The Price of Complexity in Financial Networks*

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Today

Why do we need to care about Financial Interdependencies?
- Globalisation
- Systemic Risk
- Regulatory Framework

Methodology to compute the Probability of Systemic Default
- Network Context
- Credit: External Assets and Interbank Loans
- Derivatives: Credit Default Swaps

Effect of Complexity of (1) contracts and (2) network
- Limits assessment capacity of regulator (Multiple Equilibria)
- Large effects of small errors
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  1. balance sheet interlocks (e.g. credit, repo, derivatives, etc.)
  2. indirectly via exposures to common assets
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Challenge

Open problem so far: Default Probability of one institution in a networked system.

(Greenwald, 2003), (Stiglitz, 2009), (Gai and Kapadia, 2010), (Cont et al., 2012), (Battiston et al., 2012),
(Gourieroux et al., 2013), (Ota, 2014).
This work

- Contribution of this work
  1. Develop methodology to compute the default probabilities \textit{ex-ante}
  2. Show conditions for \textit{systemic risk uncertainty} in an interconnected financial systems: \textit{multiple equilibria}
  3. \textbf{Small errors} on the network and/or nature of contracts can lead to \textbf{large errors on the probability of systemic defaults}

- Policy Implications
  1. Risk is more "systemic" in nature
  2. Confidence effects are sharper
  1. Activity supervision
  2. Data collection and confidentiality
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- Policy Implications
  - More interconnected financial system:
    1. Risk is more “systemic” in nature
    2. Confidence effects are sharper
  - Large errors on estimation of probability of systemic events
    1. Activity supervision
    2. Data collection and confidentiality
The Model

- **Builds on method à la** (Eisenberg and Noe, 2001), (Cifuentes et al., 2005)
- **Generic Approach** (Gai et al., 2011), (Beale et al., 2011), (Arinaminpathy et al, 2012)
- **Focus on Default Probability** (Gourieroux et al., 2013), (Ota, 2014)
The Model

**Time 1**  Banks allocate assets and liabilities

**Time 2**  Shocks hit external assets, some banks may default and this affects counterparties including payoff from derivative contracts
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Balance Sheet

- **Derivative Market**
- **Interbank Market**
- **External Markets**
Model set-up

External assets at time 2

\[ a_i^E(2) = a_i^E(1) \sum_k E_{ik} x_k^E(2) = a_i^E(1)(1 + \mu + \sigma u_i) \]

- \( \mu_i \): expected return
- \( \sigma_i \): standard deviation
- \( u_i \): a r.v. with mean 0 and variance 1
- \( p(u_1, ..., u_n) \): joint probability distribution of shocks
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Interbank assets at time 2
\[ a_i^B(2) = a_i^B(1) \sum_j B_{ij} x_j^B(2) \]
- \( B_{ij} \): fraction of \( i \)'s interbank assets invested at time 1 in the liability of \( j \)
- \( x_j^B \): unitary value of \( j \)'s interbank liability

\[
\begin{align*}
x_j^B(1) &= 1 \forall j & \text{and} & \ x_j^B(2) = \begin{cases} R & \text{if bank } j \text{ default} \\ 1 & \text{else} \end{cases}
\end{align*}
\]
Default condition

Negative Equity

\[ e_i(2) = a_i(2) - \ell_i < 0 \]
\[ = a_i^E(1)(1 + \mu + \sigma u_i) + a_i^B(1) \sum_j B_{ij} x_j^B(2) - \ell_i < 0 \]
Default condition

Negative Equity

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Rewrite in relative terms: \( e_i(2) < 0 \) if \( \frac{e_i(2)}{e_i(1)} < 0 \)

\[ \varepsilon_i(1 + \mu + \sigma u_i) + \beta_i \sum_j B_{ij} x_j^B(2) - \lambda_i < 0 \]

where

- \( \varepsilon \) leverage over external assets
- \( \beta \) leverage over interbank assets
- \( \lambda \) leverage (debt/equity), \( \lambda_i = \varepsilon_i + \beta_i - 1 \)
Default condition

Express default as a function of the external shock

\[ u_i < \theta_i \equiv \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i \mu + \beta_i(1 - \sum_j B_{ij}\chi_j^B(\chi_j) - 1)) \]

where:

- \( \chi_j \) is a default indicator

\[ \chi_j = \begin{cases} 
1 & \text{if bank } j \text{ default} \\
0 & \text{else} 
\end{cases} \]

Extreme cases

1. Case no bank defaults \( \theta_i = \theta_i^- = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i \mu + 1) \)
2. Case all banks default \( \theta_i = \theta_i^+ = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i \mu - \beta_i(1 - R) + 1) \)
Default Condition - Introducing Derivatives

OTC Derivative contract = triplet \( \{i, j, k\} \)

\[
\theta_i = \frac{1}{\varepsilon_i \sigma_i} (\lambda_i - \varepsilon_i (1 + \mu_i) - \beta_i \sum_j B_{ij} x_j^B (\chi_j) - \delta \sum_{jk} D_{ijk} y_{ijk}(P_k, P_j))
\]

Where

- \( \delta \): leverage over derivatives
- \( D_{ijk} \): relative investment of the contract between \( i \) and \( j \) with reference entity \( k \)
- \( P_k \): initial belief on the Default Probability of \( k \)

Seller:

\[
y_{ijk} = -P_k N(1 - R) + (1 - P_k)sN
\]

Buyer:

\[
y_{ijk} = P_k N(1 - R) - (1 - P_k)sN
\]
For a given combination of shocks \( u = \{u_1, \ldots, u_n\} \)

\[ \forall i \quad \chi_i = \Theta(\theta_i(\chi_1, \ldots, \chi_n) - u_i), \]

where

- \( \Theta \) is a Heaviside function (step function)

A solution of the system above is denoted as \( \chi^* \) (Equilibrium)
Default Probability

Individual Default Probability of bank $i$, $P_i$

$$
\forall i \quad P_i = \int \chi_i^*(u) p(u) \, du
$$
Default Probability

Individual Default Probability of bank $i$, $P_i$

$$\forall i \quad P_i = \int \chi_i^*(u) \ p(u) \ du$$

Systemic default probability $P^{sys}$

$$P^{sys} = \int \chi^{sys}(u) \ p(u) \ du$$

$$= \int \Pi_i \chi_i^*(u) \ p(u) \ du \quad \text{(Example)}$$

with $p(u)$ joint density function of shocks
Simple Example

System of 2 banks lending and borrowing form each other

2-Dimensional State Space

\[ \theta_i = \begin{cases} 
\theta_i^- & \text{when } j \text{ defaults} \\
\theta_i^+ & \text{when } j \text{ does not default}
\end{cases} \]
Proposition: **Multiple Equilibria**

Consider the case of $N$ banks, with: recovery rate $R_i < 1$; interbank leverage $\beta_i > 0$; external leverage $\varepsilon_i$ and shock variance $\sigma_i$ positive and finite; shock mean $\mu$ finite.

Multiple equilibria exist if and only if:

1. there exists a **cycle** $C_k$ of credit contracts along $k \geq 2$ banks
2. for each bank $i$ and its borrowing counterparty $i + 1$ along the cycle $C_k$, it holds $\hat{\theta}_i(\chi_{i+1} = 0) \neq \hat{\theta}_i(\chi_{i+1} = 1)$

where $\hat{\theta}_i = \min\{\max\{\theta_i, -1\}, 1\}$
Second Order Approximation

Overcome the multiplicity via expected values:

\[
\chi_i = \begin{cases} 
1 & \text{if } u_i < \theta_i \equiv \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i \mu + \beta_i(1 - \sum_j B_{ij}x_j^B) - 1)) \\
0 & \text{else}
\end{cases}
\]

\[
x_j^B = \begin{cases} 
R & \text{if } u_j < \theta_j \equiv \frac{1}{\varepsilon_j \sigma}(-\varepsilon_j \mu + \beta_j(1 - \sum_j B_{jk}E[x_k]) - 1) \\
1 & \text{else}
\end{cases}
\]

where

\[
E[x_k] = RP_k + (1 - P_k)
\]
Results: Errors on contracts’ nature

- Start from initial state (True state)
  note: empirically tuned parameters
- For each variable
  - Introduce error term
  - Compute the $P^{sys}$
  - Store Maximum and Minimum values
- Increase error term
- **Envelope** - deviation from True state
Results: Errors on contracts’ nature

- Errors on nature of contracts leads to large error on systemic default probability
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- Errors on nature of contracts leads to large error on systemic default probability
Results: Errors on contract network

- Start from initial state with isolated nodes
  note: empirically tuned parameters
- Increase the link density
  - Generate all possible network configurations
  - Compute the $P^{sys}$ of each configuration
  - Store Maximum and Minimum values
- Envelope
Results: Errors on contract network

- Errors on contracts network structure leads to large error on systemic default probability
Conclusions

- Increase in interconnections and types of contracts impacts complexity and quality of assessment for financial stability
- New methodology to compute analytically the default probabilities of N banks in a network of contracts
- Multiple equilibria arise very easily in a financial system even with only “mechanistic” properties: **Confidence matters!**
- Small errors on network and/or nature of contracts: large errors on the probability of systemic defaults
Thank You!