Externalities and Contagion in Banking Networks

Regina Martinez
George Washington University

ram76@gwu.edu

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Hamilton (1781), 1st Secretary of the Treasury, said:

”Banks are the happiest engines that ever were invented for spurring economic growth”

- Mobilize resources from savers to investors
- Deal with information asymmetries
- Provide liquidity and manage risk

But when things go wrong...

- Systemic banking crises can have devastating effects
► Last global financial crisis

► **Systemic risk** is of critical importance (Bernanke, 2011)

► **Contagion** is at the core of the systemic risk concept

► Trade-off in **interconnected** systems:
  ▶ individual incentives
  ▶ systemic implications
Goal of the paper

Propagation of a bank’s failure through a bank network

Two research questions:

▷ What is the relation between contagion and network structure?

▷ Which networks emerge when banks choose their links strategically?
Why is it relevant?

- Need to better understand systemic financial crises
- Consequences of higher interconnectivity
- No consensus about the effects of financial interconnections
  - Destabilizing force?
    [Vivier-Lirimont, 2004; Blume et al., 2011; Battiston et al., 2012]
  - Allow risk diversification and shock absorption?
    [Allen and Gale, 2000; Freixas et al. 2000]
This paper finds:

1. Propagation of a bank’s failure depends on\(^1\):
   - Financial parameters: determine the “critical connectivity”
   - Network structure: affects the distribution of the number of failures

Which network structures emerge when banks form links strategically?

2. Banks tend to form asymmetric structures
   - with highly connected components and disconnected nodes.
   - Presence of externalities

\(^1\)This part of the paper is coauthored with Gupta et al, 2013
Outline of the Presentation

1. Contagion analysis

2. Simulation Results

3. Strategic network formation

4. Conclusion
Contagion analysis

► Simple model of a banking network in which:
  ▶ Banks (nodes) are connected through interbank loans (links)
  ▶ One bank is shocked exogenously
  ▶ Network structure is given

► I examine the spread of the failure due to counterparty risk

► Extension of Acemoglu et al. (2015):
  ▶ They analyze complete and ring networks
  ▶ My departure: this paper study less stylized structures
The Model

► Main features:
  ▶ 3-period economy
  ▶ One single good
  ▶ Two agents: banks and depositors
  ▶ System of $N$ risk neutral and ex-ante identical banks
  ▶ Banks are endowed with equity but can’t use it to invest
  ▶ They need to borrow from depositors or from other banks
  ▶ Interbank lending generates the network: weighted and directed
    ▶ $l_{ij}$ is the loan amount that $i$ lends to $j$
    ▶ $l_{ij} \neq l_{ji}$
Timing of the model:

At $t = 0$

- Banks borrow from depositors and from each other
- Banks make their external investment

<table>
<thead>
<tr>
<th>ASSETS ($A_i$)</th>
<th>LIABILITIES ($L_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash, $c_i$</td>
<td>Deposits, $d_i$</td>
</tr>
<tr>
<td>Investments, $y_i$</td>
<td>Interbank Borrowings, $b_i$</td>
</tr>
<tr>
<td>Interbank loans, $l_i$</td>
<td>Equity, $e_i$</td>
</tr>
</tbody>
</table>

At $t = 1$

- Banks get the investment return.
- Debts have to be repaid:
  - Depositors have priority over other creditors
  - Repayment of $i$ to its creditor, $j$, is: $x_{ij}$

At $t = 2$

- Banks collect the return of the illiquid investment opportunity.
Exogenous shock (t=1)

- on the investment return, $R$.
- The shocked bank goes bankrupt
- and is unable to repay other banks in full.
- The failed bank’s creditors may also fail $\Rightarrow$ Cascade of failures

Assumptions

- Total repayment by bank $i$ can’t exceed $i$’s cash flow $= c_i + y_i + \sum_{s \neq i} x_{si}$.
- Partial recourse: banks pay to their creditors whatever cash flow they have
- Proportionality: creditors will be paid in proportion to their debts
Payment obligation (Eisenberg-Noe framework):

The repayment of bank $i$ to its $j$ creditors is:

$$x_{ij} = \frac{b_{ij}}{b_i} \max \left[ \min \left( c_i + y_i + \sum_{s \neq i} x_{si} - d_i, rb_i \right), 0 \right]$$

(1)

Eisenberg-Noe (2001) show the existence of a clearing vector of payments
Critical connectivity

- Let $k_i$ be the degree of a node

- **Critical connectivity**, $k^*$, is the degree
  - below which the connections facilitate contagion, and
  - above which the connections are enough to absorb the shock

- **Simplifying assumptions**
  1. Regular network: all nodes have the same number of links (degree)
  2. All links have lending value of 1 $\Rightarrow l_i = b_i$
  3. Leverage, $\Lambda = \frac{Assets_i}{Equity_i}$ is fixed
  4. Counterparty exposure, $f_i = \frac{l_i}{Assets_i}$ is fixed
The critical connectivity is:

\[
k^* = \frac{rf\Lambda}{(R - 1)\Lambda + (2 - R)}
\]  

(2)

Critical degree as a function of interbank exposure \( f \) and leverage factor \( \Lambda \) \((R = 1.05, \, r = 1.01)\).
Expected number of failures

**Figure:** Number of failures as a function of the degree, $k$. Regular network of $N = 20$ banks with parameter values: $R = 1.05$, $r = 1.01$, $f = .7$, $\Lambda = 20$.

→ The vertical line: critical degree, $k^*$
Figure: Sample regular networks with $k = 5$ (left) and $k = 10$ (right); the shocked bank is represented in black, the failing banks in red and the safe banks in green.

Applicable to other type of network structures?
Simulation results
Comparing the mean field approximation with the simulation results:

Expected number of failures in Erdös-Rényi (left) and Barabási-Albert (right) networks with $N = 50$ banks, for $R = 1.05$, $r = 1.01$, $f = 70\%$ and $\Lambda = 20$

- Then, the critical degree is applicable to all network structures.
But the structure matters:

▶ Second moments!

▷ Mean number of failures is similar in both topologies
▷ But prob. of catastrophic failure is larger in skewed networks

Distribution of failures for Erdös-Rényi and Barabási-Albert networks of $n = 50$ banks with mean degree $k = 4$, for $R = 1.05$, $r = 1.01$, $f = 70\%$ and $\Lambda = 20$
Strategic network formation
The network formation game

▶ Main features of the model:

▷ A player payoff depends on own links and the aggregate number of links
▷ *Playing the field game* (Goyal and Joshi, 2006)
▷ Banks’ goal is to maximize profits
▷ They compete for loans à la Cournot
▷ Cost reducing collaboration (e.g., operational costs decline)

▶ Main results:

▷ Payoffs are increasing in own links but decreasing in links of others
▷ Positive externality in own links; negative externality in others’ links
▷ A pairwise equilibrium network exists
▷ The equilibrium network can be asymmetric
Banks’ profit function:

- Banks’ profit function is \( \pi_i = pq_i - c_i q_i \), where
  - \( p \) is the market price of the loan
  - \( q_i \) is the quantity of loans produced by bank \( i \)
  - \( c_i \) is the cost of lending

The profit function is:

\[
\pi_i(g) = (\theta - c_0)q_i - q^2 - \sum_{j\neq i} q_i q_j + \lambda \sum_{j=1}^{n} g_{ij} q_i q_j
\]

- \( \theta - c_0 \) is the idiosyncratic effect
- \( \lambda \) is the peer effect coefficient
Peer effects

- When direct link, $g_{ij} = 1$, the cross-partial profitability is positive
  $\Rightarrow$ Positive externalities in own links

- When no direct link, $g_{ij} = 0$, the cross-partial profitability is negative
  $\Rightarrow$ Negative externalities in the total number of links
Equilibrium

- Pair-wise stability (Jackson and Wolinsky, 1996)
- Equilibrium exists
- Network can be asymmetric: a dominant group and disconnected nodes.
Equilibrium Networks: Symmetric structures (up); asymmetric structures (down).
Contribution

- Analytical solution for the critical connectivity
  - It only depends on financial parameters
  - Applicable to different network structures
  - But network structure affect the probability of catastrophic defaults

- Externalities lead to endogenous networks that can generate asymmetric structures
Policy implications

- Regulators can assess contagion without knowing the network
- Middle levels of connectivity are the problematic ones.
Literature

► Models of individual banking crises

► Models of contagion

► Network theory
  ▶ Inconclusive results
  ▶ Acemoglu et al. (2013) try to reconcile
[Definitions]

- **Degree of a node** is the number of links attached to the node.

- **Critical degree** is the degree under which the initial failure will extend to other banks.

- **Directed network**: network in which each node has 2 degree: in-degree (number of ingoing links) & outdegree (number of outgoing links).

- **Regular network**: all nodes have the same degree.

- **Complete network**: regular network in which all banks lend equally to each other.
Types of networks

- Undirected networks
  - If there is a link between \( i \) and \( j \), then \( \exists \) a link between \( j \) and \( i \).

- Directed networks
  - Bank \( i \) may lend to \( j \) but \( j \) may not lend to \( i \).
Undirected networks

1. **Erdös-Rényi network**: Prob. that a pair of nodes is connected by an undirected link is fixed, $p$. The network has a Poisson degree distribution with mean $k = p(n - 1)$

Figure: Erdös-Rényi network with $N = 50$ and $k = 4$ (left) and the histograms of their nodes’ degrees (right)
2. *Barabási-Albert network*: it's a growing network in which the probability of a link attaching to a node is proportional to the degree of the target (preferential attachment). It generates a degree distribution with a power-law tail.

**Figure**: Barabási-Albert network with $N = 50$ and $k = 4$ (left) and the histograms of their vertex degrees (right)
Directed networks

1. Erdös-Rényi directed network
2. Goh-Kahng-Kim "static" directed network
Figure: Number of failures and level of connectivity ($k$)

Number of failures as a function of the degree, $k$. Regular network of $N = 20$ banks. $R = 1.05$, $r = 1.01$, $f = .7$, $\Lambda = 20$.

▷ Vertical line: critical degree, $k^*$ ➞ critical level of connectivity
Critical degree as a function of interbank exposure $f$ and leverage factor $\Lambda$ (with $R = 1.05$, $r = 1.01$). Right: critical degree as a function of $R$ and $r$ (with $f = 70\%$ and $\Lambda = 20$)
Compute expected number of failures $\langle F \rangle$ for more general networks

$\triangleright$ For $R = 1.05$, $r = 1.01$, $f = 70\%$, and $\Lambda = 20$

Erdös-Rényi network (left), Barabási-Albert network (right).
Black node: shocked bank; red node: failed bank; green node: safe