The Price of Complexity in Financial Networks*

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Endogenous Financial Networks and Equilibrium Dynamics
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Today

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- Why do we need to care about Financial Interdependencies?
  - Globalisation
  - Systemic Risk
  - Regulatory Framework

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- Methodology to compute the Probability of Systemic Default
  - Network Context
  - Credit: External Assets and Interbank Loans
  - Derivatives: Credit Default Swaps

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- Methodology to compute the Probability of Systemic Default
  - Network Context
  - Credit: External Assets and Interbank Loans
  - Derivatives: Credit Default Swaps
- Effect of Complexity of (1) contracts and (2) network
  - Limits assessment capacity of regulator (Multiple Equilibria)
  - Large effects of small errors

Motivation

Since the beginning of Great Recession

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- Regulators warning
  No satisfactory framework yet to deal with too-big-to-fail institutions and systemic events of distress in the financial system

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  1. balance sheet interlocks (e.g. credit, repo, derivatives, etc.)
  2. indirectly via exposures to common assets

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Challenge

Open problem so far: Default Probability of one institution in a networked system.

(Greenwald, 2003), (Stiglitz, 2009), (Gai and Kapadia, 2010), (Cont et al., 2012), (Battiston et al., 2012), (Gourieroux et al., 2013), (Ota, 2014).

This work

- Contribution of this work
  1. Develop methodology to compute the default probabilities **ex-ante**
  2. Show conditions for **systemic risk uncertainty** in an interconnected financial systems: **multiple equilibria**
  3. **Small errors** on the network and/or nature of contracts can lead to **large errors on the probability of systemic defaults**

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  1. Develop methodology to compute the default probabilities **ex-ante**
  2. Show conditions for **systemic risk uncertainty** in an interconnected financial systems: **multiple equilibria**
  3. **Small errors** on the network and/or nature of contracts can lead to **large errors on the probability of systemic defaults**
- Policy Implications
  - More interconnected financial system:
    1. Risk is more “systemic” in nature
    2. Confidence effects are sharper
  - Large errors on estimation of probability of systemic events
    1. Activity supervision
    2. Data collection and confidentiality

The Model

- Builds on method à la (Eisenberg and Noe, 2001), (Cifuentes et al., 2005)
- Generic Approach (Gai et al., 2011), (Beale et al., 2011), (Arinaminpathy et al, 2012)
- Focus on Default Probability (Gourieroux et al., 2013), (Ota, 2014)

The Model

Time 1 Banks allocate assets and liabilities

Time 2 Shocks hit external assets, some banks may default and this affects counterparties including payoff from derivative contracts

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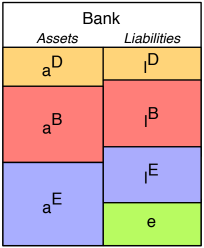
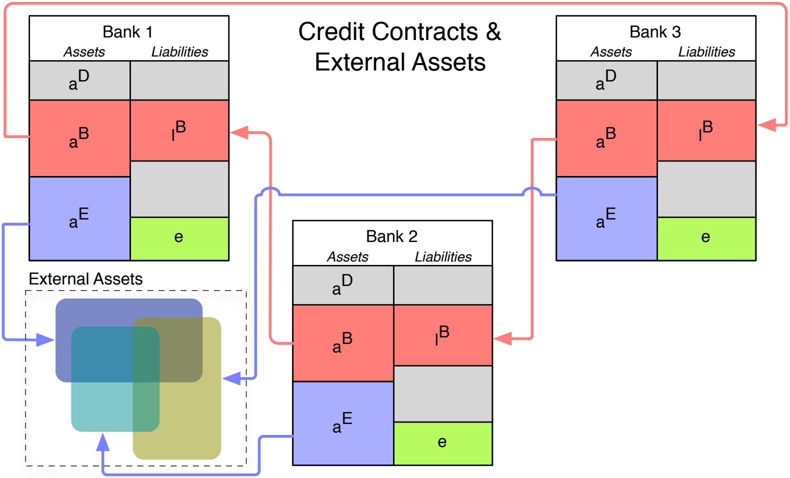
Balance Sheet



- Derivative Market
- Interbank Market
- External Markets

Interbank Credit Market



Model set-up

External assets at time 2

- $a_i^E(2) = a_i^E(1) \sum_k E_{ik} x_k^E(2) = a_i^E(1)(1 + \mu + \sigma\, u_i)$
    - μ_i: expected return
    - σ_i: standard deviation
    - u_i: a r.v. with mean 0 and variance 1
    - $p(u_1, ..., u_n)$: joint probability distribution of shocks

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Interbank assets at time 2

- $a_i^B(2) = a_i^B(1) \sum_j B_{ij} x_j^B(2)$
  - B_{ij}: fraction of i's interbank assets invested at time 1 in the liability of j
  - x_j^B: unitary value of j's interbank liability

$$x_j^B(1) = 1 \forall j \quad \text{and} \quad x_j^B(2) = \begin{cases} R \text{ if bank j default} \\ 1 \text{ else} \end{cases}$$

Default condition

Negative Equity

$$\begin{aligned} e_i(2) =& a_i(2) - \ell_i < 0 \\ =& a_i^E(1)(1 + \mu + \sigma\, u_i) + a_i^B(1) \sum_j B_{ij} x_j^B(2) - \ell_i < 0 \end{aligned}$$

Default condition

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Rewrite in relative terms: $e_i(2) < 0$ if $\frac{e_i(2)}{e_i(1)} < 0$

$$\varepsilon_i(1 + \mu + \sigma\, u_i) + \beta_i \sum_j B_{ij} x_j^B(2) - \lambda_i < 0$$

where

- ε leverage over external assets
- β leverage over interbank assets
- λ leverage (debt/equity), $\lambda_i = \varepsilon_i + \beta_i - 1$

Default condition

Express default as a function of the external shock

$$u_i < \theta_i \equiv \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i\, \mu + \beta_i(1 - \sum_j B_{ij} x_j^B(\chi_j) - 1))$$

where:

- χ_j is a default indicator

$$\chi_j = \begin{cases} 1 \text{ if bank } j \text{ default} \\ 0 \text{ else} \end{cases}$$

Extreme cases

1. Case no bank defaults $\theta_i = \theta_i^- = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i\, \mu + 1)$
2. Case all banks default $\theta_i = \theta_i^+ = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i\, \mu - \beta_i(1 - R) + 1)$

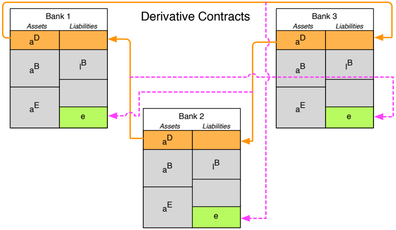
Default Condition - Introducing Derivatives

OTC Derivative contract = triplet $\{i, j, k\}$

$$\theta_i = \frac{1}{\varepsilon_i \sigma_i}(\lambda_i - \varepsilon_i(1+\mu_i) - \beta_i \sum_j B_{ij} x_j^B(\chi_j) - \delta \sum_{jk} D_{ijk} y_{ijk}(P_k, P_j))$$

Where

- δ: leverage over derivatives
- D_{ijk}: relative investment of the contract between i and j with reference entity k
- P_k: initial belief on the Default Probability of k



Seller:

$y_{ijk} = -P_k N(1-R) + (1-P_k)sN$

Buyer:

$y_{ijk} = P_k N(1-R) - (1-P_k)sN$

Equation System

For a given combination of shocks $u = \{u_1, ..., u_n\}$

$$\forall i \quad \chi_i = \Theta(\theta_i(\chi_1, ..., \chi_n) - u_i),$$

where

- Θ is a Heaviside function (step function)

A solution of the system above is denoted as χ^* (**Equilibrium**)

Default Probability

Individual Default Probability of bank i, P_i

$$\forall i \quad P_i = \int \chi_i^*(u)\, p(u)\, du$$

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Systemic default probability P^{sys}

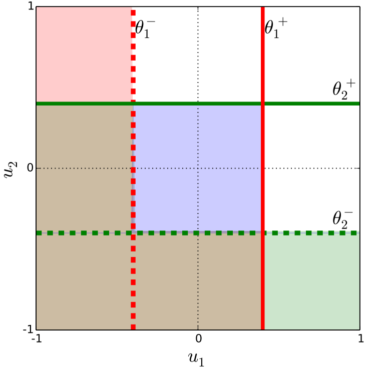
$$\begin{aligned} P^{sys} &= \int \chi^{sys}(u)\, p(u)\, du \\ &= \int \Pi_i \chi_i^*(u)\, p(u)\, du \qquad \text{(Example)} \end{aligned}$$

with $p(u)$ joint density function of shocks

Simple Example

System of 2 banks lending and borrowing form each other

2-Dimensional State Space



$$\theta_i = \begin{cases} \theta_i^- & \text{when } j \text{ defaults} \\ \theta_i^+ & \text{when } j \text{ does not default} \end{cases}$$

Results: Multiple Equilibria

From (Roukny, Battiston and Stiglitz, 2015)

Proposition: Multiple Equilibria

Consider the case of N banks, with: recovery rate $R_i < 1$; interbank leverage $\beta_i > 0$; external leverage ε_i and shock variance σ_i positive and finite; shock mean μ finite.

Multiple equilibria exist if and only if:

1. there exists a **cycle** C_k of credit contracts along $k \geq 2$ banks
2. for each bank i and its borrowing counterparty $i+1$ along the cycle C_k, it holds $\hat{\theta}_i(\chi_{i+1} = 0) \neq \hat{\theta}_i(\chi_{i+1} = 1)$

where $\hat{\theta}_i = min\{max\{\theta_i, -1\}, 1\}$

Second Order Approximation

Overcome the multiplicity via expected values:

$$\chi_i = \begin{cases} 1 & \text{if} \quad u_i < \theta_i \equiv \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i \, \mu + \beta_i(1 - \sum_j B_{ij} x_j^B) - 1)) \\ 0 & \text{else} \end{cases}$$

$$x_j^B = \begin{cases} R & \text{if} \quad u_j < \theta_j \equiv \frac{1}{\varepsilon_j \sigma}(-\varepsilon_j \, \mu + \beta_j(1 - \sum_j B_{jk} E[x_k]) - 1) \\ 1 & \text{else} \end{cases}$$

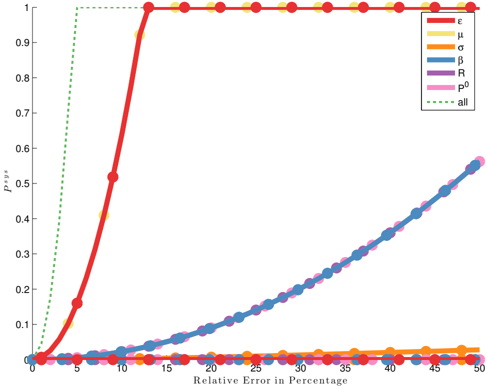
where

$$E[x_k] = RP_k + (1 - P_k)$$

Results: Errors on contracts' nature

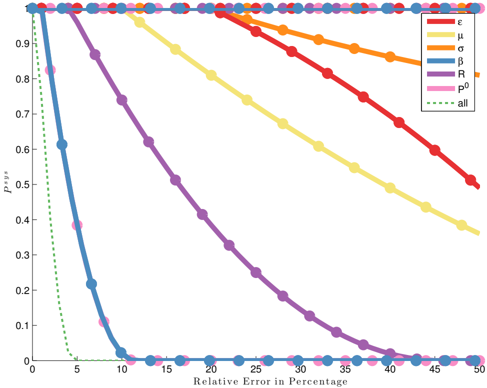
- Start from initial state (True state)
  note: empirically tuned parameters
- For each variable
  - Introduce *error term*
  - Compute the P^{sys}
  - Store Maximum and Minimum values
- Increase *error term*
- **Envelope** - deviation from True state

Results: Errors on contracts' nature



- Errors on nature of contracts leads to large error on systemic default probability

Results: Errors on contracts' nature

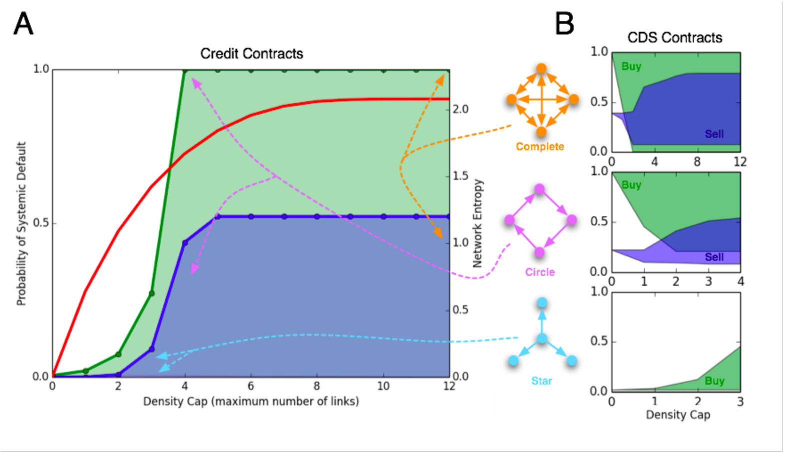


- Errors on nature of contracts leads to large error on systemic default probability

Results: Errors on contract network

- Start from initial state with isolated nodes
  note: empirically tuned parameters
- Increase the link density
  - Generate all possible network configurations
  - Compute the P^{sys} of each configuration
  - Store Maximum and Minimum values
- **Envelope**

Results: Errors on contract network



- Errors on contracts network structure leads to large error on systemic default probability

Conclusions

- Increase in interconnections and types of contracts impacts complexity and quality of assessment for financial stability
- New methodology to compute analytically the default probabilities of N banks in a network of contracts
- Multiple equilibria arise very easily in a financial system even with only "mechanistic" properties: **Confidence matters!**
- Small errors on network and/or nature of contracts: large errors on the probability of systemic defaults

Thank You!