The formation of a core periphery structure in heterogeneous financial networks

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joint with

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Banque de France, 10 July 2015
Motivation

Since 2008, important progress has been made on the role of the network of interbank exposures, i.e. the financial network, on financial stability.

- Theoretical and simulation analysis of financial contagion (Allen & Gale, 2000; Gai & Kapadia, 2010; Elliott et al., 2014; Acemoglu et al., 2015).
- Stress tests on empirically derived network structures (Upper, 2011).
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- Stress tests on empirically derived network structures (Upper, 2011).

In these analyses it is assumed that the financial network is exogenously fixed. However, trading partners are consciously chosen by profit-maximizing banks. Hence the network structure can change over time, depending on the global financial circumstances.
It is important to obtain a better understanding on the factors driving the formation of financial networks, in particular:

- What are the incentives (costs and benefits) for banks to create or delete links?
- How does the financial network structure change if incentives or market circumstances change?

Answering these questions will help us better understand:
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- What are the incentives (costs and benefits) for banks to create or delete links?
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Answering these questions will help us better understand:

- What is the risk of systemic liquidity hoarding?
- How is the structure of the financial network affected by financial regulation?
- Taking into account the effect of the network structure, what would optimal financial regulation look like?
Introduction

In this paper, we perform a **theoretical network formation analysis** of an interbank market in order to understand the formation of a **core-periphery network structure in financial networks**.
Example of a core-periphery network
Introduction

It turns out that many financial networks have a core-periphery network structure.

- Germany: Craig & von Peter (JFI, 2015),
- Netherlands: in ’t Veld & van Lelyveld (JBF, 2014),
- Italy (eMID interbank market): Fricke & Lux (CE, 2015).
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After the crisis the fit became less.

We try to understand why this is the case.
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We ask ourselves if the formation of this network structure can be explained by the benefits and costs of *intermediation/brokerage in trading networks*. We consider a model with the following elements.

- Trade opportunities can be realized directly or indirectly through an intermediator (broker).
- Imperfect competition between intermediators.
  - The more intermediators, the lower the intermediation benefits.
- Free entry of intermediators.
Main Results

Our trade intermediation model is intuitively simple and contains effectively just 3 parameters. In this model we show that:

- If agents are **homogeneous**, then a core-periphery network is **not** an equilibrium outcome.

- A core-periphery network structure is an equilibrium outcome, if the banks in the core become bigger, that is, have **more valuable trade opportunities** than banks in the periphery.
Related Literature

Our model is related to network formation theory in general, see the textbooks of Jackson (2008, *Social and Economic Networks*) and Goyal (2009, *Connections*); and more specifically to e.g. Goyal & Vega-Redondo (JET, 2007) and Farboodi (2014).
Trade network model

Consider $n$ agents (banks). Two stages:

1. At $t = 0$, agents form an **undirected network**, $g$, of long-term trading relationships. $g_{ij} = g_{ji} = 1$ denotes a link (trading relationship), and $g_{ij} = g_{ji} = 0$ the absence of a link.

2. There are infinite trading periods, $t = 1, 2, \ldots$. In each trading period each bank $i$ has a small **probability of a negative liquidity shock** $\rho \alpha_i (\rho \to 0)$, and a similar small probability of a positive liquidity shock. If $i$ has a liquidity shortage and $j$ a liquidity surplus, then $i$ and $j$ have an **opportunity to trade liquidity** ($i$ borrows from $j$), which would generate a **total surplus of 1** to $i$ and $j$ (e.g. the wedge between deposit and lending rate of central bank) in that particular period $t$. 
Trade and distribution

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- if $i$ and $j$ are indirectly connected by $m_{ij}$ **intermediators**, then the shares of the surplus for $i$, $j$ and the intermediators depend on the **level of competition** $\delta$
  - $i$ and $j$ each receive $f_e(m_{ij}, \delta)$
  - each of the $m_{ij}$ intermediators receive $f_m(m_{ij}, \delta)$
  - by definition, shares add up: $2f_e(m, \delta) + mf_m(m, \delta) = 1$
Trade and distribution

Consider intermediated trade for some pair \((i, j)\). We assume

- if there is one intermediator, then the three parties split the surplus evenly.

\[ f_e(1, \delta) = f_m(1, \delta) = \frac{1}{3}. \]
Trade and distribution

Trade between $i$ and $j$: $m = 3$, $\delta = 0$

Special case of full collusion between intermediators.
Trade and distribution

Trade between $i$ and $j$: $m = 3$, $\delta = 3/5$

Moderate level of competition.
Trade and distribution

Trade between $i$ and $j$: $m = 3$, $\delta = 1$

Special case of perfect competition.
Agents participate in a financial trading network $g$. We assume that

- Trade opportunities arise randomly between each pair of players $(i, j)$ with probability $\alpha_{i,j} = \rho^2 \alpha_i \alpha_j$. 
Payoff function

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- Benefits from participating in network $g$ is the net present value from the individual trading benefits (with discount factor $1 - \rho^2$).
- Trading relationships are mutual and maintaining such a relationship involves a cost $c$ for both partners.
Payoff function

Hence, the payoff function, $\pi_i(g)$, for an agent $i$ is:

$$\pi_i(g) = \sum_{j \in N^1_i} \left( \frac{1}{2} \alpha_{ij} - c \right) + \sum_{j \in N^2_i} \alpha_{ij} f_e(m_{ij}, \delta) + \sum_{k, l \in N^1_i \mid g_{kl} = 0} \alpha_{kl} f_m(m_{kl}, \delta)$$

where $N^r_i$ denotes the set of nodes at distance $r \geq 1$ from $i$ in $g$ and $n^r_i = |N^r_i|$ its size.
Interpretation in interbank market

- Links are **long-term trade relationships** (Cocco et al., 2009; Fecht & Braüning, 2013; Afonso et al., 2013). No relationship means no (direct) trade.
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- The cost involved in maintaining a relationship might be combination of search costs and monitoring costs.
- Intermediation benefits result from a bargaining process.
  - See Siedlarek (2011) for an explicit process, such that

\[
\begin{align*}
    f_e(m, \delta) &= \frac{m - \delta}{m(3 - \delta) - 2\delta} \\
    f_m(m, \delta) &= \frac{1 - \delta}{m(3 - \delta) - 2\delta}
\end{align*}
\]
Network formation theory analyses networks in equilibrium.

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- A network $g$ is unilaterally stable if there is no agent $i$, such that simultaneously
  - agent $i$ strictly benefits from deleting (some of) its existing links or proposing new links involving $i$
  - none of the agents to which $i$ proposes a new link is worse off
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Unilateral stability allows for entry of intermediators

- a network is unilaterally unstable if an agent has an incentive to create many links to other agents to become a broker.
A star network

\[ \alpha_{ij} = 1 \quad \forall i, j, \quad \delta = \frac{3}{5}, \quad c = \frac{1}{6}. \]

Is a star network unilaterally stable?
A complete core-periphery network

$\alpha_{ij} = 1 \forall i, j, \delta = 3/5, c = 1/6.$

As $7/3 > 5/3$ and $11/6 > 5/3$, star is not unilaterally stable.
Core-Periphery network

Definition
A network $g$ is a core-periphery network $g^{CP}$ if there is a set of agents $K \subset N$ with $k = |K| : 2 \leq k \leq n - 2$, such that

- the core agents $K$ form a completely connected clique
  - $\forall i, j \in K, i \neq j : g_{ij} = 1$
- there are no links between periphery agents $N \setminus K$
  - $\forall i, j \in N \setminus K : g_{ij} = 0$
- each core agent is connected to at least one periphery agent and vice versa
  - $\forall i \in K \exists j \in N \setminus K : g_{ij} = 1$ and $\forall j \in N \setminus K \exists i \in K : g_{ij} = 1$
Complete core-periphery network

Definition
A core-periphery network $g^{CP}$ is a **complete core-periphery network** if in addition

- every core agent is linked to all periphery agents
  - $\forall i \in K, j \in N \setminus K : g_{ij} = 1$
Example of a core-periphery network
Example of a complete core-periphery network
Core-periphery network with homogeneous agents

Consider first the case of homogeneous agents

- trade surplus is 1: $\alpha_{ij} = 1 \forall i, j$

**Theorem (1.a)**

*For any $c$ and $\delta$, any complete core-periphery network of size $n$ and core size $k$ with $2 \leq k \leq n - 2$ is not unilaterally stable.*
Suppose a complete core-periphery network is unilaterally stable.
It is better for agents 3 and 4 to trade indirectly through the 2 core agents.
Intuition Proof

But then it should be better for agents 1 and 2 to trade indirectly through 4 agents.
Incomplete core-periphery networks

Can incomplete core-periphery networks be unilaterally stable?
Incomplete core-periphery networks

Can incomplete core-periphery networks be unilaterally stable? In special cases, yes. However, if $n$ is large enough, then any core-periphery network becomes unilaterally unstable.

Theorem (1.b)

*For any $c, \delta$ and $k$, there exists a $\bar{n}$, such that any (complete or incomplete) core-periphery network of size $n > \bar{n}$ and core size $k$ is not unilaterally stable.*

Intuition: **intermediation benefits** of core members *increase quadratically* with $n$, whereas **linking costs increase linearly*. Hence, for large $n$ intermediation benefits exceed linking costs, and peripheral banks have incentives to become a core bank.
Stable networks

If the core-periphery network is not (unilaterally) stable, what kind of networks are?

We consider a dynamic process:

- Initial network is the empty network.
- Round-robin **best response** dynamics (Kleinberg et al., 2008).
- This process always converges for any $c$ and $\delta$, and the resulting network is unilaterally stable.
- There may be more unilateral stable networks to which the network does not converge.
Absorbing states dynamic process. $n = 8$. 

![Graph showing the relationship between absorbing states and network types.](image)
Example of convergence to a multipartite network

\[ \delta = \frac{3}{5}, \ c = \frac{1}{6}. \]

The dynamics lead to a core periphery network with \( k = 2 \). This network is unstable: 1 removes his link with 2. The result is a stable bipartite network with groups \( \{1, 2\} \) and \( \{3, 4, 5, 6\} \).
Extensions

Why are core-periphery networks unstable in the framework above? What additional assumptions are necessary to explain the existence of core-periphery networks?
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Why are core-periphery networks unstable in the framework above? What additional assumptions are necessary to explain the existence of core-periphery networks?

- Instability follows from the **homogeneity of banks**
  - If banks in the periphery want to trade indirectly through core banks, then banks in the core want to trade indirectly through periphery banks as well!
Consider now a case in which banks are heterogeneous

- Two types of banks: \( k \) big banks and \( n - k \) small banks
- Big banks generate more trading opportunities \( \alpha_{ij} \):

\[
\begin{array}{c|cc}
\alpha_{ij} & L & S \\
\hline
L & \alpha^2 & \alpha \\
S & \alpha & 1 \\
\end{array}
\]

with some \( \alpha > 1 \).
Theorem (2)

For a sufficiently large level of heterogeneity $\alpha > 1$, the complete core periphery networks with $k$ big banks is **unilaterally stable** for $c \in (\underline{c}, \overline{c})$, where $\underline{c}$ and $\overline{c}$ depend on $n$, $k$, $\alpha$ and $\delta$.

Intuition:

- if $\alpha$ is very large, then it is more attractive for core banks to trade directly with each other.
Dynamic process with growing size heterogeneity

Do core-periphery networks also arise as part of a dynamic process? We consider a similar dynamic process as before, but add that **profits feed back on bank size:**

- Initial empty network with homogeneous banks \( \alpha_{ij} = 1 \ \forall i, j \).
- Round-robin best response dynamics.
- After the process converges to a unilaterally stable network, with resulting profits \( \pi_i \), bank size is updated as \( \alpha_{ij} = \pi_i \pi_j / \min[\pi]^2 \).
- The attained network may be unstable. If so, round-robin best response dynamics follow until a new stable network is attained.
- The process is stopped if the \( \alpha_{ij} \)'s do not alter any more (given a tolerance level \( tl \)) or has altered more than \( H \) times.
Absorbing states dynamic process. $n = 8$, $t_l = 0.01$, $H = 1$. 
Absorbing states dynamic process. $n = 8$, $tl = 0.01$, $H = 25$. 
A calibration to the Dutch Interbank market

We calibrate $n$, $k$ and $\alpha$ to the Dutch interbank market

- In ’t Veld & van Lelyveld (2014) find in the interbank market with around $n = 100$ banks a core with around $k = 15$ banks.
- Large heterogeneity in asset size. We consider a fixed $\alpha = 10$.
- We look at the absorbing states of the dynamic process (without feedback of profits).
Absorbing states dynamic process. $n = 100, k = 15, \alpha = 10.$
Conclusions

In this paper we ask ourselves: why do financial networks have a core-periphery structure? We focus on the role of intermediation benefits. We find that:

- A core-periphery network is not stable if agents are homogeneous
- If there is enough heterogeneity in trade surplus, then a core-periphery network with big banks in the core can be stable
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In this paper we ask ourselves: why do financial networks have a core-periphery structure? We focus on the role of intermediation benefits.

We find that:

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This result suggests that we cannot abstract away heterogeneity in banking size if we want to understand the effect of heterogeneity in financial network structure (e.g. Acemoglu et al. 2015).
Future research

In future research we would like to

- Introduce default probabilities in order to understand the role of network formation on systemic risk and financial stability.