The formation of a core periphery structure in heterogeneous financial networks

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joint with
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seo economic research

Motivation

Since 2008, important progress has been made on the role of the **network of interbank exposures**, i.e. **the financial network**, on **financial stability**.

- Theoretical and simulation analysis of financial contagion (Allen & Gale, 2000; Gai & Kapadia, 2010; Elliott et al., 2014; Acemoglu et al., 2015).
- Stress tests on empirically derived network structures (Upper, 2011).

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- Stress tests on empirically derived network structures (Upper, 2011).

In these analyses it is assumed that the **financial network is exogenously fixed**. However, trading partners are consciously chosen by profit-maximizing banks. Hence the network structure can **change over time**, depending on the global financial circumstances.

Motivation

It is important to obtain a better understanding on the factors driving the **formation of financial networks**, in particular

- What are the incentives (costs and benefits) for banks to create or delete links?
- How does the financial network structure change if incentives or market circumstances change?

Answering these questions will help us better understand:

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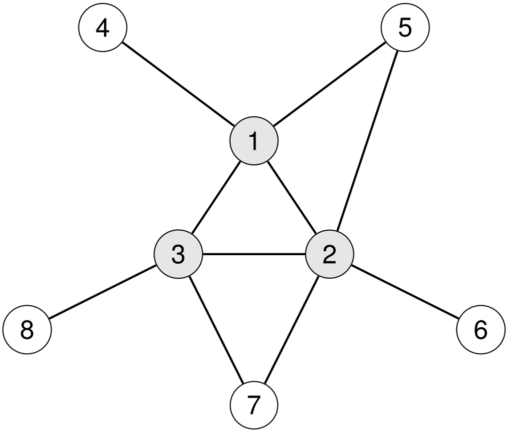
Answering these questions will help us better understand:

- What is the risk of systemic liquidity hoarding?
- How is the structure of the financial network affected by financial regulation?
- Taking into account the effect of the network structure, what would optimal financial regulation look like?

Introduction

In this paper, we perform a **theoretical network formation analysis** of an interbank market in order to understand the formation of a **core-periphery network structure in financial networks**.

Example of a core-periphery network



Introduction

It turns out that many financial networks have a core-periphery network structure.

- Germany: Craig & von Peter (JFI, 2015),
- Netherlands: in 't Veld & van Lelyveld (JBF, 2014),
- Italy (eMID interbank market): Fricke & Lux (CE, 2015).

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- Italy (eMID interbank market): Fricke & Lux (CE, 2015).
- After the crisis the fit became less.

We try to understand why this is the case.

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We consider a model with the following elements.

- Trade opportunities can be realized directly or indirectly through an intermediator (broker).
- Imperfect competition between intermediators.
  - The more intermediators, the lower the intermediation benefits.
- Free entry of intermediators.

Main Results

Our trade intermediation model is intuitively simple and contains effectively just 3 parameters.

In this model we show that:

- If agents are **homogeneous**, then a core-periphery network is **not** an equilibrium outcome.
- A core-periphery network structure is an equilibrium outcome, if the banks in the core become bigger, that is, have **more valuable trade opportunities** than banks in the periphery.

Related Literature

Our model is related to network formation theory in general, see the textbooks of Jackson (2008, *Social and Economic Networks*) and Goyal (2009, *Connections*);
and more specifically to e.g. Goyal & Vega-Redondo (JET, 2007) and Farboodi (2014).

Trade network model

Consider n agents (banks). Two stages:

1. At $t = 0$, agents form an **undirected network**, g, of long-term trading relationships. $g_{ij} = g_{ji} = 1$ denotes a link (trading relationship), and $g_{ij} = g_{ji} = 0$ the absence of a link.
2. There are infinite trading periods, $t = 1, 2, \ldots$. In each trading period each bank i has a small **probability of a negative liquidity shock** $\rho\alpha_i$ ($\rho \to 0$), and a similar small probability of a positive liquidity shock.

   If i has a liquidity shortage and j a liquidity surplus, then i and j have an **opportunity to trade liquidity** (i borrows from j), which would generate a **total surplus of 1** to i and j (e.g. the wedge between deposit and lending rate of central bank) in that particular period t.

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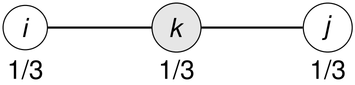
- if *i* and *j* are indirectly connected by m_{ij} **intermediators**, then the shares of the surplus for *i*, *j* and the intermediators depend on the **level of competition** δ
    - *i* and *j* each receive $f_e(m_{ij}, \delta)$
    - each of the m_{ij} intermediators receive $f_m(m_{ij}, \delta)$
    - by definition, shares add up: $2f_e(m, \delta) + mf_m(m, \delta) = 1$

Trade and distribution

Consider intermediated trade for some pair (i,j).
We assume

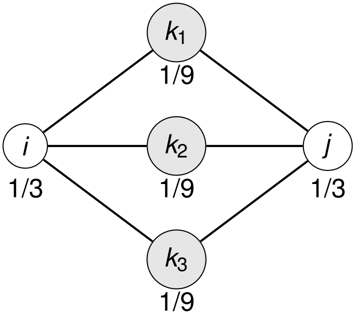
- if there is one intermediator, then the three parties split the surplus evenly.

$$f_e(1,\delta) = f_m(1,\delta) = 1/3.$$



Trade and distribution

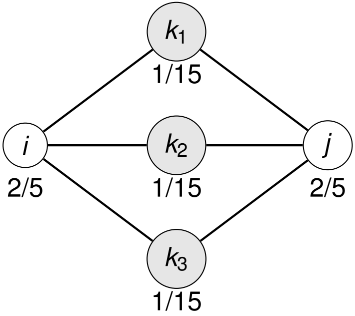
Trade between i and j: $m = 3$, $\delta = 0$



Special case of full collusion between intermediators.

Trade and distribution

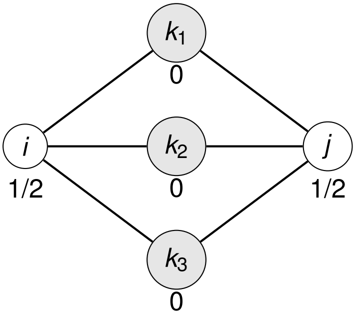
Trade between i and j: $m = 3$, $\delta = 3/5$



Moderate level of competition.

Trade and distribution

Trade between i and j: $m = 3$, $\delta = 1$



Special case of perfect competition.

Payoff function

Agents participate in a financial trading network g. We assume that

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- Benefits from participating in network g is the net present value from the individual trading benefits (with discount factor $1 - \rho^2$).
- Trading relationships are mutual and maintaining such a relationship involves a cost c for both partners.

Payoff function

Hence, the payoff function, $\pi_i(g)$, for an agent i is:

$$\pi_i(g) = \underbrace{\sum_{j \in N_i^1} (\tfrac{1}{2}\alpha_{ij} - c)}_{\text{direct trade}} + \underbrace{\sum_{j \in N_i^2} \alpha_{ij} f_e(m_{ij}, \delta)}_{\text{indirect trade}} + \underbrace{\sum_{k,l \in N_i^1 | g_{kl}=0} \alpha_{kl} f_m(m_{kl}, \delta)}_{\text{brokerage benefits}}$$

where N_i^r denotes the set of nodes at distance $r \geq 1$ from i in g and $n_i^r = |N_i^r|$ its size.

Interpretation in interbank market

- Links are **long-term trade relationships** (Cocco et al., 2009; Fecht & Braüning, 2013; Afonso et al., 2013). No relationship means no (direct) trade.

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- The cost involved in maintaining a relationship might be combination of search costs and monotoring costs.
- Intermediation benefits result from a bargaining process.
  - See Siedlarek (2011) for an explicit process, such that

$$f_e(m,\delta) = \frac{m-\delta}{m(3-\delta)-2\delta} \text{ and } f_m(m,\delta) = \frac{1-\delta}{m(3-\delta)-2\delta}$$

Unilateral stability

Network formation theory analyses networks in **equilibrium**.

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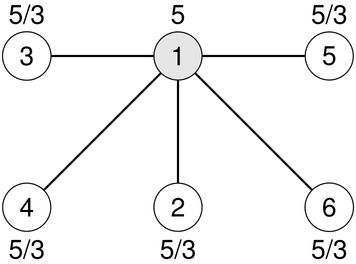
Unilateral stability allows for **entry of intermediators**

- a network is unilaterally unstable if an agent has an incentive to create many links to other agents to become a broker.

A star network

$\alpha_{ij} = 1 \ \forall i, j$, $\delta = 3/5$, $c = 1/6$.



Is a star network unilaterally stable?

A complete core-periphery network

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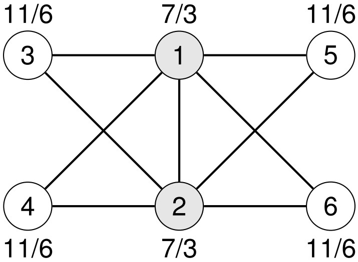


As $7/3 > 5/3$ and $11/6 > 5/3$, star is not unilaterally stable.

Core-Periphery network

Definition

A network g is a **core-periphery network** g^{CP} if there is a set of agents $K \subset N$ with $k = |K| : 2 \leq k \leq n-2$, such that

- the core agents K form a completely connected clique
  - $\forall i, j \in K, i \neq j : g_{ij} = 1$
- there are no links between periphery agents $N \backslash K$
  - $\forall i, j \in N \backslash K : g_{ij} = 0$
- each core agent is connected to at least one periphery agent and vice versa
  - $\forall i \in K \exists j \in N \backslash K : g_{ij} = 1$ and $\forall j \in N \backslash K \exists i \in K : g_{ij} = 1$

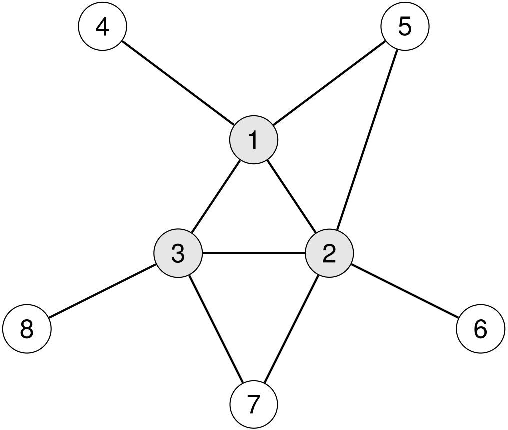
Complete core-periphery network

Definition

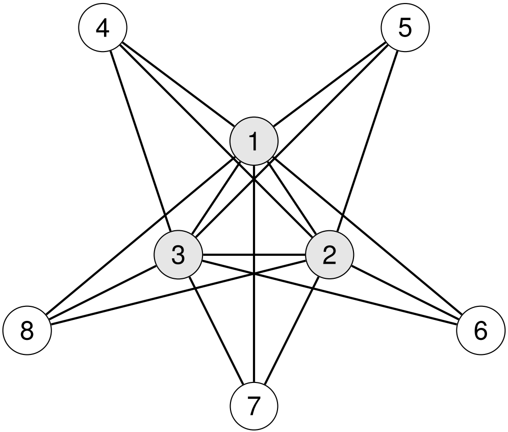
A core-periphery network g^{CP} is a **complete core-periphery network** if in addition

- every core agent is linked to all periphery agents
  - $\forall i \in K, j \in N \backslash K : g_{ij} = 1$

Example of a core-periphery network



Example of a complete core-periphery network



Core-periphery network with homogeneous agents

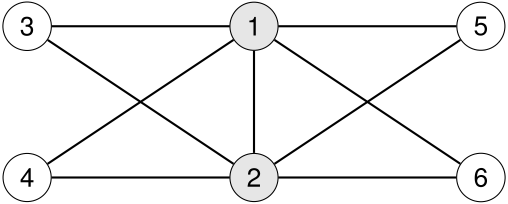
Consider first the case of **homogeneous agents**

- trade surplus is 1: $\alpha_{ij} = 1 \ \forall i, j$

Theorem (1.a)

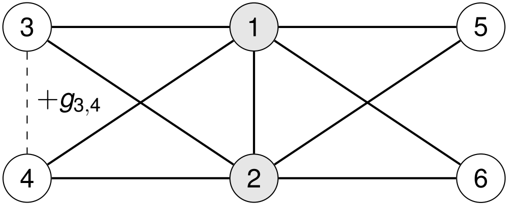
*For any c and δ, any **complete** core-periphery network of size n and core size k with $2 \leq k \leq n-2$ is **not** unilaterally stable.*

Intuition Proof



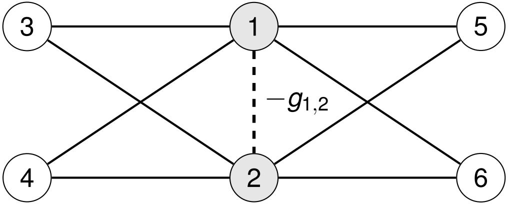
Suppose a complete core-periphery network is unilaterally stable.

Intuition Proof



It is better for agents 3 and 4 to trade indirectly through the 2 core agents.

Intuition Proof



But then it should be better for agents 1 and 2 to trade indirectly through 4 agents.

Incomplete core-periphery networks

Can incomplete core-periphery networks be unilaterally stable?

Incomplete core-periphery networks

Can incomplete core-periphery networks be unilaterally stable?
In special cases, yes. However, if n is large enough, then any core-periphery network becomes unilaterally unstable.

Theorem (1.b)

*For any c, δ and k, there exists a $\overline{n}$, such that any (complete or incomplete) core-periphery network of size $n > \overline{n}$ and core size k is **not** unilaterally stable.*

Intuition: **intermediation benefits** of core members **increase quadratically** with n, whereas **linking costs increase linearly**.
Hence, for large n intermediation benefits exceed linking costs, and peripheral banks have incentives to become a core bank.

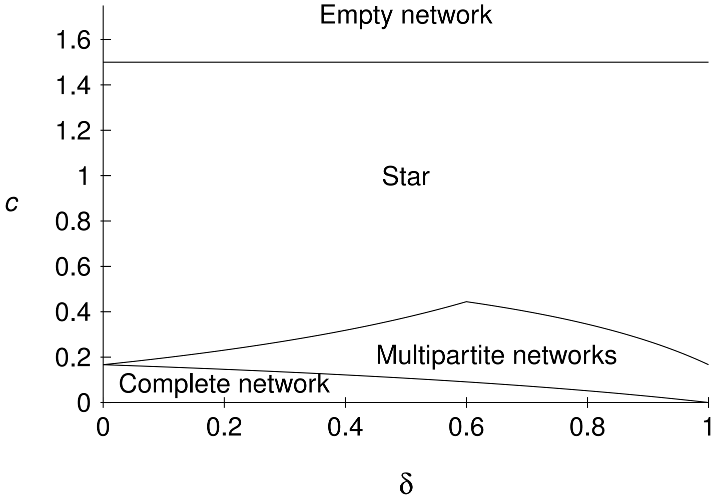
Stable networks

If the core-periphery network is not (unilaterally) stable, what kind of networks are?

We consider a dynamic process:

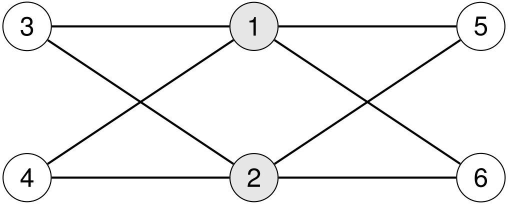
- Initial network is the empty network.
- Round-robin **best response** dynamics (Kleinberg et al., 2008).
- This process always converges for any c and δ, and the resulting network is unilaterally stable.
- There may be more unilateral stable networks to which the network does not converge.

Absorbing states dynamic process. $n = 8$.



Example of convergence to a multipartite network

$\delta = 3/5$, $c = 1/6$.



The dynamics lead to an core periphery network with $k = 2$. This network is unstable: 1 removes his link with 2. The result is a **stable bipartite network** with groups $\{1, 2\}$ and $\{3, 4, 5, 6\}$.

Extensions

Why are core-periphery networks unstable in the framework above? What additional assumptions are necessary to explain the existence of core-periphery networks?

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Why are core-periphery networks unstable in the framework above? What additional assumptions are necessary to explain the existence of core-periphery networks?

- Instability follows from the **homogeneity of banks**
  - If banks in the periphery want to trade indirectly through core banks, then banks in the core want to trade indirectly through periphery banks as well!

Heterogeneous agents

Consider now a case in which banks are heterogeneous

- Two types of banks: k big banks and $n-k$ small banks
- Big banks generate more trading opportunities α_{ij}:

α_{ij}	L	S
L	α^2	α
S	α	1

with some $\alpha > 1$.

Heterogeneous agents

Theorem (2)

For a sufficiently large level of heterogeneity $\alpha > 1$*, the complete core periphery networks with* k *big banks is* ***unilaterally stable*** *for* $c \in (\underline{c}, \overline{c})$*, where* $\underline{c}$ *and* $\overline{c}$ *depend on* n*,* k*,* α *and* δ*.*

Intuition:

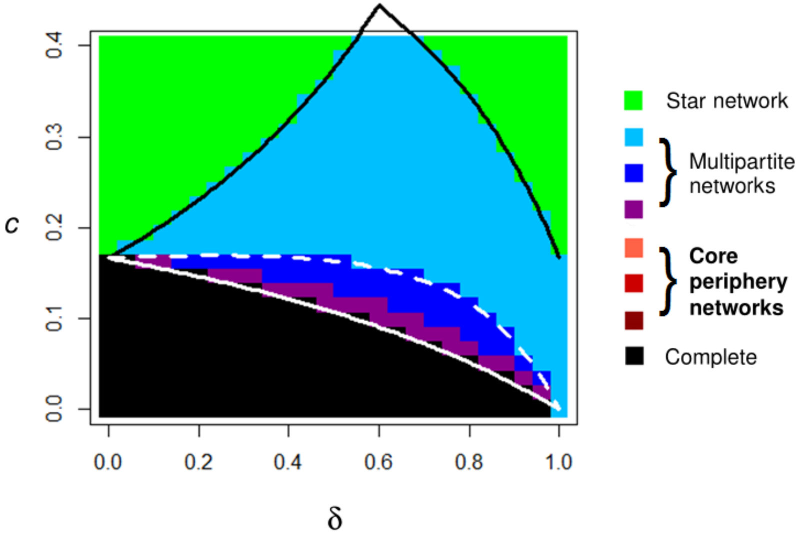
- if α is very large, then it is more attractive for core banks to trade directly with each other.

Dynamic process with growing size heterogeneity

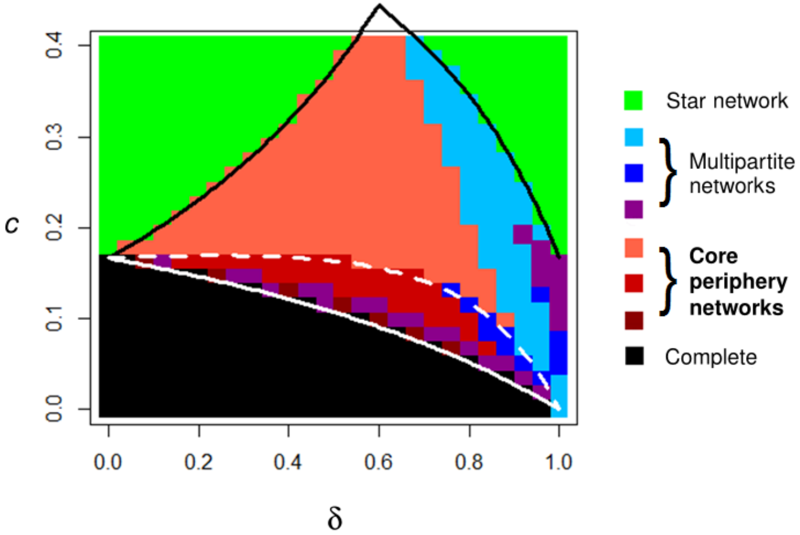
Do core-periphery networks also arise as part of a dynamic process?
We consider a similar dynamic process as before, but add that **profits feed back on bank size**:

- Initial empty network with homogeneous banks ($\alpha_{ij} = 1 \; \forall i, j$).
- Round-robin best response dynamics.
- After the process converges to a unilaterally stable network, with resulting profits π_i, bank size is updated as $\alpha_{ij} = \pi_i \pi_j / \min[\pi]^2$.
- The attained network may be unstable. If so, round-robin best response dynamics follow until a new stable network is attained.
- The process is stopped if the α_{ij}'s do not alter any more (given a tolerance level *tl*) or has altered more than *H* times.

Absorbing states dynamic process. $n = 8$, $tl = 0.01$, $H = 1$.



Absorbing states dynamic process. $n = 8$, $tl = 0.01$, $H = 25$.

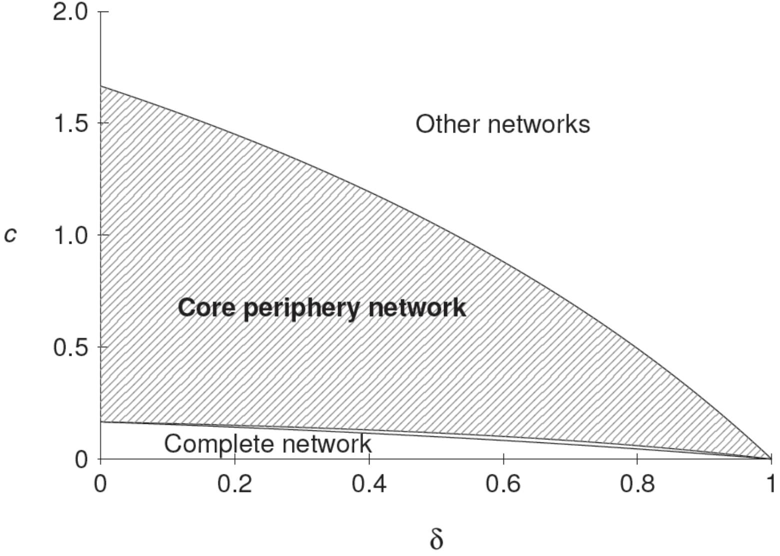


A calibration to the Dutch Interbank market

We calibrate n, k and α to the Dutch interbank market

- In 't Veld & van Lelyveld (2014) find in the interbank market with around $n = 100$ banks a core with around $k = 15$ banks.
- Large heterogeneity in asset size. We consider a fixed $\alpha = 10$.
- We look at the absorbing states of the dynamic process (without feedback of profits).

Absorbing states dynamic process. $n = 100$, $k = 15$, $\alpha = 10$.



Conclusions

In this paper we ask ourselves: why do financial networks have a core-periphery structure? We focus on the role of intermediation benefits.

We find that:

- A core-periphery network is not stable if agents are homogeneous
- If there is enough heterogeneity in trade surplus, then a core-periphery network with big banks in the core can be stable

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This result suggests that we cannot abstract away heterogeneity in **banking size** if we want to understand the effect of heterogeneity in **financial network structure** (e.g. Acemoglu et al. 2015).

Future research

In future research we would like to

- Introduce default probabilities in order to understand the role of network formation on systemic risk and financial stability.