The formation of a core periphery structure in heterogeneous financial networks

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Abstract

Recent empirical evidence suggests that financial networks exhibit a core periphery network structure. This paper aims at giving an economic explanation for the emergence of such a structure using network formation theory. Focusing on intermediation benefits, we find that a core periphery network cannot be unilaterally stable when agents are homogeneous. The best-response dynamics converge to a unique unilaterally stable outcome ranging from an empty to denser networks as the costs of linking decrease. A core periphery network structure can form endogenously, however, if we allow for heterogeneity among agents in size. We show that our model can reproduce the observed core periphery structure in the Dutch interbank market for reasonable parameter values.

Keywords: financial networks, core periphery structure, network formation models
JEL classifications: D85, G21, L14

1. Introduction

The extraordinary events of 2007 and 2008 in which the financial system almost experienced a global meltdown, have led to an increased interest in the role of the interconnectedness of financial institutions and markets on systemic risk, the risk that liquidity or solvency problems in one financial institution spreads to the whole financial sector. This interest

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followed on earlier studies of financial networks that arose after the bailout of LTCM in 1998, in which one hedge fund, LTCM, had such a large trading position that its failure threatened to bring down the whole financial sector.

Starting with Allen and Gale (2000), a number of authors have argued that there is a non-monotonic relation between the interconnectedness of the financial system and systemic risk. For low connectivity a bank default leads to a local cascade, but not a global one. For high connectivity, diversification has a dampening effect on contagion, but for intermediate connectivity the default of one bank can lead to the failure of the whole financial system. This non-monotonic effect of connectivity has been extended to different types of financial contagion mechanisms, see e.g. Gai and Kapadia (2010), Gai et al. (2011), Amini et al. (2012), Battiston et al. (2012) and Georg (2013). In a survey on systemic risk and financial contagion, Chinazzi and Fagiolo (2013) summarises the main finding in this literature as the 'robust-yet-fragile' property of the financial system. Connections can serve at the same time as shock-absorbers and shock-amplifyers.

Apart from theoretical and simulation analysis, there has been increasing research on stress tests of actual financial systems, see Upper (2011) for a review. However, both the theoretical, simulation and stress test analysis assumes that the network of financial interconnections is *exogenously fixed*. This assumption has helped to focus on the causal relation between the financial network structure, that is, the pattern of financial interconnections, and systemic risk. Nevertheless, the assumption ignores the fact that financial networks do not come out of the blue. Financial interconnections are formed consciously by financial institutions who borrow, lend and trade financial assets which each other in order to maximize profits.

This raises the question what kind of network structure financial networks have, and how financial networks are formed. Regarding the first question, recent empirical evidence suggests that interbank markets have a *core periphery structure* (Craig and von Peter, 2014; Fricke and Lux, 2014; Van Lelyveld and in 't Veld, 2012). In such networks the core is highly interconnected, while peripheral banks have only few connections, mostly with banks in the core. In this paper we aim to give insight into the second question: why does the financial network have a core periphery network structure? That is, we aim to give theoretical foundations for the empirical finding of a core periphery network structure, in order to increase our understanding of the economic forces shaping financial networks.

We present a network formation model of a financial market with an explicit role for *intermediation* or *brokerage*. The model extends the results of Goyal and Vega-Redondo (2007) and Babus (2013). Unlike those papers, we follow Siedlarek (2012a) in the possibility of imperfect competition between intermediators, and allow for free entry of intermediators. The aim is to explore whether the core periphery structure can be explained by interbank intermediation of core banks between periphery banks.

We identify which networks arise in equilibrium if banks optimally form links with other banks for interbank trading. We first consider the case in which banks are homogeneous. We find that the efficient 'star' network with one central counterparty becomes unstable if the costs for linking decrease, because peripheral banks have an incentive to replicate the position of the central player in order to receive intermediation benefits. We show that best-response dynamics converge to a unique stable outcome that ranges intuitively from an empty network (if linking costs are high) to a complete network (if linking costs are low).

Although the set of stable networks under different parameter values is rich, we find that the core periphery network cannot be stable. The intuition for this result is that the core periphery network creates an unsustainable *inequality between core and periphery banks*. Core banks receive large benefits from intermediating between periphery banks, especially if the number of banks in the system is large. Periphery banks have therefore often an incentive to enter the core, even if competition between intermediators reduces their benefits. Higher linking costs may deter periphery banks from replicating the core banks' position, but at the same time diminish the incentives of core banks to hold relationships with all other banks in the core.

We then introduce heterogeneity in our model. We consider two types of banks, big banks and small banks, and allow big banks to have more frequent trading opportunities. We find that for sufficiently large differences between big and small banks, it becomes beneficial for large banks to have direct lending relationships with all other large banks in the core, such that the core periphery network becomes a stable structure.

This paper is related to two main strands of literature. First the literature on network formation, see Goyal (2009) for a general overview. Most closely related to our paper is Goyal and Vega-Redondo (2007). They consider a model where every pair of players undertakes a transaction, possibly through intermediation by other players. It is shown that a star network arises with a single broker intermediating between all other traders. However, Goyal and Vega-Redondo (2007) use a limit case in which two or more brokers cannot earn intermediation rents simultaneously from a pair of traders: hence a core periphery network with multiple brokers is not found to be stable. Babus (2013) specifies a more detailed model for the interbank market, but also finds as the main outcome a star network with one central counterparty. These models mimic the observed inequality in the distribution of trades, but are too stylised to come close to empirically observed structures.[1]

Other network formation papers have found more complex structures, but typically ones that are (unrealistically) regular, that is, where all banks have (almost) the same number of links. Buskens and van de Rijt (2008) and Kleinberg et al. (2008) both find multipartite graphs using two different underlying mechanisms. Acemoglu et al. (2013) use an alternative formation scheme in which bank post contingent debt contracts to all other potential counterparties, and find that equilibrium networks have rings.

A recent, independent paper by Farboodi (2014) shows that a core-periphery network emerges endogenously in a model for financial intermediation, a result similar to the one in our paper. However, Farboodi (2014) assumes from the start an exogenous distinction in banks with and without potential risky investment. Our analysis starts off from the situation of homogeneous banks as in the model of Goyal and Vega-Redondo (2007). After showing that under homogeneity core periphery networks can not be stable, we introduce heterogeneity in bank size.[2]

The second relevant branch of literature is work on trading networks. Corominas-Bosch (2004) and Kranton and Minehart (2001) are early examples that assume a bipartite network. This approach rules out possible intermediation. Gale and Kariv (2007) are the first to model trading networks that do take intermediation into account. Blume et al. (2009) is a related paper that uses a similar distribution of trading surpluses as Goyal and Vega-Redondo (2007), and finds that equilibrium outcomes are always efficient. Gofman (2011) however shows that equilibrium pricing outcomes for an exogenous network are generally not efficient, unless players make a take-it-or-leave-it offer (as in Gale and Kariv, 2007), or the network is complete.[3]

This paper is organized as follows. In Section 2 we introduce our model with the pay-off function and the stability concepts. Section 3 treats the basic structures and includes the

[1]Bedayo et al. (2013) extend the result of Babus (2013) by heterogeneous discount rates: weak (or impatient) players have a lower discount rate than strong players. The authors find that any stable network is a core periphery network in which the core consists of all weak players. In our paper we focus on heterogeneity in size, and assume that big banks have more frequent trading opportunities. We show under which conditions a core periphery network with a core of big banks can form.

[2]Also in the social science literature some papers explain core periphery networks. These network formation models are typically concerned with optimal effort levels to account for peer effects. Galeotti and Goyal (2010) and Hiller (2010) provide conditions under which core periphery networks are the only stable network structure. See also Persitz (2012) who adopts heterogeneity in the connections model of Jackson and Wolinsky (1996). This literature cannot so easily be translated to financial networks, because of the different interpretation of links as (channels for) financial transactions. See Cohen-Cole et al. (2011) and König et al. (2014) who propose related models with nestedness for interbank networks.

[3]See Choi et al. (2013) for a recent contribution to the literature on trading networks.

formal definition of a core periphery network. Our main results are presented separately for homogeneous traders (Section 4) and heterogeneous traders (Section 5). In Section 6 we apply our model to the interbank market in the Netherlands. Section 7 concludes.

2. Model

Consider an undirected network g between a set of agents N, having cardinality $n = |N|$. Denote by $g_{ij} = g_{ji} = 1$ the existence of a trading relationship, and by $g_{ij} = g_{ji} = 0$ the absence of it. Network g results in a payoff vector $\{\pi_i(g)\}_{i \in N}$ to each agent i in N, as follows.

2.1. Payoffs in a financial network with intermediators

Each pair ij in the network creates a *potential trade surplus* of α_{ij}. This trade surplus can only be realized if i and j are connected directly, or indirectly through a path with one middleman. It is possible that multiple of these short paths (of length 2) exist: the number of middlemen linking i and j is denoted as m_{ij}. The trade surplus α_{ij} does not depend on whether i and j are directly or indirectly connected, or on the number of middlemen m_{ij}.

If the path length between i and j is more than 2, that is, if i and j can only be connected through a chain of at least two middlemen, we assume that trade can*not* be realized. This assumption keeps the analysis tractable. Also, intermediation over longer routes in financial networks seems empirically and intuitively less relevant.[4] Our main result – core periphery networks are generally unstable under homogeneity ($\alpha_{ij} = 1$ for all $i, j \in N$), but can be stable if agents are heterogeneous – does not depend on this assumption as shown in Appendix C and Appendix D.

We assume that, if realized, the trade surplus of i and j is divided as follows. If i and j are directly connected, then i and j each receive half of the surplus. If i and j are indirectly connected by m_{ij} middlemen, then the endnodes i and j each receive a share of $f_e(m_{ij}, \delta)$, whereas each of the m_{ij} middlemen receives a share of $f_m(m_{ij}, \delta)$. Note that by definition:

$$2f_e(m_{ij}, \delta) + m_{ij} f_m(m_{ij}, \delta) = 1.$$

Apart from the number of middlemen, m_{ij}, the division also depends on a parameter $\delta \in [0, 1]$. This variable captures the level of *competition* between the number of middlemen m_{ij} of a certain trade between i and j, and leads to the following payoffs.

[4]The same assumption was made by Kleinberg et al. (2008) and Siedlarek (2012b).

If there is one middleman, $m_{ij} = 1$, then i, j, and the middleman split the surplus evenly, hence,

$$f_e(1,\delta) = f_m(1,\delta) = \frac{1}{3}.$$

If there is more than only one middleman, then the share that the agents get depends on the amount of competition. If there is no competition, $\delta = 0$, the middlemen collude, and i and j obtain the same share as if there was one intermediator. The middlemen share the intermediation benefits evenly, that is, for all $m_{ij} \in \{2,\dots,n-2\}$:

$$f_e(m_{ij},0) = \frac{1}{3} \text{ and } f_m(m_{ij},0) = \frac{1}{3m_{ij}}.$$

In the case of perfect competition, $\delta = 1$, between $m_{ij} > 1$ intermediaries, their payoffs will always disappear, hence

$$f_e(m_{ij},1) = \frac{1}{2} \text{ and } f_m(m_{ij},1) = 0.$$

The same happens in the limit of the number of middlemen to infinity. A further straightforward assumption is monotonicity with respect to the competitiveness and number of middlemen.

Table 1 summarizes the assumed dependencies of the functions to the parameters. Figure 1 gives examples of payoff shares received by different agents involved in a trade between i and j for different levels of competition δ and different numbers of middlemen $m_{i,j}$.

Share of payoffs for:	endnodes	middlemen
Level of competition δ	$f_e(.,0) = \frac{1}{3}$ $\frac{\partial f_e}{\partial \delta} > 0$ $f_e(.,1) = \frac{1}{2}$	$f_m(.,0) = \frac{1}{3m}$ $\frac{\partial f_m}{\partial \delta} < 0$ $f_m(.,1) = 0$
Number of middlemen m	$f_e(1,.) = \frac{1}{3}$ $\frac{\partial f_e}{\partial m} > 0$ $\lim_{m\to\infty} f_e(m,.) = \frac{1}{2}$	$f_m(1,.) = \frac{1}{3}$ $\frac{\partial f_m}{\partial m} < 0$ $\lim_{m\to\infty} f_m(m,.) = 0$

Table 1: Assumptions about the payoff shares $f_e(m_{ij},\delta)$ and $f_m(m_{ij},\delta)$.

Finally, we assume that each link between i and j involves a cost c to both i and j. The payoff function for an agent i in graph g is then the sum of benefits agent i obtains from

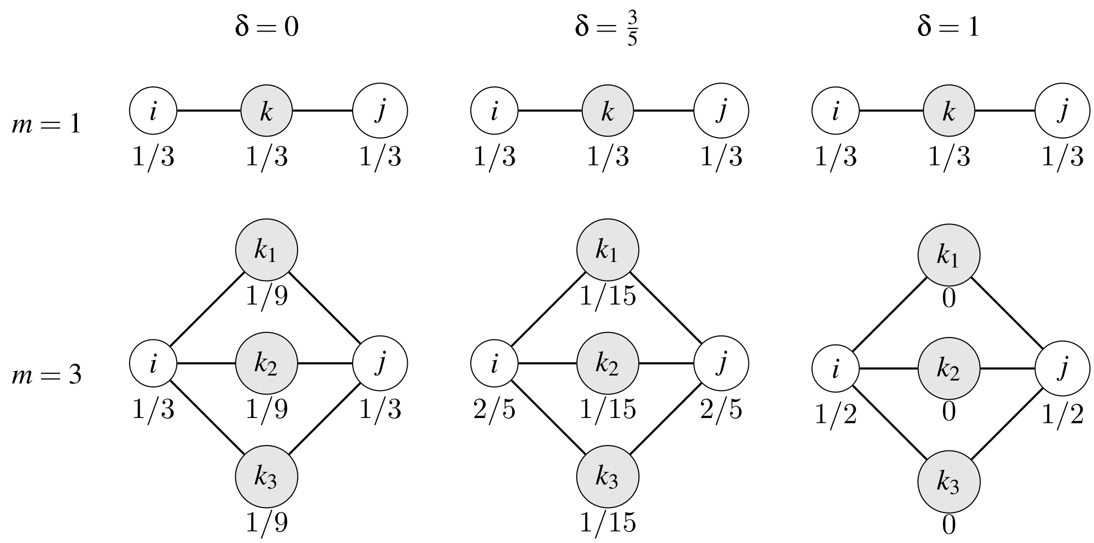


Figure 1: Examples of payoff shares received by endnodes i and j and intermediaries k, depending on the parameters δ and m under the specification of f_e and f_m in equation (2), as in Siedlarek (2012a).

the trades it is involved in, minus the cost of linking:

$$\pi_i(g) = \sum_{j \in N_i^1(g)} \left(\frac{1}{2}\alpha_{ij} - c \right) + \sum_{j \in N_i^2(g)} \alpha_{ij} f_e(m_{ij}, \delta) + \sum_{j,l \in N_i^1(g) | g_{jl}=0} \alpha_{jl} f_m(m_{jl}, \delta), \tag{1}$$

where $N_i^r(g)$ denotes the set of nodes at distance r from i in network g. Here the first term denotes the benefits of direct trades minus the cost of maintaining a link, the second term denotes the benefits of indirect trades through middlemen, and the third term denotes intermediation benefits.[5]

We now comment on the interpretation of these payoffs, in particular how trade surpluses α_{ij}, links g_{ij}, linking costs c and surplus distributions f_e and f_m can be translated back to an interbank market setting. First, trade possibilities in the interbank market arise when one bank has a liquidity shortage and another bank has a liquidity surplus, relative to its targeted liquid asset holdings. These shortages and surpluses arise expectedly and unexpectedly to the banks through their daily operations. The trade surpluses in our model should be seen as the net present values of the benefits from the realization of arising trade opportunities. In Appendix A we formalize this interpretation of trade surpluses.

[5] The payoff function of Goyal and Vega-Redondo (2007) is a special case of ours with homogeneity ($\alpha_{ij} = 1$ $\forall\ i, j$) and perfect competition ($\delta = 1$).

Second, the undirected links in this model should be interpreted as established preferential trading relationships. We assume that trade opportunities can only be realized if the agents are linked directly or indirectly through intermediation by mutual trading relationships. This is a strong, but not implausible assumption. The existence of preferential trading relationships has been shown by Cocco et al. (2009) in the Portuguese interbank market and by Braüning and Fecht (2012) in the German interbank market. Afonso and Lagos (2012) document how some commercial banks act as intermediaries in the U.S. federal funds market. A bank that attempts to borrow outside its established trading relationships may signal that it is having difficulties to obtain liquidity funding and, as a consequence, may face higher borrowing costs. Hence, banks have incentives to use its established trading relationships.

Third, it is assumed that the preferential trading relationship comes at a fixed cost. This cost follows from establishing mutual trust and from monitoring, i.e. assessing the other bank's risks. In principle, it is possible that these costs are not constant over banks, e.g. economies of scale may decrease linking costs in the number of relationships that are already present. The possible heterogeneity in linking costs is most likely smaller than the heterogeneity in trading surpluses, and we therefore assume that all banks pay an equal(ly small) cost for a trading relationship.

Fourth, the division of the trade surplus and intermediation benefits in our model follow from a bargaining protocol. We do not model this bargaining process explicitly. However, our assumptions on $f_e(\cdot)$ and $f_m(\cdot)$ generalize an explicit Rubinstein-type of bargaining process developed by Siedlarek (2012a). From his bargaining protocol, in which δ has the interpretation of a discount factor, the distribution of the surplus is given by

$$f_e(m,\delta) = \frac{m-\delta}{m(3-\delta)-2\delta} \text{ and } f_m(m,\delta) = \frac{1-\delta}{m(3-\delta)-2\delta}. \tag{2}$$

It is easily checked that (2) satisfies the assumptions we made on $f_e(\cdot)$ and $f_m(\cdot)$. We use this explicit function to illustrate our (more general) results by Figures 7, 8 and 9. In Appendix B, we give a detailed explanation of the specification of payoffs by Siedlarek (2012a).

2.2. Network stability concepts

Given the setup of the payoffs discussed above, we analyze which networks arise if agents form links strategically. Here we assume that for the establishment of a link between two agents, both agents have to agree, and both agents face the cost of a link, a version of network formation that is called *two-sided network formation*. The tools for this analysis come from network formation theory, see Goyal (2009) for a text book discussion. Network formation theory has developed stability or equilibrium concepts to analyze the *stability* of a network. Here stability does not refer to systemic risk, but to the question whether an agent or a pair of agents has an incentive and the possibility to modify the network in order to receive a higher payoff.

There are many stability concepts, which differ in the network modifications allowed. For an overview of these stability concepts, we refer to Jackson (2005) or Goyal (2009). For our purposes we consider two stability concepts.

The first concept is *pairwise stability*, originally introduced by Jackson and Wolinsky (1996). A network is pairwise stable if for all the links present, no player benefits from deleting the link, and for all the links absent, one of the two players does not want to create a link. Denote the network $g+g_{ij}$ as the network identical to g except that a link between i and j is added. Similarly, denote $g-g_{ij}$ as the network identical to g except that the link between i and j is removed. Then the definition of pairwise stability is as follows:

Definition 1. *A network g is* pairwise stable *if for all $i, j \in N$, $i \neq j$:*

(a) if $g_{ij}=1$, then $\pi_i(g) \geq \pi_i(g-g_{ij}) \wedge \pi_j(g) \geq \pi_j(g-g_{ij})$;

(b) if $g_{ij}=0$, then $\pi_i(g+g_{ij}) > \pi_i(g) \Rightarrow \pi_j(g+g_{ij}) < \pi_j(g)$.

The concept of pairwise stable networks only allows for deviations of one link at a time. This concept is often too weak to draw distinguishable conclusions, i.e. in many applications including ours, there are many networks that are pairwise stable.

In our application, we consider it relevant that agents may consider to propose many links simultaneously in order to become an intermediary and establish a client base. The benefits from such a decision may only become worthwhile if the agent is able to create or remove more than one link. This leads us naturally to the concept of *unilateral stability*, originally proposed by Buskens and van de Rijt (2008).

A network is unilaterally stable if no agent i in the network has a profitable unilateral deviation: a change in its links by either deleting existing links such that i benefits, or proposing new links such that i and all the agents to which it proposes a new link benefit. Denote g^{iS} as the network identical to g except that all the links between i and every $j \in S$ are altered by $g^{iS}_{ij} = 1 - g_{ij}$, i.e. are added if absent in g or are deleted if present in g.

Definition 2. *A network g is* unilaterally stable *if for all i and for all subsets of players $S \subseteq N \backslash \{i\}$:*

(a) *if* $\forall j \in S : g_{ij} = 1$, *then* $\pi_i(g) \geq \pi_i(g^{iS})$;

(b) *if* $\forall j \in S : g_{ij} = 0$, *then* $\pi_i(g^{iS}) > \pi_i(g) \Rightarrow \exists j : \pi_j(g^{iS}) < \pi_j(g)$.

Note that unilateral stability implies pairwise stability, that is, a network that is unilaterally stable is also pairwise stable, but not vice versa. This can be easily verified by considering subsets S that consist of only one node $j \neq i$.[6]

Apart from analyzing the stability of networks, for policy considerations it is also relevant to consider efficient networks. As usual in the literature, we define a network efficient, if it maximizes the total sum of payoffs of the agents.

Definition 3. *A network g is* efficient *if there is no other network g', such that*

$$\sum_{i \in N} \pi_i(g') > \sum_{i \in N} \pi_i(g).$$

3. Basic structures

Our analysis revolves around a couple of relevant network structures, which we define here. Denote the *empty network*, g^e, as the network without any links, i.e. $\forall i, j \in N : g_{ij} = 0$, and the *complete network*, g^c, as the network with all possible links, i.e. $\forall i, j \in N : g_{ij} = 1$. A *star network*, g^s (see Figure 2), has a single player, the center of the star, that is connected to all other nodes, and no other links exist: $\exists i$ such that $\forall j \neq i : g_{ij} = 1$ and $\forall j, k \neq i : g_{jk} = 0$.

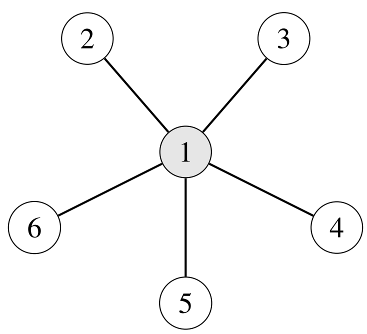


Figure 2: A star network with $n = 6$ players

A *core periphery network* is a network, in which the set of agents can be partitioned in a

[6] Our definition of unilateral stability is slightly less restrictive than the definition in Buskens and van de Rijt (2008). While we are following Buskens and van de Rijt (2008) in the simultaneity in deleting *or* proposing multiple links, we do not allow simultaneous deleting *and* proposing of multiple links. This adaptation eases the exposition, but does not affect our results qualitatively.

core and a periphery, such that all agents in the core are completely connected within and are linked to some periphery agents, and all agents in the periphery have at least one link to the core, but no links to other periphery agents.

Definition 4. *A network g is a core periphery network, if there exists a set of core agents $K \subset N$ and periphery agents $P = N \setminus K$, such that:*

(a) $\forall i, j \in K : g_{ij} = 1$, *and* $\forall i, j \in P : g_{ij} = 0$;

(b) $\forall i \in K : \exists j \in P$ *with* $g_{ij} = 1$, *and* $\forall j \in P : \exists i \in K$ *with* $g_{ij} = 1$.

See Figure 3 for an example. A special case of a core periphery network is the *complete core periphery network*, where each agent in the core K is linked to *all* agents in the periphery P: $\forall i \in K$ and $\forall j \in P$ it holds that $g_{ij} = 1$. See Figure 4 for an example. We denote a complete core periphery network with $k = |K|$ agents in the core as $g_{com}^{CP(k)}$.[7]

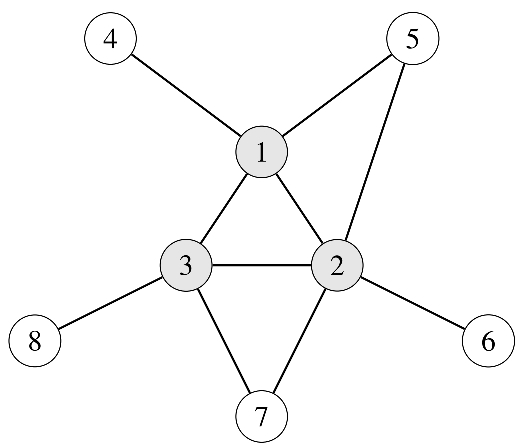


Figure 3: A core periphery network with $n = 8$ players, of which $k = 3$ are in the core

Finally, a *complete multipartite network* is a network, in which the agents can be partitioned into q groups, $N = \{K_1, K_2, \ldots, K_q\}$, such that nodes do not have links within their group, but are completely connected to all nodes outside their own group. Formally, in a complete multipartite network it holds that $\forall m \in \{1, 2, \ldots, q\} : \forall i \in K_m$ we have $\forall j \in K_m : g_{ij} = 0$ and $\forall j \notin K_m : g_{ij} = 1$. Complete multipartite networks will be denoted as $g_{k_1, k_2, \ldots, k_q}^{mp(q)}$, where $k_m \equiv |K_m|$ is the size of the m-th group. Multipartite networks are called

[7]By Definition 4, empty, star and complete networks are special cases of core periphery networks with cores of size $k = 0$, $k = 1$ and $k = n$ respectively. A complete core periphery networks with $k = n - 1$ is also identical to a complete network. In discussing our results we will make clear when we are speaking of non-trivial core periphery networks with $k \in \{2, 3, ..., n - 2\}$.

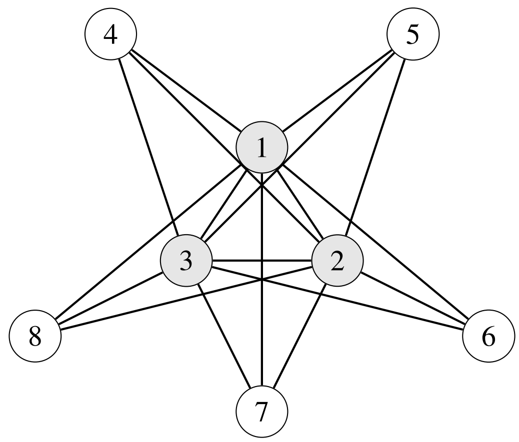


Figure 4: A complete core periphery network with $n = 8$ players, of which $k = 3$ are in the core

balanced if the group sizes are as equal as possible, i.e. $|k_m - k_{m'}| \leq 1$ for all m, m'. Figure 6 in Subsection 4.3 presents examples of complete multipartite networks that arise in our model.

4. Results for homogeneous traders

We now analyze the model for the baseline homogeneous case where all pairs generate the same trade surplus, $\alpha_{ij} = 1$ for all $i, j \in N$ with $i \neq j$.[8] After briefly discussing efficient networks, we analyze the model using the two stability concepts described above, pairwise and unilateral stability. In Section 4.2 we develop the main result that core periphery networks are not unilaterally stable under the assumption of homogeneity. To find what structures arise in this homogeneous case, if not core periphery networks, we investigate a best-response dynamic process in Section 4.3.

4.1. Efficient networks

We start with a description of the efficient network. Following the results of Goyal and Vega-Redondo (2007), minimally connected networks, i.e. with $n-1$ links, are efficient for $c < \frac{n}{4}$. Networks are efficient if all trade surpluses are realized irrespective of the distribution of these trading surpluses. For higher c it is efficient to have no network at all, i.e. an empty network. Because of the assumption that two agents only trade if they are at distance 1 or 2, the network should not have a maximal distance higher than 2, leaving the star as the unique efficient minimally connected network. This is summarized in the following theorem.

Theorem 1. *If the payoff function $\pi(g)$ is given by equation (1) with $\alpha_{ij} = 1$ for all $i, j \in N$, then:*

(a) If $c \geq n/4$, then the empty network *is efficient*

(b) If $c \leq n/4$, then the star network *is efficient*

(c) No other network structure than the empty or star network is efficient.

Theorem 1 implies that core periphery networks with $k \geq 2$ are not efficient.

4.2. Stability of basic structures

A natural starting point of the analysis is the stability of the empty, star and complete networks. The easiest way to find under which conditions these networks are unilaterally stable is by introducing the best feasible (unilateral) action of a certain player i. An action

[8] The normalization $\alpha = 1$ in the homogeneous case goes without loss of generality. If $\alpha_{ij} = \alpha$ for all i, j with $\alpha > 0$, then the payoffs in equation (1) are proportional to α when costs c are considered as a fraction of α.

of player i is feasible if its proposed links to every $j \in S$ are accepted by every $j \in S$, or if i deletes all links with every $j \in S$. This action changes the network g into g^{iS}. The best feasible action is formally defined as follows.

Definition 5. *A feasible action for player i is represented by a subset $S \subseteq N \backslash \{i\}$ with:*

(a) $\forall j \in S : g_{ij} = 1$, *or:*

(b) $\forall j \in S : g_{ij} = 0$ *and* $\pi_j(g^{iS}) \geq \pi_j(g)$.

The best feasible action S^* *is the feasible action that gives i the highest payoffs:*

$$\forall S \subseteq N \backslash \{i\}: \ \pi_i(g^{iS^*}) \geq \pi_i(g^{iS}).$$

In an empty network, the best feasible action for a player is to connect either to all other players or to none, as shown in the following lemma.

Lemma 1. *In an empty network that is not unilaterally stable, the best feasible action for a player is to add links to all other nodes.*

Proof. The marginal benefits[9] of adding l links to an empty network are:

$$M_i(g^e, +l) = l(\frac{1}{2} - c) + \binom{l}{2}\frac{1}{3}. \tag{4}$$

As this function is convex in l, maximising the marginal benefits over l results in either $l^* = 0$ or $l^* = n-1$. □

The star is unstable for low costs c, because periphery members gain incentives to add links. The gain of having a direct (rather than an intermediated) trade are in this case $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. However, it is even better for one periphery member to propose multiple links, thereby creating more chains of neighbors through which it intermediates.

[9] Throughout the paper, marginal benefits of an action S by player i for player j (not necessarily equal to i) are defined as:

$$M_j(g, S) \equiv \pi_j(g^{iS}) - \pi_j(g). \tag{3}$$

To simplify the notation, we will informally denote the action S as $+l$ or $-l$, where $l = |S|$ is the number of links that is either added or deleted in the network. From the text it will be clear which player i and which set S are considered.

Lemma 2. *Consider a star network that is not unilaterally stable because a peripheral player i can deviate by adding one or more links. Then the best feasible action is to add links to all $l = n-2$ other periphery players. In other words, it is never a best feasible action to add $0 < l < n-2$ links to some other peripheral nodes.*

Proof. Consider the deviation of a peripheral player i to add l links. The marginal benefits consist of making trades direct instead of intermediated by the center of the star, and of creating middlemen benefits:

$$M_i(g^s, +l) = l(\frac{1}{6} - c) + \binom{l}{2} f_m(2, \delta) \tag{5}$$

As in Lemma 1 this function is convex in l, so the maximum is reached at $l^* = 0$ or $l^* = n-2$. □

Lemmas 1 and 2 imply that to find the stability conditions of empty and star networks, it is sufficient to check whether adding or deleting all possible links is beneficial in these networks. Proposition 1 below specifies the stability conditions for empty, star and complete networks under homogeneous agents. Figure 7 in Section 4.3 depicts the stability regions I, II and III of the empty, star and complete networks in the (δ, c)-space for $n = 4$ and $n = 8$.

Proposition 1. *In the homogeneous baseline model with $\alpha_{ij} = 1$ for all $i, j \in N$ the following networks are stable:*

I: *The empty network is unilaterally stable if and only if $c \geq \frac{1}{2} + \frac{1}{6}(n-2)$.*

II: *The star network is unilaterally stable if and only if*

$$c \in \left[\frac{1}{6} + (n-3)\min\{\frac{1}{2} f_m(2, \delta), f_e(2, \delta) - \frac{1}{3}\}, \frac{1}{2} + \frac{1}{6}(n-2)\right]$$

III: *The complete network is unilaterally stable if and only if $c \leq \frac{1}{2} - f_e(n-2, \delta)$.*

Proof. As in Goyal and Vega-Redondo (2007), the center of the star earns more than the periphery members, but receives less profit per link. The star therefore becomes unilaterally unstable if the center of the star would benefit from deleting all its links, i.e.:

$$\begin{aligned} M_i(g^s, -(n-1)) = -\pi_i(g^s) \quad &< \quad 0 \\ \Rightarrow c \quad &> \quad \frac{1}{2} + \frac{1}{6}(n-2). \end{aligned} \tag{6}$$

If this condition holds the empty network is unilaterally stable.

Lemma 2 states that if a peripheral player i in a star network wants to deviate by adding links, this player will become a member of a newly formed core. In other words, a complete core periphery network arises with $k = 2$. Player i becomes directly connected to all other players, and also receives intermediation gains which depend on δ. For the new core member i to have positive marginal benefits of supporting all new links, it is required that

$$\begin{aligned} M_i(g^s, +(n-2)) &> 0 \\ \Rightarrow c &< \frac{1}{6} + \frac{1}{2}(n-3) f_m(2, \delta), \end{aligned} \tag{7}$$

and every remaining peripheral player $j \neq i$ requires

$$\begin{aligned} M_j(g^s, +(n-2)) = \frac{1}{6} - c + (n-3)(f_e(2,\delta) - \frac{1}{3}) &\geq 0 \\ \Rightarrow c &\leq \frac{1}{6} + (n-3)(f_e(2,\delta) - \frac{1}{3}). \end{aligned} \tag{8}$$

Conversely, the star network is stable if the deviation of player i is *not* beneficial to i and/or a peripheral player $j \neq i$, i.e. if c exceeds the minimum of the two values in equations (7) and (8):

$$c \geq \frac{1}{6} + (n-3) \min\{\frac{1}{2} f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}. \tag{9}$$

A complete network can become unstable when any two nodes decide to remove a mutual link. Note that the trade between those two links is sustained by intermediation through all other nodes. Formally we have therefore a core periphery network with $k = n-2$ core players. The complete network is therefore stable if:

$$c \leq \frac{1}{2} - f_e(n-2, \delta) \tag{10}$$

□

Given the stability conditions for empty, star and complete networks in Proposition 1, we see that these networks cannot be stable at the same time, except for borderline cases such as $c = \frac{1}{2} + \frac{1}{6}(n-2)$. In Subsection 4.3, we will show that if these networks are stable, they are also the unique outcome of a best-response dynamic process.

We now consider the stability of core periphery networks, starting with complete core periphery networks. The profits in a complete core periphery network, in which k core members are connected to all other nodes, are:

$$\pi_i(g_{com}^{CP(k)}) = \begin{cases} (n-1)(\frac{1}{2}-c) & + \quad \binom{n-k}{2} f_m(k,\delta) \qquad \text{if } i \in C \\ \\ k(\frac{1}{2}-c) & + \quad (n-k-1) f_e(k,\delta) \quad \text{if } i \in P \end{cases} \tag{11}$$

The reason we focus on this special type of complete core periphery networks is that they are formed naturally as peripheral agents try to improve their position by avoiding payments to core players and linking to other peripheral players. As it was optimal in an empty graph and in a star network to link immediately to all players (Lemmas 1 and 2), if a peripheral agent will add a link it is optimal to connect to all other players, thereby entering in the core himself.

Lemma 3. *Consider a complete core periphery network with $k \in \{2,3,...,n-2\}$ that is not unilaterally stable because a peripheral player i can deviate by adding one or more links. Then the best feasible action is to add links to all $l = n-k-1$ other periphery players. In other words, it is never a best feasible action to add $0 < l < n-k-1$ links to some other peripheral nodes.*

Proof. The marginal benefits for replicating the position of the center of the star was given in Lemma 2. It can easily be generalized to complete core periphery networks with k core members:

$$M_i(g_{com}^{CP(k)}, +l) = l(\frac{1}{2} - f_e(k,\delta) - c) + \binom{l}{2} f_m(k+1,\delta)) \tag{12}$$

The maximum of this function is reached at $l^* = 0$ or $l^* = n-k-1$. □

Lemmas 2 and 3 show that peripheral players have an incentive to replicate the position of core players to raise their profits. Stable networks must therefore exhibit limited inequality between players in different network positions. This intuitive economic phenomenon, introduced in our model by using unilateral stability as stability concept, will drive the results for the core periphery network below.

Although there is an incentive for periphery players to become a new core member if the cost c are low enough, the resulting complete core periphery network is not stable. We now show that under fairly general conditions in the baseline homogeneous model *any* core periphery network (complete or incomplete) is not stable.

Proposition 2. *Let the payoff function be homogeneous ($\alpha_{ij} = 1$ for all $i,j \in N$) and let $c > 0$ and $0 < \delta < 1$ be given. Then:*

(a) *complete core periphery networks with $k \geq 2$ core members are not pairwise stable, and thus also not unilaterally stable, for all $n > k+2$;*

(b) *incomplete core periphery networks with $k \geq 1$ core member(s) are not unilaterally stable if n is sufficiently large. More precisely, there is a function $F(c,\delta,k)$ such that if $n > F(c,\delta,k)$, the result holds.*

Proof. See Appendix C. □

We emphasize that this result does not depend on the assumption that trade surpluses between i and j are only realized if the path length between i and j is less than 3, as shown in Appendix C. This assumption is made to keep the analysis of our dynamic process tractable (see Section 4.3).

The intuition for Proposition 2 is that for homogeneous agents core periphery networks create an unsustainable inequality between core and periphery players. The proof of Proposition 2 relies on the idea that the differences in payoff between the core and periphery cannot be too large. Consider for example the payoffs for complete core periphery networks in equation (11). As the network size n increases for a fixed core size k, core members get more and more intermediation benefits (at a quadratic rate), while periphery payoffs fall behind (as it grows at a linear rate).

We now compare this result with that of Goyal and Vega-Redondo (2007). They show that for $c > \frac{1}{6}$, $\delta = 1$ and n sufficiently large, the star network (i.e. a core periphery network with $k = 1$) is the unique non-empty *bilaterally* stable network, stressing the importance of the star structure. Proposition 2 refines this result and indicates that for $\delta < 1$ core periphery networks with k core players *including* star networks with $k = 1$ are *not unilaterally* stable, even for values of δ arbitrary close but below 1.

We argue that the crucial assumption for our result is imperfect competition, that is, $\delta < 1$. Imperfect competition creates profitable deviations for peripheral players to circumvent (large) intermediation fees to core players and to start receiving intermediation benefits. A further important difference in this paper compared to Goyal and Vega-Redondo (2007) is the stability concept: in the proof of Proposition 2, we make explicit use of the fact that periphery players may add multiple links at the same time, which is not allowed in Goyal and Vega-Redondo (2007). Because we allow for unilateral deviations, peripheral players can replicate the position of core players in order to benefit from intermediation. Inequality between core and periphery players makes that core periphery networks cannot be stable for sufficiently large n.

4.3. A dynamic process

So far, we have analyzed the unilateral stability of empty, star, complete and core periphery architectures. Core periphery networks were shown to be generally unstable under homogeneous agents. To investigate what other networks can plausibly be formed, we consider round-robin best-response dynamics as in Kleinberg et al. (2008), starting from an empty graph. We order nodes $1,2,\ldots,n$ and in this order nodes consecutively try to improve their position by taking best feasible actions. After node n the second round starts again with node 1. The process is stopped if $n-1$ consecutive players can not improve their position, thereby agreeing to the deviation made by the last player.

The assumptions about the dynamic process are made primarily to ease the exposition and to simplify the analytical analysis. The advantage of starting in an empty network is that initially nodes can only add links to the network. A fixed round-robin order limits the number of possibilities that has to be considered for every step in the process.[10] The outcome of the dynamic process must be a unilaterally stable network. We find that the best-response dynamics converge to a unique unilaterally stable network for every choice of the model parameters (except for borderline cases).[11]

Theorem 2 below shows which network structures result from the best-response dynamics. The parameter regions for which empty, star and complete networks are the attracting steady state coincide with conditions I, II and III in Proposition 1 for which these networks are stable. In the remaining parameter regions the attracting steady state cannot be a core periphery network (in line with Proposition 2) and turns out to be a multipartite network. The theorem singles out one special type of multipartite networks, called maximally unbalanced bipartite networks, for parameters satisfying condition IV, while parameters under condition V can lead to various types of multipartite networks. Figure 7 plots the parameter regions I to IV for $n=4$ (for which V$=\emptyset$) and I to V for $n=8$ in the (δ,c)-space.

Theorem 2. *Consider the homogeneous baseline model with $\alpha_{ij}=1$ for all $i,j\in N$. From an empty graph, the round-robin best-response dynamics converge to the following unilaterally stable equilibria:*

I: *for* $c>\frac{1}{2}+\frac{1}{6}(n-2)$

[10] Houy (2009) considers a dynamic framework for the model of Goyal and Vega-Redondo (2007), in which randomly drawn agents make best responses to the network. Simulations for our model indicate that the results in Theorem 2 also hold for a random order of agents.

[11] In principle, a dynamic process may lead to cycles of improving networks (cf. Jackson and Watts, 2002; Kleinberg et al., 2008), but Theorem 2 shows that this is not the case in our model.

the empty network,

II: *for* $c \in \left(\frac{1}{6} + (n-3)\min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}, \frac{1}{2} + \frac{1}{6}(n-2)\right)$
the star network,

III: *for* $c < \frac{1}{2} - f_e(n-2,\delta)$
the complete network.

IV: *for* $c \in \left(\frac{1}{2} - f_e(2,\delta) + (n-4)\min\{\frac{1}{2}f_m(3,\delta), f_e(3,\delta) - f_e(2,\delta)\}, \frac{1}{6} + (n-3)\min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}\right)$
the maximally unbalanced bipartite network $g^{mp(2)}_{2,n-2}$,

V: *for* $c \in \left(\frac{1}{2} - f_e(n-2,\delta), \frac{1}{2} - f_e(2,\delta) + (n-4)\min\{\frac{1}{2}f_m(3,\delta), f_e(3,\delta) - f_e(2,\delta)\}\right)$
a multipartite network $g^{mp(q)}_{k_1,k_2,\ldots,k_q}$ *with* $q \geq 2$ *and* $|k_m - k_{m'}| < n-4$ *for all* $m, m' \in \{1,2,..,q\}$.

Proof. See Appendix C. □

Figure 5 shows the possible routes of best-response dynamics for $n = 4$, depending on the remaining parameters c and δ. For such a low n, the only possible multipartite network is one that consists of $q = 2$ groups of 2 nodes, which coincides with a ring of all 4 players. It is noteworthy that for large n the set of attained multipartite networks is quite diverse. Possible outcomes for $n = 8$ are the maximally unbalanced bipartite network $g^{mp(2)}_{2,6}$, but also, for example, a less than maximally balanced bipartite network $g^{mp(2)}_{3,5}$ or a balanced multipartite network $g^{mp(3)}_{2,3,3}$ consisting of 3 groups. Figure 6 presents these three examples of multipartite networks and the values of δ and c for which they are formed.

Figure 7 illustrates the parameter regions specified by Theorem 2 for $n = 4$ and $n = 8$ under the specification of f_e and f_m in equation (2), as in Siedlarek (2012a). The possible network outcomes range intuitively from empty to complete networks as the cost of linking decreases. The star is an important outcome in between empty and complete networks, but cannot be a unilaterally stable outcome for intermediate competition δ and relatively low c. As explained in Section 4.2, for intermediate δ the incentives to enter the core and the incentives for periphery members to accept the proposal of the new core member are both high. The parameter area between complete networks and stars gives multipartite networks as the stable outcomes, and this area increases with n. Different types of multipartite networks can arise including the maximally unbalanced bipartite network with group sizes of 2 and $n-2$.

A comparison with the results of Goyal and Vega-Redondo (2007) can easily be made by setting $\delta = 1$. We observe that their assumption of perfect competition $\delta = 1$ is a special

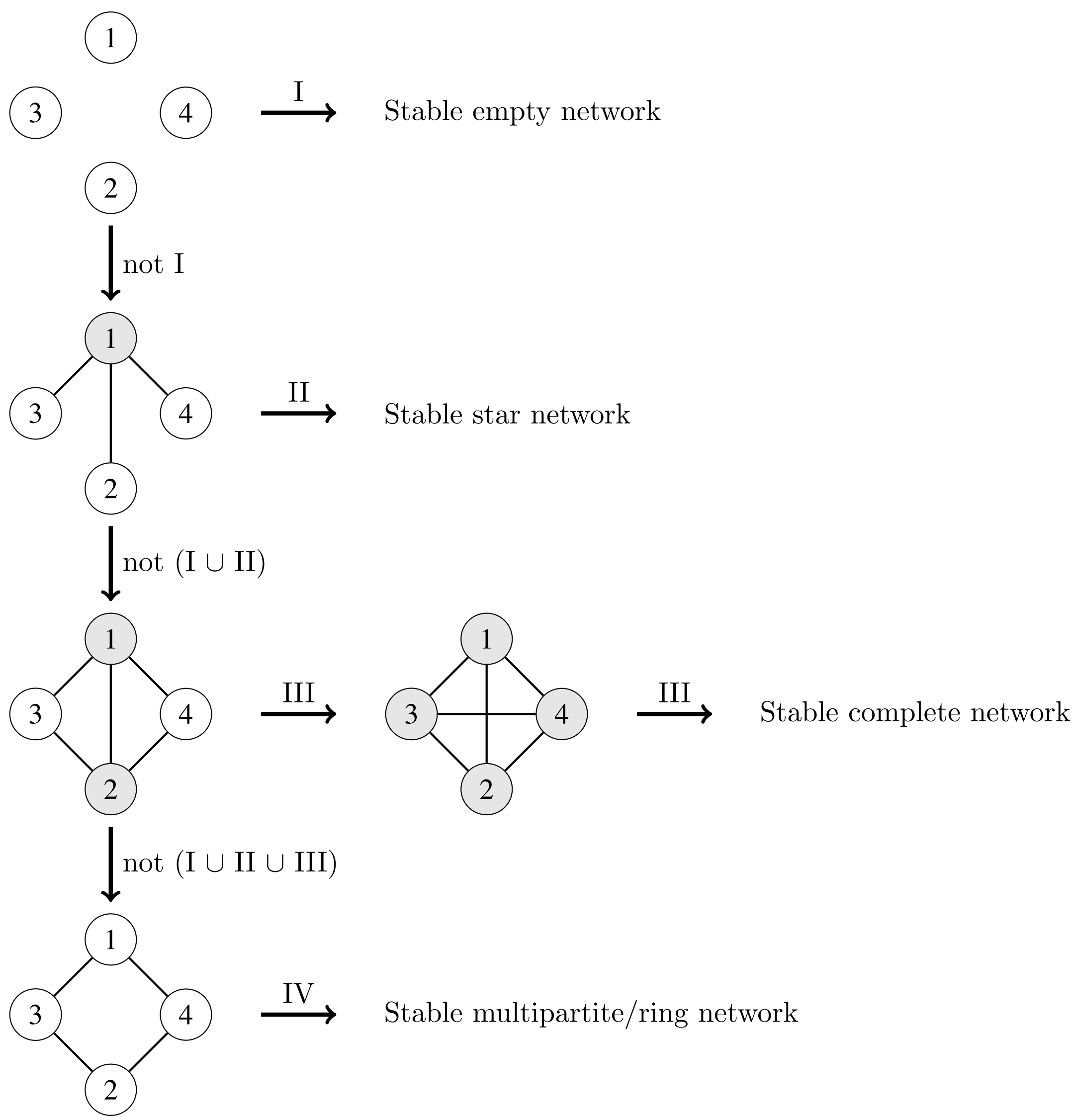
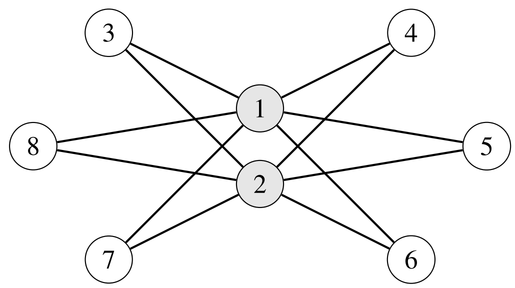
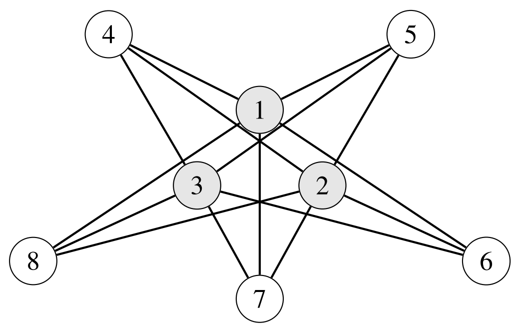


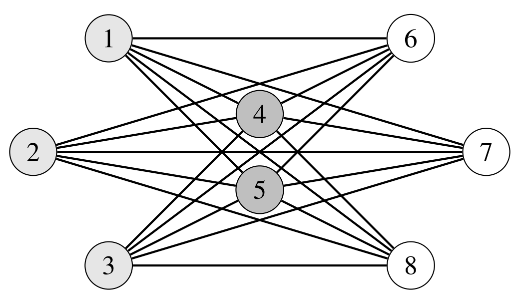
Figure 5: Map of best-response dynamics from an empty graph for $n = 4$ leading to one out of 4 stable possible structures. The roman numbers correspond with the conditions as in Theorem 2 under which the best-response route follows the direction of the arrows.



(a) Black square: $(\delta, c) = (0.8, 0.3)$. The maximally unbalanced bipartite network: $g^{mp(2)}_{2,6}$.



(b) Black circle: $(\delta, c) = (0.4, 0.15)$. A less than maximally unbalanced bipartite: $g^{mp(2)}_{3,5}$.



(c) Black star: $(\delta, c) = (0.8, 0.06)$. A balanced multipartite network: $g^{mp(3)}_{2,3,3}$

Figure 6: Examples of attained complete multipartite networks after best-response dynamics from an empty graph for $n = 8$ players. The symbols (black square, black circle and black star) correspond to locations in the (δ, c)-space in Figure 7.

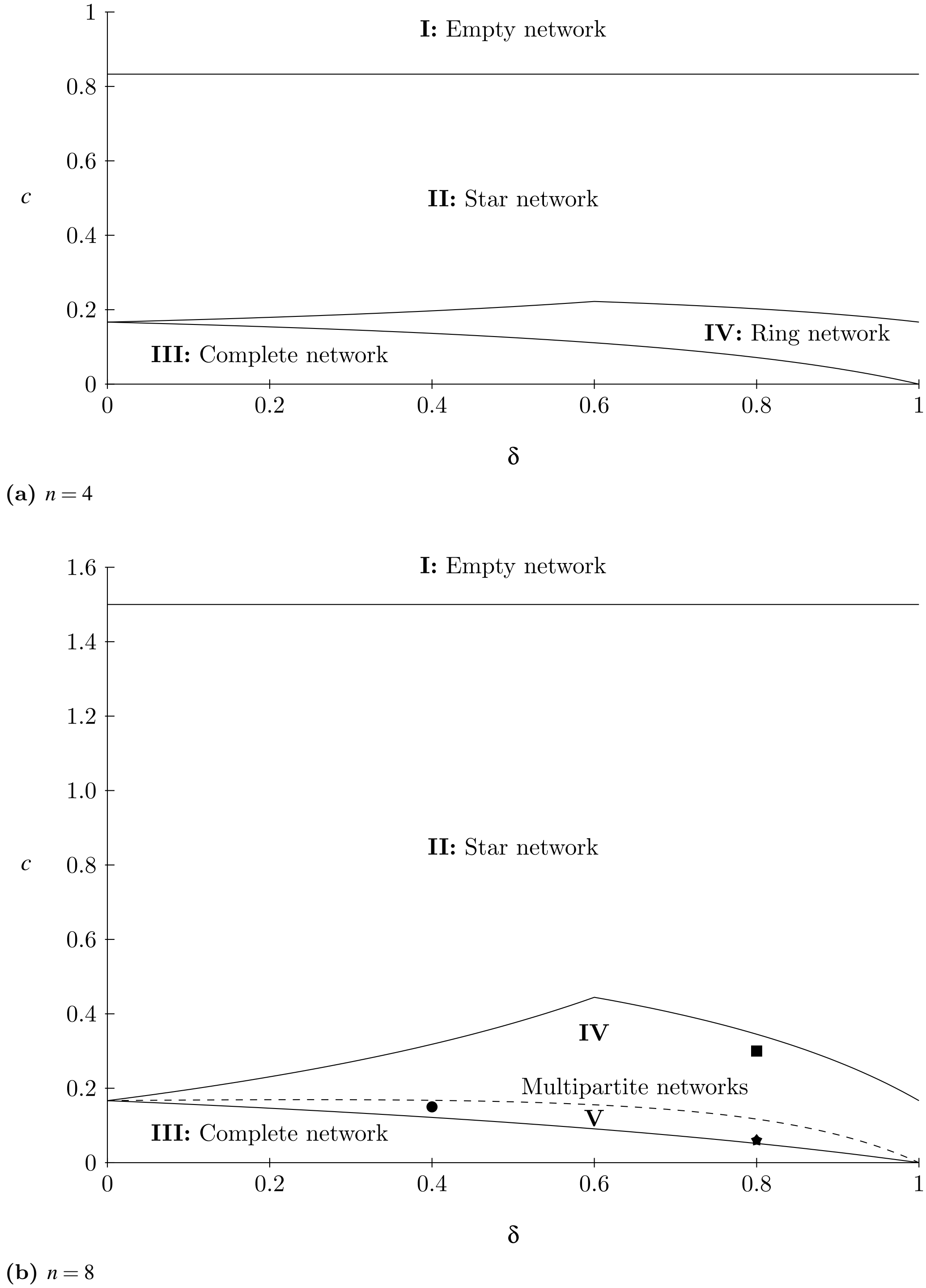


Figure 7: Attained equilibria after best-response dynamics from an empty network in (δ, c)-space for $\alpha_{ij} = 1$ for all $i, j \in N$ and $n \in \{4, 8\}$ under the specification of f_e and f_m in equation (2). The roman numbers correspond with those in Theorem 2. The symbols (black square, black circle and black star) correspond to examples of multipartite networks in Figure 6.

case for which a complete network is not stable. In their model agents in a complete network always have incentives to remove links even for arbitrary low linking costs, because intermediated trades along multiple middlemen gives them exactly the same share of the surplus.

More importantly, we show that the star is less stable when competition is imperfect compared to the case of Goyal and Vega-Redondo (2007). For $\delta < 1$ and a relatively low c multipartite networks arise instead of stars. Multipartite networks arise for larger regions of parameter choices if n increases; see Figure 9a for the results for $n = 100$. Interestingly, multipartite networks were found by Buskens and van de Rijt (2008) and Kleinberg et al. (2008) as the main equilibrium architecture. The current analysis shows that empty, star, complete and multipartite networks can all arise within a network formation model with intermediation and imperfect competition. Our model thus reproduces earlier results of homogeneous network formation models and places them in a more general perspective.

The results indicate a conflict between stability and efficiency in our network formation model with intermediation benefits. Whenever the star network is attained, it is the efficient network. However, for low costs under conditions III, IV or V, attracting networks are either multipartite or complete networks, and these structures are denser than is efficient. The upperbound on c for which stable networks are not efficient is increasing in n as shown in Figure 9a. So for relatively large network sizes, stable networks can be expected to be overconnected.

5. Results for heterogeneous traders

We found that complete core periphery networks were not stable, not because such networks are inefficient, but because the large inequalities in payoffs between core and periphery players were not sustainable. Instead other overconnected networks are formed, namely multipartite networks, in which banks are connected to members of other groups, but not within their own group. In real interbank markets the core banks are typically the biggest banks, and they have a strong incentive to have tight connections within the core as well as to the periphery for intermediation reasons.

For this reason we analyze the consequences of exogenous heterogeneity in the size of banks within our model.[12] We introduce two types of banks, k big banks and $n-k$ small banks. The difference in size is captured by a parameter $\alpha \geq 1$ quantifying the relative size

[12] In doing so we also add to a small literature analyzing network formation with heterogeneous agents, e.g. Galeotti et al. (2006), Persitz (2012) and Bedayo et al. (2013).

of a big bank. Trade is assumed to be proportional to size as motivated in Appendix A. The trade surplus between two big banks i and i' will be weighted as $\alpha_{ii'} = \alpha^2$, and the trade surplus between big bank i and small bank j with $\alpha_{ij} = \alpha$. The trade surplus between two small banks j and j' remains $\alpha_{jj'} = 1$. Our model does not explain why some banks are big or small, but rather takes their size as exogenously given. Given heterogeneity in the size of banks, we will show that a stable core periphery network can form.

The number of big banks, k, has become a new exogenous parameter, and we consider a complete core periphery network where the core consists of the k big banks. To investigate the stability of such a structure, we start with possible deviations of a peripheral player by adding links to other periphery players. Note that the change in payoffs by these deviations do not depend on α, because they only concern trade surpluses between small banks. Hence Lemma 3 holds also in this heterogeneous setting: if adding one link improves the payoff of a small periphery player, the best feasible action is to connect to all other small banks.

Starting from a complete core periphery network with the k big banks in the core, if one peripheral player adds links to all other periphery players, the core is extended to $k+1$ players, k big banks and 1 small bank. For the new core member i to have positive marginal benefits of supporting all new links, it is required that

$$\begin{aligned} M_i(g_{com}^{CP(k)}, +(n-k-1)) &> 0 \\ \Rightarrow c &< \frac{1}{2} - f_e(k,\delta) + \frac{1}{2}(n-k-2) f_m(k+1,\delta) \end{aligned} \tag{13}$$

and every remaining peripheral player $j \neq i$ requires

$$\begin{gathered} M_j(g_{com}^{CP(k)}, +(n-k-1) \geq 0 \\ \Rightarrow c \leq \tfrac{1}{2} - f_e(k,\delta) + \tfrac{1}{2}(n-k-2) + (n-k-2)(f_e(k+1,\delta) - f_e(k,\delta)). \end{gathered} \tag{14}$$

Conversely, the deviation of player i is *not* a best feasible action if c exceeds the minimum of the two values in equations (13) and (14):

$$c \geq \frac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\frac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}. \tag{15}$$

If this condition is fulfilled, the complete core periphery network can be unilaterally stable for large enough heterogeneity α, as stated in the following proposition.

Proposition 3. *If c exceeds the minimum level to restrain periphery links given by (15), there exists an $\overline{\alpha}$, such that for all $\alpha > \overline{\alpha}$ the complete core periphery network with k big banks is unilaterally stable. In other words, for a sufficiently large level of heterogeneity*

$\alpha > 1$, *the complete core periphery network with k big banks is unilaterally stable under the following condition:*

$$\text{VII:} \qquad c \in \Bigg(\quad \tfrac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\tfrac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\},$$
$$\min\Big\{ \quad \alpha^2(\tfrac{1}{2} - f_e(n-2,\delta)), \alpha(\tfrac{1}{2} - f_e(k-1,\delta)) + \tfrac{1}{2}(n-k)(n-k-1) f_m(k,\delta),$$
$$\min_{l \leq k}\big\{\alpha(\tfrac{1}{2} - f_e(k-l,\delta)) + \tfrac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))\big\}\Big\}\Bigg).$$

Proof. See Appendix C. □

For limit values of the parameters δ and k, the required level of heterogeneity $\overline{\alpha}$ is arbitrarily close to 1, as shown in Appendix C. In combination with Proposition 2, this shows that heterogeneity is crucial in understanding core periphery networks. Complete core periphery networks are never stable under homogeneous players, but can be stable for arbitrary small levels of heterogeneity.

Heterogeneity also affects the results of the best-response dynamics. Starting from an empty graph, complete core periphery networks can arise because players replicate the position of the central players and become intermediators for periphery players. If the relative size α is sufficiently large and condition VII of Proposition 3 is fulfilled, the result will be unilaterally stable. Theorem 3 below specifies the seven possible attracting stable networks for the special case of $n = 4$ and $k = 2$ big banks. In this dynamic process, it is assumed that the k large banks are the first in the round-robin order. Given $n = 4$, $k = 2$ and any choice of the other parameters c, δ and α (except for borderline cases), the theorem shows that the dynamics converge to a unique unilaterally stable network.[13]

Theorem 3. *Consider the model with $n = 4$, of which $k = 2$ big banks having size $\alpha > 1$ and $n - k = 2$ small banks having size 1. From an empty graph, the round-robin best-response dynamics starting with the two big banks converge to the following unilaterally stable equilibria:*

I: *for* $c > \max\left\{\tfrac{1}{2}\alpha^2, \tfrac{1}{6}\alpha^2 + \tfrac{5}{9}\alpha + \tfrac{1}{9}\right\}$
the empty network,

[13] Under heterogeneous players the results of the dynamic process depend on the order in which agents make best responses. If agents are randomly drawn to make best responses, multiple attracting steady states may exist. Using simulations, we found that for certain parameter values in VII a multipartite ring network can arise. However, for a large subset of the parameter region VII, the complete core periphery remains the unique attracting steady state even under a random order of agents.

II: *for* $c \in \left(\frac{1}{6}\alpha + \min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}, \min\left\{\frac{5}{6}\alpha + \frac{1}{6}, \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}\right\}\right)$
the star network,

III: *for* $c < \frac{1}{2} - f_e(2,\delta)$
the complete network,

IV: *for* $c \in \left(\min\left\{\alpha^2(\frac{1}{2} - f_e(2,\delta)), \frac{1}{6}\alpha + f_m(2,\delta), \frac{1}{6}\alpha + f_e(2,\delta) - \frac{1}{3}\right\}, \frac{1}{6}\alpha + \min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}\right)$
the multipartite (ring) network $g^{mp(2)}_{2,2}$

V: *(other multipartite networks do not exist for* $n = 4$*),*

VI: *for* $c \in \left(\min\left\{\frac{5}{6}\alpha + \frac{1}{6}, \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}\right\}, \max\left\{\frac{1}{2}\alpha^2, \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}\right\}\right)$
the 'single pair' network g *with* $g_{12} = 1$ *and* $g_{ij} = 0$ *for all* $(i, j) \neq (1,2)$*,*

VII: *for* $c \in \left(\frac{1}{2} - f_e(2,\delta), \min\left\{\alpha^2(\frac{1}{2} - f_e(2,\delta)), \frac{1}{6}\alpha + f_m(2,\delta), \frac{1}{6}\alpha + f_e(2,\delta) - \frac{1}{3}\right\}\right)$
the complete core periphery network $g^{CP(2)}_{com}$
(other core periphery networks do not exist for $n = 4$*).*

Proof. See Appendix C. □

Figure 8 illustrates the different network outcomes for $n = 4$, $k = 2$ and two levels of heterogeneity $\alpha \in \{1.5, 2\}$. Theorem 3 introduces a new (simple) type of network, called a 'single pair' network, in which only the two big banks are linked and the small banks have no connections. The parameter region VI is nonempty if the level of heterogeneity is sufficiently high: $\alpha > \frac{1}{6}(5 + \sqrt{37}) \approx 1.85$. As observed in Figure 8a, for $\alpha = 1.5$ the regions of empty and star networks share a border in the (δ, c)-diagram: $c = \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}$. For a larger value like $\alpha = 2$ in Figure 8b, the single pair network arises under condition VI. This type of network structure in which only part of the nodes is connected intuitively arises because some connections are more worthwhile.

Let us now discuss the main network outcome of interest, namely core periphery networks. The condition under which the complete core periphery network is the attracting steady state is in Figure 8 indicated by the shaded regions. As expected, this region increases with the level of heterogeneity α. Also observe that for complete core periphery networks to arise it is necessary that competition is less than fully perfect, i.e. $\delta < 1$. For the special case of $\delta = 1$ (as considered by Goyal and Vega-Redondo, 2007) core periphery networks are never unilaterally stable, not even under large heterogeneity. Finally, complete core periphery networks arise for a larger set of parameters if the number of players

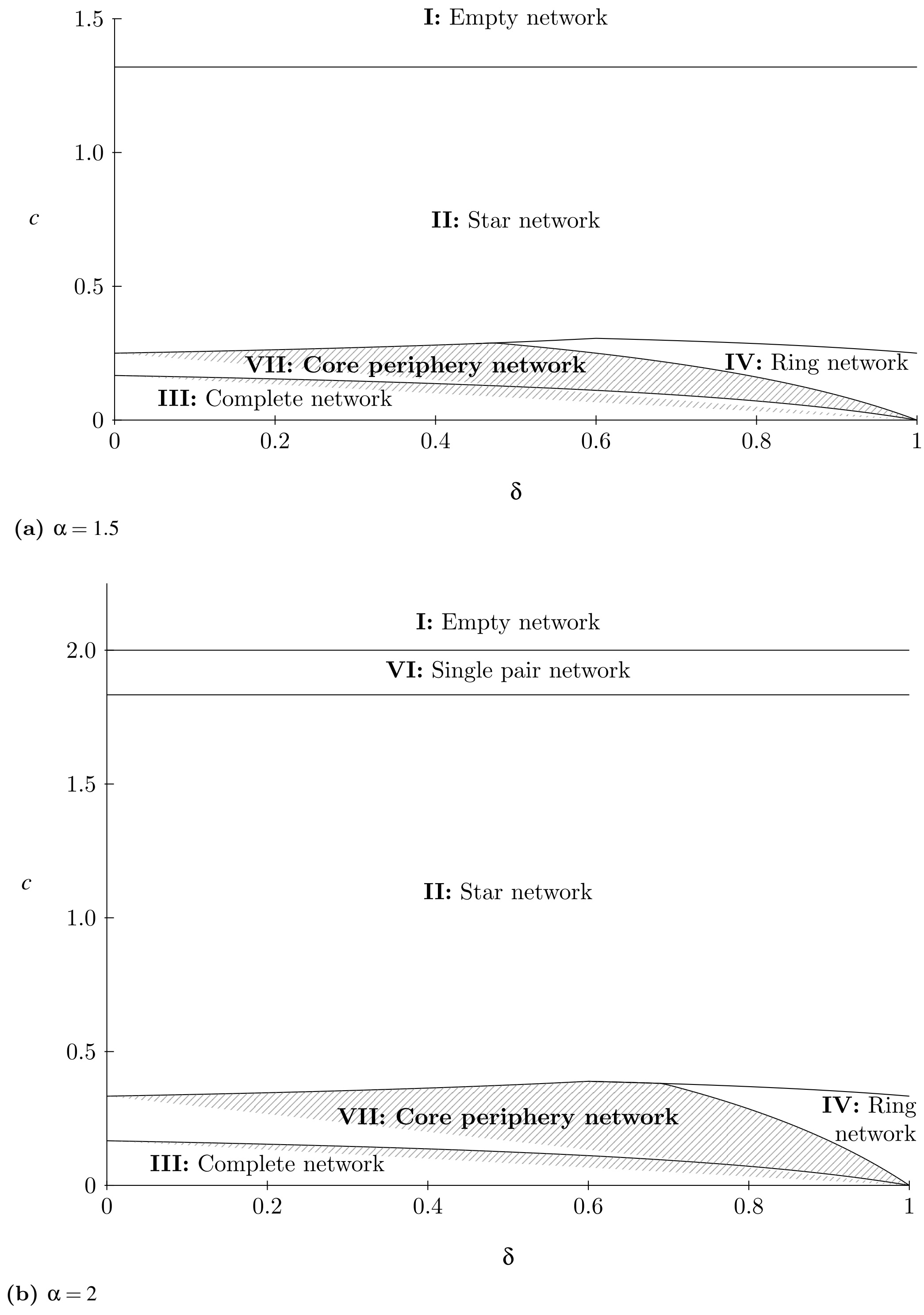


Figure 8: Attained equilibria after best-response dynamics from an empty graph in (δ, c)-space for $n = 4$, $k = 2$ and $\alpha \in \{1.5, 2\}$ under the specification of f_e and f_m in equation (2). The roman numbers correspond with those in Theorem 3. In the shaded area, the complete core periphery network with $k = 2$ big banks is the unique unilaterally stable outcome of the dynamics.

is larger than the minimal network size of $n = 4$. See Figure 9b for the shaded region VII given $n = 100$, $k = 15$ and $\alpha = 10$.

6. Applying the model to the Dutch interbank market

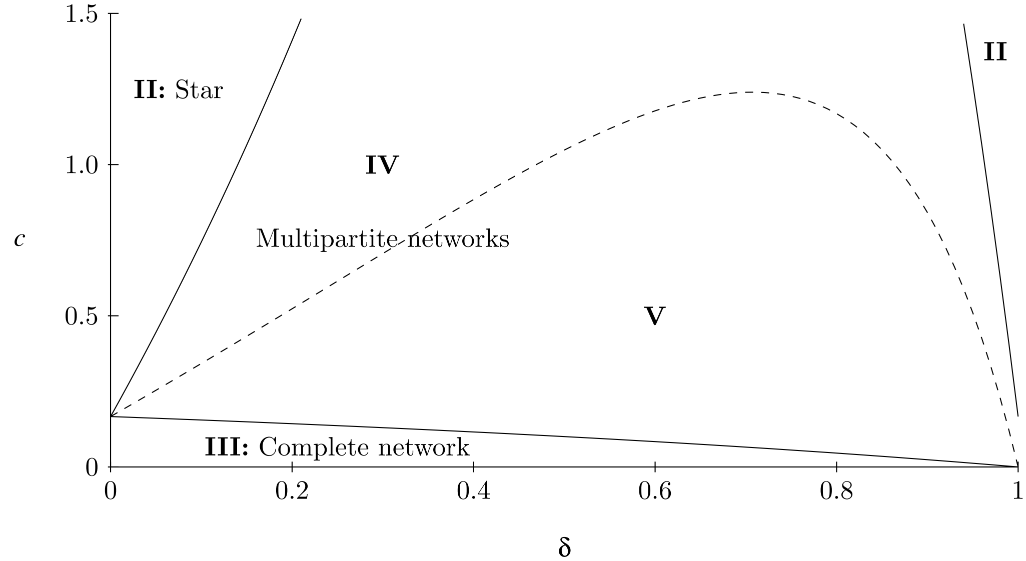
To gain some insight into the general applicability of the model, we calibrate the model to the Dutch interbank market. Van Lelyveld and in 't Veld (2012) investigate the network structure of the interbank market in the Netherlands, a relatively small market with approximately a hundred financial institutions. The attracting networks in the dynamic homogeneous model with $n = 100$ are indicated in Figure 9a. Under homogeneous banks the model predicts multipartite networks for most parameter values. In contrast, Van Lelyveld and in 't Veld (2012) found that the observed network contains a very densely connected core with around $k = 15$ core banks.

We choose a level of heterogeneity $\alpha = 10$ to capture in a stylised way the heterogeneity of banks in the Netherlands. In reality banks in the core as well as in the periphery of the Dutch banking system are quite diverse. A few very large banks reach a total asset size of up to €1 *trillion*, while the asset value of some investment firms active in the interbank market may not be more than a few million euro. The median size of a core bank in Van Lelyveld and in 't Veld (2012) lies around €8 billion. For periphery banks the median size is approximately €300 million (see Figure 11 in Van Lelyveld and in 't Veld, 2012, for the plotted distribution of total asset size over core and periphery banks). Ignoring the exceptionally large size of some core banks, a relative difference of $\alpha = 10$ seems a reasonable order of magnitude.

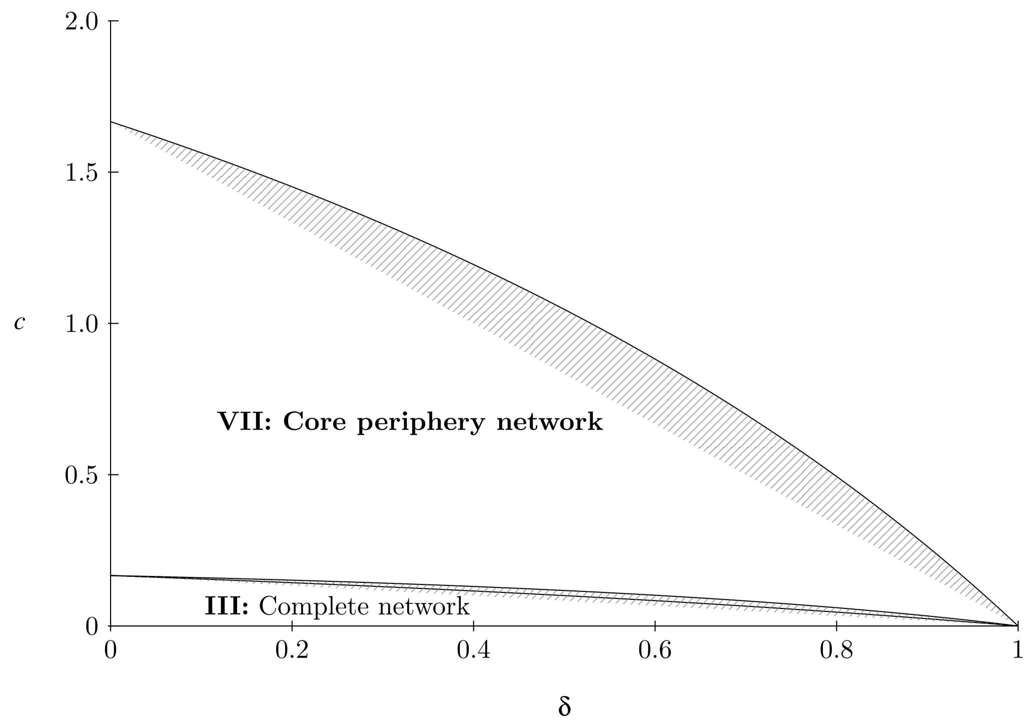
Figure 9b shows the parameter region given by Proposition 3 for which a complete core periphery network of the fifteen big banks is a unilaterally stable network. The stability of the complete network does not depend on α as also indicated in Figure 9b. These complete and complete core periphery networks would also be the outcome of best-response dynamics starting with the big banks.[14] The observed core periphery structure in the Netherlands can be reproduced for many reasonable choices of linking costs c and competitiveness δ.

This application suggests that our model is very suitable to explain stylised facts of national or perhaps even international interbank networks. It should be noted that observed core periphery networks are not necessarily complete core periphery networks, which can

[14] A full description of the outcomes of these best-response dynamics would require a generalisation of Theorem 3 for all $n \geq 4$. Simulations show that for linking costs so high that CP networks are not stable, many different structures arise. As none of them are core periphery networks, we restrict ourselves to Proposition 3 and Theorem 3.



(a) Homogeneous banks ($\alpha_{ij} = 1$ for all $i, j \in N$)



(b) Heterogeneous banks: $k = 15$ big banks (with relative size $\alpha = 10$) and $n - k = 85$ small banks

Figure 9: Application of the model to the Dutch interbank market. Attained equilibria after best-response dynamics from an empty graph in (δ, c)-space for $n = 100$, $k = 15$ and $\alpha \in \{1, 10\}$ under the specification of f_e and f_m in equation (2). The roman numbers correspond with those in Theorem 2 and Proposition 3. In the shaded area, the complete core periphery network with $k = 15$ big banks is the unique unilaterally stable outcome of best response dynamics starting with the large banks.

be explained as follows. Empirical studies of interbank markets often rely on either balance sheet data measuring the total exposure of one bank on another, or overnight loan data specifying the actual trades. We interpret the undirected links in our model as established preferential lending relationships, which are typically difficult to observe directly in practice. Given a theoretically complete core periphery network of lending relationships, trades and exposures are executed on the same structure of connections; see Appendix A for the interpretation of trade surpluses in our model. The empirically observed core periphery structure can have less than complete connections between core and periphery depending on the realisations of trade opportunities. In any case, the densely connected core of a subset of the banks is a well-documented empirical fact that is reproduced by our model.

7. Conclusion

In this paper we have proposed a way to explain the formation of financial networks by intermediation. We have focused on the core periphery network because it is found to give a fair representation of the complex empirical structures, while at the same time being relatively simple and intuitively appealing. In our model brokers strive to intermediate between their counterparties and compete with each other. Our results suggest that heterogeneity is crucial, that is, the core periphery structure of the interbank network cannot be understood separately from the heterogeneity and inequality in the intrinsic characteristics of banks.

Naturally our analysis can be extended in many ways. It seems interesting to endogenize heterogeneity in our model by updating the size of each bank with the payoffs received from trades in the network. Better connected banks receive higher payoffs by intermediation, which could feed back on the balance sheet and thus on future trade opportunities of these banks. This is consistent with a recent empirical finding by Akram and Christophersen (2012) that banks with many financial linkages in the Norwegian interbank market face lower interest rates. We can expect that core periphery networks also arise endogenously in the extended model with ex ante homogeneous agents and feedback of the network structure on trade surpluses.

Another important extension is to introduce default probabilities in order to understand network formation in stress situations. This seems relevant in light of findings that the fit of the core periphery network in the interbank market deteriorated during the recent financial crisis (Van Lelyveld and in ’t Veld, 2012; Fricke and Lux, 2014). The present paper provides an economic model for financial networks with high empirical relevance, and suggests a potentially fruitful road of finding policies reducing systemic risk.

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Appendix A. Trade surpluses in the interbank market

Our paper uses a stylized model where transactions are executed between every pair of agents. We now give a formal interpretation of the trade surpluses α_{ij} in a simple two-period framework. In the first period, banks establish relationships in the interbank market to insure themselves against liquidity shocks. In the second period, after the shocks have been revealed, each bank uses these relationships to rebalance its liquidity position. If it is unable to do so in the interbank market, the bank has to trade with the central bank under less favorable conditions. The value α_{ij} is the ex ante surplus of the trade relationship between i and j.

We consider a bank i that has a preferential lending relationship with bank j. Each bank i faces exogenous future liquidity shocks x_i: x_i positive implies bank i has excess liquidity and x_i negative implies that bank i has a liquidity shortage. Assume that the CB offers lending and borrowing facilities to banks at rates $\overline{p}$ and $\underline{p}$. Without interbank transactions, the values attributed to shocks are:

$$\Pi_i^{CB} = \begin{cases} \underline{p}x_i > 0 & \text{if } x_i \geq 0 \\ \overline{p}x_i < 0 & \text{if } x_i < 0 \end{cases} \tag{A.1}$$

These CB facilities represent the outside options of banks.

Using its relationship with j, bank i can try to receive more favorable conditions for lending and borrowing, by agreeing on a rate $p > \underline{p}$ when he had a positive liquidity shock, or $p < \overline{p}$ in case he is short of liquidity. Conversely, bank j would also be happy to trade for some $p \in (\underline{p}, \overline{p})$, as long as the shocks are of opposite sign. In this case the ex post value of the trade surplus between i and j is therefore:

$$TS_{ij} = (\overline{p} - \underline{p}) \min\{|x_i|, |x_j|\} \tag{A.2}$$

For simplicity, assume that two random banks receive a shock of size 1 and that these shocks are of opposite sign. When the shocks hit banks i and j, these two banks can trade and create a total surplus of $(\overline{p} - \underline{p})$. Assume that the probability of receiving a shock for i is λ_i. The total ex ante value of a trading relationship between i and j is then:

$$\alpha_{ij} = \lambda_i \lambda_j (\overline{p} - \underline{p}) \tag{A.3}$$

In the baseline model with homogeneous banks, we have imposed that probabilities of receiving liquidity shocks are equal for all banks ($\lambda_i = \lambda_j \ \forall \ i, j$), and normalized α_{ij} to 1. For heterogeneous banks, the assumptions on trade surpluses α_{ij} made in Section 5 boils down to assuming that a big bank i has a higher probability of receiving liquidity shocks than a small bank j in a fixed proportion $\lambda_i = \alpha \cdot \lambda_j$. Normalizing the surplus of two small banks j and j' to $\alpha_{jj'} = 1$, gives that the higher trade surplus between one big bank i and one small bank j is $\alpha_{ij} = \alpha$ and between big banks i and i' is $\alpha_{ii'} = \alpha^2$.

Appendix B. Siedlarek's payoff function

Siedlarek (2012a) applies the bargaining protocol introduced by Merlo and Wilson (1995) to derive the distribution of a surplus for a trade that is intermediated by competing middlemen. The underlying idea is that one involved agent is selected to propose a distribution of the surplus. If the proposer succeeds in convincing the other agents to accept his offer, the trade will be executed. Otherwise, the trade will be delayed and another (randomly selected) agent may try to make a better offer. A common parameter $0 \leq \delta \leq 1$ is introduced with which agents discount future periods, that results in the level of competition as used in our paper. As a special case, for $\delta = 1$ the surplus is distributed equally among the essential players as in Goyal and Vega-Redondo (2007).

More formally, assume that the set of possible trading routes is known to all agents (i.e. complete information). Each period a route is selected on which the trade can be intermediated, and additionally one player (the 'proposer') on this route that proposes an allocation along the entire trading route.[15] Any state, which is a selection of a possible path and a proposer along that path, is selected with equal probability and history independent. The question now is: what is the equilibrium outcome, i.e. the expected distribution of the surplus, taking into account that every agent proposes optimally under common knowledge of rationality of other possible future proposers? Siedlarek (2012a) shows that the unique Markov perfect equilibrium is characterized by the following payoff function for any player i in a certain state:

$$f_i = \begin{cases} 1 - \sum_{j \neq i} \delta E_j[f_j] & \text{if } i \text{ is the proposer in this state} \\ \\ \delta E_i[f_i] & \text{else if } i \text{ is involved in this state} \end{cases} \tag{B.1}$$

This equation shows that the proposer can extract all surplus over and above the outside option value given by the sum of $E_j[f_j]$ over all other players $j \neq i$. All (and only) the players along the same route have to be convinced by offering exactly their outside option.

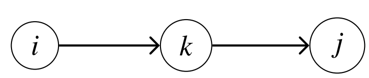


Figure B.10: A trading route with one intermediary

[15] Siedlarek (2012a) assumes that only *shortest* paths are considered, an assumption not explicitly made by Goyal and Vega-Redondo (2007). However, in the model Goyal and Vega-Redondo (2007) only essential players, who by definition are part of shortest paths, receive a nonzero share of the surplus.

In the simple example of intermediary k connecting i and j (see Figure B.10), each of the three players has equal probability of becoming the proposer and proposes an equal share to the other two, so the equilibrium distribution of the surplus will simply be the equal split $(f_i, f_k, f_j) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ for all δ.

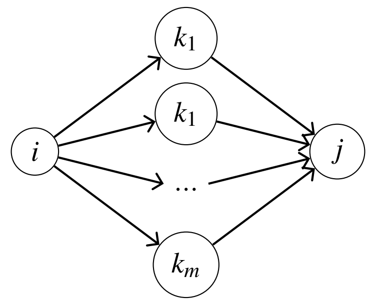


Figure B.11: A trading network with m competing intermediaries

In case m intermediaries compete for the trade between i and j (as in Figure B.11), the extracted intermediation rents are reduced. Siedlarek (2012a) shows that:

$$f_i = f_j = f_e(m,\delta) \equiv \frac{m-\delta}{m(3-\delta)-2\delta} \tag{B.2}$$

$$f_k = f_m(m,\delta) \equiv \frac{1-\delta}{m(3-\delta)-2\delta} \tag{B.3}$$

As mentioned before, this distribution of payoffs satisfies our assumptions on $f_e(m,\delta)$ and $f_m(m,\delta)$. The distribution of Siedlarek (2012a) is used as the leading example in the results. All results hold for general $f_e(m,\delta)$ and $f_m(m,\delta)$ satisfying the assumptions of Table 1.

In a general network with longer intermediation chains $d > 2$, the payoffs cannot be calculated directly, but depend on specifics of (part of) the network. The vector of payoffs $\overrightarrow{F}$ can be calculated indirectly from the network g using the following notation:

- s: The number of shortest paths that support the trade,
- d: The length of the shortest paths (i.e. the number of required intermediaries plus one),
- $\overrightarrow{P}$: $(n \times 1)$-vector of shortest paths that run through any player,
- K: $(n \times n)$-diagonal matrix of times that any player receives an offer,
- S: $(n \times n)$-off-diagonal matrix of times that two players share a path.

As in any given state the distribution of profits is as given in equation (B.1), the vector of payoffs that averages over all possible states of the world is:

$$\overrightarrow{F} = \frac{1}{s \cdot d}(\overrightarrow{P} - \delta(S-K)\overrightarrow{F}) \tag{B.4}$$

And if $\delta < 1$ this can be solved to:

$$\overrightarrow{F} = (s \cdot d \cdot I_n + \delta(S-K))^{-1}\overrightarrow{P} \tag{B.5}$$

In Appendix D we use this general formula to investigate whether core periphery networks can be stable when intermediation paths of lengths $d = 3$ are allowed.

Appendix C. Proofs

Proof of Proposition 2. The intuition of the second proof is *limited inequality between core and periphery players.* We will provide sufficient conditions under which at least one agent will be able improve its position in any (complete or incomplete) core periphery network.

(a) Complete core periphery networks. Consider the deviation of a core player i to delete one link to another core member. The marginal benefit of doing so is:

$$M_i(g_{com}^{CP(k)}, -1) = c - (\frac{1}{2} - f_e(n-2, \delta)) \tag{C.1}$$

The marginal benefit for a peripheral player j to add a link in the periphery is:

$$M_j(g_{com}^{CP(k)}, +1) = (\frac{1}{2} - f_e(k, \delta)) - c \tag{C.2}$$

In order for the network to be pairwise stable, it is required that $M_i(g_{com}^{CP(k)}, -1) < 0$ and $M_j(g_{com}^{CP(k)}, +1) < 0$, so:

$$\begin{array}{ccccc} \frac{1}{2} - f_e(k,\delta) & < & c & < & \frac{1}{2} - f_e(n-2,\delta) \\ f_e(n-2,\delta) & < & \frac{1}{2} - c & < & f_e(k,\delta) \end{array}$$

However, as $\frac{\partial f_e}{\partial k} > 0$ and $k < n-1$, this interval of c is empty. Complete core periphery networks are never pairwise stable, and therefore also not unilaterally stable.

(b) Incomplete core periphery networks. We will determine the benefits of a periphery player to propose links to all other periphery players. In Lemma 3 we showed that this is a best feasible action, if there exists a profitable deviation by adding links at all, in *complete*

core periphery networks. In a general core periphery network, adding to all other periphery players is not necessarily a best feasible action, but it will help to find the sufficient condition for instability of the networks, as claimed in the Proposition.

Consider a periphery player i proposing $l_i = n-k-1$ links in order to reach all other periphery players. The minimal marginal benefits for i of this action are bounded by:

$$M_i(g^{CP(k)}, +l_i) > -(n-k-1)c + \binom{n-k-1}{2} f_m(k+1,\delta). \tag{C.3}$$

This lower bound for the marginal benefits consists of two parts. The first part, $-(n-k-1)c$, denotes the direct costs for adding the l_i links and is linear in n. The second part, $\binom{n-k-1}{2} f_m(k+1,\delta)$, denotes the minimal intermediation benefits from becoming a new intermediator between the $n-k-1$ remaining periphery members and is quadratic in n. In general these intermediation benefits for i can be higher if there are fewer $m_{jl} < k$ intermediators in the original network $g^{CP(k)}$ between $j,l \in P$. Benefits from trade (direct or indirect) do not have to be considered because, obviously, adding links to the network can only increase the access from i to other players, and the benefits from trade therefore weakly increase.

Let the parameters $c > 0$, $0 < \delta < 1$ and $k \geq 1$ be given. Then the marginal benefit function (C.3) is strictly convex in n and will always become bigger than 0 for large enough n: the quadratic intermediation part of the expression will always dominate the linear cost part for strictly positive intermediation benefits $f_m(k+1,\delta) > 0$. If c is large M_i may be negative for small n, but eventually the intermediation benefits will become so large that the deviation will become beneficial. More precisely, positive marginal benefits for i imply:

$$\begin{aligned} M_i(g^{CP(k)}, +l_i) &> 0 && \text{(C.4)} \\ \Rightarrow c &< \frac{1}{2}(n-k-2) f_m(k+1,\delta) && \text{(C.5)} \\ \Leftrightarrow n &> F_i(c,\delta,k) \equiv k+2+\frac{c}{\frac{1}{2} f_m(k+1,\delta)} && \text{(C.6)} \end{aligned}$$

For the deviation of l_i links to be executed, the peripheral players $j \in (P \setminus i)$ also have to agree with the addition, that is, they should not receive a lower payoff. The marginal benefits for j depend on its number of connections n_j, but are bounded by:

$$M_j(g^{CP(k)}, +l_i) > -c + (n-k-2) \min_{n_j \leq k} \{ f_e(m+1,\delta) - f_e(m,\delta) \}. \tag{C.7}$$

The second part of these marginal benefits indicates that the number of intermediation paths from j to any other periphery member $l \in (P \setminus i, j)$ strictly increases, and therefore the benefits from trade strictly increase.[16] This function is similarly convex, and positive marginal benefits for j imply:

$$\begin{aligned} M_j(g^{CP(k)}, +l_i) &\geq 0 && \text{(C.8)} \\ \Rightarrow c &< (n-k-2) \min_{n_j \leq k} \{f_e(m+1,\delta) - f_e(m,\delta)\} && \text{(C.9)} \\ \Leftrightarrow n &> F_j(c,\delta,k) \equiv k+2+\frac{c}{\min_{n_j \leq k}\{f_e(m+1,\delta) - f_e(m,\delta)\}} && \text{(C.10)} \end{aligned}$$

Combining the conditions for i and j, a sufficient lower bound for the network size n for any core periphery network to be unstable is:

$$n > F(c,\delta,k) \equiv k+2+\frac{c}{\min\left\{\frac{1}{2} f_m(k+1,\delta), \min_{n_j \leq k}\{f_e(m+1,\delta) - f_e(m,\delta)\}\right\}} \qquad \text{(C.11)}$$

□

Remarks on Proposition 2. In the derivation of this sufficient lower bound on n, we have not made any assumptions about the path lengths on which trade is allowed. It is sufficient to consider the parts of the marginal benefits for i and j that depend on the constant costs c and the new intermediation route is formed between $j, l \in (P \setminus i)$.

A core periphery network is generally unstable because the inequality between core and periphery becomes large for increasing n, and a periphery player can always benefit by adding links to all other players. An important assumption for this result is therefore that multiple links can be added at the same time. It is crucial that $\delta > 0$ because for $\delta = 0$ intermediated trade always generates $f_e(m,0) = \frac{1}{3}\ \forall m$, i.e. additional intermediation paths do not increase profits for endnodes. Moreover it is crucial to impose $\delta < 1$ because for $\delta = 1$ intermediation benefits disappear, i.e. $f_m(m,1) = 0\ \forall m > 1$.

Proof of Theorem 2. By Lemma 1, after the move of player $i = 1$ the network is either empty or a star. Obviously, in case it is empty it is stable, as all other nodes face the same decision as $i = 1$. If it is a star network and none of the other nodes wants to add a subset of links, this is the final stable outcome. Otherwise, by Lemmas 2 and 3, each next node $i = 2, \ldots, k$ adds links to all nodes not yet connected to i. There is a third and final possibility that

[16] It is possible that player i pays intermediation benefits to $j \in P$ if it can access some $l \in C | g_{il} = 0$ better. The lower bound on M_j given in C.7 is sufficient for the proof.

the dynamic process is ended before the second round, namely if node $i = n-1$ fulfills the complete network by linking to node n. The conditions I, II and III for convergence to the empty, star or complete network coincide with the stability conditions of these three networks as in Proposition 1.

For parameters not satisfying I, II or III, the first round of best responses results in a complete core periphery network with $1 < k < n-1$ core members. This complete core periphery network is not stable by the part *(a)* of Proposition 2. Because the $k+1$-th node did not connect to other periphery nodes, adding links in the periphery cannot be beneficial. Therefore the first core bank $i = 1$ must have an incentive to delete at least one within-core link. The marginal benefit of deleting l core links in the network is:

$$M_i(g_{com}^{CP(k)}, -l) = l(c - \frac{1}{2} + f_m(n-l-1, \delta)). \tag{C.12}$$

As $M_i(g_{com}^{CP(k)}, -0) = 0$ and $M_i(g_{com}^{CP(k)}, -1) > 0$, there is a unique choice $l_1^* > 0$ of the optimal number of core links to delete. It will become clear that for parameters outside I $\cup$ II $\cup$ III, attracting networks are multipartite networks of various sorts, depending on the choice l_1^*.

An important case is that a complete core periphery network with $k = 2$ has arisen after the first round. For $k = 2$ the only possible solution is $l_1^* = 1$. A complete bipartite network arises with a small group of 2 players and a large group of $n-2$ players, the maximal difference in group size possible for bipartite networks. We denote such a network as $g_{2,n-2}^{mp(2)}$.

The case of $k = 2$ happens if the third player $i = 3$ does not enter the core, either because entering is not beneficial for himself or because some other periphery players j does not accept the offer of i. For $k = 2$ a positive marginal benefit of entering the core implies:

$$\begin{aligned} M_i(g_{com}^{CP(2)}, +(n-2)) &> 0; \\ \Rightarrow c &< \frac{1}{2} - f_e(2,\delta) + \frac{1}{2}(n-4) f_m(3,\delta), \end{aligned} \tag{C.13}$$

and the periphery player j has an incentive to accept the offer if

$$\begin{aligned} M_j(g_{com}^{CP(2)}, +(n-2)) &\geq 0; \\ \Rightarrow c &\leq \frac{1}{2} - f_e(2,\delta) + (n-4)(f_e(3,\delta) - f_e(2,\delta)); \end{aligned} \tag{C.14}$$

So if after the first round the result is $k = 2$ and the third player has not entered the core, it must be the case that (compare equation (9)):

$$c \geq \frac{1}{2} - f_e(2,\delta) + (n-4)\min\{\frac{1}{2} f_m(3,\delta), f_e(3,\delta) - f_e(2,\delta)\}. \tag{C.15}$$

The parameter values for which the maximally unbalanced network $g^{mp(2)}_{2,n-2}$ is the attracting steady state are given by condition IV.

Alternatively, under the remaining condition V, the first round has resulted in a complete core periphery network with $k > 2$. This network cannot be stable, and the first core bank $i = 1$ has an optimal choice of $0 < l^*_1 \leq k-1$ links to delete depending on the parameters. First consider that the core bank deletes all its within-core links, i.e. $l^*_1 = k-1$. This action necessarily implies that the linking costs exceed the loss in surplus associated with having an indirect connection to other core banks via the periphery rather than a direct connection:

$$c > \frac{1}{2} - f_e(n-k, \delta). \tag{C.16}$$

Given this high level of linking costs, the next core banks $i = \{2, ..., k\}$ have the same incentive to delete all their within-core links. The attracting network is therefore a complete multipartite network $g^{mp(2)}_{k_1,k_2}$ with two groups of size $k_1 = k$ and $k_2 = n-k$.

Finally, consider the case $0 < l^*_1 < k-1$.[17] Denote $K_1 \subset K$ as the set of core banks with at least one missing within-core link after the best response of $i = 1$, including players $i = 1$ itself. The size of this set of banks is $k_1 = l^*_1 + 1$. The best response of $i = 1$ necessarily implies:

$$c > \frac{1}{2} - f_e(n-k_1, \delta). \tag{C.17}$$

Given this high level of linking costs, all next banks $i \in K_1$ with connections to some other $j \in K_1$ with $j > i$ have the same incentive to delete every link g_{ij}. These banks do not have an incentive to remove any further link, because the number of intermediators for indirect connections to banks $j \in K_1$ would become less than $n - k_1$. If the latter were worthwhile, $i = 1$ would have chosen a higher number l^*_1. Therefore K_1 becomes a group of players not connected within their group, but completely connected to all players outside their group.

Players $i \in (K \setminus K_1)$ are connected to all other players $j \in N$. Because of the lower bound on c in equation (C.17), these players have an incentive to remove links to some j as long as the number of intermediators for the indirection connections to j stays above $n - k_1$. This will lead to other groups $K_2, ..., K_{q-1}$ of players not connected within their group, but completely connected to all players outside their group. The remaining group K_q was the

[17]This is the only case in which the best response is not unique in a parameter region with nonzero measure: the first core banks $i = 1$ has $\binom{k}{l^*_1}$ best responses deleting l^*_1 links from the core. The resulting multipartite networks can consist of different sets $K_1, K_2, ..., K_{q-1}$, but all are isomorphic.

original periphery at the end of the first round. The result of the best-response dynamics if $0 < l_1^* < k-1$ is a multipartite network $g^{mp(q)}_{k_1,k_2,\dots,k_q}$ with $q \geq 3$.

For all parameters under condition V, the resulting network is multipartite with $q = \lfloor \frac{k}{k_1} \rfloor + 1$ groups. These multipartite networks are more balanced than $g^{mp(2)}_{2,n-2}$, i.e. have group sizes $|k_m - k_{m'}| < n-4$ for all $m, m' \in \{1,2,..,q\}$. $\square$

Proof of Proposition 3. We will derive a $\overline{\alpha}$, such that for all $\alpha > \overline{\alpha}$ the complete core periphery network with k big banks is unilaterally stable. First observe that no player wants to add any links. In a complete core periphery network only periphery players can add links, but given a sufficiently high c satisfying (15) this is not beneficial. We therefore only need to consider the possible deletion of one or multiple links by either core or periphery players.

First, consider a core player i. Player i can delete links with other core players and/or links with periphery players. The marginal benefit of deleting l^c core links and l^p periphery links is:

$$\begin{aligned} M_i(g^{CP(k)}_{com}, -(l^c+l^p)) &= l^c\Big(c - \alpha^2\big(\frac{1}{2} - f_e(n-l^c-l^p-1,\delta)\big)\Big) \\ &\quad + l^p\Big(c - \alpha\big(\frac{1}{2} - f_e(k-l^c-1,\delta)\big) - (2n-2k-l^p-1)f_m(k,\delta)\Big) \\ &\equiv M_i^c + M_i^p \end{aligned} \tag{C.18}$$

The marginal benefit of deleting $l^c + l^p$ links can be separated in benefits from deleting links with the core M_i^c and benefits from deleting links with the periphery M_i^p. The cross-over effects of deleting links with both groups of banks are negative: M_i^c is decreasing in l^p and M_i^p is decreasing in l^c. To find the conditions under which the core player does not want to delete any link, it is therefore sufficient to consider deletion of links in each group separately.

The marginal benefit of deleting l^c core links is:

$$M_i(g^{CP(k)}_{com}, -l^c) = l^c\Big(c - \alpha^2\big(\frac{1}{2} - f_e(n-l^c-1,\delta)\big)\Big) \tag{C.19}$$

If $M_i(g^{CP(k)}_{com}, -l^c) > 0$, it must hold that $M_i(g^{CP(k)}_{com}, -1) > 0$, because $f_e(n-l^c-1,\delta)$ decreases in l^c. Player i thus has a beneficial unilateral deviation if

$$\begin{aligned} & M_i(g^{CP(k)}_{com}, -l^c) > 0 \\ \Rightarrow\quad & M_i(g^{CP(k)}_{com}, -1) > 0 \\ \Leftrightarrow\quad & \alpha < \sqrt{c/(\frac{1}{2} - f_e(n-2,\delta))} \end{aligned} \tag{C.20}$$

The marginal benefit for a core player i of deleting l^p links with the periphery is :

$$M_i(g_{com}^{CP(k)}, -l^p) = l^p\Big(c - \alpha\big(\frac{1}{2} - f_e(k-1,\delta)\big) - (2n-2k-l^p-1)f_m(k,\delta)\Big) \tag{C.21}$$

If $M_i(g_{com}^{CP(k)}, -l^p) > 0$, it must be a best feasible action to choose $l^p = n-k$ and delete all links with the periphery, as the function is convex in l^p. Player i thus has a beneficial unilateral deviation if

$$\begin{aligned} & M_i(g_{com}^{CP(k)}, -l^p) > 0 \\ \Rightarrow \quad & M_i(g_{com}^{CP(k)}, -(n-k)) > 0 \\ \Leftrightarrow \quad & \alpha < \frac{c - \frac{1}{2}(n-k)(n-k-1)f_m(k,\delta)}{\frac{1}{2} - f_e(k-1,\delta)} \end{aligned} \tag{C.22}$$

Second, consider a player $i \in P$ in the periphery. This marginal benefit for a periphery bank $i \in P$ to delete l links is positive if:

$$\begin{aligned} & M_i(g_{com}^{CP(k)}, -l) = l\Big(c - \alpha\big(\frac{1}{2} - f_e(k-l,\delta)\big)\Big) - (n-k-1)(f_e(k,\delta) - f_e(k-l,\delta)) > 0 \\ \Leftrightarrow \quad & \alpha < \frac{c - \frac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))}{\frac{1}{2} - f_e(k-l,\delta)} \end{aligned} \tag{C.23}$$

The network is unilaterally stable if neither of these three deviations is beneficial, i.e. if α exceeds all values given in equations (C.20), (C.22) and (C.23):

$$\begin{aligned} \overline{\alpha} = \max\Big\{ \quad & \sqrt{c/(\frac{1}{2} - f_e(n-2,\delta))}, \frac{c - \frac{1}{2}(n-k)(n-k-1)f_m(k,\delta)}{\frac{1}{2} - f_e(k-1,\delta)}, \\ & \max_{l \le k}\Big(\frac{c - \frac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))}{\frac{1}{2} - f_e(k-l,\delta)}\Big)\Big\} \end{aligned} \tag{C.24}$$

By rewriting we get given a sufficiently large level of heterogeneity $\alpha > 1$ the following condition for unilaterally stable core periphery networks in terms of linking costs:

$$\begin{aligned} c \in \Bigg(\quad & \tfrac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\tfrac{1}{2}f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}, \\ & \min\Big\{ \quad \alpha^2(\tfrac{1}{2} - f_e(n-2,\delta)), \alpha(\tfrac{1}{2} - f_e(k-1,\delta)) + \tfrac{1}{2}(n-k)(n-k-1)f_m(k,\delta), \\ & \qquad \min_{l \le k}\big\{\alpha(\tfrac{1}{2} - f_e(k-l,\delta)) + \tfrac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))\big\}\Big\}\Bigg). \end{aligned}$$

$\square$

Remarks on Proposition 3. Notice that, for given c and δ and for n sufficiently large, the level of heterogeneity $\overline{\alpha}$ as given in (C.24) equals

$$\sqrt{c/(\frac{1}{2} - f_e(k-1,\delta))}. \tag{C.25}$$

For such a value of α, the complete core periphery network with k big banks in the core is unilaterally stable if condition (15) is satisfied. To minimise $\overline{\alpha}$ we take the smallest vale of c satisfying (15). A lower bound for $\overline{\alpha}$, given values of δ, k and sufficiently large n, is thus given by:

$$\overline{\alpha} \geq \overline{\alpha}_{min} \equiv \sqrt{\frac{\frac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\frac{1}{2}f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}}{\frac{1}{2} - f_e(k-1,\delta)}}. \tag{C.26}$$

Using the assumptions we made about the distribution of intermediated trades in Section 2.1, one can verify that

$$\lim_{k\to\infty} \overline{\alpha}_{min} = \lim_{\delta\downarrow 0} \overline{\alpha}_{min} = \lim_{\delta\uparrow 1} \overline{\alpha}_{min} = 1, \tag{C.27}$$

showing that an arbitrary small level of heterogeneity can be sufficient to have an unilaterally stable core periphery network.

Proof of Theorem 3. Starting in an empty network, the first big bank $i = 1$ has two relevant options: either connect to all players or connect only to big bank 2. Linking to only part of the small banks cannot be a best feasible action, because linking to an additional small bank pays off positive additional intermediation benefits (similar to the proof of Lemma 1). Player 1's payoffs depending on its action S are:

$$\pi_1(S) \quad = \begin{cases} 0 & \text{if } S = \emptyset \\ \frac{1}{2}\alpha^2 - c & \text{if } S = \{2\} \\ \frac{1}{2}\alpha^2 - c + 2(\frac{1}{2}\alpha - c) + 2\frac{1}{3}\alpha + \frac{1}{3} & \\ \quad = \frac{1}{2}\alpha^2 + \frac{5}{3}\alpha + \frac{1}{3} - 3c & \text{if } S = \{2,3,4\} \end{cases} \tag{C.28}$$

Under condition I, player 1's best feasible action is not to add any links. The resulting network is empty. In this case the network must be unilaterally stable. The reason is that second big bank faces the same decision as $i = 1$, and small banks have strictly lower payoffs from adding links, so no player will decide to change the network structure.

Under condition VI, the best feasible action is to add only one link to big bank 2. The second big can choose from the same resulting networks as $i = 1$ could, so does not add or remove links. Small banks have strictly lower payoffs from adding links and also do not change the structure. So under condition VI the resulting network with only one link, namely $g_{12} = 1$, is stable.

If the first player adds links to all three other players, a star network is formed. Then the best response for the second big bank $i = 2$ is either to do nothing or to add links to both periphery players. If 2 does nothing, the periphery nodes 3 and 4 will likewise decide not to add links, and the star network is the final, stable outcome. For 2 to have positive marginal benefits of supporting two links, it is required that

$$\begin{aligned} M_2(g^s, +2) &> 0 \\ \Rightarrow c &< \frac{1}{6}\alpha + \frac{1}{2} f_m(2, \delta), \end{aligned} \tag{C.29}$$

and each peripheral player $j \in \{3, 4\}$ requires

$$\begin{aligned} M_j(g^s, +2) &\geq 0 \\ \Rightarrow c &\leq \frac{1}{6}\alpha + f_e(2, \delta) - \frac{1}{3}. \end{aligned} \tag{C.30}$$

Conversely, the star network is stable if the deviation of player 2 is *not* beneficial to 2 and/or a peripheral player j, i.e. if c exceeds the minimum of the two values in equations (C.29) and (C.30):

$$c \geq \frac{1}{6}\alpha + \min\{\frac{1}{2} f_m(2, \delta), f_e(2, \delta) - \frac{1}{3}\}, \tag{C.31}$$

leading to condition II.

Consider the case that both big banks have added all possible links. Then there is a possibility that the dynamic process lead to a complete network if node $i = 3$ links to 4. This happens if c is so small that the share $f_e(2, \delta)$ from intermediated trade between 3 and 4 can be raised to $\frac{1}{2}$ by creating a direct link. The complete network is therefore stable under condition III.

For parameters not satisfying I, II, III or VI, the first round of best responses results in a complete core periphery network with 2 core members. This complete core periphery network can be stable because of the heterogeneity between core and periphery banks with $\alpha > 1$. As stated in proposition 3, this occurs under condition VII. For $n = 4$ and $k = 2$

condition VII reduces to:

$$\begin{aligned} c \in \Bigg(& \tfrac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\tfrac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}, \\ & \min\Big\{ \alpha^2(\tfrac{1}{2} - f_e(n-2,\delta)), \alpha(\tfrac{1}{2} - f_e(k-1,\delta)) + \tfrac{1}{2}(n-k)(n-k-1) f_m(k,\delta), \\ & \qquad \min_{l \le k}\{\alpha(\tfrac{1}{2} - f_e(k-l,\delta)) + \tfrac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))\}\Big\}\Bigg) \\ \in & \left(\tfrac{1}{2} - f_e(2,\delta), \min\{\alpha^2(\tfrac{1}{2} - f_e(2,\delta)), \tfrac{1}{6}\alpha + f_m(2,\delta), \tfrac{1}{6}\alpha + f_e(2,\delta) - \tfrac{1}{3}\}\right). \end{aligned}$$

Notice that when this condition is fulfilled, the second player always adds the two links to the periphery (cf. condition (C.31)).

Finally, in the remaining region IV, the complete core periphery network cannot be stable. Because the node $i = 3$ node did not connect to the last periphery node, adding links cannot be a best response. Therefore the first core bank $i = 1$ must have an incentive to delete the link with 2. The attracting steady state is the multipartite (ring) network consisting of the groups $\{1,2\}$ and $\{3,4\}$. □

Appendix D. Longer intermediation chains

In this appendix we generalize the model to allow for intermediation paths of lengths longer than two. For simplicity, we will analyze the general model for lengths up to distance three and under homogeneity (i.e. $\alpha_{ij} = 1$ for all i, j). These simplifications make it possible to check that our main result – a core periphery structure can form in *heterogeneous* financial networks only – holds in the most general model.

Before rewriting a more general form of the payoff function (1) formally, we repeat that the proof of Proposition 2 does not require any assumptions on the path lengths on which trade is allowed; see the remarks in Appendix C on this proposition. Proposition 2 states that complete core periphery networks are not pairwise (or unilaterally) stable, and that incomplete core periphery networks are not unilaterally stable if n is sufficiently large. Also Proposition 3, specifying a level of heterogeneity sufficient for a complete core periphery network to be unilaterally stable, holds for longer intermediation paths.

We introduce new, more general notation for intermediation over longer path lengths: $F_e(g,\{i,j\},\delta)$ denote the shares for the endnodes in the pair i and j; and $F_m(g,\{i,j,k\},\delta)$ denotes how much middleman k receives. In Appendix B it is explained how such an distribution can be derived for long intermediation chains in the example of Siedlarek (2012a). Note that if i and j at length three are assumed to generate a surplus, middlemen involved in a trade between i and j do not necessarily earn the same: if k lies on more of the shortest paths than k', k will earn more than k'. For this reason (part of) the graph g must be given as an argument in the function F_e and F_m. The payoff function becomes:

$$\pi_i(g) = n_i(\frac{1}{2} - c) + \sum_{j \in N_i^3(g)} F_e(g, \{i, j\}, \delta) + \sum_{\substack{k,l \in N_i^2(g) \\ g_{kl}=0, d_{kl} \leq 3}} F_m(g, \{k, l, i\}, \delta), \tag{D.1}$$

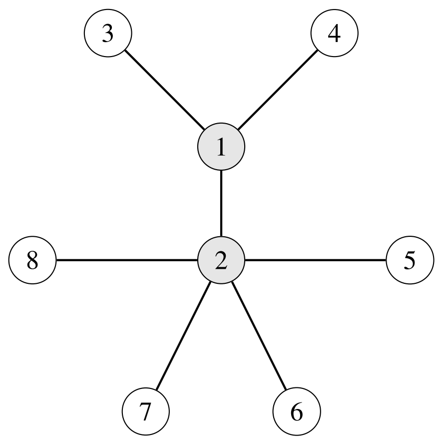
where $N_i^r(g)$ denotes the set of nodes at distance r from i in network g, $n_i = |N_i^1(g)|$ the number of direct connections of i, and d_{kl} the distance between nodes k and l.

First note that the results for the stability of the star network does not depend on the assumption of maximum intermediation chains of two. Obviously, in the star all pairs are at distance one or two of each other. If links to one or more periphery players are deleted, the distance to them becomes non-defined (because they become isolated nodes outside the main component). If links are added, the distance between pairs of nodes can only decrease. So considerations of intermediation chain of length three do not play a role for the star.

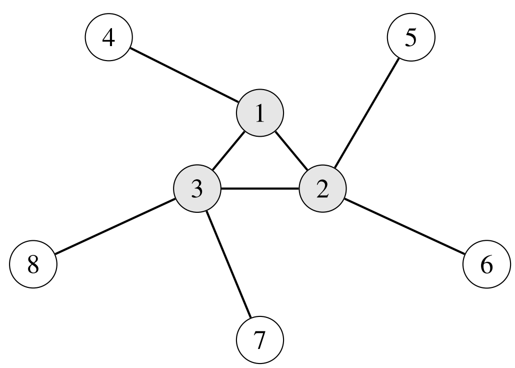
In incomplete core periphery networks, shortest paths of three may exist between some periphery players, which were previously assumed not to generate any trading surplus. By allowing intermediation chains of three we found that some core periphery networks can become unilaterally stable. For $n = 8$, we found that $k = 2$ and $k = 3$ are the only possibly stable core sizes. See Figure D.12 for two examples of networks that are stable for the given parameter values when paths of three are allowed. These two network structures are not stable in the baseline model.

We can safely interpret these examples as low-dimensional exceptions to the rule that the core periphery structure in homogeneous networks is generally unstable. The examples in Figure D.12 show that for $n = 8$ incomplete core periphery networks with core sizes of $k = 2$ and $k = 3$ can be stable. For higher network sizes, however, core periphery networks are always unstable as stated by Proposition 2.

Moreover, even though exceptionally for small n core periphery networks can be unilaterally stable, they are never the outcome of a dynamic process as described in Subsection 4.3. For $n = 8$, the core periphery networks with $k = 2$ and $k = 3$ were found to be stable in parameter regions where the star networks is stable, cf. region II in Figure 7b. Exploring the parameter space by simulations, we found that this was always the case for such stable core periphery networks. By Lemma 1, the star is created as a first step in the dynamic process whenever the initial empty network is not stable. Therefore the star network is the outcome of a dynamic process even when exceptional (low-dimensional) networks are



(a) $(\delta, c) = (0.8, 1.1)$. A minimally connected core periphery network with $k = 2$.



(b) $(\delta, c) = (0.5, 0.9)$. A minimally connected core periphery network with $k = 3$.

Figure D.12: Examples of unilaterally stable core periphery networks after allowing for intermediation chains of length 3, for $n = 8$ players and given (δ, c).

unilaterally stable as well given the parameter values. This implies that the dynamic results of Theorem 2 do not depend on the assumption of maximal intermediation paths of length two, as was already shown for the static results of Propositions 2 and 3.