Fundamental Disagreement

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The views expressed here are the authors’ and are not representative of ones of the Banque de France, the Bundesbank, the Eurosystem, the Federal Reserve Bank of New York or the Federal Reserve System.
Disagreement About Future Economic Outcomes

- Well documented in every survey of financial analysts, households, professional forecasters, FOMC members...

- At odds with full information rational expectation setup.

- Heterogeneous beliefs: important role in models with info. frictions
  - Macro: Mankiw-Reis (2002), Sims (2003), Woodford (2003), Lorenzoni (2009), Mackowiak-Wiederholt (2009), Angeletos-Lao (2013), ...
  - Finance: Scheinkman-Xiong (2003), Nimark (2009) ...

- Empirical properties of disagreement informative about such models?
This Paper

- Document some new facts about forecasters’ disagreement.
- Evaluate models’ ability to match the facts.
- Consider two popular models of imperfect info:
  - **Noisy information model**: Sims (2003), Woodford (2003).
  - **Sticky information model**: Mankiw and Reis (2002).
- Consider setup where forecasts do not impact data generating process.
New Facts

- Use Blue Chip Financial Forecasts (BCFF) survey:
  - Forecasts about output growth, inflation, fed-funds rate.
  - Up to 6-11 years ahead horizon.

- We document:
  1. Forecasters disagree about fundamental (long-horizon) outcomes.
  2. Term-structure of disagreement differs markedly across variables.
  3. LT disagreement varies over time and co-varies across variables.
Replicating the Facts

Independent of the model of info. frictions, key ingredients:

1. Current state of the economy *imperfectly observed.*
2. Unobserved *permanent* and *transitory* shocks.
3. *Dynamic interactions* between variables.

Minimal departure from full information setup: *symmetric* agents

- No information advantage (consistent with evidence that hard to beat the consensus).
- Forecasters agree on the parameters of the ‘true’ model but disagree about the unobserved states.
The Blue Chip Financial Forecasts Survey

- ~ 50 professional forecasters.

- We look at forecasts for RGDP growth \((g)\), CPI inflation \((\pi)\), FFR \((i)\).

- Sample period is 1986:Q1-2013:Q2.

- For 1Q, 2Q, 3Q, 4Q: observe individual forecasts.

- For 2Y, 3Y, 4Y, 5Y and long-term (6-to-11Y): observe average forecasts, top 10 average forecasts, and bottom 10 average forecasts.

- Our measure of disagreement: top 10 average — bot 10 average.
  - strongly correlated with cross-section stdev or IQR at short horizons.
The Term Structure of Disagreement in the BCFF
The Time Series of Long Run Disagreement
Model
Underlying state

- True state $z = \{g, \pi, i\}$ where

\[
\begin{align*}
  z_t &= (I - \Phi)\mu_t + \Phi z_{t-1} + v_t^z, \\
  \mu_t &= \mu_{t-1} + v_t^\mu,
\end{align*}
\]

with $v_t^z \sim iid \ N(0, \Sigma^z)$ and $v_t^\mu \sim iid \ N(0, \Sigma^\mu)$.

- Parameters: $\theta = (\Phi, \Sigma^z, \Sigma^\mu)$
Model

Information Friction: Noisy Information

- Forecaster $j$ observes:

$$y_{jt} = z_t + \eta_{jt}$$

with $\eta_{jt} \sim iid \ N(0, \Sigma^\eta)$, $\Sigma^\eta$ diagonal.

- Individual $j$’s optimal forecast computed using the Kalman filter.

- Model parameters: $(\theta, \Sigma^\eta)$.

- Disagreement driven by variance of observation errors $\Sigma^\eta$. 
Model

Information Friction: Sticky Information

- At each date, a forecaster $j$ observes $k^{th}$ element of $y_t$ with a fixed probability $\lambda_k$; otherwise sticks to latest observation of that variable.

- Individual $j$’s optimal forecast computed using the Kalman filter with missing observations.

- Disagreement driven by different individual sequences of observations.

- Same number of parameters as in noisy info with $\lambda$’s instead of $\Sigma^\eta$. 

Calibration via Penalized MLE

Principle

- Sample: realizations 1955Q1-2013Q2; survey 1986Q1-2013Q2.

- We minimize the **Likelihood** associated to true state + ...

- ... a **penalty function** measuring the distance between model implied moments (simulations) and their survey data counterpart.

- We choose 15 moments:
  - Std-dev of consensus forecasts for Q1, Q4, Y2 and Y6-11.
  - Disagreement about Q1 forecasts **only**.
Summary of Parameter Estimates

- Common parameters ($\theta$) robust to type of info. friction considered.

- Long-run vol. ($\Sigma^\mu$) much lower than short-run vol. ($\Sigma^z$).

- FFR is perfectly observed:
  - Noisy: observation error ($\Sigma^\eta$) for FFR is zero.
  - Sticky: probability of observing FFR ($\lambda_i$) is one.

- RGDP/CPI: degree of info frictions consistent with previous studies.
Data and Model-implied Term Structures of Disagreement
Noisy and Sticky

Figure 3: Term Structure of Disagreement
Noisy and Sticky Information Models
This figure displays the model-implied (time) average of disagreement across different horizons for the generalized noisy information model (dark blue) and the generalized sticky information model (light blue) calibrated with $\alpha = 50$ along with the Blue Chip Financial Forecasts survey (red). Open circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, \ldots, S_T)$. 
Disagreement and Consensus Volatility
Noisy (very similar patterns for sticky version)

The first column displays the model-implied disagreement for the generalized noisy information model calibrated with $\alpha = 50$ (blue) and the noisy information model without shifting endpoints calibrated with $\alpha = 50$ (green) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open white circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, \ldots, S_T)$ for the model without shifting endpoints. Open white and light blue circles designate survey moments used to form the penalization term for the generalized noisy information model. Model-implied 95% confidence intervals for the model with and without shifting endpoints are designated by shaded regions and dotted lines, respectively.

**Real Output Growth**

**CPI Inflation**

**Federal Funds Rate**

**Figure 4: Disagreement and Standard Deviation of Forecasts**
Noisy Information Model

The first column displays the model-implied disagreement for the generalized noisy information model calibrated with $\alpha = 50$ (blue) and the noisy information model without shifting endpoints calibrated with $\alpha = 50$ (green) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding standard deviation of consensus forecasts. Open white circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, \ldots, S_T)$ for the model without shifting endpoints. Open white and light blue circles designate survey moments used to form the penalization term for the generalized noisy information model. Model-implied 95% confidence intervals for the model with and without shifting endpoints are designated by shaded regions and dotted lines, respectively.
Role of Key Ingredients

- Decomposition into permanent and transitory components:
  - Needed to explain disagreement beyond the short/medium term.

- Multivariate model:
  - Needed to explain disagreement about future FFR even though perfectly observed.
  - Needed to generate upward-sloping disagreement.
Time Variation & Co-movement in Disagreement

Noisy

The first column displays the model-implied (time) variance of disagreement for the generalized noisy information model calibrated with $\alpha = 50$ (blue) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding correlation of disagreement between variables. Model-implied 95% confidence intervals are designated by shaded regions.

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<thead>
<tr>
<th>Forecast Horizon</th>
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Time Variation & Co-movement in Disagreement

Sticky

Figure 8: Second Moments of Disagreement

Sticky Information Model

The first column displays the model-implied (time) variance of disagreement for the generalized sticky information model calibrated with $\alpha = 50$ (blue) along with the Blue Chip Financial Forecasts survey (red). The second column displays the corresponding correlation of disagreement between variables. Model-implied 95% confidence intervals are designated by shaded regions. Results are based on 2,500 simulations.

<table>
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<tr>
<th>Forecast Horizon</th>
<th>Real Output Growth</th>
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Data

Model
Is Disagreement about FFR Consistent with a Taylor Rule?

- Generate ind. FFR forecasts given model ind. forecasts of $g$ and $\pi$

$$i_t = \rho \cdot i_{t-1} + (1 - \rho) \cdot i_t^* + \epsilon_t$$

$$i_t^* = \bar{i}_t + \varphi_{\pi} \cdot (\pi_t - \bar{\pi}_t) + \varphi_g \cdot (g_t - \bar{g}_t)$$

- Find parameters $\{\rho, \varphi_{\pi}, \varphi_g\}$ giving best fit of FFR term structure of disagreement obtained with previous reduced form model.

  - $\rho = 0.98$, $(1 - \rho)\varphi_{\pi} = .26$, $(1 - \rho)\varphi_g = 0.30$.

- Compare with various parametric restrictions.

  - Std Taylor rule parameters: $\rho = 0.9$, $\varphi_{\pi} = 2$, $\varphi_g = 0.50$.
  - No policy smoothing $\rho = 0$.
  - Restrictions on time varying components of $\bar{i}_t$. 
‘Standard’ Taylor Rule
Role of Components in the Time-Varying Intercept

Figure 6: Monetary Policy Rules
Noisy Information Model

This figure shows the results of the analysis discussed in Section 4.1.1. The top chart displays model-implied disagreement for different values of $(\rho, \phi, \pi, \phi, g)$ along with the Blue Chip Financial Forecasts survey (red). The "standard rule" is given by $(\rho, \phi, \pi, \phi, g) = (0.9, 2, 0.5)$. The bottom chart shows model-implied disagreement for different specifications of $i_t$. Open circles designate survey moments used to form the penalization term $P(\theta_1, \theta_2; S_1, ..., S_T)$. 

Data
Model
Model w/ $i_t = i$
Model w/ $i_t = r + \pi_t$
Model w/ $i_t = -400 \cdot \log(\beta) + g_t + \pi_t$
Conclusion

- Present new facts about forecaster disagreement.
  - Identify key features needed to replicate them.

- Models of imperfect info can account for complex facts for sound parameter values.
  - Minimal departure from REH: agents know and agree on true model/params.

- Disagreement informative about both degree of imperfect info and underlying DGPs.
  - Results underline importance of uncertain long-run component in macro-variable (implications for macro & finance).
  - Informative about perceived structural relationships (e.g. Taylor rule).
Extensions / Future Research

- Accounting for time variance of disagreement:
  - Stochastic volatility.
  - State-dependent imperfect info / inattention.

- More structure...
## Noisy Information Model

### Table 1: Results of Calibration for $\alpha = 50$

<table>
<thead>
<tr>
<th>$\Phi$</th>
<th>$\Sigma^z$</th>
<th>$\sqrt{\text{diag}(\tilde{\Sigma}^\eta)}$</th>
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</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 0.378 &amp; -0.503 &amp; -0.153 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 3.419 &amp; -0.019 &amp; 0.561 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 2.592 \end{bmatrix}$</td>
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<tr>
<td>$\begin{bmatrix} 0.125 &amp; 0.974 &amp; -0.033 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -0.019 &amp; 0.645 &amp; 0.365 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1.429 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\begin{bmatrix} 0.147 &amp; 0.104 &amp; 0.924 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.561 &amp; 0.365 &amp; 0.632 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0.000 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

| $|\text{eig}(\Phi)|$ | $\Sigma^\mu$ | $\sqrt{\text{diag}(\Sigma^\eta)}$ |
|-----------------|--------------|---------------------------------|
| $\begin{bmatrix} 0.920 \end{bmatrix}$ | $\begin{bmatrix} 0.008 & 0.014 & 0.026 \end{bmatrix}$ | $\begin{bmatrix} 4.317 \end{bmatrix}$ |
| $\begin{bmatrix} 0.711 \end{bmatrix}$ | $\begin{bmatrix} 0.014 & 0.024 & 0.045 \end{bmatrix}$ | $\begin{bmatrix} 2.731 \end{bmatrix}$ |
| $\begin{bmatrix} 0.646 \end{bmatrix}$ | $\begin{bmatrix} 0.026 & 0.045 & 0.085 \end{bmatrix}$ | $\begin{bmatrix} 0.000 \end{bmatrix}$ |
## Sticky Information Model

### Table 1: Results of Calibration for $\alpha = 50$

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<tbody>
<tr>
<td>\begin{bmatrix} 0.392 &amp; -0.478 &amp; -0.142 \ 0.122 &amp; 0.939 &amp; -0.024 \ 0.146 &amp; 0.087 &amp; 0.931 \end{bmatrix}</td>
<td>\begin{bmatrix} 3.736 &amp; -0.065 &amp; 0.564 \ -0.065 &amp; 0.911 &amp; 0.347 \ 0.564 &amp; 0.347 &amp; 0.635 \end{bmatrix}</td>
<td>\begin{bmatrix} 2.586 \ 1.355 \ 0.000 \end{bmatrix}</td>
</tr>
</tbody>
</table>

| $|\text{eig}(\Phi)|$ | $\Sigma^\mu$ | $\lambda$ |
|-----------------|--------------|--------|
| \begin{bmatrix} 0.920 \\ 0.674 \\ 0.674 \end{bmatrix} | \begin{bmatrix} 0.007 & 0.012 & 0.022 \\ 0.012 & 0.021 & 0.039 \\ 0.022 & 0.039 & 0.073 \end{bmatrix} | \begin{bmatrix} 0.260 \\ 0.260 \\ 1.000 \end{bmatrix} |
ST/LT Disagreement

Real Output Growth

CPI Inflation

Federal Funds Rate
Long-Term Forecasts: GDP/GNP

Real GDP 6−11 Years Ahead Forecasts

- Consensus
- Top
- Bottom
Long-Term Forecasts: CPI

Appendix

CPI 6–11 Years Ahead Forecasts

Consensus
Top
Bottom

Appendix

Long-Term Forecasts: FFR

Federal Funds Rate 6–11 Years Ahead Forecasts

- Consensus
- Top
- Bottom

- Federal Funds Rate 6–11 Years Ahead Forecasts


- Consensus

- Top

- Bottom

- 29
Calibration via Penalized MLE
Details (1/2)

- Consider realizations as signals about $z_t$: $\mathcal{Y}_t = z_t + \tilde{\eta}_t$ with $\tilde{\eta}_t \sim iid \ N(0, \tilde{\Sigma}^\eta)$.

- $-\mathcal{L} \left( \mathcal{Y}_1, \cdots, \mathcal{Y}_T; \theta, \tilde{\Sigma}^\eta \right) = \text{likelihood obtained with Kalman filter.}$
Calibration via Penalized MLE

Details (2/2)

- Given \((\theta, \Sigma^\eta)\), we generate individual forecasts \(f^h_{jt}\) and minimize distance between simulated moments \(S(\theta, \Sigma^\eta)\) and their survey data counterparts \(S_t\).

- distance between model implied expectation moments and their survey data counterpart, \(\mathcal{P}(S_1, \cdots, S_T; \theta, \Sigma^\eta)\)

\[
\mathcal{P} = \left[ S(\theta, \Sigma^\eta) - \frac{1}{T} \sum_t S_t \right]' W \left[ S(\theta, \Sigma^\eta) - \frac{1}{T} \sum_t S_t \right]
\]

- We minimize the penalized likelihood:

\[
\mathcal{C} \left( \theta, \Sigma^\eta, \tilde{\Sigma}^\eta \right) = \mathcal{L} \left( Y_1, \cdots, Y_T; \theta, \tilde{\Sigma}^\eta \right) + \alpha \mathcal{P}(S_1, \cdots, S_T; \theta, \Sigma^\eta).
\]
Calibration in Practice

- Sample: realizations 1955Q1-2013Q2; survey 1986Q1-2013Q2.

- Simulate $R = 100$ histories of shocks $\epsilon_t$ and observation noises $\eta^j_t$ with $T = 120$ (nb of dates) and $N = 50$ (nb of forecasters).

- We choose 15 moments:
  - Std-dev of consensus forecasts for Q1, Q4, Y2 and Y6-11.
  - Disagreement about Q1 forecasts only.
  - Strong weight on Q1 disagreement in the penalty criterion ("normalisation").

- Penalty parameter $\alpha = 50$. 
Robustness

- Various penalty parameters $\alpha = 1$, $\alpha = 10$.
- Initial conditions.
- Econometrician perfectly observing the state ($\tilde{\Sigma}^\eta = 0$).