

$$\mathbb{E}[\theta_j|\pi_i \geq 0] = \frac{\int_{R^a_{-i}} \int_{\bar{R}^a_i(R^a_{-i})}^{\infty} \theta_j(\mathbf{R})\mathrm{d}G(\mathbf{R})}{\int_{R^a_{-i}} \int_{\bar{R}^a_i(R^a_{-i})}^{\infty} \mathrm{d}G(\mathbf{R})}$$

This weight is extremely intuitive: due to limited liability, bank i does not care about the repayment of a loan to bank j in states of the world in which bank i itself is in default, since all value is transferred to creditors. The right-hand side highlights three different costs of lending:

1. f_i', the direct cost of funds.
2. $\phi\omega_\ell\mu_i$, the cost of a binding leverage constraint.
3. $\kappa_{ij}'(\ell_{ij})$, marginal lending fees.

2.4.1 Equilibrium

We are now ready to define a full rational expectations equilibrium in this economy.

Definition 2.1. *An Interbank Equilibrium consists of a collection of portfolio allocations:* $\mathbf{L} \in \mathbb{R}^{N\times N}$, $\mathbf{c} \in \mathbb{R}^N$*; prices* $\mathbf{r} \in \mathbb{R}^N$*; and a repayment protocol* $\boldsymbol{\theta}(\mathbf{R};\mathbf{L},\mathbf{c}) \in \mathbb{R}^N$ *such that:*

1. *Given expected repayments* $\mathbb{E}[\theta(\mathbf{R};\mathbf{L},\mathbf{c})]$ *and prices* $\mathbf{r}$*, the portfolio allocations* $\mathbf{L},\mathbf{c}$ *solve (17) for each representative bank* $i \in N$.
2. *Prices are such that all* N *interbank credit markets clear*

$$B_i = \sum_{j\neq i} \ell_{ji}, \forall i \in N$$

and defined as $r_i = f_i'$ *whenever* $B_i = 0$.

3. *For each joint realization of long-term asset payoffs* $\mathbf{R} \sim G(\mathbf{R})$, $\boldsymbol{\theta}(\mathbf{R};\mathbf{L},\mathbf{c})$ *constitutes a repayment equilibrium.*

At $t = 0$, banks maximize their profits taking both prices and expected repayments as given. By acknowledging that $\boldsymbol{\theta}$ is a function of $\mathbf{R}$, they also account for the fact that it depends on $\mathbf{L}$, but they do not account for the impact of their own actions in the network. At $t = 1$, equilibrium repayments are consistent with the realized shocks $\mathbf{R}$ and the network structure that emerged in the earlier period.

2.5 Equilibrium Analysis

I now proceed to discuss some aspects of optimal bank behavior in equilibrium. Three distinct, but closely interrelated phenomena arise in equilibrium: interest rate dispersion, a risk-shifting motive, and intermediation. The first is a consequence of market segmentation (through different levels of risk and physical costs). The risk-shifting motive, that leads banks to "herd" (correlate their investments and states of default), is a consequence of the interaction between limited liability and negative externalities of failure. Finally, intermediation arises as a consequence of interest rate dispersion and market segmentation, and is shown to be closely influenced by bank herding in this model. I define the following notation:

$$i \rightsquigarrow j \text{ if and only if } \ell_{ij} > 0$$

2.5.1 Interest Rate Dispersion

Taking the definition of the interbank rate of borrowing for bank i, $r_i = f_i'$, we can conclude from the first-order condition (13) that a necessary condition for $\ell_{ij} > 0$ is

$$r_j \geq r_j \mathbb{E}[\theta_j | \pi_i \geq 0] = f_i' + \phi\omega_\ell \mu_i + \kappa_{ij}'(\ell_{ij}) \geq r_i$$

That is, bank i only lends to bank j if the cost of funds of bank j is at least as great as the cost of funds for bank i. This inequality is strict if any of the following are true

1. $\mu_i > 0$: the leverage constraint is binding for bank i.
2. $\kappa_{ij}'(\ell_{ij}) > 0$: marginal lending fees are strictly positive.
3. $\mathbb{E}[\theta_j | \pi_i \geq 0] < 1$: there exists a measurable set of states of the world in which bank j defaults but bank i does not;

in which case the lending contract is established with a strictly positive interest rate differential. Since no lending relationship $i \rightsquigarrow j$ can be established when $r_j < r_i$, we conclude that all intermediation and lending paths will feature (weakly) increasing prices along the chain. The first two sources of interest rate dispersion are relatively mechanical. The third source is discussed in greater detail in the remainder of the section.

Proposition 2.3. *In equilibrium, bank i lends to bank j only if $r_j \geq r_i$. The inequality is strict if: bank i's leverage constraint binds; lending fees are strictly positive; and/or there exists a measurable set of states of the world in which bank j defaults while bank i does not default.*

Proof. See above. □

2.5.2 Risk-Shifting and Herding

An immediate conclusion from the third source of interest rate dispersion is that bank i has greater incentives to lend to banks that it perceives as being safer. The not so obvious insight comes from what it means to be "safer": from the point of view of bank i, it means that repayments are high *in states of the world in which bank i itself does not default*. This is regardless of the unconditional probabilities of default: even if bank j has a very high (unconditional) probability of default, it may be perceived as a safe investment by bank i if these two banks have very correlated defaults, since it then offers high repayments in the only states of the world that are payoff-relevant for bank i (those in which the bank does not default).

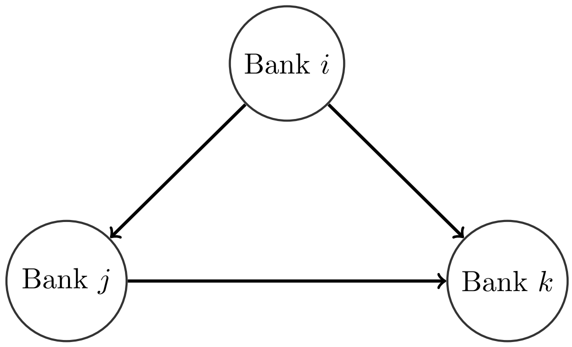
This effect, which I call *risk-shifting*, can manifest itself as a sort of strategic complementarity: by lending greater volumes to bank j, bank i is directly correlating its own defaults with those of bank j. This, in turn, may increase the perceived payoff of lending to bank j, since it may appear to become a safer investment due to this risk-shifting motive. This effect is very similar to what Acharya (2009) calls a "systemic risk-shifting incentive" or "bank herding": the interaction between limited liability and negative externalities of failure leads banks to bet on the upside and undertake correlated investments. The risk-shifting motive I have presented is precisely this: banks endogenously correlate the states of the world in which they are solvent.

2.5.3 Intermediation

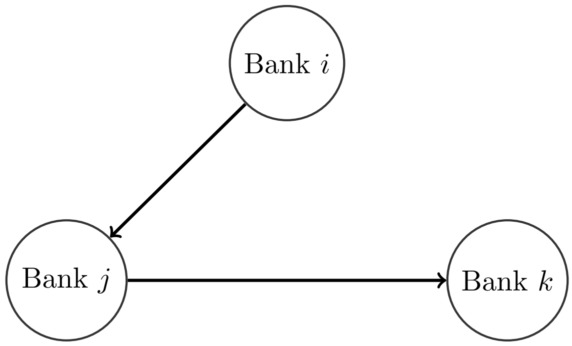
This analysis is important to understand the emergence of endogenous intermediation in this model. I focus on two types of intermediation: *partial* and *pure*. Consider the potential intermediation chain $i \rightsquigarrow j \rightsquigarrow k$, that is, where bank i lends to bank j, which in turn lends to bank k. I call this a partial intermediation chain whenever $\ell_{ik} > 0$, bank i also lends to bank k, and call it pure whenever $\ell_{ik} = 0$. In the case of pure intermediation, bank j serves as a pure intermediary between banks i and k, and the latter do not interact directly. This distinction is depicted in figure 4.

Figure 4: Types of Intermediation

(a) Partial Intermediation



(b) Pure Intermediation



In order to have $\ell_{ij}, \ell_{jk} > 0$, we need the respective FOCs to bind

$$r_j \mathbb{E}[\theta_j | \pi_i \geq 0] - r_i - \phi \omega_\ell \mu_i - \kappa'_{ij}(\ell_{ij}) = 0$$
$$r_k \mathbb{E}[\theta_k | \pi_j \geq 0] - r_j - \phi \omega_\ell \mu_j - \kappa'_{jk}(\ell_{jk}) = 0$$

We can combine these two conditions with the FOC that determines whether i lends to k to assess when is pure intermediation possible. This yields

$$\begin{aligned}
&\mathbb{E}[\theta_j | \pi_i \geq 0] \mu_j \\
&+ \mathbb{E}[\theta_j | \pi_i \geq 0] \kappa'_{jk}(\ell_{jk}) + \kappa'_{ij}(\ell_{ij}) - \kappa'_{ik}(\ell_{ik}) \\
&+ r_k \left(\mathbb{E}[\theta_k | \pi_i \geq 0] - \mathbb{E}[\theta_k | \pi_j \geq 0] \mathbb{E}[\theta_j | \pi_i \geq 0] \right) \leq 0
\end{aligned}$$

Pure intermediation arises whenever this inequality is strict. The condition is written in three different lines to highlight the three main forces that may contribute for and against the existence of pure intermediation: the first two lines represent relatively mechanical forces. First, intermediation is less likely if the leverage constraint for bank j is binding, since this raises the bank's cost of funds and makes it less likely to lend. Second, if marginal lending fees for $i \rightsquigarrow k$ are substantially higher than the combined marginal costs of forming an intermediation chain $i \rightsquigarrow j \rightsquigarrow k$, pure intermediation is more likely to arise.

The third line is the most interesting and less obvious force, and is connected with the previously discussed risk-shifting motive that is induced by limited liability. Assume that leverage constraints do not bind and there are no lending costs. The pure intermediation condition can be written as

$$\mathbb{E}[\theta_k|\pi_i \geq 0] \leq \mathbb{E}[\theta_k|\pi_j \geq 0]\mathbb{E}[\theta_j|\pi_i \geq 0]$$

That is, intermediation through j is more likely when the chain of expected repayments through j is expected to exceed the direct repayment from bank k in case bank i does not default. This is easily seen through a concrete example: assume that banks i and k have negatively correlated defaults, so that bank k defaults whenever bank i does not default and vice-versa. Then, $\mathbb{E}[\theta_k|\pi_i \geq 0]$ is likely to be low. Assume now that bank j survives in some of the states in which bank i survives and in some of the states in which bank k survives. Then, expected repayments by bank k are likely to be higher from the point of view of bank j, as well as expected repayments by bank j from the point of view of bank i. Risk-shifting becomes the main driver of intermediation in the absence of marginal lending fees. More generally, this risk shifting motive implies that banks are more likely to lend to other banks that have more correlated defaults.

Proposition 2.4. *A necessary condition for the pure intermediation chain $i \rightsquigarrow j \rightsquigarrow k$ to arise in equilibrium is*

$$\begin{aligned}
&\mathbb{E}[\theta_j|\pi_i \geq 0]\mu_j \\
&+ \mathbb{E}[\theta_j|\pi_i \geq 0]\kappa'_{jk}(\ell_{jk}) + \kappa'_{ij}(\ell_{ij}) - \kappa'_{ik}(\ell_{ik}) \\
&+ r_k \left(\mathbb{E}[\theta_k|\pi_i \geq 0] - \mathbb{E}[\theta_k|\pi_j \geq 0]\mathbb{E}[\theta_j|\pi_i \geq 0]\right) \leq 0
\end{aligned}$$

Proof. See above. □

2.6 Numerical Example

Unfortunately, due to the lack of closed forms for the fixed point repayments, the amount of information that can be extracted from a purely analytical discussion of the model is limited. For this reason, and to gain further insights on the forces at play, I calibrate the model and solve for the equilibrium numerically.

2.6.1 Solution Method and Algorithm

The solution method consists of solving for the equilibrium in each period iteratively. It takes advantage of the fact that the equilibrium conditions can be alternatively represented as a nonlinear complementarity problem (NCP). Given expected repayments and prices, each bank solves its portfolio allocation problem. This yields allocations that allow us to update expected repayments and prices, and the process is repeated until convergence is achieved. Formally, the algorithm is:

1. Guess initial conditional expected repayments $\mathbb{E}_0[\theta_j|\pi_i \geq 0], \forall i, j \in N$

2. Given these conditional expected repayments, find the equilibrium at $t = 0$ by solving a $N \times (N+2)$ NCP. This system is composed of $N+2$ Kuhn-Tucker conditions for each bank: $N-1$ FOC for lending, 1 FOC for cash reserves, 1 leverage constraint and 1 reserve requirements constraint. The system can then be solved for $N+2$ variables for each bank: $N-1$ lending decisions $\{\ell_{ij}\}_{j\neq i}$, 1 cash decision c_i and 2 multipliers (μ_i, λ_i).

$$\begin{aligned} r_j\mathbb{E}_0[\theta_j|\pi_i \geq 0] - r_i - \phi\omega_\ell\mu_i - \kappa'_{ij}(\ell_{ij}) \leq 0 \perp \ell_{ij} \geq 0, \forall j \neq i \\ 1 - r_i + \lambda_i \leq 0 \perp c_i \geq 0 \\ \phi\left(\omega_a a_i + \omega_\ell \sum_{j\neq i} \ell_{ij}\right) - (a_i - d_i) \leq 0 \perp \mu_i \geq 0 \\ \tau d'_i - c_i \leq 0 \perp \lambda_i \geq 0 \end{aligned}$$

where $r_i = f'_i\left(s_i + B_i - c_i - \sum_{j\neq i}\ell_{ij}\right)$ and $B_i = \sum_{j\neq i}\ell_{ji}$.

3. Given the portfolio allocation decisions, compute new conditional expected repayments $\mathbb{E}_1[\theta_j|\pi_i \geq 0], \forall i, j \in N$ and iterate until these expectations converge.

Conditional expected repayments are computed using Monte Carlo simulations. First, a sufficiently large sequence for the joint realization of long-term asset payoffs $\mathbf{R}$ is drawn. Then, given portfolio allocations, repayment equilibria can be computed for each possible realization of the shock, and conditional expectations are computed for each bank via numerical integration. Standard methods are used to solve the NCP; I use the `ncpsolve` function provided in the CompEcon toolbox by Miranda and Fackler (2002) [7].

2.6.2 Calibration

While the goal of this paper is not to provide a full quantitative description of the financial system, I attempt to choose parameters that reflect the current institutional and technological features of the banking sector in developed economies. The baseline calibration is presented in table 1.

[7] This function solves the NCP by transforming the original problem in an approximate semismooth system of equalities and solving it with a standard Newton-Raphson method. Several robustness checks were performed, by independently solving the problem with other methods, and no substantial differences were found.

Table 1: Summary of Calibration

Parameter	*Value*	*Description*
d	$0.9 \times a$	Initial Deposits/Assets
ϕ	0.08	Maximum Leverage
ω_a	1	Risk-weight on a
ω_ℓ	0.2	Risk-weight on Interbank Lending
τ	0.1	Reserve Requirements
δ	0.2	Default Costs
$f_i(v)$	$v(\alpha - 0.5\Psi v)$	Outside Market
α	1.01	Baseline Rate
Ψ	0.2	Outside Market Elasticity
$\kappa_{ij}(\ell_{ij})$	$\frac{0.05}{(a_i a_j)^2}\ell_{ij}(1+\ell_{ij})$	Lending Fees
$g(\mathbf{R})$	$\mathcal{U}[\underline{R}, 1.05]^N$	Asset Returns

Most banking parameters are calibrated to match the commercial banking sector in the United States. Initial deposits are 90% of initial assets to match the average leverage of 10 for US commercial banks. Note that deposits are interpreted in a broad sense and taken to be, more generally, any source of short-term senior or collateralized funding, such as repo contracts or other short-term core liabilities. Regulatory parameters are calibrated to their institutional counterparts: ϕ is chosen to match the Basel II and Basel III minimum ratio of Tier 1 + Tier 2 equity to risk-weighted assets of 8%. Risk-weights are equal to 1 for most bank assets such as corporate and retail loans, and a significant part of the trading portfolio [8] and equal to 0.2 for exposures to other depository institutions, motivating the choices of ω_a, ω_ℓ. Finally, reserve requirements in the United States follow a step function peaking at 10% for institutions with large net transactions accounts (in excess of $79.5 million), justifying the choice of τ. Default costs are assumed to be 20% of the short-fall between junior debt and residual assets in case of a default. They are taken to be greater than the 10% measure of asset value discussed previously since they only apply on to the shortfall between junior debt and residual assets, and not to the total debt shortfall.

The outside market function is chosen to be quadratic. This is an useful simplification that satisfies all of the required properties up to $v = \Psi^{-1}\alpha > 0$, as it becomes decreasing thereafter. Due to reserve requirements, however, we know that interest rates are bounded below by 1, meaning that it is never optimal to invest more than $\bar{v} = \Psi^{-1}(\alpha - 1) < \Psi^{-1}\alpha$ in the outside market, thus making this problem immaterial [9]. The two parameters in this function deserve further discussion: I call α the *baseline rate*. Note that the cost of

[8] This is a lower bound for the trading portfolio of banks, as many securities are allotted a risk weight in excess of 100% under the Basel III framework, such as most publicly traded equities.

[9] Alternatively, one could redefine the function as being equal to $f_i(v)$ for $v \leq \Psi^{-1}(\alpha - 1)$ and 1 thereafter. This changes none of the results.

funds for bank i can be written as

$$f_i' = \alpha - \Psi v_i \tag{14}$$

So that α is the cost of borrowing for a bank that has a zero outside position. One can think of this as the interest rate target set by the Central Bank (i.e. the Federal Funds rate). As the outside position becomes negative, the cost of funds for the bank exceeds this baseline rate. In the next sections I discuss the role of this parameter as a tool of monetary policy and regulation. Ψ is the elasticity of the cost of funds to the outside position, and its interpretation is clear from (14): it measures the rate of change of the cost of funds with respect to the outside position of the bank. This parameter could, in principle, vary across banks, but is set to be the same in order to discipline the results. This parameter is chosen to generate reasonable levels of interest rate dispersion.

Lending fees $\kappa_{ij}(\ell_{ij})$ are chosen to be a convex function of lending volumes [10], scaled by a pair-specific constant that takes the form $\frac{\kappa}{(a_i \cdot a_j)^\beta}$. This choice is motivated by the presence of economies of scale in the establishment of platforms for access to payments and settlement systems such as CHIPS or Fedwire without introducing explicit fixed costs. Larger banks (as measured by the size of their balance sheets) are more likely to pay these fixed costs of access. Smaller banks that choose not to access payments systems due to their costs often rely on relationships with larger banks, called "correspondents", who participate in these systems on their behalf [11]. These features are captured by the size interaction term: it is relatively cheaper for two large institutions to establish links than for two small institutions. Similarly, the marginal cost of interacting with a larger counterparty is lower for a smaller bank, than to interact directly with a counterparty of similar size. A comprehensive description of some of the facts that motivate this choice can be found in CPSS (2012).

Finally, long-term asset payoffs are assumed to be independent and uniformly distributed between $\underline{R}$ and 1.05, where the upper bound is the average return on assets by the banking sector in the US in the $2000-2014$ time period. The reason the average return is the upper bound is in order to study the impact of increasing risk in this economy. I will, in general, consider different scenarios for the lower bound $\underline{R}$.

Bank Size Distribution The basic numerical examples are undertaken for $N = 15$. While I undertake numerical examples with equally sized banks, to highlight some of the model's mechanisms, I then calibrate the bank size distribution to the US banking sector in 2014, with data taken from FRED (Federal Reserve Economic Data, from the Federal Reserve Bank of St. Louis). Table 2 summarizes this data.

[10] This simplifies the nonlinear nature of lending fees, see Fedwire (2014) for pricing details.

[11] As an example, out of more than $15,000$ depository institutions in the United States, around $9,000$ participate in Fedwire and CHIPS has 51 participants as of 2014.

Table 2: Bank Size Distribution for the US, March 2014

Asset Value	*Frequency*	*Relative Freq.*
Up to \$300M	4062	69.9%
\$300M-\$1B	1223	21.1%
\$1B-\$10B	437	7.5%
\$10B-\$20B	33	0.6%
More than \$20B	53	1.0%
Total	5814	100%

For simplicity, I discard the smallest category of banks and collapse the two intermediate categories to one. This leaves us with roughly 70% "small" banks (assets in the \$300M-\$1B range), 25% "medium banks" (assets in the \$1B-\$20B range) and 5% "large banks". For $N = 15$, I adapt these ratios to generate 10 small banks, 3 medium sized banks and 2 large banks. I normalize the size of small banks to $a_i^S = 1$, setting the size of medium banks to $a_i^M = 10$ and the size of large banks to $a_i^L = 20$.

Liquidity Shocks Due to sampling noise, it is impossible to generate a series of finite and independent liquidity shocks that do not result in either a surplus or a deficit of aggregate liquidity in the system. Furthermore, since I assume that initial deposits are a constant fraction of initial assets, equally distributed liquidity shocks would have very different implications for small and large banks: if liquidity shocks are, on average, of the same size, they could correspond to large shocks for small banks and small shocks for very large banks.

In an attempt to address these issues, I draw liquidity shocks for each bank as

$$d_i' \sim \mathcal{U}[(1-\sigma)d_i, (1+\sigma)d_i]$$

That, is shocks are uniformly distributed around d_i, and their variance is controlled by the parameter σ. Setting $\sigma \leq 1$ ensures that no bank ever receives negative deposits. The shock scales with the size of the balance sheet, thereby avoiding the differential size impact problem. While the sampling noise problem cannot be solved in an obvious manner, I report the value of the aggregate liquidity shock $S \equiv \sum_{i=1}^{N} s_i 0$. Different exercises for different levels of S_i do not seem to reflect in substantially different dynamics (as long as this value is not to large in absolute value), and are reflected mostly in the level of the interest rates (that correlate negatively with aggregate liquidity). Whenever $S < 0$, aggregate liquidity is negative, one can think of an exogenous flight from bank short-term debt. I set $\sigma = 0.5$ for most simulations (unless otherwise noted).

2.6.3 Results

Equally Sized Banks I simulate an economy with $N = 6$ equally sized banks. Holdings of long-term assets are normalized to $a_i = 1$. To focus on the role of risk as a source of intermediation, I eliminate lending fees from the model in the examples that follow. Figure 5 plots the resulting network, and some summary statistics can be found in table 3. I set the lower bound of the return distribution to $\underline{R} = 0.95$ in the baseline case.

Figure 5: Low Risk, No Lending Fees, $N = 6$ Network

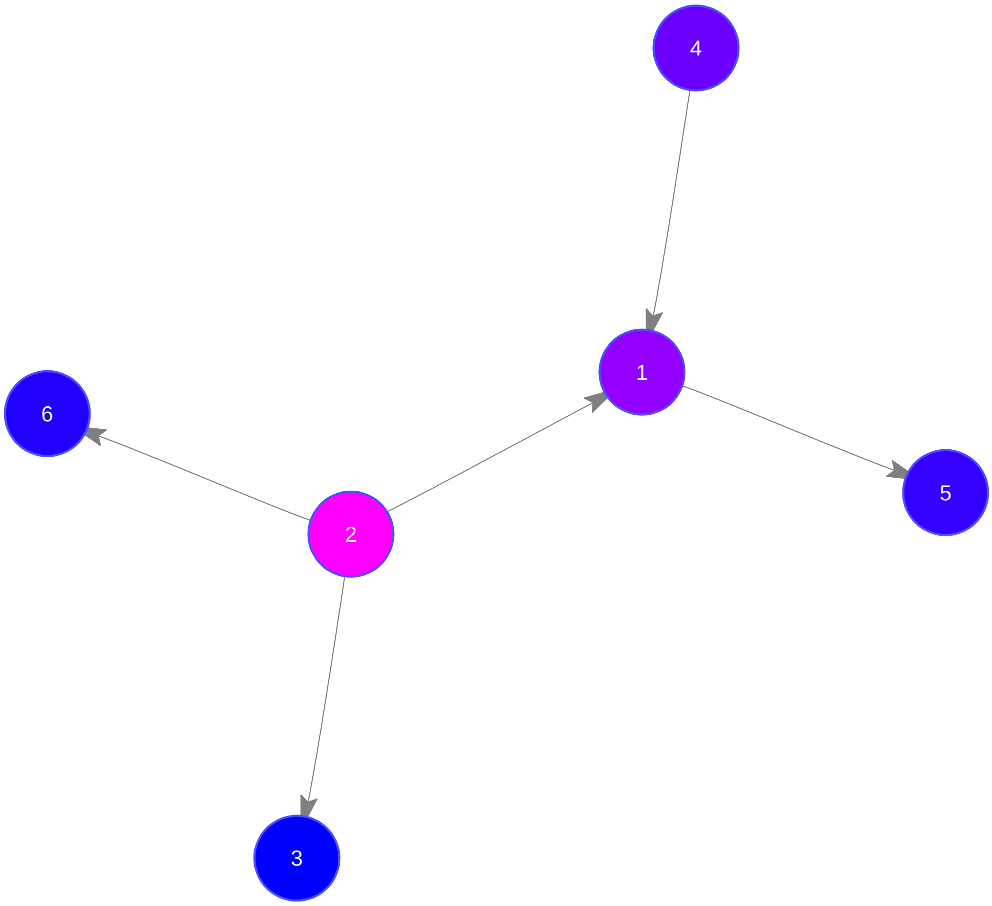


Table 3: Low Risk - Statistics

Network Results						
Bank	s_i	B_i	r_i	Outdegree	Indegree	Prob. Def.
1	-0.07	0	9.05%	2	0	0%
2	0.20	0	9.05%	2	0	0%
3	-0.45	0.23	9.05%	0	3	0%
4	-0.18	0	9.05%	1	0	0%
5	-0.32	0.11	9.05%	0	1	0%
6	-0.37	0.15	9.05%	0	1	0%

This table reports summary statistics for a simulated network with 6 equally-sized banks. Long-term assets are set to a_i and the lower bound on the i.i.d. long-term asset returns is set to $\underline{R} = 0.95$. s_i is the liquidity shock; B_i is total interbank borrowing; r_i is the cost of funds; Outdegree is the number of outgoing links, or number of banks that bank lends to; Indegree is the number of incoming links, or number of banks that bank is borrowing from; Prob. Def. is the unconditional probability of default.

Bank 2 is the only bank that received a positive liquidity shock, so it arises as the main supplier of liquidity in the network, lending to banks 6 and 3. Even though banks 1 and 4 received negative liquidity shocks, the respective low magnitudes make this banks find worthwhile to borrow from the outside market and lend to banks 3 and 5. In the absence of lending costs, default risk, and binding constraints, interest rates are fully equalized across the network. Liquidity flow from the banks with positive to those with negative endowments. The reference interest rate, computed by eliminating interest rates below the 23% percentile, and above the 77% percentile, and taking the average of the rest, is 9.05%.

A natural experiment is to observe what happens if each counterparty becomes (exogenously) riskier. I study this by setting $\underline{R} = 0.9$ and simulating the same network, fixing all remaining parameters (including liquidity shocks). Figure 6 plots the resulting network, and some summary statistics can be found in table 4.

Figure 6: High Risk, No Lending Fees, $N = 6$ Network

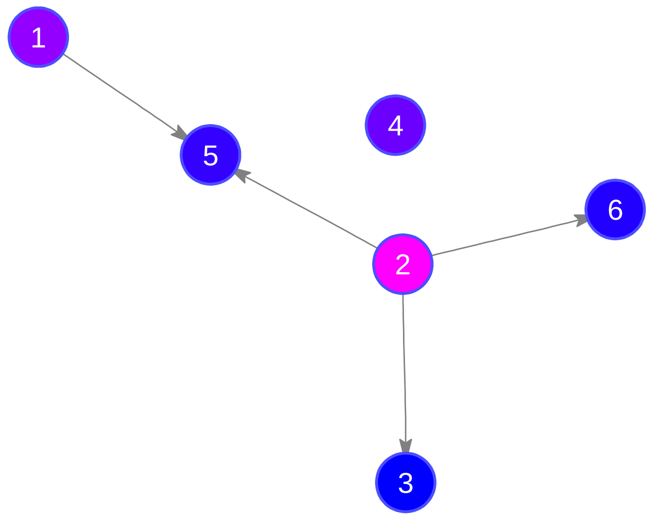


Table 4: High Risk - Statistics

Network Results						
Bank	s_i	B_i	r_i	Outdegree	Indegree	Prob. Def.
1	-0.07	0.02	7.58%	1	1	1.00%
2	0.20	0	7.31%	3	0	4.00%
3	-0.45	0.17	10.65%	0	1	26.00%
4	-0.18	0	8.50%	0	0	8.00%
5	-0.32	0.07	10.33%	0	1	17.00%
6	-0.37	0.12	9.33%	0	1	19.00%

This table reports summary statistics for a simulated network with 6 equally-sized banks. Long-term assets are set to a_i and the lower bound on the i.i.d. long-term asset returns is set to $\underline{R} = 0.9$. s_i is the liquidity shock; B_i is total interbank borrowing; r_i is the cost of funds; Outdegree is the number of outgoing links, or number of banks that bank lends to; Indegree is the number of incoming links, or number of banks that bank is borrowing from; Prob. Def. is the unconditional probability of default.

The structure of the network now changes considerably: there is, overall, less borrowing both at the intensive (magnitude of the links) and extensive margins (number of links). Bank 4, for example, stops participating in the market altogether (it was a lender in the lower risk case). Bank 2, while still lending directly to banks 3 and 6, does not find it worthwhile to lend directly to bank 5, doing so through bank 1 that now acts as an intermediary. Bank 1 borrows from the main source of liquidity, bank 2, and lends to banks 5. Since no constraints are binding, and there are no lending fees, this is a result of pure risk-based intermediation: bank 2 deems bank 5 to be too risky for a direct link to be established (compared to the interest rates at which this banks are willing to borrow). Bank 1, however, is willing to take the risk and intermediate the funds. In fact, one can check that

$$\mathbb{E}[\theta_1|\pi_2 \geq 0] = 0.9974$$
$$\mathbb{E}[\theta_5|\pi_2 \geq 0] = 0.9725$$

and

$$\mathbb{E}[\theta_5|\pi_1 \geq 0] = 0.9751$$

So that it is safer to lend to bank 1 than directly to bank 5, from the point of view of bank 2. This highlights how uninformative unconditional default probabilities can be, and how little relevant they end up being for the analysis of intermediation. The reference rate is now given by 8.73%, lower than in the low risk situation. As risk increases, so does interest rate dispersion - but dispersion increases asymmetrically as banks that previously lent now tend to hoard liquidity. This causes their own cost of funding to decrease in a disproportionate manner, bringing reference interest rates down.

To study the impact of the networked structure of the market, I consider two measures that, albeit crude, provide us with interesting insights:

1. Risk Sharing: the difference between the counterfactual probability of default that would prevail in the absence of the Interbank market (allowing for optimal portfolio choice over cash reserves and outside investment) and the unconditional probability of default;

2. Risk Exposure: the difference between the unconditional probability of default and the counterfactual probability of default that would prevail if all of the banks' counterparties repaid their loans in full.

While I delay a detailed discussion of systemic risk measures to the following section, these variables offer some interesting insights. These two measures are presented in table 5. The measure of risk-sharing is in general positive, meaning that the existence of an Interbank market allows for banks to share their risks and reduce their default probabilities. If this measure is negative, the risk-sharing benefits of liquidity trade are being dominated by risk-shifting incentives [12]. Note that this measure tends to be greater for banks with very negative liquidity shocks that manage to fund themselves in the interbank market. The second column is a measure of *risk-shifting*, and accounts for the additional default probability that is a direct consequence of counterparty risk. It is, by construction, positive; it is always greater for lenders and equal to zero for banks that do not lend (since they do not expose themselves to counterparty risk).

[12] This should not be seen as a normative statement against risk-shifting: even an extremely risk-averse agent may find it worthwhile to engage in some risk. The measures I consider regard probabilities of default only, and do not consider expected gains and losses.

Table 5: High Risk - Risk Measures

Bank	Risk Sharing	Risk Exposure
1	3.00%	1.00%
2	−4.00%	4.00%
3	2.00%	0.00%
4	0.00%	0.00%
5	0.00%	0.00%
6	2.00%	0.00%

This table reports two measures of risk in the simulated network for 6 *equally sized banks and* $\underline{R} = 0.9$. *Risk-Sharing is the difference between counterfactual probability of default that would arise in autarky (allowing the bank to optimize over cash reserves and outside investment only) and the unconditional probability of default. Risk Exposure measures the difference between the probability of default and the counterfactual probability of default that would arise if, fixing the network exposure, all counterparties fully repaid their contractual liabilities.*

3 Policy and Welfare

There are two main sources of externalities in this model. The first is limited liability, which induces banks to (optimally) take excessive risks from the perspective of their debtors. This leads us to the second source of externalities, which originates in the networked structure of the economy: not only do banks not account for their counterparties' losses in the event of default, nor they do account for the losses that may potentially be inflicted on their counterparties' counterparties. Excessive risk-taking purely caused by limited liability may be exacerbated by the herding motive, through which banks have an incentive to correlate their defaults and potentially magnify total risk in this economy. Since defaults are costly, this may not be in the interests of a welfare-maximizing regulator.

This suggests that the regulator can potentially decrease the total amount of risk-taking in the economy through the policy constraints. Both constraints act, either directly or indirectly, as limits on total leverage that can be taken by banks. The previous numerical examples highlight the trade-off faced by the regulator: on one hand, the interbank market can help banks share risks, and allows banks with liquidity surpluses to transfer these funds to banks that need liquidity (at lower costs than they would otherwise face in the respective outside markets). By constraining banks' activity, the regulator is preventing liquidity to flow from deficit to surplus banks.

3.1 Welfare

Since a meaningful discussion of the costs and benefits of different policies require a precise of definition of welfare, I make some assumptions regarding the other agents in the model. I assume that depositors, outside investors and the fee-receiving institution are risk-neutral agents who value consumption at $t = 1$ only. I assume that depositors have access to an initial endowment $Y > \max\left\{\sum_{i=1}^{N} d_i, \sum_{i=1}^{N} d'_i\right\}$ and a risk-free storage technology with unity return. This allows me to ignore the initial liquidity shock and reallocation of deposits, for simplicity. Given these assumptions, and absent costs of default, bank defaults are purely redistributive phenomena, and I assume that the planner weighs all agents equally. The planner, or policymaker, seeks to maximize *ex-ante* welfare, prior to the realization of the liquidity shock. The previous assumptions allow me to write *ex-ante* welfare as

$$W = Y + \mathbb{E}_{\mathbf{S},\mathbf{R}}\left[\sum_{i=1}^{N}\left\{\pi_i^+ + (\theta_i - 1)\sum_{j\neq i}\kappa_{ij}(\ell_{ij}) + v_i - f(v_i)^+ + \theta_i(-f(v_i))^+ + (\varphi_i - 1)d'_i\right\} - \gamma\Psi^2\right] \tag{15}$$

where $\mathbf{S}$ is the $N \times 1$ vector of liquidity shocks and φ_i is the fraction of new deposits that is effectively repaid by bank i[13]. The first term is the endowment. The second term is the sum, over each bank, of the net returns the bank generates: to equity holders, to outside claimants (outside investors, whether they are borrowing or lending, and the entity that collects lending fees) and to depositors. The last term represents costs of active interventions, such as the provision of liquidity facilities and bailouts. Ψ is the total spending with the program, and γ is a parameter that controls/scales the deadweight loss costs of taxation. I assume that these welfare costs of taxation are convex, hence entering quadratically in the welfare function. This is an useful reduced form for modeling the costs of government interventions, and is standard in the literature. Note that welfare is measured as total profits, regardless of their sign, since defaults are merely redistributive in the absence of costs. I assume that changes in policy are costless from a resource point of view, so as to focus on the trade-off between risk-sharing and risk-shifting incentives.

3.2 Systemic Risk

While welfare as defined in (15) will be used as the main criterion for the desirability of different policies, it is also interesting to look at the endogenous build-up of risk in the model through the lenses of some commonly used measures of systemic risk. While the disclaimer that this model does not aim at being a fully quantitative description of the financial system is still valid, the process of endogenous risk formation suggests that the model can be used as some sort of benchmark against which applicable measures of systemic risk can be evaluated.

Unfortunately, not all commonly used measures of systemic risk can be applied to the current model. This is mostly a consequence of specific data requirements concerning variables about which the model is silent. An example is the Systemic Expected Shortfall proposed by Acharya et al. (2012b). For the interested reader, a conceptual and concise review of commonly used measures is provided by Hansen (2013). A much more extensive and technical review is undertaken by Bisias et al. (2012).

3.2.1 Model-Specific Measures

The model itself suggests some heuristic measures, such as the ones presented in the previous section. Recall that the risk-sharing measure, Γ_i^{RS} measures the difference between the unconditional default probability if the bank were subject to the same liquidity shock but had no access to the interbank network and the unconditional default probability of bank i in the interbank equilibrium .

$$\Gamma_i^{RS} = \Pr(\pi_i < 0 | \ell_{ij}, \ell_{ji} = 0, \forall j \neq i) - \Pr(\pi_i < 0)$$

[13]Recall that bank i's salvage value may be less than d_i', in which case only a fraction of senior debt is repaid. I denote this fraction by $\varphi_i \in [0, 1]$.

If $\Gamma_i^{RS} > 0$, bank i is benefitting from the risk-sharing role of the interbank market and reducing its probability of default by interacting with counterparties. If $\Gamma_i^{RS} < 0$, the risk-shifting motive is so strong that it could be socially efficient to prevent bank i from participating in the market.

Another already discussed measure is the difference between the unconditional probability of default and the counterfactual probability of default assuming that all counterparties fully repay their obligations in all states of the world. This can be seen as a measure of risk exposure, Γ_i^{RE}, and is by construction positive.

$$\Gamma_i^{RE} = \Pr(\pi_i < 0) - \Pr(\pi_i < 0 | \theta_j = 1, \forall j \neq i : \ell_{ij} > 0)$$

It will tend to be particularly large whenever the risk-shifting motive is very strong, in which case the bank is unconditionally exposing itself to a substantial amount of risk.

3.2.2 The Role of the Network

To understand the role of the network in magnifying systemic risk, I construct a counterfactual scenario in which a centralized exchange (a clearinghouse) redistributes losses from default in a manner similar to the one described in Dubey et al. (2005). I take banks' portfolio allocation decisions as given, and construct counterfactual default probabilities in which the central exchange equally redistributes losses from default amongst all participating banks. That is, all banks receive as a repayment a bank-independent fraction θ of their interbank claims. θ is such that it solves

$$\text{total repayments} = \theta(\text{total nominal claims})$$

or, using our repayment notation

$$\sum_{i=1}^{N} x_i = \theta \sum_{i=1}^{N} Q_i$$

This allows us to express

$$\theta(\mathbf{x}, \mathbf{Q}) = \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} Q_i}$$

and to adapt the standard fixed-point repayment problem as

$$x_i = \min \left\{ Q_i, \max\{(1+\delta) \max\{e_i + \theta(\mathbf{x}, \mathbf{Q}) \sum_{j \neq i} r_j \ell_{ij} - d_i', 0\} - \delta Q_i, 0\} \right\}$$

It is still possible to show that, under similar conditions, this problem has a unique greatest fixed point, the one we are interested in. For a simple proof, consider the following algorithm in fictitious default time:

1. Start with $\theta_0 = 1$
2. Compute the vector of feasible repayments $\mathbf{x}_0$
3. Update $\theta_1 = \frac{\sum_{i=1}^{N} x_{i,0}}{\sum_{i=1}^{N} Q_i}$
4. Repeat until convergence

Since there are N banks, and x_i can only decrease, this algorithm converges in at most N iterations. I then compare the counterfactual probabilities of default that emerge to the ones that are generated by the networked structure.

It can be argued that this is not a "fair" counterfactual, as banks chose their portfolios under the presumption of a certain repayment protocol at $t = 0$. Therefore, these repayments are not consistent with the rational expectations that are taken by banks when allocating their lending decisions. There are several reasons why I choose not to fully re-solve the model with the centralized exchange, and I present two of them: first and foremost, lending fees lose their meaning, and it becomes complicated to model their centralized equivalent. Removing them altogether from the centralized model would make the comparison even less fair, as there would one less source of costs for banks. This would make results biased towards greater amplification of risk in the networked market structure. Secondly, the fact that only one interest rate prevails in this centralized exchange can change considerably the optimal decisions of the agents, and the resulting balance sheets can differ considerably from those that emerge in the networked structure. This makes the default probabilities not comparable for any purpose.

In principle, these "centralized" default probabilities can be greater or smaller than the network default probabilities. Whenever smaller, this means that the networked structure of the interbank market does contribute to the build-up of risk in this economy. If greater, this is suggestive that the network structure may, in fact, be attenuating the emergence of systemic risk.

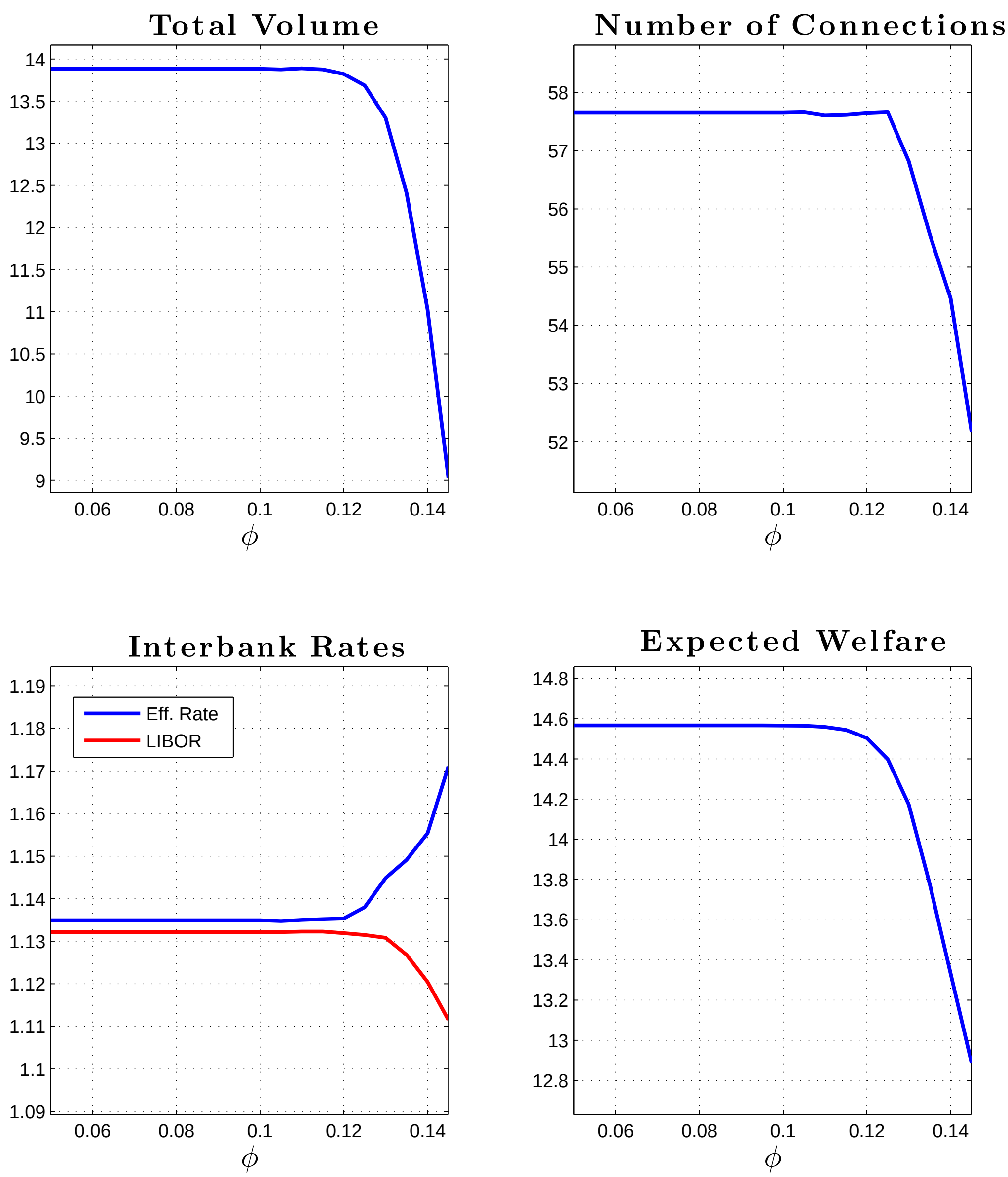
3.3 Policy and Regulation

I now proceed to discuss the costs and benefits of some policies. Policy analysis is undertaken in a very simple way, and I focus mostly on comparative statics. To evaluate each policy I use a network of $N = 20$ banks with the baseline calibration in table 1 $\underline{R} = 0.85$. The only difference with respect to the baseline calibration is that I set $d = 0.85a$, initial leverage to less than 10. The initial size distribution features 13 small banks, 5 medium banks and 2 large banks. For each parameter value, I simulate and compute the network equilibrium for a large number of potential liquidity shocks. I then present some plots of average statistics of the network, as functions of the parameter value. I dedicate particular attention to welfare and systemic risk measures.

3.3.1 Capital Adequacy Ratio

The capital adequacy ratio, controlled by the parameter ϕ, acts as a direct constraint on leverage. By raising it, the regulator is limiting the total amount of lending that each bank can extend. By forcing banks to reduce lending, the regulator is curbing their ability to share risks, but is also limiting risk-taking and risk-shifting incentives. Results are shown in figures 7 and 8.

Figure 7: Changes in ϕ



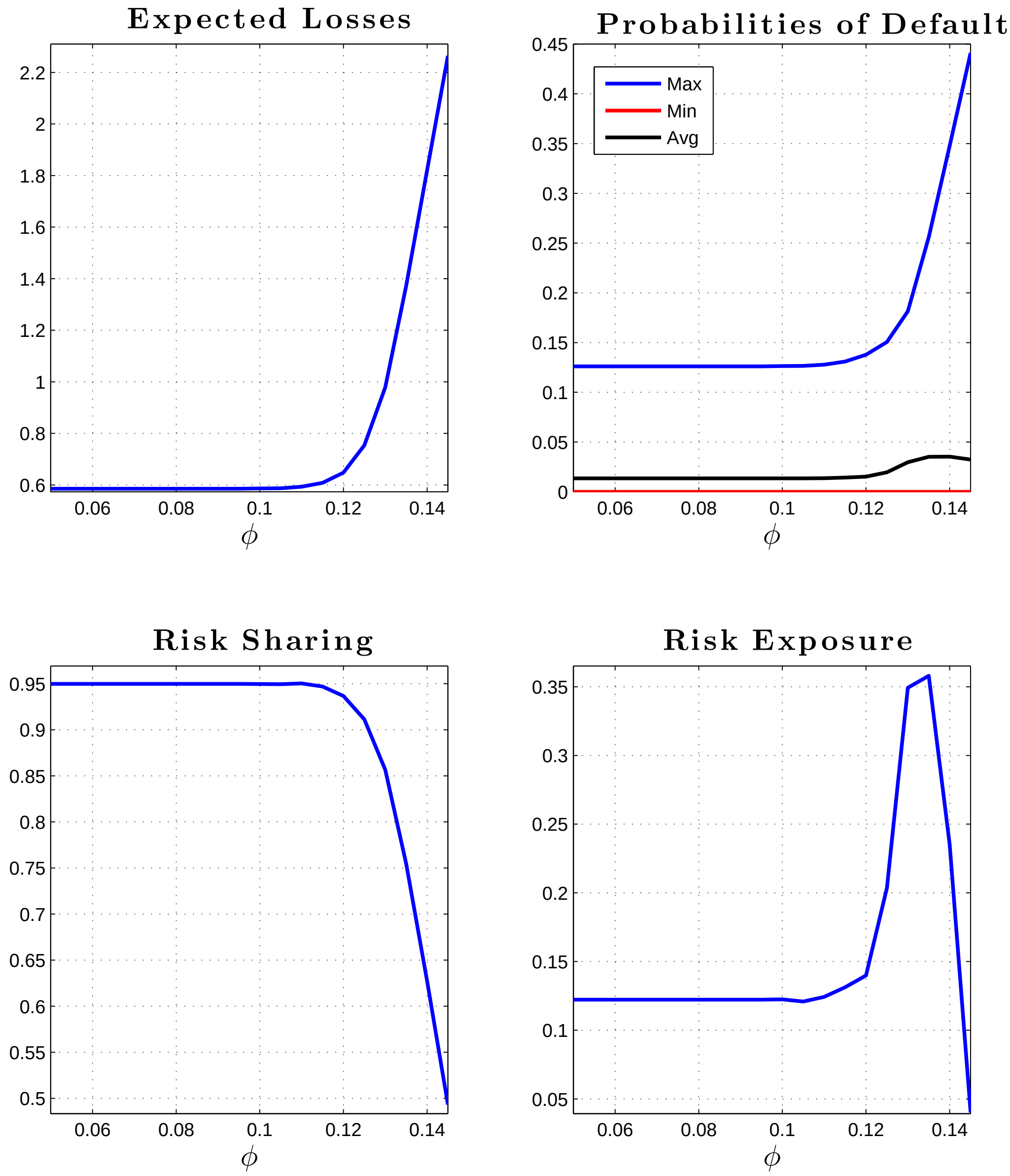
The first two panels of figure 7 represent basic network and market statistics: total interbank lending volume and total number of connections. The reaction of market volume to an increase in ϕ is as expected: by constraining leverage, the regulator is directly constraining the ability of banks to lend, so volumes decrease. What may not be so obvious is the behavior of the other measure: the number of connections can actually (slightly) increase in response to an increase in leverage requirements, and will tend to decrease at a lower rate than total volume. This happens because as lenders become constrained in how much they can lend, borrowers look for other sources of funding. While it may not seem so (due to the scale of the plot), the number of connections is actually relatively stable when compared with the number of banks. Since borrowers' willingness to pay for liquidity is high (because their former lenders are lending less), potential lenders that would otherwise not find it worthwhile to establish a connection now start lending. This increases both the total number of connections and the number of links that are associated with pure intermediation. The third panel describes the behavior of the effective interest and reference interest rates. These rates are computed as

$$r^{eff} = \frac{\sum_{i=1}^{N} r_i \sum_{j\neq i} \ell_{ji}}{\sum_{i=1}^{N} \sum_{j\neq i} \ell_{ji}}$$

$$r^{ref} = \frac{\sum_{i=1}^{N} r_i \mathbb{1}[r_i \text{ is between the 23 and 77 percentiles}]}{\sum_{i=1}^{N} \mathbb{1}[r_i \text{ is between the 23 and 77 percentiles}]}$$

That is, the effective rate can be seen as the average rate at which trades take place, and the reference rate is computed exactly like the LIBOR: by taking the trimmed mean of the cost of funds for each bank at the 23% and 77% percentiles. While the effective rate increases, as we would expect, the reference rate actually decreases - this is due to the fact that, as banks lend lower amounts, their cost of funds actually decreases, and this effect tends to dominate for banks with costs of funding that are around the median. The last panel of figure 7 computes expected welfare. Expected welfare is constant for most of the ϕ space, suggesting that either the leverage constraint is not binding on this region, or if it is, banks can easily substitute between sources of funding, and this does not translate into greater probabilities of default. . We know that the explanation is the second one, as declining total volumes reflect binding leverage constraints. The first panel of figure 8 helps us understand the behavior of expected welfare, by plotting expected losses due to default. The decline in welfare for high values of ϕ reflects the increase in expected losses due to default: banks that need liquidity face greater costs of funding as $\phi \uparrow$, thereby increasing their probability of default, and the expected costs associated with defaults.

Figure 8: Changes in ϕ



The second panel of figure 8 plots the minimum, average and maximum probabilities of default. Once again, they are relatively constant for most of the ϕ space, increasing sharply as banks become unable to satisfy their funding needs in the interbank market. This pushes them to the outside market, thereby increasing costs of funding and the likelihood of default. The regulator is effectively pushing the interbank market to autarky by raising ϕ. The third panel is the risk-sharing measure discussed in the previous section, and is generally declining due to the fact that the market is being forcefully dissolved by the regulator as ϕ increases. Thus, by construction, this parameter must decrease as constraints bind and banks stop trading. The risk-exposure measure displays an interesting nonlinear behavior, as it is low for low values of ϕ, increases and then falls. The fall is mechanical, as banks reduce their exposure to counterparties. The increase, however, is not obvious, and is mainly related to increasing interest rates, that generates greater exposures in terms of value.

Alternatively, the regulator can also change ω_ℓ, the risk-weight on interbank lending. The analysis is very similar, and the results are plotted in figures 9 and 10 for reference.

Figure 9: Changes in ω_ℓ

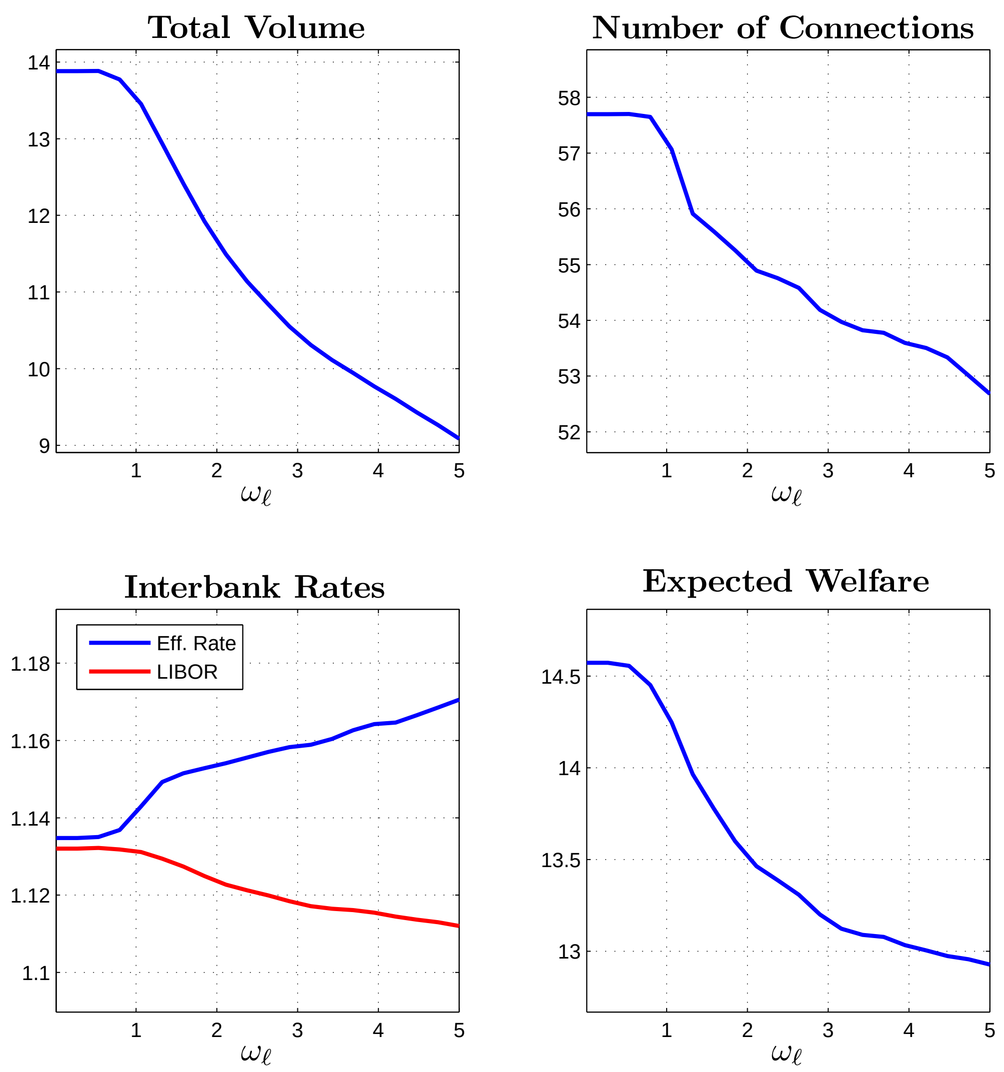
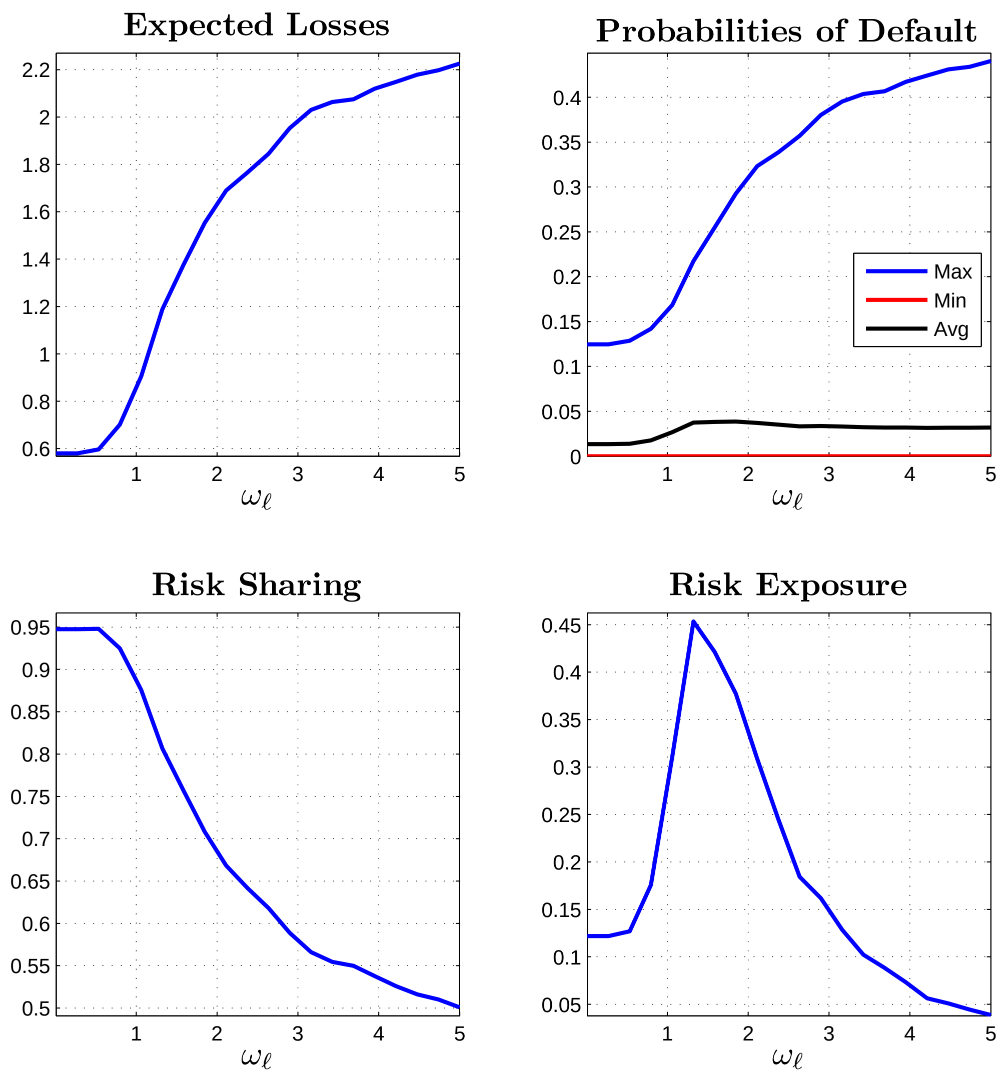


Figure 10: Changes in ω_ℓ



3.3.2 Reserve Requirements

As discussed previously, reserve requirements can act as an indirect leverage constraint: by forcing banks to hold additional risk-free reserves, the regulator is effectively distorting the bank's portfolio allocation problem, biasing it towards safe assets. The use of this measure as a constraint on risk-taking can lead to some problems, however: by forcing banks to hold additional cash reserves, the regulator is imposing a greater penalty on banks that are already liquidity constrained, further raising their cost of funding. This can be directly seen from equation (12), the first-order condition for cash reserves. The cost of funding for bank i is equal to one plus the Lagrange multiplier on the reserve requirements constraint. The tighter this constraint binds, the greater the cost of funding.

This effect is closely related to the historical role of reserve requirements as a tool for interest rate setting by Central Banks. Results are shown in figures 11 and 12.

Figure 11: Changes in τ

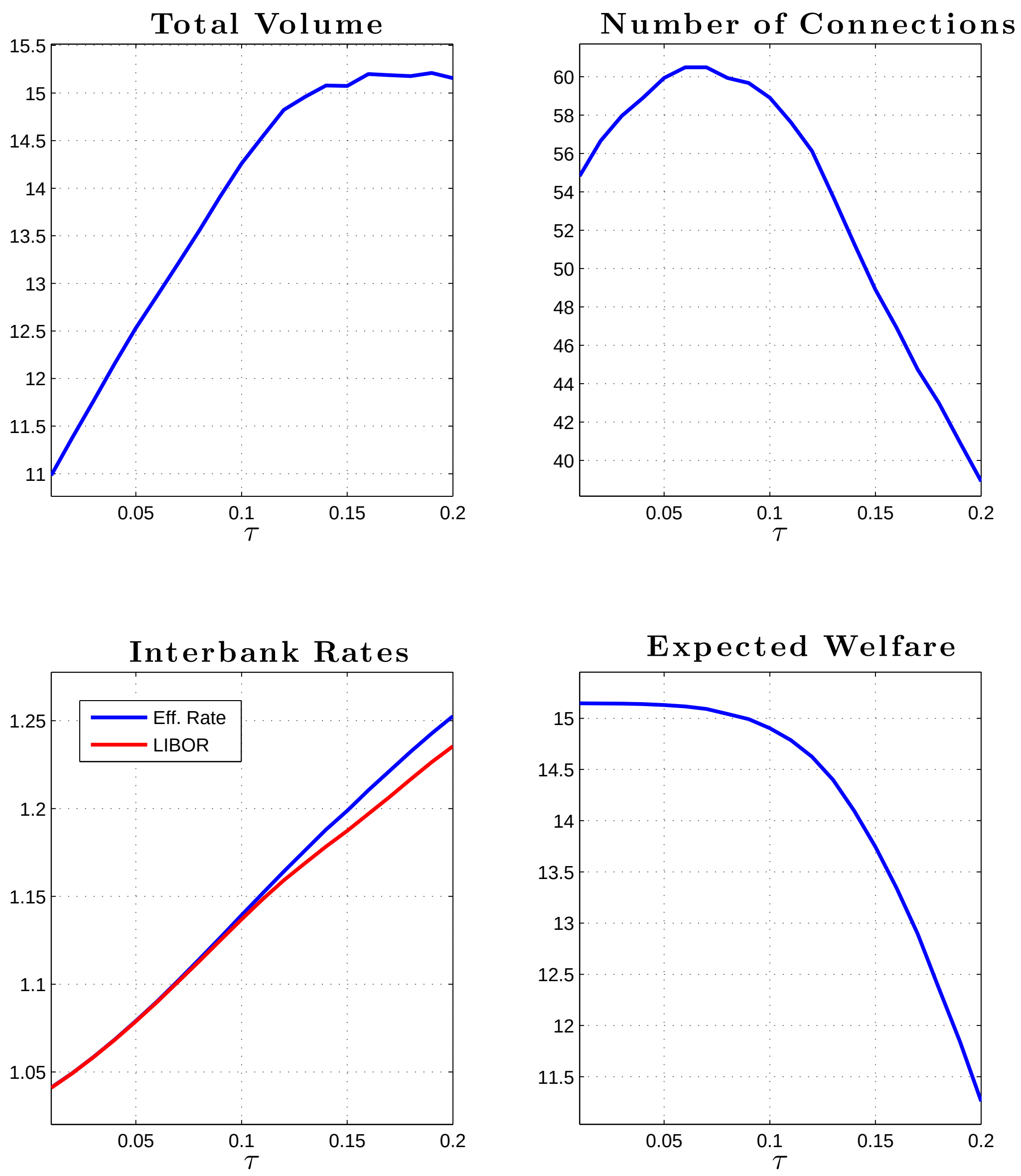
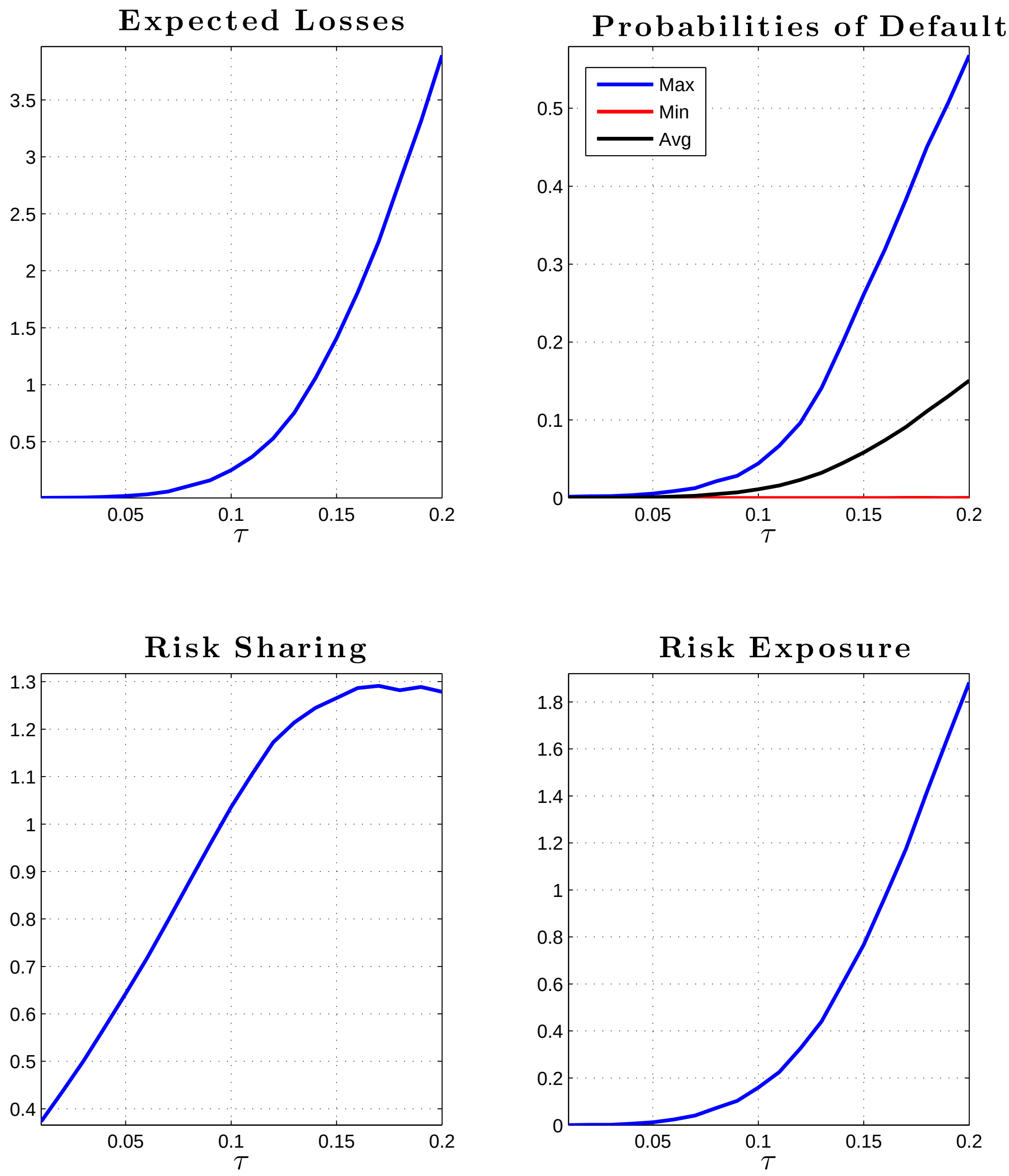


Figure 12: Changes in τ



Surprisingly, an increase in reserve requirements leads to increased lending volume. This is mainly due to rising interest rates: banks with negative liquidity shocks have their cost of funding disproportionately affected (positively) by rising reserve requirements. This creates incentives for other banks to lend. Also, as it can be seen in the final panels of figure 12, the measure of risk exposure also rises, further creating incentives to lend. On the other hand, risk-sharing also increases since bank activity in autarky would be affected by increased reserve requirements as well, and the existence of the market allows them to better share the risks. Risk-shifting does appear to dominate over risk-sharing, and leads to decreased welfare through increasing expected losses from default. Due to risk-shifting, lenders tend to concentrate their exposures in correlated counterparties, thereby increasing the contribution of the network structure for systemic risk.

3.3.3 Lending and Liquidity Facilities

I now assume that the Central Bank can provide liquidity in unlimited amounts at a fixed interest rate $\bar{r}$. By changing $\bar{r}$, which effectively becomes the outside option for banks, the Central Bank is able to directly affect banks' cost of funding. The indirect effects of this policy are, however, distinct from reserve requirements in the sense that changes in $\bar{r}$ do not have, in principle, a disproportionate impact on banks that are already liquidity-constrained. On the contrary, this policy will disproportionately benefit these banks.

To model the introduction of central lending facilities, I assume that banks have access to an additional source of funding $b_i \geq 0$ that requires a repayment $\bar{r}b_i$ at $t = 1$. Note that the discount window interest rate $\bar{r}$ will now bound above the cost of funding for each bank. I assume that the Central Bank becomes a junior creditor, *pari passu* with other banks and outside investors. Central Bank borrowing enters the capital adequacy ratio as a liability:

$$\frac{a_i + \sum_{j\neq i} \ell_{ij} + c_i + v_i - B_i - d_i' - b_i}{\omega_a a_i + \omega_\ell \sum_{j\neq i} \ell_{ij}} \geq \phi$$

and enters the flow of funds constraint as an inflow

$$\sum_{j\neq i} \ell_{ij} + c_i + v_i = B_i + s_i + b_i$$

It is then easy to see that the first-order condition implied by this (riskless) source of funding is given by

$$f_i' \leq \bar{r}$$

thus implying that we will have $r_i \leq \bar{r}$ at all times. In order to generate a meaningful trade-off, I assume that Central Bank lending entails costs equal to $\Psi = \bar{r}\sum_{i=1}^N b_i$.

I assume that $\bar{r} > 1$, as otherwise the Central Bank could subsidize a free arbitrage between lending facilities and reserves. While this seems to be an obvious technical assumption, it merits some discussion. Acharya and Steffen (2013) discuss the role of what they call "the greatest carry trade ever" in the unfolding of the European banking and sovereign debt crisis. The authors argue that cheap provision of liquidity by the European Central Bank induced Eurozone banks to engage in a carry trade with sovereign bonds of the peripheral GIPSI (Greece, Ireland, Portugal, Spain and Italy) [14]. While the Central Bank could choose to allow implicit recapitalizations in my model, I choose not to pursue this analysis for technical reasons, as well as the static nature of the model.

[14] Commentators suggest that the ECB has maintained the incentives for the carry trade in place with its long-term refinancing operations program (among others) as an attempt to implicitly recapitalize the Eurozone banking system.

This policy has several advantages with respect to the previously considered interventions: first, as mentioned, it mechanically benefits more banks that are more liquidity-distressed, by bounding above their cost of funding. Second, by bounding above the interest rates of the natural borrowers, it can help prevent excessive risk-shifting. Note that the main cause of risk-shifting were the extremely high interest rates offered by banks with negative liquidity shocks. By controlling prices, the Central Bank discourages natural lenders from taking excessive risks. Note that in the presence of lending fees and counterparty risk, the introduction of such facilities leads to a collapse of the interbank market for $\bar{r}$ low enough. To see this, suppose that $\bar{r}$ is low enough such that it is equal to the cost of funding for all banks. Then, our previous analysis implies that $\ell_{ij} > 0$ is only possible if bank i is not leverage constraint, marginal lending fees are zero and $\mathbb{E}[\theta_j|\pi_i \geq 0] = 1$, bank j fully repays in all states of the world in which bank i does not default. If any of these conditions fail, bank i demands a strictly positive differential between r_j and r_i, which is impossible since all banks face the same cost of funding. In this environment, the network becomes a *star*, with the Central Bank in the middle and all banks connecting to the Central Bank only. The market is effectively dismantled, and risk-sharing benefits are lost: this is obvious from the fact that banks with liquidity surpluses will no longer lend, and will simply invest in risk-free assets (reserves and outside investment).

The obvious way to implement lending facilities without causing the interbank market to collapse is then to set $\bar{r}$ high enough, so that a significant fraction of banks still have incentives to access the outside market. In this case, only banks with very large liquidity deficits will access the lending facilities. This works, effectively, as a discount window, and sets a ceiling on the cost of capital in the market. Banks with not so large liquidity shocks still find it optimal to operate in the interbank market, and are able to enjoy the risk-sharing benefits provided by this market. The results of changing $\bar{r}$ are shown in figures 13 and 14.

Figure 13: Changes in $\bar{r}$

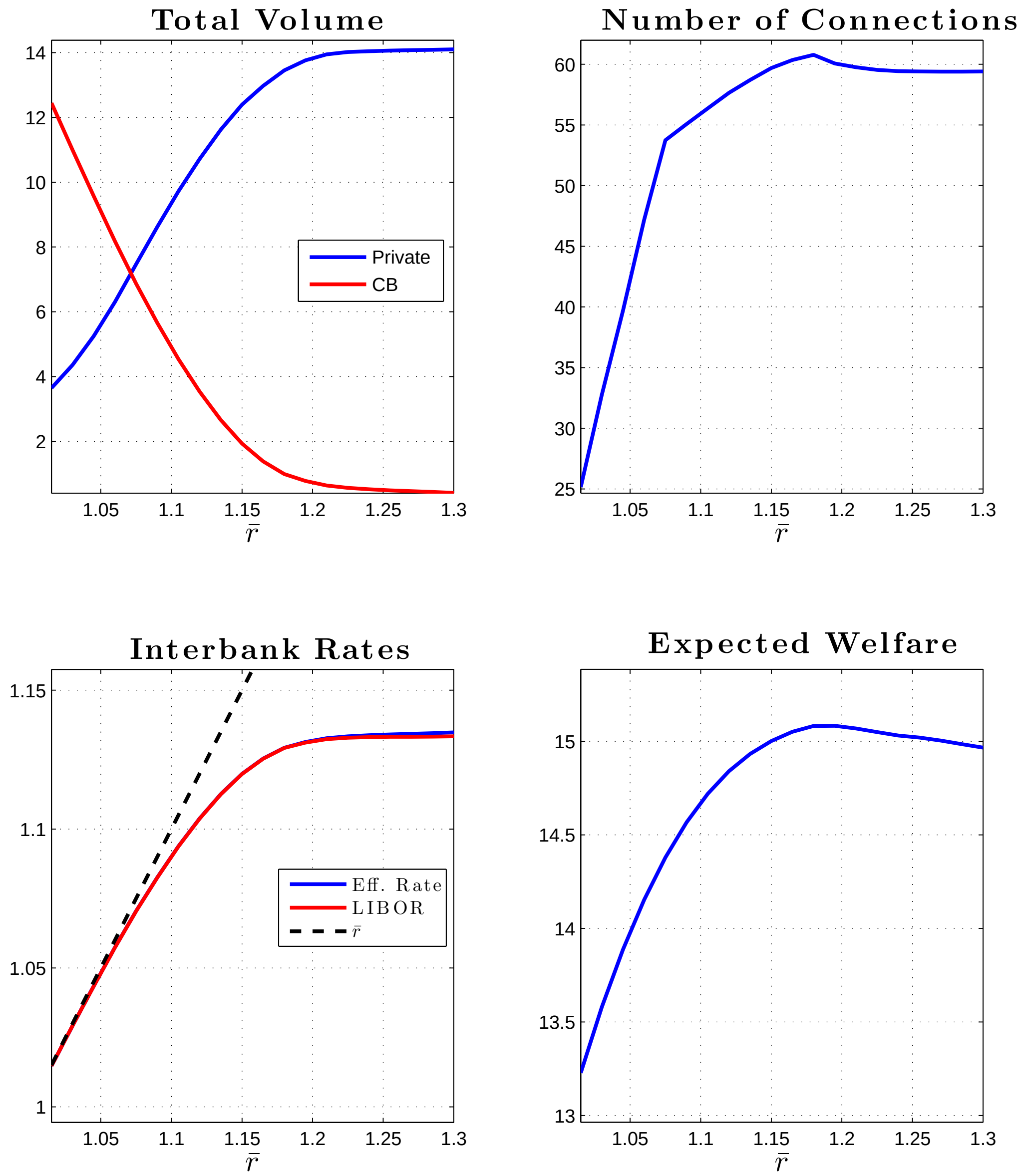
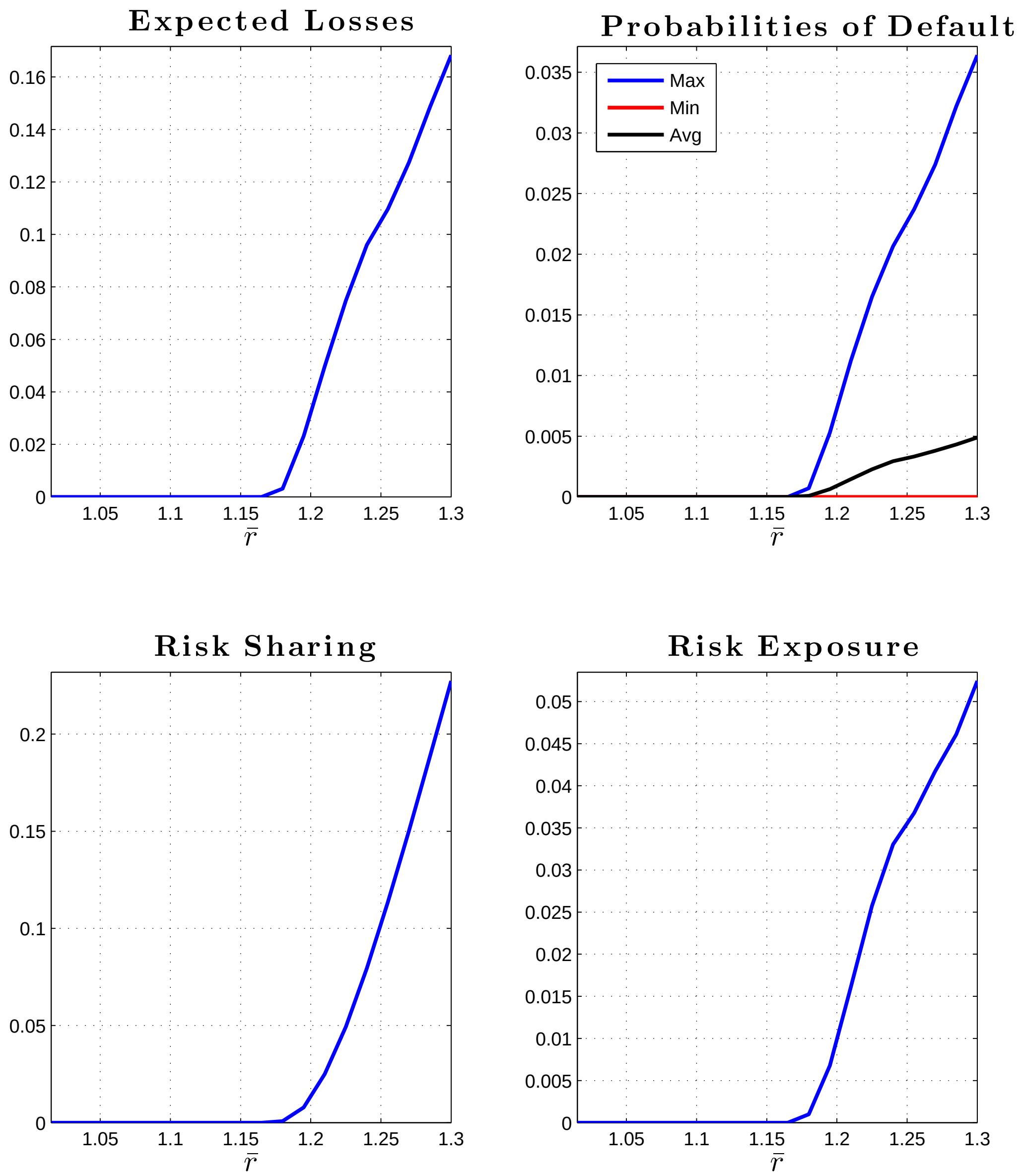


Figure 14: Changes in $\bar{r}$



The first panel of figure 13 is not surprising: for low values of the discount window rate, the private market is crowded out by central bank lending. This is because the discount rate imposes a ceiling on private costs of funding, and private lenders may not be willing to lend at such low rates. So banks that require liquidity can find it at the Central Bank. As $\bar{r}$ increases, the private market "resumes operations", and total volumes as well as the total number of connections increase. The lower usage of the discount window is also reflected in the third panel of figure 13: for low values of $\bar{r}$, the behavior of private rates is completely dominated by the discount rate. As it increases, however, both effective and reference rates depart from the 45-degree line: as borrowing from the Central Bank becomes more expensive, banks start resorting to other sources of funding, thus stabilizing interest rates. Finally, expected welfare is nonlinear due to the choice of $\gamma = 0.01$: depending on the cost parameter, welfare can be either strictly increasing or strictly decreasing. In particular, for $\gamma = 0$, low discount rates have an extremely positive impact on welfare: by crowding out the private market and making borrowers finance themselves at the unlimited liability central bank only, this central institution is absorbing all risk in the economy. Since the risk does not propagate (due to the Central Bank never failing), this policy is extremely welfare-enhancing.

This positive welfare impact can be seen in figure 14. A low discount window rate has two very positive effects: first, it bounds above the cost of funding for banks, thus reducing their default probabilities mechanically. Second, by completely crowding out the market, and since the Central Bank never fails, it is completely absorbing all counterparty risk in the economy, and shutting off any potential for contagion. This is evident in the fourth panel: the risk exposure measure is basically zero for very low values of $\bar{r}$, and then increasing with this parameter.

Without the fiscal cost of funds, lending facilities are welfare-improving in my model. This is because they contribute to simultaneously correct the main two sources of externalities: limited liability (that induces counterparty risk) and the network structure (that amplifies this risk by allowing for contagion). This large benefits of having a lender of last resort are reminiscent of Bagehot (1873)'s analysis, who urges the Bank of England to use the discount rate in a liberal manner, that is for the reserve bank to "lend freely, boldly". It should be noted that due to the focus of my model in these two features (limited liability and the networked structure of the market), this policy comes at very little cost. I do not explicitly model the benefits of having an efficiently functioning private interbank market. It then comes at little surprise that the Central Bank may wish to overtake the private sector in providing liquidity for deficit banks.

3.3.4 Credit Guarantees and Bailouts

The second type of active policy intervention that I consider are debt repayment guarantees issued by the regulator/planner. In this type of intervention, the regulator becomes liable for compensating any repayment shortfall to the insolvent bank's creditors. I also assume that the regulator is forced to pay any default costs that would otherwise arise (that is, default costs are still computed as the shortfall between the contractual junior liabilities and total residual value of the bank after senior creditors - depositors - have been repaid). Contrary to lending facilities, which can be seen as an *ex-ante* $t = 0$ intervention, this is an *ex-post* intervention that is only triggered in case of insolvency. For that reason, this can be seen as a crude way to introduce and model financial sector bailouts in this environment, and allows to discuss issues such as "too-big-to-fail", while accounting for the behavioral response of agents to the introduction of this policy.

The fact that this is an *ex-post* intervention raises issues of commitment: it is well known and studied in the literature that the commitment to bail out agents reinforces the risk-taking incentive put in place by limited liability (in fact, limited liability is not even necessary for this effect, and serves only as an amplification mechanism). Jeanne and Korinek (2013), for example, discuss this issue at length, by studying the trade-offs between macroprudential regulation and *ex-post* interventions (bailouts), and how these different policies distort the incentives of private agents. Since the design of optimal policy is beyond the scope of this paper, I assume away any time consistency issues, and impose that the regulator commits to bailing out creditors in case of bank insolvency. Other relevant work in this topic has been developed by Farhi and Tirole (2012).

To see that a policy trade-off still arises, it should be taken into account that while the regulator furthers the risk-taking incentives, it is also containing contagion and the propagation of default shocks in the interbank network. I model two types of credit guarantees: *expected* and *unexpected*. Expected credit guarantees function exactly as described above: the regulator commits to bailing out in case of default, and so banks simply set $\mathbb{E}[\theta_j | \pi_i \geq 0] = 1, \forall i \neq j$ when allocating their portfolios. I then compute the fiscal costs of the bailout as the shortfall that ensues in case the bank defaults. Unexpected credit guarantees, on the other hand, involve agents optimizing as in the decentralized equilibrium with no intervention, and the regulator unexpectedly bailing out in case of default.

I present the results in table 6. This table compares some summary measures for three cases: no intervention, expected bailouts and unexpected bailouts. To discipline the results, I maintain $\gamma = 0.01$ as in the analysis of the lender of last resort.

Table 6: Bailouts: Expected and Unexpected

Variable	*No Bailout*	*Unexpected Bailout*	*Expected Bailout*
Volume	14.27	14.27	14.50
# Connections	58.54	58.54	62.46
r^{eff}	14.01%	14.01%	13.92%
r^{ref}	13.59%	13.59%	13.91%
Welfare	14.86	14.57	14.88
Spending	0	0.55	0.26
$\mathbb{E}$[defaults]	0.012	0.004	0.002

This table reports simulation results for $N = 20$ with varying sizes and lending costs, comparing: a) Decentralized equilibrium with no intervention; b) ex-post, unanticipated bailout; c) ex-post, anticipated bailout. The first row reports average traded volume across simulations; the second row reports the average number of connections (lending links); the third row is the effective interest rate in the interbank market, computed as the weighted average of the interest rates at which transactions are undertaken; the fourth row is the reference interest rate, computed as the average cost of funding after excluding the 23% and 77% percentiles; the fifth row is total welfare; the sixth row is total spending with the bailout; the seventh row is the expected number of defaults for each policy.

First, all $t = 0$ measures are identical for the decentralized equilibrium and the unexpected bailout, by construction. The expected bailout is computed by setting $\mathbb{E}[\theta_j|\pi_i \geq 0] = 1, \forall i \neq j$. Since banks perceive less risk when expecting a bailout in case of failure, there is more lending volume and more connections are established. There is more activity at both the intensive and extensive margins. Also, effective (traded) interest rates are lower, reflecting less perceived risk, and tend to be more equalized across banks: this can be understood from a higher reference interest rate, since this measure tends to decrease as interest rate dispersion increases.

Lower perceived risk translates in lower interest rates, which seems to greatly contain the propagation of risk in the economy. This can be seen by the fact that an expected bailout reduces the number of expected defaults more than an unexpected bailout does. Also, an expected bailout involves less public spending: this may seem counter-intuitive at first, as it seems to counter the usual collective moral hazard logic. The reason behind this counterintuitive result is that by decreasing the perceived amount of risk in the economy, the expectation of a bailout makes banks lend at lower (and more equalized rates). This decreases the incentives to risk-shift and engage in risk-based intermediation, thereby decreasing the amount of endogenous risk that is created in the economy. Therefore, the expectation of the bailout, by itself, makes the bailout "unnecessary". This logic goes in the opposite direction of most of the literature in this topic that, through collective moral hazard arguments, establishes that the expectation of a bailout tends to make the bailout occur more often than it would occur if it was completely unexpected. I can therefore establish that, for reasonable parameter values, there is no time consistency problem in the model: the expectation of the bailout makes the bailout unnecessary, thereby increasing welfare.

3.4 Discussion

The previous analyses suggest that well-intended policies aimed at curbing systemic risk may have undesirable consequences from a social point of view. Constraining leverage and interbank lending directly leads agents to lend less, but to more different counterparties, thereby exposing themselves to different sources of risk. Since the network becomes more connected, shocks propagate more easily from one bank to another. Increases in reserve requirements and benchmark rates have a disporportionate impact on banks that are liquidity constrained: this leads to their costs of funding to increase substantially. Not only this increases their probabilities of default, but also creates incentives for other banks to expose themselves to these defaults. Risk is magnified, especially because some banks that would otherwise not lend to these liquidity constrained banks now voluntarily expose themselves to this risk. If counterparty risk is substantial, risk-shifting incentives may arise, further promoting exposure.

These results allow us to draw two main conclusions: first, accounting for the behavioral response of agents to changes in policy is crucial, as the potential magnification of risk due to policies aimed at curbing risk is neither mechanical nor obvious. Second, current regulatory policies appear to be relatively ineffective at controlling systemic risk in a systems context. The results suggest that more sensible policies would involve limiting the number of counterparties each bank can interact with, or applying differential risk-weights to lending to different counterparties. The first measure is discussed at length (and advocated) in Farboodi (2014). The second measure is partly in line with the regulatory measures to be implemented in the new Basel III framework. The new protocol for standardized risk-weights applies differentially for exposures to foreign banks, with the risk-weight depending on the credit rating of the respective sovereign. However, within the U.S., for example, exposures to financial institutions that are (conditionally) guaranteed by the FDIC or the National Credit Union Administration still receive a uniform risk-weight equal to 20%.

Finally, active (interventionary) policies seem to have a much more welfare-friendly impact, and are far superior when it comes to mitigating risk. Lending facilities reduce risk through two channels: a direct channel, by imposing a ceiling on banks' cost of funds, and an indirect channel, by making the Central Bank become exposed to counterparty risk (in lieu of private parties) and absorbing the risk that would otherwise propagate. For my calibration, bailouts also mitigate risk, by lowering interest rates and containing risk-shifting incentives. In particular, and contrary to most of the literature, I find that there is no time consistency problem with bailouts: by making banks expect full repayment on their lending, the expectation of the bailout reduces interest rates and, therefore, risk-taking and risk-shifting incentives. Thus the expectation of the bailout, by itself, makes the bailout unnecessary [15].

[15] A qualification is in order: my results on bailout optimality should be seen as an *extremely optimistic* view on fiscal interventions on the banking sector. In particular, it is easy to change the parameter γ so that bailouts no longer become optimal. The result that an expected bailout dominates an unexpected one is, however, robust to changes in the parameters.

4 Conclusion

In this paper, I developed a model of endogenous formation of a network of mutual exposures between financial institutions. The interaction between the network structure, limited liability and different liquidity endowments leads to heterogeneous attitudes towards risk: while there are unambiguous gains from trade that allow banks to fund themselves at lower costs, perverse risk-shifting incentives may arise that lead to the endogenous build-up of risk in the economy. Since banks face limited liability, they only value repayments in states of the world in which they do not default. This creates an incentive for banks to correlate their investments. Due to the networked structure of the interbank market, this translates in greater propagation of shocks and exacerbates the potential for inefficient defaults.

The abundance of externalities creates room for policy. I focus on three policy measures: capital adequacy ratios, reserve requirements and benchmark interest rates. These measures constrain banks' activity and trades, but have the potential to constrain the endogenous formation of risk by doing so. I find that the benefits from trade tend to outweigh the costs of constraining agents' activities: by limiting leverage and interbank lending, for example, incentives to diversify may arise, and making the network more connected may actually lead to increased exposure to counterparty risk. On the other hand, forcing banks to invest in risk-free reserves hurts liquidity-constrained banks in a disproportionate manner. This leads to an overall rise in interest rates that creates further incentives to lend and engage in risk-shifting behavior. Active policies, such as the provision of liquidity at fixed rates by the Central Bank, seem to be much more effective at improving welfare and containing the endogenous formation of risk. In my model, banking sector bailouts do not involve a time consistency problem for reasonable parameter values: the expectation of a bailout reduces perceived risk in the economy, thereby reducing risk-taking incentives and overall formation of risk.

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A Appendix

A.1 Proofs

Proof of Proposition 2.1. I start by noting that the operator $\mathbf{\Phi}$ defined in (8) is bounded above by $\mathbf{Q} = (Q_1, \ldots, Q_N)^T$. I show that the operator is monotone: if $\mathbf{x}_0 \leq \mathbf{x}_1$, then $\mathbf{\Phi}(\mathbf{x}_0) \leq \mathbf{\Phi}(\mathbf{x}_1)$. To see this, note first that if $[\mathbf{\Phi}(\mathbf{x}_0)]_i = Q_i$, then we know that

$$\begin{aligned} Q_i &\leq e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_0]_j - d'_i \\ &\leq e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_1]_j - d'_i \end{aligned}$$

given our assumption that $\mathbf{x}_0 \leq \mathbf{x}_1$. Then, $[\mathbf{\Phi}(\mathbf{x}_1)]_i = Q_i$, and the claim is true. Now, assume that

$$[\mathbf{\Phi}(\mathbf{x}_0)]_i = \max \left\{ (1-\delta) e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_0]_j - d'_i, 0 \right\}$$

This implies that $Q_i > e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_0]_j - d'_i$ (as otherwise the repayment would be Q_i). Now, we know that

$$e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_1]_j - d'_i \geq e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_0]_j - d'_i$$

Meaning that either bank i does not default under $\mathbf{x}_1$, in which case $[\mathbf{\Phi}(\mathbf{x}_1)]_i = Q_i$ and the claim is true, or we have that

$$\begin{aligned} [\mathbf{\Phi}(\mathbf{x}_1)]_i &= \max \left\{ (1-\delta) e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_1]_j - d'_i, 0 \right\} \\ &\geq \max \left\{ (1-\delta) e_i + \sum_{j \neq i} \Pi_{ji} [\mathbf{x}_0]_j - d'_i, 0 \right\} \\ &= [\mathbf{\Phi}(\mathbf{x}_0)]_i \end{aligned}$$

so the claim is also true in this case. This establishes monotonicity of the operator. This allows us then to apply Tarski's Fixed Point Theorem, implying that the set of fixed points of $\mathbf{\Phi}$, call it $\mathbf{x}^*$, is a complete lattice: nonempty, with a greatest and least element. This establishes existence and uniqueness of the greatest equilibrium repayment vector. □

Proof of Proposition 2.2. Let π_i denote revenues net of costs (recall that profits are $\pi_i^+ = \max\{0, \pi_i\}$), we can rewrite the objective function as

$$\mathbb{E}_{\mathbf{R}}[\pi_i^+] = \int_{\mathbf{R}:\pi_i(\mathbf{R})<0} 0 \mathrm{d}G(\mathbf{R}) + \int_{\mathbf{R}:\pi_i(\mathbf{R})\geq 0} \pi_i(\mathbf{R}) \mathrm{d}G(\mathbf{R})$$

That is, due to limited liability, the bank only cares about profits in states of the world where it does not default. To make this more tractable, let $R^a_{-i} = \{R^a_j\}_{j\neq i}$ denote a joint realization of long-term asset returns for all banks other than i, and define

$$\bar{R}^a_i(R^a_{-i}) = \frac{r_i B_i + d'_i + \sum_{j\neq i} \kappa_{ij}(\ell_{ij}) - f_i(v_i) - c_i - \sum_{j\neq i} \theta_j(R^a_{-i}) r_j \ell_{ij}}{a_i} \tag{16}$$

as the lowest realization of the long-term asset payoff that bank i can experience without defaulting, given portfolio choices and a joint realization of long-term asset payoffs for all other banks. By construction, $\pi_i[\bar{R}^a_i(R^a_{-i}), R^a_{-i}] = 0$. This allows us to write the objective function as

$$\mathbb{E}_{\mathbf{R}}[\pi_i^+] = \int_{R^a_{-i}} \int_{\bar{R}^a_i(R^a_{-i})}^{\infty} \pi_i(\mathbf{R}) \mathrm{d}G(\mathbf{R})$$

where I abuse notation by letting $\int_{R^a_{-i}}$ be the sequence of multiple integrals over all possible realizations of other banks' long-term asset returns. Note that $\bar{R}^a_i(R^a_{-i})$ can potentially be ill-defined, since it depends on $\boldsymbol{\theta}(\mathbf{R})$, which could in turn depend on R^a_i if it is low enough for bank i to default. The crucial observation is that $\bar{R}^a_i(R^a_{-i})$ is only defined for states of the world in which bank i does not default, and hence θ_i is implicitly assumed to be equal to 1 over all these states.

Since v_i is allowed to be positive or negative, we can eliminate it from the problem by plugging the flow of funds constraint in the objective function and the leverage constraint, and thereby eliminating one constraint. The rewritten problem becomes

$$\begin{aligned}
&\max_{c_i,\{\ell_{ij}\}_{j\neq i},B_i} \int_{R^a_{-i}} \int_{\bar{R}^a_i(R^a_{-i})}^{\infty} \left[R^a_i a_i + c_i + f_i\left(B_i + s_i - \sum_{j\neq i} \ell_{ij} - c_i\right) + \sum_{j\neq i} \theta_j(\mathbf{R}) r_j \ell_{ij} \right. \\
&\left. -r_i B_i - d'_i - \sum_{j\neq i} \kappa_{ij}(\ell_{ij}) \right] \mathrm{d}G(\mathbf{R}) \\
&\text{subject to} \\
&a_i - d_i \geq \phi\left(\omega_a a_i + \omega_\ell \sum_{j\neq i} \ell_{ij}\right) \\
&c_i \geq \tau d'_i
\end{aligned} \tag{17}$$

where I have used the fact that $s_i = d'_i - d_i$. Note that the objective function is potentially convex due to limited liability (see, for example, Gollier et al. (1993)), emphasizing the technical role of the leverage constraint to ensure that the problem is well-defined.

Let μ_i denote the Lagrange multiplier in the capital adequacy ratio, and λ_i be the multiplier associated with the reserve requirement. For notational ease, I define

$$P_i \equiv \Pr(\pi_i \geq 0) = \int_{R^a_{-i}} \int_{\bar{R}^a_i(R^a_{-i})}^{\infty} \mathrm{d}G(\mathbf{R})$$

as one minus the probability of default for bank i. The first-order conditions with respect to each of the controls can be written as

$$\begin{aligned}
(B_i) &: P_i(f_i' - r_i) \leq 0 \\
(c_i) &: P_i(1 - f_i' + \lambda_i) \leq 0 \\
(\ell_{ij}) &: \int_{R^a_{-i}} \int_{\bar{R}^a_i(R^a_{-i})}^{\infty} \theta_j(\mathbf{R}) r_j \mathrm{d}G(\mathbf{R}) - P_i[f_i' + \phi\omega_\ell\mu_i + \kappa_{ij}'(\ell_{ij})] \leq 0, \forall j \neq i
\end{aligned}$$

where $f_i' = f_i'(v_i)$. The first-order conditions acquire relatively simple forms due to the fact that the perceived impact of the bank's own actions on $\bar{R}^a_i(R^a_{-i})$ is of second order, and so drops out. Since all controls are restricted to be non-negative, all first-order conditions are potential inequalities (with the exception of c_i as long as $\tau d_i' > 0$). $\square$

A.2 Extensions

In this section, I present some possible extensions of the model.

A.2.1 Market Power

Under some assumptions, the model can be extended to allow for imperfect competition in the interbank market. This is done by assuming that bilateral interest rates are the outcome of a Generalized Nash Bargaining process, in which the two parties commit to dividing the (marginal) surplus generated by the lending contract. A tractable model of imperfect competition can be obtained by assuming that the lender has full bargaining power in the bilateral bargaining process. The resulting environment is one in which lenders compete *à la* Cournot in each of the segmented markets, and borrowers take prices as given (due to their lack of bargaining power), but account for their monopsony power when allocating their lending portfolios. The structure that emerges is reminiscent of the environment developed by Nava (2013).

I now assume that r_{ij} is the outcome of a bilateral Nash bargaining process, where β stands for the bargaining weight of the lender, and $1-\beta$ is the bargaining weight of the borrower. For simplicity, let x_i denote the vector of strategies (optimal portfolio allocations) of bank i [16], and let $x_i + \Delta_{ij}$, with some abuse of notation, denote the same vector

[16] Formally, $x_i = [\ell_{i1}, \ldots, \ell_{iN}, B_i, c_i, v_i]^T$.

of strategies, but where ℓ_{ij} is replaced by $\ell_{ij} + \Delta_{ij}$, where $\Delta_{ij} > 0$ is a potentially small increment. The interest rate can then be defined as

$$r_{ij} = \arg\max \left\{\mathbb{E}[\pi_i(x_i + \Delta_{ij})^+] - \mathbb{E}[\pi_i(x_i)^+]\right\}^{\beta} \left\{\mathbb{E}[\pi_j(x_j + \Delta_{ij})^+] - \mathbb{E}[\pi_j(x_j)^+]\right\}^{1-\beta}$$

Note that banks are not maximizing the expected joint surplus, but rather the joint expected surpluses of the contract. This is crucial to obtain a tractable a solution and is consistent with self-interested behavior. Letting $\mathcal{S}_i \equiv \mathbb{E}[\pi_i(x_i + \Delta_{ij})^+] - \mathbb{E}[\pi_i(x_i)^+]$ denote the surplus of bank i (in this case, the lender), it is easy to see that the first-order condition of the Nash program is given by

$$\beta \mathcal{S}_j \frac{\partial \mathcal{S}_i}{\partial r_{ij}} = -(1-\beta)\mathcal{S}_i \frac{\partial \mathcal{S}_j}{\partial r_{ij}}$$

This expression is, in principle, very complicated to analyze. I make two assumptions:

1. Bargaining takes place over the division of surplus for a marginal unit of lending, $\Delta_{ij} \to 0$
2. The lender has full bargaining power, $\beta \to 1$

Given the second assumption, the first-order condition becomes

$$\mathcal{S}_j \frac{\partial \mathcal{S}_i}{\partial r_{ij}} = 0$$

We can show that

$$\lim_{\Delta_{ij}\to 0} \frac{\mathcal{S}_j}{\Delta_{ij}} = \int_{R_{-j}} \int_{\bar{R}_j(x_j)}^{\infty} [f_j'(x_j) - r_{ij}] \mathrm{d}G(\mathbf{R})$$

and, after some algebra, it is also possible to show that

$$\lim_{\Delta_{ij}\to 0} \frac{1}{\Delta_{ij}} \frac{\partial \mathcal{S}_i}{\partial r_{ij}} = \int_{R_{-i}} \int_{\bar{R}_i(x_i)}^{\infty} [\theta_j g(\mathbf{R}) \mathrm{d}R_i^a + (r_{ij}\theta_j - \mu_i \phi \omega_\ell - f_i'(x_i) - \kappa_{ij}'(x_i)) g(\bar{R}_i(x_i), R_{-i}) \frac{\theta_j \ell_{ij}}{a_i} \mathrm{d}R_{-i}$$

This is the sum of two terms: the first is weakly positive, since $\theta_j \in [0,1]$ for any realization, and the second must be strictly positive, as otherwise bank i would not lend to bank j in the first place. This allows us to cancel the derivative from the expression, leaving us with the following first-order condition for the bargaining problem

$$r_{ij} = f_j'$$

The main difference from the baseline model is that, now, banks will account for the impact of their actions on interest rates. To avoid further complications, I still assume that banks take repayments $\boldsymbol{\theta}(\mathbf{R})$ as given. We can reformulate the banks' problem as...

[TO BE COMPLETED]

A.2.2 Endogenous Asset Choice and Fire-Sales

Assume now that banks are allowed to rebalance their long-term asset positions at $t = 0$, when deciding their portfolio allocation problem. I assume, as in the main text, that banks start with a_i long-term assets in their balance sheet, and are allowed to sell or purchase these long-term assets at price p, retaining a final position equal to a_i'. For simplicity, I assume that all assets are *ex-ante* identical and fungible: they can all be traded in a centralized exchange at a single price p. One can think of this long-term asset as being an imperfectly diversified market portfolio, that may yield different returns at $t = 1$.

[TO BE COMPLETED]