## Bank Networks: Contagion, Systemic Risk & Prudential Policy

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**Endogenous Financial Networks and Equilibrium Dynamics**

Banque de France
Motivation

- Aftermath of crisis $\implies$ channels of contagion
  - credit interconnections
  - overlapping portfolios / fire sales
  - liquidity hoarding
  - rollover risk / information coordination

- Interaction between channels
- Role of networks

Most importantly

- Quest for an “appropriate” regulatory framework
- Trade off: efficiency (maximize banks’ investment in non-liquid risky assets) and financial stability (minimize systemic risk).
This paper

- Optimizing heterogeneous risk-averse banks, s.t. regulatory requirements
- Two endogenous price mechanisms
  - Interbank market (pre-shock)
  - Non-liquid assets market (post-shock)
- Interbank matching to replicate stylized facts about real world interbank networks
- Study effects of changes in prudential policy (evaluate trade-off)
Channels of financial contagion (risk transmission):

1. *Credit interlinkages (network externalities)*
2. *Fire sale of common non-liquid assets (pecuniary externalities)*
3. *Liquidity hoarding*

Systemic risk is due to the spreading of defaults through these channels.
The connections of bank $i$

\[
c_i + n_i p + \sum_{j=1}^{N} l_{ij} = d_i + \sum_{j=1}^{N} b_{ij} + e_i \quad (BSI)
\]
Shock to non-liquid assets $\Rightarrow \downarrow p$

Exogenous drop of the price of nla. Shock on all the banks holding nla

Banks 1 and 3 violate capital requirement. They start selling nla

1

2

3

4
Self-reinforcing downward pressure on price of nla

Further (endogenous) drop of the price of nla.

Non liquid assets

Reserves

Deposits

Banks i violates capital requirement. She starts selling nla

1

2

3

4

\begin{align*}
\text{Reserves} & \quad c(i) \\
\text{Deposits} & \quad d(i) \\
\text{Non liquid assets} & \quad p_n(i)
\end{align*}
Collapse in mkt value of banks’ assets might lead to default

Further (endogenous) drop of the price of nla.

Non liquid assets

Reserves

Deposits

Banks i violates capital requirement. She starts selling nla

Banks 1 and 3 default. Bank i experience further losses

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Preview of main results

**Simulation exercise**

- *Systemic risk* (S.R.) not closely related to bal. sheet items (exc. TA)
- S.R. and *systemic importance* (S.I.) indicators do not deliver consistent message

**Policy analysis**

- Increasing the liquidity requirement reduces *systemic risk*, but with significant cost in terms of *efficiency*
- Increasing the equity requirement also reduces *systemic risk* (particularly over an initial range) without a considerable reduction in *efficiency*, but significantly reduces *interbank market activity*
A bird’s eye view & related literature

Banks optimize

ib tâtonnement

Optimal quantities obtained \( (c_i, n_i, l_i, b_i) \)

Matching

Financial System complete

Shock hits

Compute network metrics & systemic importance

Compute systemic risk

ib transmission

Fire sales of nla

nla tâtonnement

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\[\text{ib tâtonnement} \]

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Compute systemic risk

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Compute systemic risk

Important: banks choose overall exposure to network (Glasserman & Young ‘14)

Alves et al. ‘13
Langfield & Soramäki ‘14

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Important: banks choose overall exposure to network (Glasserman & Young ‘14)
The problem of the bank

- Banks’ preferences: CRRA utility of profits ⇒ $U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma}$
- Choose $c_i, n_i, l_i, b_i$ to maximize expected utility of profits (second order Taylor approximation around $E[\pi_i]$):

$$E[U(\pi)] \approx \frac{E[\pi]^{1-\sigma}}{1-\sigma} - \frac{\sigma}{2} E[\pi]^{(1+\sigma)} \left( n_i^2 \sigma_r^2 - (b_i r_i^l)^2 \xi^2 (1 - \xi E[pd_i])^{-4} \sigma_{pd_i}^2 \right)$$

- Subject to (BSI), liquidity and equity requirements (+ n.n.c.)

$$c_i \geq \alpha d_i$$  \hspace{1cm} (LR)

$$\frac{c_i + n_i p + l_i - d_i - b_i}{\omega n p n_i + \omega l l_i} \geq \gamma + \tau$$  \hspace{1cm} (ER)
Tâtonnement on the interbank market

- Why? Demand and supply will not be mutually consistent after initial optimization (given starting value of $r^l$)
- Auctioneer evaluates total demand ($B$) and supply ($L$) of ib loans
- If $B > L$ ($B < L$) $\implies$ $\uparrow r^l$ ($\downarrow r^l$)
  - Let banks optimize again given the new $r^l$
  - continue until equilibrium is achieved
- We obtain two vectors $l = [l_1, l_2, ..., l_N]$ and $b = [b_1, b_2, ..., b_N]$ that are mutually consistent, such that $B = L$
- But ...who is lending to whom and who is borrowing from whom? (i.e. how does the matrix of ib exposures look like?)
Matching and the formation of the network

To answer this we use a Closest Matching (or minimum distance) Algorithm (CMA)
- Associates closest demand and supply
- Order the vectors $\mathbf{l}$ and $\mathbf{b}$ in descending order
- Assign transaction $x_{ij} = \min\{l_i, b_j\}$

The algorithm determines the topology of the network.

By construction, CMA yields very low density
Life after a shock: \( nla \) mkt tâtonnement

- Pre-shock, \( p = 1 \)
- Post-shock, supply and price of \( nla \) are affected
- Banks sell \( nla \) to fulfill ER
- \( s_i'(p) < 0 \implies s_n'(p) < 0 \)
- CFS inverse demand
  \[ p = \exp(-\beta d_n) \]
- Equilibrium \( s_n = d_n \)
  \[ \Theta(p) = \exp(-\beta s(p)) \]

**Source:** Cifuentes et al. (2005)
Systemic importance and systemic risk

- *Ex ante* measures of vulnerability: network centrality measures (degree (in, out), closeness, betweenness, eigenvector)
- *Ex post* measure: ratio of the value of assets of defaulting banks (grouped in the set $\Omega$) to total assets:

\[
\Phi = \frac{\sum_{\Omega} \text{assets}_\Omega}{\sum_i \text{asset}_i}
\]

- Contribution of each bank to systemic risk → *Shapley value*:

\[
\Xi_i(\Psi) = \frac{1}{N!} \sum_{P \in \pi_N} (\psi(\text{Pre}_i(P) \cup i) - \psi(\text{Pre}_i(P)))
\]

- In practice approximated by taking $k$ randomly sampled permutations:

\[
\hat{\Xi}_i(\Psi) = \frac{1}{k} \sum_{P \in \pi_k} (\psi(\text{Pre}_i(P) \cup i) - \psi(\text{Pre}_i(P)))
\]
## Baseline Calibration

<table>
<thead>
<tr>
<th>Par./Var.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of banks in the system</td>
<td>20</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Liquidity requirement ratio</td>
<td>0.10</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Risk weight on non-liquid assets</td>
<td>1</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>Risk weight on interbank lending</td>
<td>0.20</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Equity requirement ratio</td>
<td>0.08</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Desired equity buffer</td>
<td>0.01</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Bank deposits</td>
<td>Top20 EA</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Bank equity</td>
<td>Top20 EA</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Bank risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Loss given default</td>
<td>0.5</td>
</tr>
<tr>
<td>$E[pd]$</td>
<td>Expected default probability</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma^2_{pd}$</td>
<td>Variance of default probability</td>
<td>0.003</td>
</tr>
<tr>
<td>$r^n_i$</td>
<td>Return on non-liquid assets</td>
<td>$U(0, 0.15)$</td>
</tr>
<tr>
<td>$\sigma^2_{r^n_i}$</td>
<td>Variance of $r^n_i$</td>
<td>$\frac{1}{12}(\max(r^n_i) - \min(r^n_i))^2$</td>
</tr>
<tr>
<td>$\Psi_i$</td>
<td>Shocks to non-liquid assets</td>
<td>$\mathcal{N}(5, 25 \times 1)$</td>
</tr>
</tbody>
</table>
## Network metrics

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (%)</td>
<td>7.37</td>
</tr>
<tr>
<td>Average Degree</td>
<td>1.40</td>
</tr>
<tr>
<td>Average Path Length</td>
<td>2.60</td>
</tr>
<tr>
<td>Betweenness Centrality (Av.)</td>
<td>7.10</td>
</tr>
<tr>
<td>Eigenvector Centrality (Av.)</td>
<td>0.13</td>
</tr>
<tr>
<td>Clustering Coefficient (Av.)</td>
<td>0.03</td>
</tr>
<tr>
<td>Assortativity</td>
<td></td>
</tr>
<tr>
<td>out-in degree</td>
<td>-0.15</td>
</tr>
<tr>
<td>in-out degree</td>
<td>0.26</td>
</tr>
<tr>
<td>out-out degree</td>
<td>-0.31</td>
</tr>
<tr>
<td>in-in degree</td>
<td>-0.44</td>
</tr>
<tr>
<td># Intermediaries</td>
<td>9</td>
</tr>
<tr>
<td># Core Banks</td>
<td>3</td>
</tr>
<tr>
<td>Interbank Assets/Total Assets (%)</td>
<td>23.68</td>
</tr>
<tr>
<td>Equilibrium Interbank Rate (%)</td>
<td>2.98</td>
</tr>
</tbody>
</table>

**Table 1**: Network characteristics - Baseline setting
Figure 1: Contribution to systemic risk (mean SV) by bank
Shapley value vs. bank characteristics
Shapley value vs. bank characteristics

Strong relationship?
- A bit with total assets
- Not much with the rest ...
Shapley value vs. centrality measures
Shapley value vs. centrality measures

Consistent ranking?
- Not at all...

Bottom line
- Systemic risk and systemic importance indicators deliver different messages
Changes in the liquidity requirement ($\alpha$)

**Systemic Risk**

**Ib lending/Assets**

**NLA/Equity**

**Ib Interest Rate**

**Leverage**

**Density**
Changes in the equity requirement ($\gamma$)

- Systemic Risk
- Ib lending/Assets
- NLA/Equity

**Systemic Risk**

**Ib Interest Rate**

**Leverage**

**Density**

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Model Comparison

- **Model 1**: benchmark model (risk-averse banks, fire sales & network externalities)
- **Model 2**: risk neutral banks (linear objective function, i.e. \( \sigma = 0 \))
- **Model 3**: *Model 1* without fire sale channel (\( n_i \) no longer choice variable but calibrated to *Model 1* values, only network externalities)
- **Model 4**: *Model 3* without risk aversion (\( \sigma = 0 \))
Model Comparison - Results

Systemic Risk

Changes in LR ($\alpha$)

- M1: Benchmark
- M2: Risk neutral
- M3: M1-fire sales
- M4: M3+c =0

Changes in ER ($\gamma$)

- M1: Benchmark
- M2: Risk neutral
- M3: M1-fire sales
- M4: M3+c =0

Ib lending/Assets

NLA/Equity

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Current extension

- Incorporate risk coming from the liability side
- News about a given banks health
  \[\Rightarrow\] might lead to deposit withdrawal if above a given threshold
  \[\Rightarrow\] information based runs: the sudden withdrawals of liquidity might force the bank, hit by the news shock, to fire-sale/default
- Interaction with other transmission channels
- Incorporation into banks’ optimization problem
- Focus on liquidity regulation
THANK YOU!

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Appendix

Obtaining the objective function

- Banks profit = return on ib lending (A) - cost of ib borrowing (B) + return on nla investment (C)

- (A) By lending $l_{ij}$ to $j$, $i$ expects to get

  $$\underbrace{(1 - pd_j) \left( r^l + r^p_j \right)}_{\text{with no default}} l_{ij} + \underbrace{pd_j \left( r^l + r^p_j \right) (1 - \zeta) l_{ij}}_{\text{with default}}$$

- Without the possibility of default, $i$ gets the risk-free rate

  $$l_{ij} r^l$$

- Equate 2 and 3 to solve for the fair risk premium charged to counterparty $j$:

  $$r^p_j = \frac{\zeta pd_j}{1 - \zeta pd_j} r^l$$

Back to bank problem.
Plugging the premium back into 2 and summing over all counterparties yields the expected return on interbank lending

$$l_i r^l$$

(B) As a borrower bank, $i$ must pay the premium associated to its own default probability; then cost of borrowing is:

$$r^b_i b_i = (r^l + r^p_i) b_i = \frac{1}{1 - \xi p d_i} r^l b_i$$

(C) Gains from nla investment: $r^n_i n_i$

Back to bank problem.
Obtaining the objective function (cont.)

- Bank profits given by
  \[
  \pi_i = r_i^n \frac{n_i}{p} + r^l l_i - \frac{1}{1 - \xi pd_i} r^l b_i
  \]  
  (7)

- Bank’s preferences → CRRA utility function:
  \[
  U(\pi_i) = \frac{(\pi_i)^{1-\sigma}}{1-\sigma}
  \]  
  (8)

- Second order Taylor approximation of 8 in the neighborhood of the expected value of profits \(E[\pi_i]\) yields Eq. 1, with the variance of profits given by
  \[
  \sigma^2_{\pi} = \text{Var} \left( r_i^n n_i + r^l l_i - \frac{1}{1 - \xi pd_i} r^l b_i \right) = 
  \left( \frac{n_i}{p} \right)^2 \sigma^2_{r_i^n} - (b_i r^l)^2 \text{Var} \left( \frac{1}{1 - \xi pd_i} \right) + 2n_i r^l b_i \text{cov} \left( r_i^n, \frac{1}{1 - \xi pd_i} \right)
  \]