Externalities and Contagion in Banking Networks

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Hamilton (1781), 1^{st} Secretary of the Treasury, said:

"Banks are the happiest engines that ever were invented for spurring economic growth"

- Mobilize resources from savers to investors
- Deal with information asymmetries
- Provide liquidity and manage risk

But when things go wrong...

- Systemic banking crises can have devastating effects

- Last global financial crisis
- Systemic risk is of critical importance (Bernanke, 2011)
- Contagion is at the core of the systemic risk concept
- Trade-off in interconnected systems:
  - individual incentives
  - systemic implications

Goal of the paper

Propagation of a bank's failure through a bank network

- Two research questions:
  - ▷ What is the relation between **contagion and network structure?**
  - ▷ Which **networks emerge** when banks choose their links strategically?

Why is it relevant?

- Need to better understand systemic financial crises
- Consequences of higher interconnectivity
- No consensus about the effects of financial interconnections
  - Destabilizing force?
    [Vivier-Lirimont, 2004; Blume et al., 2011; Battiston et al., 2012]
  - Allow risk diversification and shock absorption?
    [Allen and Gale, 2000; Freixas et al. 2000]

This paper finds:

1. Propagation of a bank's failure depends on[1]:

   ▷ Financial parameters: determine the "critical connectivity"

   ▷ Network structure: affects the distribution of the number of failures

Which network structures emerge when banks form links strategically?

2. Banks tend to form asymmetric structures

   ▷ with highly connected components and disconnected nodes.

   ▷ Presence of externalities

[1] This part of the paper is coauthored with Gupta et al, 2013

Outline of the Presentation

1. Contagion analysis
2. Simulation Results
3. Strategic network formation
4. Conclusion

Contagion analysis

- Simple model of a banking network in which:
  - Banks (nodes) are connected through interbank loans (links)
  - One bank is shocked exogenously
  - Network structure is given
- I examine the spread of the failure due to counterparty risk
- Extension of Acemoglu et al. (2015):
  - They analyze complete and ring networks
  - My departure: this paper study less stylized structures

The Model

- Main features:
  - 3-period economy
  - One single good
  - Two agents: banks and depositors
  - System of N risk neutral and ex-ante identical banks
  - Banks are endowed with equity but can't use it to invest
  - They need to borrow from depositors or from other banks
  - Interbank lending generates the network: weighted and directed
    - l_{ij} is the loan amount that i lends to j
    - $l_{ij} \neq l_{ji}$

- Timing of the model:

At $t = 0$

- Banks borrow from depositors and from each other
- Banks make their external investment

ASSETS (A_i)	LIABILITIES (L_i)
Cash, c_i	Deposits, d_i
Investments, y_i	Interbank Borrowings, b_i
Interbank loans, l_i	Equity, e_i

At $t = 1$

- Banks get the investment return.
- Debts have to be repaid:
    - Depositors have priority over other creditors
    - Repayment of i to its creditor, j, is: x_{ij}

At $t = 2$

- Banks collect the return of the illiquid investment opportunity.

- Exogenous shock (t=1)
  - on the investment return, R.
  - The shocked bank goes bankrupt
  - and is unable to repay other banks in full.
  - The failed bank's creditors may also *fail* $\Rightarrow$ Cascade of failures

Assumptions

- Total repayment by bank i can't exceed i's cash flow $= c_i + y_i + \sum_{s \neq i} x_{si}$.
- Partial recourse: banks pay to their creditors whatever cash flow they have
- Proportionality: creditors will be paid in proportion to their debts

- Payment obligation (Eisenberg-Noe framework):

  - The repayment of bank i to its j creditors is:

$$x_{ij} = \frac{b_{ij}}{b_i} max \left[min \left(c_i + y_i + \sum_{s \neq i} x_{si} - d_i,\ rb_i \right),\ 0 \right] \tag{1}$$

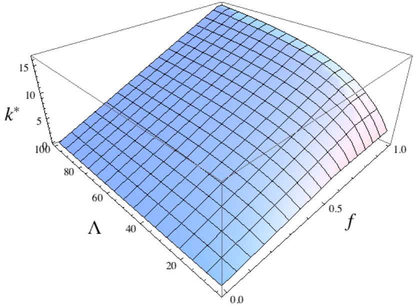
Eisenberg-Noe (2001) show the existence of a clearing vector of payments

Critical connectivity

- ▷ Let k_i be the degree of a node
- ▷ **Critical connectivity, $\mathbf{k}^*$**, is the degree
  - ▷ below which the connections facilitate contagion, and
  - ▷ above which the connections are enough to absorb the shock
- ▷ *Simplifying assumptions*
  1. Regular network: all nodes have the same number of links (degree)
  2. All links have lending value of 1 $\Rightarrow l_i = b_i$
  3. Leverage, $\Lambda = Assets_i / Equity_i$ is fixed
  4. Conterparty exposure, $f_i = l_i / Assets_i$ is fixed

- The critical connectivity is:

$$\boxed{k^* = \frac{rf\Lambda}{(R-1)\Lambda + (2-R)}} \tag{2}$$



Critical degree as a function of interbank exposure f and leverage factor Λ ($R = 1.05$, $r = 1.01$).

▶ Expected number of failures

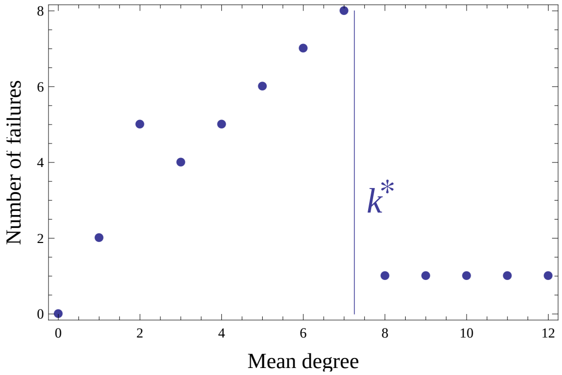


Figure: Number of failures as a function of the degree, k. Regular network of $N = 20$ banks with parameter values: $R = 1.05$, $r = 1.01$, $f = .7$, $\Lambda = 20$.

$\longrightarrow$ **The vertical line: critical degree, $\mathbf{k}^*$**

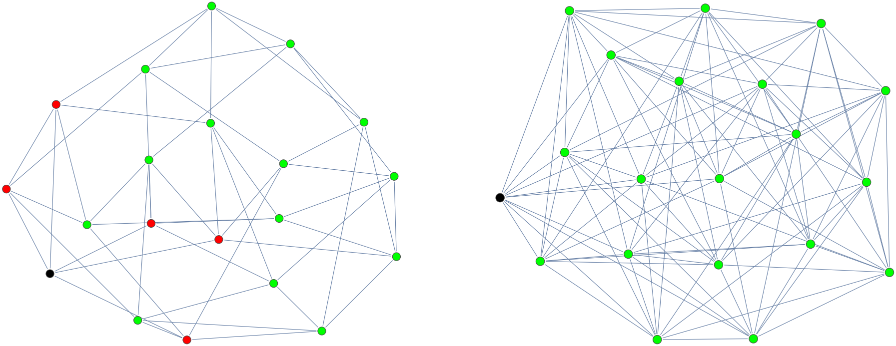
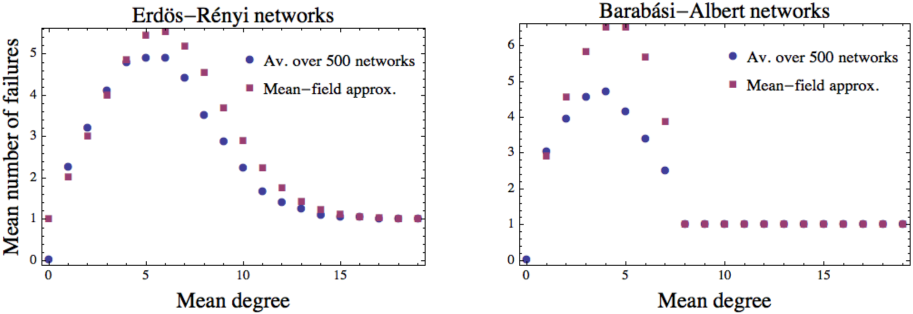


Figure: Sample regular networks with $k = 5$ (left) and $k = 10$ (right); the shocked bank is represented in black, the failing banks in red and the safe banks in green.

- **Applicable to other type of network structures?**

Simulation results

Comparing the mean field approximation with the simulation results:

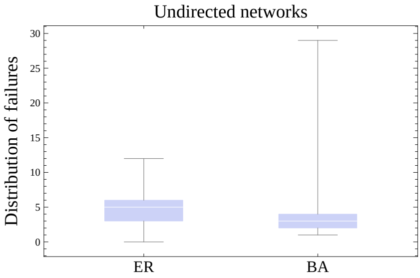


Expected number of failures in Erdös-Rényi (left) and Barabási-Albert (right) networks with $N = 50$ banks, for $R = 1.05$, $r = 1.01$, $f = 70\%$ and $\Lambda = 20$

- Then, the critical degree is applicable to all network structures.

But the structure matters:

- Second moments!
  - Mean number of failures is similar in both topologies
  - But prob. of catastrophic failure is larger in skewed networks



Distribution of failures for Erdös-Rényi and Barabási-Albert networks of $n = 50$ banks with mean degree $k = 4$, for $R = 1.05$, $r = 1.01$, $f = 70\%$ and $\Lambda = 20$

Strategic network formation

The network formation game

- Main features of the model:
  - A player payoff depends on own links and the aggregate number of links
  - *Playing the field game* (Goyal and Joshi, 2006)
  - Banks' goal is to maximize profits
  - They compete for loans à la Cournot
  - Cost reducing collaboration (e.g., operational costs decline)

- Main results:
  - Payoffs are increasing in own links but decreasing in links of others
  - Positive externality in own links; negative externality in others' links
  - A pairwise equilibrium network exists
  - The equilibrium network can be asymmetric

Banks' profit function:

- Banks' profit function is $\pi_i = pq_i - c_i q_i$, where
  - p is the market price of the loan
  - q_i is the quantity of loans produced by bank i
  - c_i is the cost of lending

- The profit function is:

$$\pi_i(g) = \underbrace{(\theta - c_0)q_i - q^2}_{idiosyncratic} \underbrace{- \sum_{j \neq i} q_i q_j + \lambda \sum_{j=1}^{n} g_{ij} q_i q_j}_{\text{peer effect}}$$

Peer effects

- When direct link, $g_{ij} = 1$, the cross-partial profitability is positive

$\Rightarrow$ Positive externalities in own links

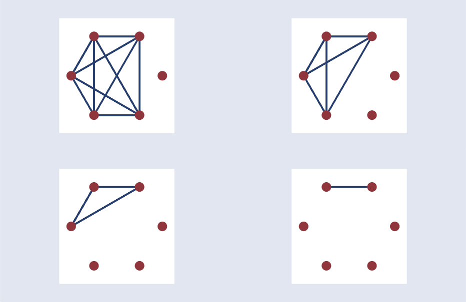
- When no direct link, $g_{ij} = 0$, the cross-partial profitability is negative

$\Rightarrow$ Negative externalities in the total number of links

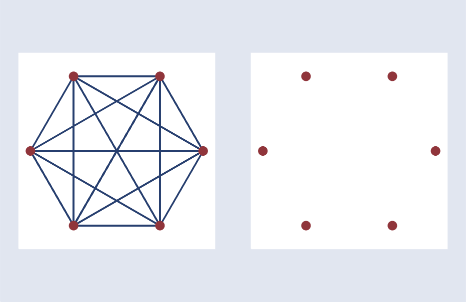
Equilibrium

- Pair-wise stability (Jackson and Wolinsky, 1996)
- Equilibrium exists
- Network can be asymmetric: a dominant group and disconnected nodes.

Asymmetric Networks



Symmetric Networks



Contribution

- Analytical solution for the critical connectivity
  - It only depends on financial parameters
  - Applicable to different network structures
  - But network structure affect the probability of catastrophic defaults
- Externalities lead to endogenous networks that can generate asymmetric structures

Policy implications

- Regulators can assess contagion without knowing the network
- Middle levels of connectivity are the problematic ones.

APPENDIX

Literature

- Models of individual banking crises
  - Diamond and Dybvig (1983), Santomero (1984)
- Models of contagion
  - Kaminsky and Reinhart (1999), Allen and Gale (2000)
- Network theory
  - Inconclusive results
  - Acemoglu et al. (2013) try to reconcile

[Definitions]

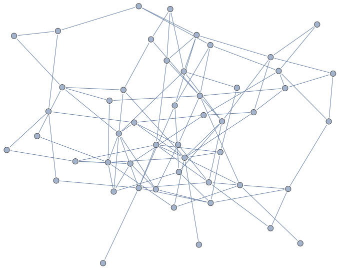
- *Degree of a node* is the number of links attached to the node
- *Critical degree* is the degree under which the initial failure will extend to other banks
- *Directed network*: network in which each node has 2 degree: in-degree (number of ingoing links) & outdegree (number of outgoing links)
- *Regular network*: all nodes have the same degree
- *Complete network*: regular network in which all banks lend equally to each other

[Types of networks]

- Type of networks
  - ▷ Undirected networks
    - ▷ If there is a link between i and j, then $\exists$ a link between j and i.
  - ▷ Directed networks
    - ▷ Bank i may lend to j but j may not lend to i.

- Undirected networks

  1. *Erdös-Rényi network*: Prob. that a pair of nodes is connected by an undirected link is fixed, p. The network has a Poisson degree distribution with mean $k = p(n-1)$



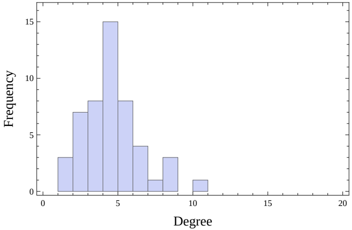


Figure: Erdös-Rényi network with $N = 50$ and $k = 4$ (left) and the histograms of their nodes' degrees (right)

2. *Barabási-Albert network*: it's a growing network in which the prob of a link attaching to a node is proportional to the degree of the target (preferential attachment). It generates a degree distribution with a power-law tail.

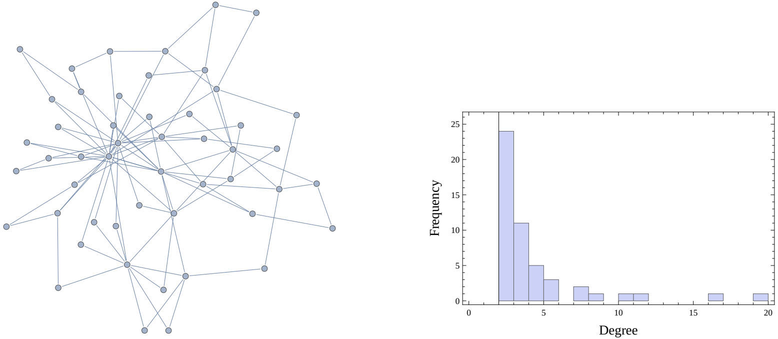
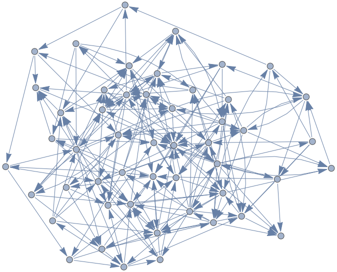
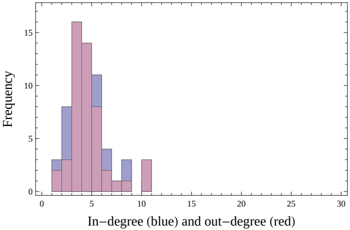


Figure: Barabási-Albert network with $N = 50$ and $k = 4$ (left) and the histograms of their vertex degrees (right)

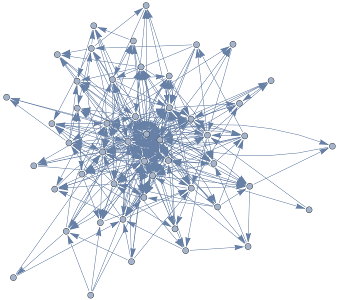
- Directed networks

1. *Erdös-Rényi directed network*





2. *Goh-Kahng-Kim "static" directed network*



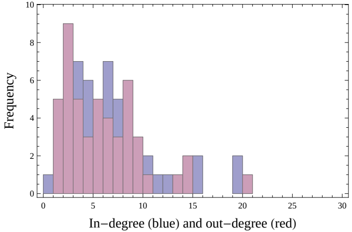
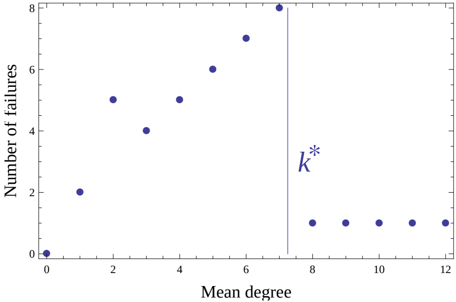
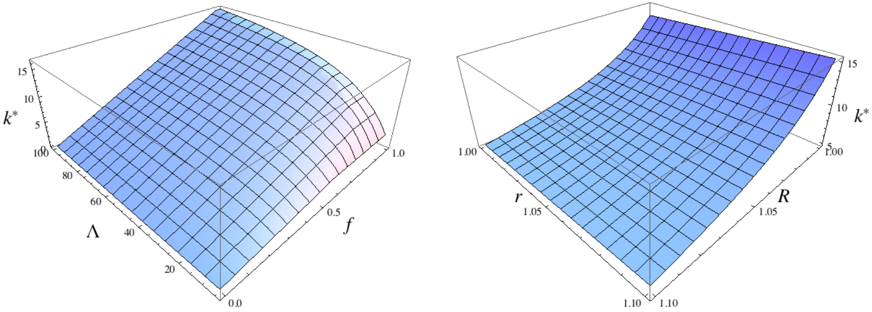


Figure: Number of failures and level of connectivity (k)



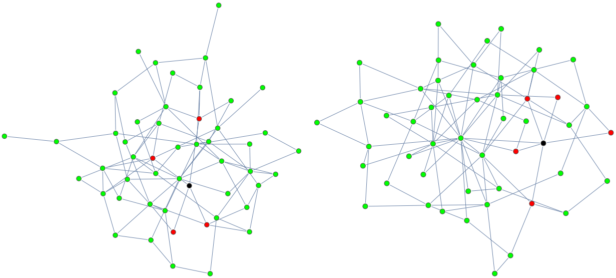
Number of failures as a function of the degree, k. Regular network of $N = 20$ banks. $R = 1.05$, $r = 1.01$, $f = .7$, $\Lambda = 20$.

▷ Vertical line: **critical degree, $\mathbf{k}^*$ $\Longrightarrow$ critical level of connectivity**



Critical degree as a function of interbank exposure f and leverage factor Λ (with $R = 1.05$, $r = 1.01$). Right: critical degree as a function of R and r (with $f = 70\%$ and $\Lambda = 20$)

- Compute expected number of failures $\langle F \rangle$ for more general networks
  - For $R = 1.05, r = 1.01,\ f = 70\%,\ and\ \Lambda = 20$



Erdös-Rényi network (left), Barabási-Albert network (right).
Black node: shocked bank; red node: failed bank; green node: safe