Servicing Securitisation through Inefficient Foreclosure*

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Abstract

How does securitisation distort foreclosure decision of non-performing mortgages? Why do mortgage servicers, who decide to foreclose or to renegotiate delinquent mortgages, seem to be given biased incentives? What role do they play in securitisation? To address these questions, we develop a model in which an impatient, informed mortgage pool owner (a bank) designs and sells a mortgage-backed security to uninformed investors, and chooses the servicing arrangement which affects the subsequent decision to foreclose or modify delinquent mortgages. By contracting with a third-party servicer, the bank is able to effectively commit to a foreclosure policy that optimally trades off the ex ante cost of securitisation under asymmetric information against the ex post cost of inefficient foreclosure. We show that securitisation leads to excessive (insufficient) foreclosures in a bad state if the mortgage pool is of low (high) quality. The servicer's incentives are thus *endogenously biased*. Our model generates novel predictions regarding foreclosure rate and mortgage servicing contract that are consistent with various empirical findings about the subprime mortgage crisis in the United States. (*JEL* D8, G21, G24)

Keywords. Security design, mortgage backed securities, mortgage foreclosure, servicer contracts, asymmetric information, commitment

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1 Introduction

The wave of mortgage foreclosures in the aftermath of the subprime mortgage crisis in the United States has raised concerns from the general public and policy makers.[1] Reports and empirical studies have argued that foreclosures often result in significant losses for both the lenders and the borrowers, and impose substantial negative externalities to the broader society.[2] In response to the unfolding foreclosure crisis started in 2008, the U.S. government has developed the Home Affordable Modification Program, a large-scale intervention to incentivise mortgage renegotiation instead of foreclosure.[3]

Recent studies about the subprime mortgage crisis have suggested that securitisation and the biased incentives of mortgage servicers, who have the discretion to foreclose or to renegotiate a mortgage when it becomes non-performing (delinquent), could contribute to the severity of the foreclosure crisis. For instance, Piskorski et al. (2010) and Agarwal et al. (2011a) show that mortgages in a securitised pool are more likely to be foreclosed than otherwise similar mortgages on bank portfolios. Thompson (2009) analyses the compensation structure of mortgage servicers and concludes that their legal and financial incentives bias them towards foreclosure instead of mortgage modification, even when foreclosure is expected to be more costly than renegotiation.[4]

Three questions follow naturally from the empirical findings. i) How does securitisation affect the decision to foreclose or to renegotiate a delinquent mortgage? ii) What is the role played by these third-party mortgage servicers? And, iii) if servicers' biased incentives have caused inefficient foreclosures, why are such compensation contracts written in the first place? This paper studies the *causal* relationship between securitisation and foreclosure policy, highlights the benefits of third-party servicing, and characterises mortgage servicers' optimal contract.

We develop a model of mortgage-backed securitisation which incorporates three

[1]According to a recent report by RealtyTrac (2015), there are more than 14 millions US properties with foreclosure filings from 2008-2014.

[2]See for example Pennington-Cross (2006) for a survey on the deadweight loss on foreclosure. Using data from 1987 to 2009 in Massachusetts, Campbell et al. (2011) estimate foreclosure discounts as large as 27 percent on average

[3]HAMP provides direct monetary incentives to mortgage servicers for each successfully renegotiated delinquent mortgage. For a detailed description and an empirical evaluation of HAMP, see Agarwal et al. (2012)

[4]See also Krueger (2014) for a empirical survey of mortgage servicers' compensation contracts.

critical elements of the industry: (1) the servicing arrangement of the loans, (2) the security design problem in securitisation under asymmetric information, and (3) the foreclosure decision when delinquency occurs. Given the key friction, namely the asymmetric information between the securitiser and the outside investors, we analyse the implications for the choice of in-house versus third-party servicing by the securitisers, and the subsequent foreclosure decision of delinquent mortgages.

Mortgage-backed securitisation is modelled as a liquidity-based security design problem à la DeMarzo and Duffie (1999) with an *endogenous* foreclosure decision after the securities are issued (or retained). A mortgage pool owner (henceforth 'securitiser') desires to raise cash by selling securities backed by the mortgages to outside investors. After the MBS are issued, however, an aggregate shock might occur, causing some mortgages to become non-performing. The securitiser must decide whether to modify or to foreclose the delinquent mortgages. These two decisions will affect the final cash flow differently: if a mortgage is modified (forbearance), the full repayment is only recovered with some probability due to the exposure of borrowers' (re-)default risk and the aggregate risk in the economy (e.g. future house prices); in contrast, foreclosing a mortgage and selling the underlying property immediately at a distressed price limits such risk exposures and generates a more stable cash flow.

When the securitiser has private information regarding the recovery probability of the delinquent mortgages, she chooses to issue a senior security, or debt, to the outside investors and retains the junior tranche to signal her positive information as in DeMarzo and Duffie (1999) and DeMarzo (2005).[5] However, since the foreclosure policy affects the riskiness of the mortgage pool cash flow, the securitisation process interacts with the equilibrium foreclosure policy.

In particular, the servicing arrangement of in the securitisation process has implications for the incentives for foreclosure. A loan originator can securitise the mortgage pool with service performed in-house, or sell the pool to a specialist securitiser while remaining as a third-party servicer under a servicing contract.[6] If

[5]Using a sample of RMBS from the pre-crisis period, Begley and Purnanandam (2013) find evidence of equity-tranche retention being used as signal of unobserved pool quality. Consistent with our result, they find lower foreclosure rate for deals with a higher level of equity tranche.

[6]According to the 2014 10-K report by Fannie Mae, "[g]enerally, the servicing of the mortgage loans that are held in our retained mortgage portfolio or that back our Fannie Mae MBS is performance by mortgage servicers on our behalf. Typically, the lenders who sell single-family

the securitiser services the mortgage in-house, she tends to grant excessive forbearance. This is because given her retained equity tranche, she benefits from riskier cash flows, creating an incentive to forbear instead of foreclose.

The first result of this paper is that, contracting with a third-party servicer improves the efficiency of securitisation under asymmetric information. An important function performed by mortgage servicers is the decision of forbearance versus foreclosure.[7] The separation of servicing right from cash flow right allows the securitiser to effectively commit to an *ex ante optimal* foreclosure policy that is *ex post inefficient*, by providing an incentive contract for the third-party servicer to implement the desired foreclosure policy. The model therefore addresses the role played by third-party mortgage servicers in the securitisation industry. This is inline with the view of Thompson (2009), who argues that the rise of the servicing industry is a by-product of securitisation.

Next, we turn to the main result of the paper – how securitisation distorts foreclosure policy in equilibrium. We show that the equilibrium foreclosure policy with a third-party servicer exhibits a two-sided distortion relative the full information (also the first-best) benchmark. Specifically, in a bad economic state, there is excessive foreclosure if the underlying mortgage pool is of low quality, but there is insufficient foreclosure if the mortgage pool is of high quality. This result implies that, conditional on a bad aggregate state with high delinquency, securitisation amplifies the effect of unobservable asset quality on the equilibrium foreclosure rate.

The intuition of the above result is as follows. The equilibrium foreclosure policy trades off the ex ante cost of securitisation under asymmetric information against the ex post cost of inefficient foreclosure. In order to reduce the expected signalling cost of retaining junior securities, the securitiser, before receiving the private information, designs servicing contracts so that a third-party servicer implements foreclosure policy that discourages the low type (the securitiser with a mortgage pool of low recovery probability) from mimicking the high type. The low type's mimicking payoff comprises the proceeds from selling the debt claim at the high

mortgage loans to us service these loans for us."

[7] The servicer performs duties including collecting the payments, forwarding the interest and principal to the lenders, and negotiating new terms if the debt is not being paid back, or supervising the foreclosure process.

type's price and her valuation of the retained cash flow. As such, by marginally foreclosing more for the low type, the retained junior tranche values less because it is convex in the final cash flow and foreclosure reduces cash flow risk. Similarly insufficient foreclosures for the high type increases cash flow risk and reduces the value of the concave debt claim.

Finally, the model predicts that, since the securitiser contracts with a third-party servicer to implement the optimal foreclosure policy discussed above, the servicers are provided with endogenously biased incentives. For mortgage pools of low quality, the compensation to the servicer is designed to lean towards foreclosure. This implements the optimal foreclosure policy which would appear to be excessive ex post, namely, the foreclosure may result in an ex post loss to the investors. This is evident in the past financial crisis. For example, Levitin and Goodman (2009) estimates that lenders lose approximately 50% of their investment in a foreclosure situation.

Our paper closely relates to the empirical works on the causal relationship between securitisation and foreclosures. Agarwal et al. (2011a), Piskorski et al. (2010), and Krueger (2014), using different data sets and identification strategies, find that private securitisation increases the foreclosure probability and decreases modification probability for delinquent mortgages. This paper provides a theoretical explanation for this causal relationship, based on the information friction in the securitisation process.

Our results contribute to the understanding of the role of servicers and their incentive contracts. Agarwal et al. (2011b) find that when servicers hold the junior tranche, they act to maximise the value of their claim potentially at the expense of investors who are the senior claimants. Levitin and Goodman (2009) and Krueger (2014) argue that servicers' financial incentives are biased towards foreclosures. In this paper, we endogenise the servicers' optimally biased incentive contracts and explain why third-party servicers are hired in the first place—to overcome the time-inconsistency problem documented in Agarwal et al. (2011b).

A recent paper by Mooradian and Pichler (2014) also studies the role of servicers in the mortgage-backed securitisation industry. The authors argue that the servicers need to be provided with incentives to exert effort to gather information following a loan default, in order to offer loan renegotiation efficiently. The authors then study the asset composition of the MBS and show that non-diversified mortgage pool can alleviate the servicer's moral hazard problem. Our paper instead focuses on the securitisation (tranching) problem under asymmetric information for a mortgage pool of given quality and shows that it is *ex ante* optimal to have an *ex post* inefficient foreclosure policy. We therefore propose a different mechanism for how securitistaion distorts foreclosures and can endogenise the hiring decision of third-party servicers and their general compensation contracts.

Our paper also relates to the study of optimal loan modification and foreclosure policy. Wang et al. (2002) show that when a lender (bank) has a high screening cost to ascertain whether a borrower is in distress, it could be optimal for the bank to randomly reject loan workout requests to deter the non-distressed borrower from opportunistically applying for a loan modification. Riddiough and Wyatt (1994) study the case in which the lender's foreclosure cost is private information and the borrowers will infer this cost from past loan foreclosure decisions and consequently decide their default decision and concession request. The lender thus may costly foreclose many loans today to reduce future expected default and loan modification costs. Gertner and Scharfstein (1991) focus on the free-riding problem among multiple creditors and show that when the cost of debt concessions is private but the benefit is shared, a creditor's incentive to grant concessions to a distressed firm is reduced. While the literature typically finds that the frictions lead to excessive foreclosure, this paper argues that securitisation can be another important factor affecting foreclosure decisions, and the distortion can go either way.

More generally this paper belongs to the growing body of literature on the incentive problems associated with mortgage securitisation. Various studies argue that securitisation relaxes the *ex ante* lending standards. Keys et al. (2010, 2012), using evidence from securitised subprime loans, show that the ease of securitisation reduces lenders' incentives to carefully screen the mortgage borrowers and that mortgages with higher likelihood to be securitised have higher default rates. Mian and Sufi (2009) find that securitisation of subprime loans is associated with credit expansion and, as a result, counties with a high proportion of subprime mortgages face a larger number of defaults. Elul (2011) also finds securitised prime loans have a higher default rates than otherwise comparable portfolio loans. Hartman–Glaser et al. (2012) and Malamud et al. (2013) study ex ante loan originators' screening effort and the design of compensation contract. Chemla and Hennessy (2014) and Vanasco (2014) analyse how securitisation and the liquidity in the MBS market affect ex ante loan originators' screening effort. Our work complements this literature by studying the decision of *ex post* mortgage foreclosures in relation to securitisation.

The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 characterises the first-best and the full-information benchmarks. Section 4 carries out the main analysis to solve for the equilibrium with third-party servicer. Section 5 studies foreclosure policy under in-house servicing and compares to the equilibrium with third-party servicer to highlight the efficiency gain of third-party servicing. Section 6 lists the model's empirical implications. Section 7 concludes.

2 Model setup

This section sets up the model and comments on the assumptions which are central to the model.

There are four dates: 0, 1, 2 and 3. The model's participants consist of two banks and a continuum of outside investors. All agents are risk neutral. The outside investors are are deep pocketed and competitive. The banks are impatient and have a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the banks' liquidity needs to raise capital by securitising part of their long term assets as they have access to some positive return investment opportunities. The outside investors have no such discount. Hence, there are gains from trade between the banks and the investors.

Mortgage pool and foreclosures

The underlying asset in our model is a pool of a continuum of ex ante identical mortgages that pays off at $t = 3$. We abstract from the ex ante selection issues of the mortgage pool, which have been studied by previous literature, to focus on the foreclosures of the mortgages when they become delinquent, as detailed below.

The mortgage performance firstly depends on the aggregate state of the economy, which realises at $t = 2$ and becomes common knowledge. The state of the economy affects the mortgage borrowers' ability to repay the mortgages. With probability π, the economy is in a good state (G) and no borrowers default. In state G, the value of the mortgage pool is V_G. With probability $1 - \pi$, the economy is in a bad state (B) and a fixed portion of the mortgages becomes delinquent. This can be interpreted as a well diversified portfolio with only a systemic component of default risk. We normalise the measure of the delinquent mortgages in the pool to 1. The remaining performing mortgages continue to repay an exogenous value of $V_B < V_G$ at $t = 2$. Since mortgage delinquency only occurs in the bad state, we will focus primarily on the sequence of events after a realisation of the bad state to study mortgage foreclosures. We make the following assumption to ensure that the bad state (B) arises with sufficient probability to guarantee its relevance, which ensures the concavity of the objective function in the analysis.

Assumption 1. $\frac{1-\pi}{\pi} \geq 1 - \delta$.

When a mortgage becomes delinquent at $t = 2$, it can be foreclosed or renegotiated.[8] In the case of foreclosure, the collateral property is repossessed and sold to outside investors. Alternatively, if the delinquent mortgage is renegotiated, it pays X with probability θ at $t = 3$ (recovery) or zero otherwise (re-default). For simplicity, we assume that the recovery of delinquent mortgages are perfectly correlated. It can be interpreted as capturing the aggregate nature of the risk of mortgage recoveries (e.g. aggregate property prices and employment opportunities for borrowers). Finally, we further assume that $V_G \geq V_B + X$, so that the value of a mortgage in the good state is at least as high as in a bad state, even if all delinquent mortgages resume payments in the bad state.

The recovery probability $\theta \in \{\theta_H, \theta_L\}$ of the mortgage pool is mortgage pool-specific and is the source of information asymmetry between the bank and outside investors, as detailed in the next section. At $t = 0$ all model participants have the prior belief that $\theta = \theta_H$ with probability γ, where $\theta_H > \theta_L$. We interpret θ_i as the "quality" of the mortgage pool (subscript "H" stands for "High" and "L" for "Low"), arising from the pool's exposure to the systematic default risk of the borrowers.[9]

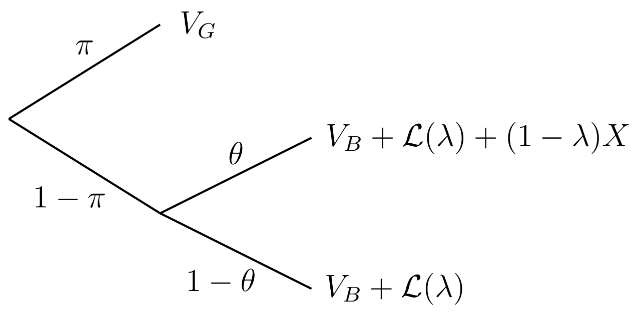
The focus of the paper is to study what proportion of the delinquent mortgages are chosen to be foreclosed in equilibrium and how securitisation affects this decision

[8] Throughout the paper, we use 'mortgage renegotiation' and 'mortgage forbearance' interchangeably.

[9] The assumption that the private information only concerns the credit risk of the delinquent mortgages is to simplify analysis and is not central to the model. Nevertheless, one interpretation could be that there is generally less public information on delinquent loans, making it more difficult to assess the recovery rate of such borrowers by investors outside of the originating bank.

variable. Importantly, we assume this decision is not contractible or else there is no role for incentives in mortgage servicing contracts. Denote $\lambda \in [0, 1]$ the fraction of delinquent mortgages foreclosed (i.e. $(1-\lambda)$ of delinquency mortgages renegotiated), and $\mathcal{L}(\lambda)$ the total liquidation proceeds from repossessed properties. The overall cash flow from mortgage pool at $t = 3$ is then $V + \mathcal{L}(\lambda) + (1-\lambda)X$ with probability θ, and $V + \mathcal{L}(\lambda)$ with probability $(1-\theta)$, as illustrated in Fig 1.

Figure 1: Mortgage pool cash flow



The exact functional form of the liquidation proceeds $\mathcal{L}(\lambda)$ depends on the characterisation of the market for distressed properties as well as the direct and indirect costs associated with foreclosures. We abstract from these considerations to keep the analysis general and make the following intuitive assumption to ensure an interior optimal foreclosure policy in the first-best case.

Assumption 2. *For $\lambda \in [0, 1]$, $\mathcal{L}(\lambda)$ is strictly increasing and concave, and $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \in [0, X]$.*

That is, first, $\mathcal{L}(\lambda)$ is strictly increasing and concave in λ. The decreasing marginal liquidation value of the foreclosed loans could be due to either scarce capital or scarce expertise in making the renovation needed to realise the value of the properties. Secondly, the marginal liquidation value of the mortgage is below the full repayment value of the mortgage $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \in [0, X]$ for any $\lambda \in [0, 1]$. Intuitively, there are costs associated with liquidating a mortgage, due to, for example, renovation and repair costs associated with investing in distressed property, as well as other outstanding liabilities such as unpaid fees and taxes.

Securitisation

Because of the liquidity discount δ, at $t = 1$, the bank who owns the mortgage pool would like to design and sell securities backed by the cash flow of the mortgage pool at $t = 3$ to outside investors. We will henceforth refer to the bank as the "securitiser" and the security as the mortgage-backed securities (MBS). The securitiser thus receives the cash from selling the MBS at $t = 1$, and retains any residual cash flow from the mortgage pool after paying off the investors at $t = 3$.

At the beginning of $t = 1$, the securitiser receives private information regarding the recovery probability of potential future delinquent loans $\theta_i \in \{\theta_H, \theta_L\}$. The source of private information could come from new information produced during the process of structuring the individual mortgages into a pool for securitisation, as in DeMarzo and Duffie (1999).

The securitiser with information θ_i then offers a security $\mathcal{F}_i$. The security $\mathcal{F}_i$ is contracted on the cash flows at $t = 3$, specifying payments to the MBS investors for each realisation of the cash flow. Observing the security on offer, the competitive investors form a posterior belief $\hat{\theta}$ regarding the private information of the securitiser, and bids the price of the security p to its fair value, taking the foreclosure policy in equilibrium as given. After repaying investors according to $\mathcal{F}_i$, the securitiser retains the residual cash flow from the mortgage pool.

We restrict our attention to monotone securities. That is, a higher realisation of the mortgage pool cash flow should leave both the outside investors and the securitiser a (weakly) higher payoff.[10]

The securitisation process could be hindered by the securitiser's private information regarding the mortgage pool, as the classic lemon's problem in Akerlof (1970). When the securitiser is uninformed, there is symmetric information between the securitiser and the investors. Under symmetric information a simple "pass-through" security or selling the whole mortgage pool to the investors is optimal as it exhausts all the gains from trade. If the securitiser is informed, however, the presence of information

[10] Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the mortgage pool if the security leaves the issuer a payoff that is not increasing in the cash flow. If such actions cannot be observed, the monotonicity assumption is without loss of generality.

asymmetry prompts the securitiser to optimally design the MBS to mitigate the information friction. We study the optimal security design problem of an informed securitiser in Section 4.2.

Servicing arrangement and contract

Besides modelling foreclosures, another novelty of this paper is to study different mortgage servicing arrangement and the terms of servicing contract.

We model the mortgage servicing arrangement game as follows. Bank 1 (the "originator") is endowed with the mortgage pool at $t = 0$ and becomes privately informed about the quality θ_i, $i \in \{H, L\}$ at $t = 1$. The originator can directly securitise the mortgage pool himself under asymmetric information at $t = 1$, and continue to service the mortgages. In particular, as a servicer he makes the foreclosure decisions of delinquent mortgages at $t = 2$. We call this "securitisation with in-house servicing".

There is another possible servicing arrangement. Bank 2, who has no stake in the mortgage pool initially, can make a take-it-or-leave-it offer of a menu of (servicing) contracts to the originator at $t = 0$ to acquire the cash flow rights of the mortgage pool. We henceforth refer to bank 2 as she and bank 1 (the originator) as he. If the originator rejects the offer, he will securitise the pool with in-house servicing as described above. If he accepts the offer on the other hand, he selects one of the contracts in the menu at $t = 1$ after receiving information regarding the type θ of the mortgage pool, and continues to service the mortgages. Independent of bank 1's contract choice, bank 2 at $t = 1$ also becomes informed about the pool's quality due to her involvement in the securitisation process, and securitises it. We call this "securitisation with third-party servicing", because bank 1, the servicer, has no direct economic interest in the mortgage pool and is solely compensated through the servicing contract.

The menu of contracts offered by bank 2 at $t = 0$ consists of two contracts $\{(\alpha_i, \beta_i, \tau_i)\}_{i \in \{H,L\}}$.[11] Each of the contracts specifies the payments to the third-party servicer, including a percentage α_i of the forbearance cash flow to be paid at $t = 3$,

[11]That the menu consists of exactly two contracts is without loss of generality, as we restrict attention to deterministic contracts and separating equilibrium. Nonetheless, the two contracts can be identical, i.e. a pooling contract.

a percentage $\alpha_i\beta_i$ of the foreclosure cash flow to be paid at $t = 3$, and a flat transfer τ_i to be paid at $t = 1$. Therefore β_i captures the servicer's relative incentive to foreclose a delinquent loan as compared to granting forbearance.

The servicing contract (in particular, β) as well as the in-house or third-party servicing arrangements are important for the foreclosure decisions because they affect the servicers' incentives and the foreclosure policy $\{\lambda_i\}$ is not contractible. Servicers thus choose the foreclosure policy that maximises their expected payoff. In other words, in-house servicers maximise the expected retained cash flow from the pool and third-party servicers maximise the expected compensation from the servicing contracts.

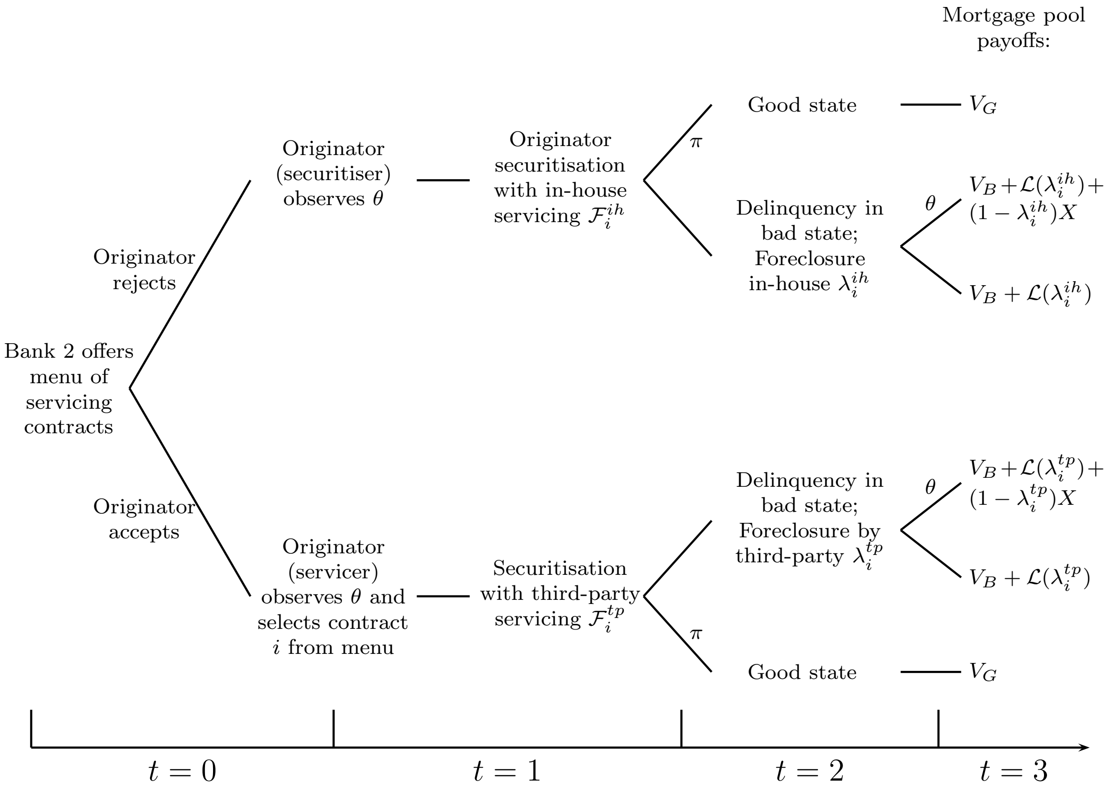
The model setup strives to capture some key elements of the mortgage servicing industry and to study the role of servicers and the interaction between foreclosures and securitisation, with minimal deviation from the security design literature. To achieve this goal, we will make the following assumptions: i) both parties can commit to the menu of the contracts once it is signed at $t = 0$; ii) the investors can observe the menu of contract offered and whether the originator accepts it at $t = 0$, i.e. whether the servicing will be done in-house or through third-party; iii) at the securitisation stage, the investors cannot observe which contract $(\alpha_i, \beta_i, \tau_i)$ is chosen by the informed third-party servicer at $t = 1$ to infer the private information. Assumptions i) and iii) ensure that the design and the choice of the servicing contracts cannot be used as informative signals, so as to keep the security design game largely unchanged, whereas Assumption ii) is realistic and avoids complex deviation of servicing contracts hence equilibrium foreclosure policies.

Finally, we assume that the securitiser incurs a transaction cost of $\kappa \geq 0$ if he agrees to be a third-party servicer in the process of securitisation. The cost κ is assumed to common knowledge. This cost does not play a significant role in our model, but only affects the willingness for the securitiser to acquire the mortgage pool from the originator. It can be interpreted as direct legal compliance costs and/or indirect unmodelled agency costs associated with contracting a third-party servicer.

Time line and the equilibrium concept

The timeline of the model is summarised in Figure 2.

Figure 2: Model timeline



The equilibrium concept in this model is the perfect Bayesian equilibrium (PBE). Formally, a PBE consists of a menu of servicing contracts, the acceptance decision of the servicing contract by the originator, the security issued by the securitiser, the foreclosure policy, the price of the security issued, and a system of beliefs such that 1) the choices made by the two banks maximise their respective objective function, given the equilibrium choices of the other agent and the equilibrium beliefs, and 2) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes' rule (whenever applicable). As there can be multiple equilibria in games of asymmetric information, we invoke the Intuitive Criterion of Cho and Kreps (1987) to eliminate equilibria with unreasonable out-of-equilibrium belief. This allows us to restrict attention to only the least cost separating equilibrium.

3 First-best and the full-information benchmarks

In this section we study two benchmark cases – first best and full information. The case of full information only differs from the first best due to the non-contractibility of the foreclosure rate λ_i. We first characterise the first-best foreclosure policy. We then analyse the equilibrium under full information, and show that the first best is achieved in the full-information equilibrium.

The first-best foreclosure policy maximises the value of the mortgage pool

$$\lambda_i^{FB} = \arg\max_{\lambda_i \in [0,1]} \pi V_G + (1-\pi)[V_B + \mathcal{L}(\lambda_i) + (1-\lambda_i)\theta_i X] \tag{1}$$

The solution is characterised by the first order condition that $\frac{\partial \mathcal{L}(\lambda_i^{FB})}{\partial \lambda} = \theta_i X$ for $i \in \{H, L\}$. That is, since the marginal value obtained from foreclosure is decreasing with the fraction of foreclosed loans, the first-best level of foreclosure is determined such that the the margin value from foreclosure is equal to the expected recovery value given forbearance.

We now characterise the equilibrium under full information. Firstly consider the optimal security issued in the securitisation process at $t = 1$. Since any retention of the cash flows by the securitiser incurs a liquidity discount, it is optimal for the security issued to be a full equity pass through security to the investors, since all securitise are fairly priced given full information. Secondly, given that the entire cash flow is securitised, the securitiser at $t = 2$ is indifferent between all foreclosure policies. As a tie break convention, we focus on the Pareto dominating equilibrium, in which the securitiser chooses the foreclosure policy that maximises the value of the mortgage pool.

The following proposition thus summarises the full-information benchmark results. All proofs are in Appendix unless stated otherwise.

Proposition 1. *In the full-information benchmark, the originator securitises the mortgage pool by issuing a pass through equity security backed by the cash flows, and chooses the first-best foreclosure policy* $(\lambda_H^{FB}, \lambda_L^{FB})$*, where* $\lambda_H^{FB} < \lambda_L^{FB}$.

Denote henceforth the first-best value of the mortgage pool of type i as $U_i^{FB} \equiv \pi V_G + (1-\pi)[V_B + \mathcal{L}(\lambda_i^{FB}) + (1-\lambda_i^{FB})\theta_i X]$.

Notice that the first-best foreclosure policy is achieved in the full-information benchmark equilibrium. Therefore any inefficiency in the foreclosure policy discussed in the subsequent sections are due to the asymmetric information problem between the securitiser and the outside investors.

In the full-information equilibrium, a high-type securitiser forecloses a smaller fraction of delinquent mortgages and obtains less liquidation proceed than a low type, $\lambda_H^{FB} < \lambda_L^{FB}$ and $\mathcal{L}(\lambda_H^{FB}) < \mathcal{L}(\lambda_L^{FB})$. This is because the high-quality mortgage pool has a higher recovery probability and the high-type securitiser is therefore less inclined towards foreclosure.

Moreover, the originator should securitise the mortgage pool himself in the first-best benchmark, so as to avoid the transaction cost $\kappa \geq 0$ of selling his mortgage pool to the securitiser. In Section 4 we will show that there is a role for securitisation with third-party servicing, when there is asymmetric information in the securitisation process.

4 Securitisation and foreclosures with third-party servicing

In this section we study the case of securitisation under asymmetric information with a third-party servicer. We solve the model backwards by first characterising the third-party servicer's foreclosure decisions at $t = 2$ for a given servicing contract $\{(\alpha_i, \beta_i, \tau_i)\}$ he chooses at $t = 1$. Then we solve the security design problem at $t = 1$ for the given foreclosure policies $\hat{\lambda}_i$. Finally we characterise the ex ante optimal foreclosure policies and the corresponding menu of servicing contracts at $t = 0$.

4.1 Foreclosure decisions by third-party servicer

Recall that the investors observe the menu of the contracts and at $t = 2$ know the mortgage pool's quality after the securitisation stage in a separating equilibrium. By studying the incentive constraint of the servicer, the investors can infer his choice of contract. Suppose type i servicer has chosen a servicing contract (α, β, τ). At $t = 2$, he chooses a foreclosure policy to maximise his expected servicing fee

$$\max_{\lambda_i \in [0,1]} \quad \alpha[\beta\, \mathcal{L}(\lambda_i) + (1 - \lambda_i)\theta_i X] \tag{2}$$

The first order condition for the above programme is $\beta\frac{\partial \mathcal{L}(\lambda_i)}{\partial \lambda} = \theta_i X$. Denote the optimal choice of foreclosure policy by the third-party servicer $\lambda_i^s(\beta)$. There following lemma states an important property of the servicing contract.

Lemma 1. *In the interior region, $\lambda_i^s(\beta)$ is strictly increasing in β, and equal to λ_i^{FB} if and only if $\beta = 1$.*

Lemma 1's result is immediate since β captures the relative payment to the servicer from foreclosure cash flows compared to recovery cash flows, the higher the relative payment, the more the servicer will bias towards foreclosures.

This simple result nonetheless highlights the important role of servicer in our model, which is to separate the foreclosure decisions from the securitisation decisions. By contracting with a third-party servicer, the securitiser delegates the foreclosure decisions to the servicer, essentially "tying her hand ex post" and allows her to commit to a foreclosure policy ex-ante.

4.2 Security design with third-party servicing

At $t = 1$, the securitiser with private information θ designs and issues an MBS backed by the cash flow of the mortgage pool. Following from the discussion in the previous subsection, the investors know the foreclosure policy $\hat{\lambda}_i$ will be implemented if the mortgage pool's quality turns out to be θ_i, even if they do not know the pool's quality at this stage.

Let's start the analysis with the securitiser who owns a mortgage pool with low recovery probability. In a separating equilibrium, the low-type securitiser always receives the fair price on any security issued. Therefore the securitiser maximises expected payoff by selling the entire cash flow from the mortgage pool to outside investors. There is no distortion in the form of inefficient retention for the low type. Given the foreclosure policy $\hat{\lambda}_L$, the expected payoff $U_L(\hat{\lambda}_L)$ for the low-type securitiser is

$$U_L(\hat{\lambda_L}) = \pi V_G + (1-\pi)[V_B + \mathcal{L}(\hat{\lambda_L}) + (1-\hat{\lambda_L})\theta_L X] \quad (3)$$

The high-type securitiser on the other hand has to design and sell a security that is not profitable for the low type to deviate and mimic. Since the security issued by a low type is a full pass through equity, we suppress the type subscript in the notation and refer to the security issued by the high-type securitiser as $\mathcal{F}$, which maps the realisation of the mortgage pool cash flows to a set of payoffs to the outside investors, as summarised in Table 1.

Table 1: Payoffs of the security issued by the high-type securitiser with third-party servicing

Realisation of cash flow	Security payoff $\mathcal{F}$
$c_1 \equiv V_G$	f_1
$c_2 \equiv V_B + \mathcal{L}(\hat{\lambda}_H) + (1-\hat{\lambda}_H)X$	f_2
$c_3 \equiv V_B + \mathcal{L}(\hat{\lambda}_L) + (1-\hat{\lambda}_L)X$	f_3
$c_4 \equiv V_B + \mathcal{L}(\hat{\lambda}_L)$	f_4
$c_5 \equiv V_B + \mathcal{L}(\hat{\lambda}_H)$	f_5

The cash flows are ranked in descending order in Table 1, if and only if $\hat{\lambda}_H \leq \hat{\lambda}_L$, since the margin foreclosure value is always (weakly) less than the full recovery value of the delinquent mortgage $\frac{\partial \mathcal{L}(\lambda)}{\partial \lambda} \leq X$. We will assume that this is the case in this subsection. But it will be verified that indeed it is the case in equilibrium in the next subsection.[12]

In the least cost separating equilibrium, for a given foreclosure policy $\{\hat{\lambda}_H, \hat{\lambda}_L\}$, the high-type securitiser maximises the proceed from securitisation plus the residual cash flow.

$$\begin{aligned}
&\max_{\mathcal{F}} && p(\mathcal{F}) + \delta(\pi(c_1 - f_1) + (1-\pi)[\theta_H(c_2 - f_2) + (1-\theta_H)(c_5 - f_5)]) \\
&s.t. \quad (MC) && p(\mathcal{F}) = \pi f_1 + (1-\pi)[\theta_H f_2 + (1-\theta_H) f_5] \\
&\quad (IC) && U_L(\hat{\lambda}_L) \geq p(\mathcal{F}) + \delta\pi(c_1 - f_1) \\
&&& \qquad + \delta(1-\pi)[\theta_L(c_3 - f_3) + (1-\theta_L)(c_4 - f_4)] \\
&\quad (LL) && f_j \leq c_j \; \forall \, j \in \{1,2,3,4,5\} \\
&\quad (MNO) && f_1 \geq f_2 \geq f_3 \geq f_4 \geq f_5 \geq 0 \\
&\quad (MNI) && c_1 - f_1 \geq c_2 - f_2 \geq c_3 - f_3 \geq c_4 - f_4 \geq c_5 - f_5 \geq 0
\end{aligned} \tag{4}$$

where (MC) is the market clearing condition that the market believes that the issuer

[12] In particular, the first-best foreclosure policies are indeed such that $\lambda_H^{FB} < \lambda_L^{FB}$ as given in Proposition 1. Hence as long as the distortions due to information asymmetry are not in the opposite direction nor too big, the equilibrium foreclosure policies should have the conjectured ranking.

of the security $\mathcal{F}$ is of the high-type, (LL) is the limited liability condition for each cash flow realisation, (IC) is the incentive compatibility constraint for the low type not to mimic the security issued by the high-type, (MMO) is the outside investors' monotonicity constraint, and (MNI) is the insider residual claim's monotonicity constraint.

The following property for the optimal security issued by the high-type securitiser is crucial for the mechanism for the paper.

Proposition 2. *For $\hat{\lambda}_H \leq \hat{\lambda}_L$, the high-type securitiser's optimal security's payoff conditional on the bad state resembles that of a debt security. Specifically, there exists a threshold $\bar{c} \in [V_B + \mathcal{L}(\hat{\lambda}_L), V_B + \mathcal{L}(\hat{\lambda}_H) + (1 - \hat{\lambda}_H)X]$ such that the payoff of the security is*

$$f_j = \begin{cases} c_j, & \text{if } c_j < \bar{c} \\ \bar{c}, & \text{if } c_j \geq \bar{c} \end{cases} \quad \forall\, j \in \{2, 3, 4, 5\} \tag{5}$$

This result is consistent with the literature on the pecking order of outside financing, e.g. Myers (1984) under asymmetric information. Since in the bad state the high-type enjoys greater upside potential $V_B + \mathcal{L}(\hat{\lambda}_H) + (1 - \hat{\lambda}_H)X > V_B + \mathcal{L}(\hat{\lambda}_L) + (1 - \hat{\lambda}_L)X$ and has a higher probability of recovery $(\theta_H > \theta_L)$, in order to discourage the low type from mimicking, the optimal security issued by the high-type resembles a debt security in the bad state.[13]

The high-type securitiser's retention of residual claims of future cash flow could be seen as a necessary signalling cost in order to separate from the low type, a well-established result in the security design literature such as Leland and Pyle (1977) and DeMarzo and Duffie (1999). In the next subsection we will show how the design of foreclosure policy can mitigate this signalling cost, which is the key novel mechanism of this paper.

[13]The optimal security is not a standard debt in general since its payoff in the good state is higher than the threshold $\bar{c}$. This is irrelevant for our analysis on foreclosures which only pertains to the bad state. In practice, since default and foreclosures lead to early repayment of the loans to the MBS holders, the effective nominal repayment is indeed reduced because of interest payment write-downs. Standard debt may be not be optimal in our model because both types have equal probability reaching the good state hence there is no information asymmetry for the cash flow realised in the good state. This is similar to the results established by Fulghieri et al. (2014) in a pooling equlibrium setting that if the information asymmetry is less severed in the higher end of the cash flow distributions than in the lower end, the optimal security needs not be debt.

4.3 Ex ante optimal foreclosure policy and servicing contract

In subsection 4.1 we show that the securitiser can essentially commit to a foreclosure policy by contracting with a third-party servicer and by delegating the foreclosure decisions to him. In this subsection, we will study how the securitiser optimally utilises this commitment device, and characterise the properties of the optimal foreclosure policy and the servicing contracts.

At $t = 0$, bank 2 (the securitiser) offers a menu of contracts $\{(\alpha_i, \beta_i, \tau_i)\}_{i\in\{H,L\}}$ to bank 1 (the originator). Denote the originator's expected payoff at $t = 1$ given his private information θ_i and his choice of contract (α, β, τ) by

$$w_i(\alpha, \beta, \tau) \equiv \tau + \delta\alpha[\beta\,\mathcal{L}(\lambda_i^s(\beta)) + (1 - \lambda_i^s(\beta))\theta_i X] \tag{6}$$

The securitiser then solves the following maximisation problem for a menu of contracts that maximises her ex ante expected payoff.

$$\begin{aligned}
\max_{\{(\alpha_i,\beta_i,\tau_i)\}_{i\in\{H,L\}}} \quad & \gamma[U_H(\lambda_H^{tp}, \lambda_L^{tp}) - w_H(\alpha_H, \beta_H, \tau_H)] \\
& \quad + (1-\gamma)[U_L(\lambda_L^{tp}) - w_L(\alpha_L, \beta_L, \tau_L)] \\
\text{s.t.} \quad (PC) \quad & \gamma w_H(\alpha_H, \beta_H, \tau_H) + (1-\gamma) w_L(\alpha_L, \beta_L, \tau_L) \geq \gamma U_H^{ih} + (1-\gamma) U_L^{ih} \\
(IC_H) \quad & w_H(\alpha_H, \beta_H, \tau_H) \geq w_H(\alpha_L, \beta_L, \tau_L) \\
(IC_L) \quad & w_L(\alpha_L, \beta_L, \tau_L) \geq w_L(\alpha_H, \beta_H, \tau_H) \\
& \lambda_i^{tp} = \lambda_i^s(\beta_i)\ \forall\, i \in \{H, L\}
\end{aligned} \tag{7}$$

$$\begin{aligned}
\text{where} \quad & U_H(\lambda_H^{tp}, \lambda_L^{tp}) \equiv (1-\delta) p(\mathcal{F}^{tp}) \\
& \quad + \delta(\pi V_G + (1-\pi)[V_B + \mathcal{L}(\lambda_H^{tp}) + (1-\lambda_H^{tp})\theta_H X]) \\
\text{and} \quad & \mathcal{F}^{tp} \text{ is given by Eq. 4 for } \hat{\lambda}_i = \lambda_i^{tp}
\end{aligned} \tag{8}$$

The securitiser's optimisation programme (Eq. 7) maximises the expected value of securitised mortgage pool $U_i(\cdot)$ less the expected compensation to the third-party servicer $w_i(\cdot)$, given a set of constraints: (PC) the *ex ante* participation constraint for the originator to be willing to accept the menu of servicing contracts, instead of securitising the pool himself with in-house servicing, taking as given the payoffs to the originator if he securitises the mortgage pool himself with in-house servicing, (U_H^{ih}, U_L^{ih}), which are solved for explicitly in Section 5 ; (IC_i) the incentive compatibility constraint for an originator of type i to prefer servicing contract i to the other contract offered in the menu; and $\lambda_i^{tp} = \lambda_i^s(\beta_i)$ takes into the type i servicer's ex post optimal foreclosures decision at $t = 2$ if his chosen contract is with β_i; lastly, Eq. 8 says that the securitiser anticipates the choice of contracts and hence foreclosure policy will affect the security design $\mathcal{F}^{tp}$.[14]

To clearly illustrate the third-party servicer's function in our model as a commitment device, we also characterise the ex ante optimal foreclosure policy of a securitiser who can commit. Denote $(\tilde{\lambda}_H, \tilde{\lambda}_L)$ as the securitiser's optimal foreclosure policy with commitment, which maximises the securitiser's expected payoff as follows

$$\begin{aligned}(\tilde{\lambda}_H, \tilde{\lambda}_L) = \arg\max_{(\lambda_H,\lambda_L)} \quad & \gamma\delta(\pi V_G + (1-\pi)[V_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)\theta_H X]) \\ & \qquad + \gamma(1-\delta)p(\mathcal{F}) + (1-\gamma)U_L(\lambda_L) \\ s.t. \quad & \mathcal{F} \text{ is given by Eq. 4 with } \hat{\lambda}_i = \lambda_i \\ & U_L(\cdot) \text{ is given by Eq. 3 with } \hat{\lambda}_L = \lambda_L \end{aligned} \tag{9}$$

The following proposition characterises the equilibrium involving third-party servicing and the properties of the equilibrium foreclosure policy, as well as a comparison with foreclosure policy with commitment.

Proposition 3. *In equilibrium the securitiser makes an offer that is accepted by the originator if and only if* $\kappa \leq [\gamma U_H(\tilde{\lambda}_H, \tilde{\lambda}_L) + (1-\gamma)U_L(\tilde{\lambda}_L)] - [\gamma U_H^{ih} + (1-\gamma)U_L^{ih}]$. *The equilibrium foreclosure policy with third-party servicing coincides with the foreclosure policy with commitment* $(\lambda_H^{tp}, \lambda_L^{tp}) = (\tilde{\lambda}_H, \tilde{\lambda}_L)$, *where*

$$\lambda_H^{tp} \leq \lambda_H^{FB} < \lambda_L^{FB} \leq \lambda_L^{tp} \tag{10}$$

That is, in the bad state, there is (weakly) insufficient foreclosure if the mortgage pool is of high quality, and (weakly) excessive foreclosure if the mortgage pool is of

[14] The above programme restricts that the securitiser only makes offers that the originator of either type accepts through (PC). This is without loss of generality, since the the securitiser is indifferent between making an offer that will not be accepted, and not making an offer at all.

low quality. The weak inequalities are strict if and only if

$$\frac{\theta_H - \theta_L}{1-\theta_H}(1-\lambda_L^{FB})X \quad > \quad [\mathcal{L}(\lambda_L^{FB}) - \mathcal{L}(\lambda_H^{FB})] + \frac{\pi}{1-\pi}\frac{1-\delta}{1-\theta_H}[\mathcal{L}(\lambda_H^{FB}) - \mathcal{L}(\lambda_L^{FB}) + (\lambda_L^{FB} - \lambda_H^{FB})X] \tag{11}$$

Proposition 3 highlights a number of important results of this model. Firstly, it formally establishes the role of third-party servicer, which is to allow the securitiser to implement the ex ante optimal foreclosure policy as if she has the commitment power to do so. It also predicts that when the benefit of having this effective commitment outweighs the cost of contracting with a third-party servicer κ, securitisation with third-party servicing arises in equilibrium.

The second part of the proposition concerns the equilibrium foreclosure policy, which is the main focus of the paper. The equilibrium foreclosure policy deviates from the first-best policy and exhibits two-sided distortions when Eq. 11 holds.This is more likely when the bad state, which is also the information sensitive state, is more relevant, i.e. π is low. In other words, when the information friction in the securitisation process is severe enough, the optimal foreclosure policy is distorted: the low type forecloses excessively and the high-type insufficiently.

The two-sided distortion in the equilibrium foreclosure policy aims to mitigate the signalling cost to be incurred at the securitisation stage. The intuition is best illustrated by studying the low type's no-mimicking, incentive compatibility constraint, which can written as

$$\underbrace{p(\mathcal{F})}_{\text{cash flow affected by } \lambda_H^{tp}} + \underbrace{\text{retained equity}}_{\text{cash flow affected by } \lambda_L^{tp}} \leq U_L(\lambda_L^{tp}) \tag{12}$$

The left-hand-side of the above inequality is the low type's mimicking payoff. It comprises of two parts – the cash proceeds $p(\mathcal{F})$ which she receives from issuing the security, and the value of the retained cash flow. The optimal foreclosure policy in equilibrium deviates from the first-best (also ex post efficient) benchmark in order to reduce the mimicking payoff, thereby relaxing the incentive compatibility constraint and reducing the retention cost of signalling in equilibrium.

This is achieved precisely through the two-sided distortion of the foreclosure policy. Recall two important features of the model. First, the optimal security issued by the good type is a debt like security in the bad state, leaving the issuer with a levered equity claim. Second, an increase in foreclosure of delinquent loans decreases the riskiness of the cash flows of the mortgage pool of a given type. The result then follows. One the one hand, the foreclosure policy of a high-type securitiser is lower than first best so as to increase the riskiness of her cash flows. By Jensen's inequality, this then reduces the value of the high-type's debt-like, concave security and the incentive for a bad type to mimic. On the other hand, the foreclosure policy of a low-type securitiser is higher than first best so as to decrease the riskiness of her cash flows. This lowers the value of her retained equity should she mimics the high-type.

Last but not least, the equilibrium servicing contracts offered to the originator appear to contain biased incentives.

Corollary 1. *The menu of two servicing contracts offered to the originator in equilibrium are such that $\beta_H^{tp} \leq 1 \leq \beta_L^{tp}$, where the inequalities are strict if and only if the condition given by Eq. 11 holds.*

To conclude this section, we would like to emphasise that the securitiser is able to effectively commit to a set of ex ante foreclosure policy through contracting with a third-party servicer. Therefore the contracts to the servicer contain endogenously biased incentives, in order to implement the desired foreclosure policy. In particular, for a mortgage pool of low quality, the servicer's contract is biased towards excessive foreclosures.

5 Comparing to securitisation with in-house servicing

This section studies the equilibrium securitisation and foreclosure policy under in-house servicing. The key difference from third-party servicing is that when the securitiser is also the servicer, his foreclosure decisions ex post will be affected by his retained claims on the mortgage pool. By finding the equilibrium payoff under in-house servicing, we can endogenise the efficiency gains from commitment via third-party servicing.

5.1 Securitisation with in-house servicing

In this section we consider the equilibrium foreclosure policy in the sub-game of securitisation with in-house servicing. At $t = 2$, the securitiser-servicer makes the foreclosure decision to maximise his retained cash flow given his type i and the security issued at $t = 1$.

For a low-type securitiser, since he securitises a full pass-through equity security in a separating equilibrium, he retains no cash flow. Maintaining the assumption that in this case that he makes the first-best foreclosure decision to maximise the value of the mortgage pool, $\lambda_L^{ih} = \lambda_L^{FB}$, the payoff to a low-type originator-securitiser is therefore equal to $U_L^{ih} = U_L^{FB}$.

The high-type securitiser chooses the optimal security $\mathcal{F}$ at $t = 1$ to maximise his expected payoff, taking into account the subsequent foreclosure policy λ_H chosen at $t = 2$ given the security issued. In the least cost separating equilibrium, the optimal security is given by

$$\begin{aligned}
&\max_{\mathcal{F}} && p(\mathcal{F}) + \delta\pi(c_1 - f_1) + \delta(1-\pi)[\theta_H(c_2 - f_2) + (1-\theta_H)(c_5 - f_5)] \\
s.t. \quad &(IC) && U_L^{ih} \geq p(\mathcal{F}) + \delta\pi(c_1 - f_1) \\
& && \qquad + \delta(1-\pi)[\theta_L(c_3 - f_3) + (1-\theta_L)(c_4 - f_4)] \\
&(IH_H) && \lambda_H = \arg\max_{\lambda} \pi(c_1 - f_1) + (1-\pi)\left[\theta_H(c_2 - f_2) + (1-\theta_H)(c_5 - f_5)\right] \\
&(IH_L) && \lambda_L' = \arg\max_{\lambda} \pi(c_1 - f_1) + (1-\pi)\left[\theta_L(c_3 - f_3) + (1-\theta_L)(c_4 - f_4)\right] \\
&(MC), && (LL), (MNO) \text{ and } (MNI) \text{ given in Eq. 4}
\end{aligned} \tag{13}$$

where the security $\mathcal{F}$ maps the realisation of the mortgage pool cash flows c_j, $j \in \{1, 2, 3, 4, 5\}$, to a set of payoffs to the outside investors, as given in Table 2, (IC) is the incentive compatibility constraint for the low type not to mimic the security issued by the high type, (IH_H) is the incentive compatibility constraint that the in-house foreclosure policy of the high type's security, λ_H, following issuing the security $\mathcal{F}$, maximises the expected value his residual claim, and (IH_L) is the incentive compatibility constraint that, if a low-type securitiser deviates to mimic the high type and issues the security $\mathcal{F}$, his subsequent in-house foreclosure policy λ_L' maximises the expected value of his residual claim.

This security design problem for the high-type securitiser with in-house servicing

Table 2: Payoffs of the security issued by the high type with in-house servicing

Realisation of cash flow	Security payoff $\mathcal{F}$
$c_1 \equiv V_G$	f_1
$c_2 \equiv V_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X$	f_2
$c_3 \equiv V_B + \mathcal{L}(\lambda'_L) + (1 - \lambda'_L)X$	f_3
$c_4 \equiv V_B + \mathcal{L}(\lambda'_L)$	f_4
$c_5 \equiv V_B + \mathcal{L}(\lambda_H)$	f_5

has two main differences compared to that for the high-type securitiser with third-party servicing (Eq. 4). Firstly, the (IC) constraint depends on the off-equilibrium foreclosure policy λ'_L when the low-type securitiser mimics, instead of his equilibrium foreclosure policy λ_L. This is because, following a deviation by the low-type securitiser to issue a security $\mathcal{F}$, his incentive to foreclose delinquent mortgages at $t = 2$ is determined by his residual stake. λ'_L thus determines the mimicking cash flows c_3 and c_4 in Table 2, instead of λ_L. Secondly, the optimisation programme given in Eq. 13 has two additional constraints, namely (IH_H) and (IH_L), compared to that with third-party servicing (Eq. 4). These two constraints reflect the fact that with in-house servicing, the securitiser does not have commitment power over the choice of his foreclosure policy. Instead, the equilibrium foreclosure policy for the high type λ_H and the off-equilibrium foreclosure policy for the low type λ'_L must be incentive compatible at $t = 2$ given the security issued.

The optimal security $\mathcal{F}^{ih}$ still resembles debt in this case. This is because Eq. 13 is a more constrained version of Eq. 4, and the solution to this constrained programme should satisfy all the properties of the more relaxed programme. Recall that Proposition 2 implies that $f_2 < V_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X$ and $f_5 = V_B + \mathcal{L}(\lambda_H)$ for any $\lambda_H \leq \lambda'_L$. By (IH_H), it follows that the optimal foreclosure policy for the high-type securitiser maximises $V_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X$. That is, he chooses zero foreclosure $\lambda_H^{ih} = 0$ as $\mathcal{L}'(0) \leq X$. The following proposition formally summarises the equilibrium foreclosure policy under in-house servicing.

Proposition 4. *If the originator declines the offer and securitises the mortgage pool with in-house servicing, the equilibrium foreclosure policy is given by* $(\lambda_H^{ih}, \lambda_L^{ih}) = (0, \lambda_L^{FB})$*. That is, there is excessive forbearance if the mortgage pool is of high quality.*

The intuition behind this proposition is similar to the classical conflict of interests between equity holders and creditors inside a financially distressed firm. As the optimal security issued by the high-type securitiser resembles a debt in the bad state, his retained stake is a levered equity. Foreclosure tends to reduce the riskiness in the mortgage pool cash flow and accrues all the proceeds to the creditors if the mortgage pool does not recover. Therefore the high-type securitiser with in-house servicing chooses zero foreclosure rate in a risk-shifting attempt. The high-type securitiser's expected payoff in equilibrium is given by

$$U_H^{ih} \equiv (1-\delta)p(\mathcal{F}^{ih}) + \delta[\pi V_G + (1-\pi)(V_B + \theta_H X)] \tag{14}$$

where $\mathcal{F}^{ih}$ is the solution to the optimisation programme Eq. 13.

5.2 Efficiency of securitisation with third-party servicing

Having characterised the equilibrium securitisation and foreclosure with in-house servicing, we now compares it with the equilibrium with third-party servicing.

The previous analysis indicates that there is *ex ante* inefficiency with in-house servicing. The high-type securitiser's zero foreclosure policy reduces the expected value of the mortgage pool and especially the MBS significantly. Anticipating this to happen in a bad state in equilibrium, investors would pay a much lower price for the MBS ex ante. In the end, the securitiser himself bears the cost of excessive forbearance and if possible, he would choose to commit to foreclose more.

Third-party servicing enable such commitment. Thus securitisation with third-party servicing is ex ante more efficient than that with in-house servicing, as stated in the following proposition.

Proposition 5. *The expected value of the mortgage pool in equilibrium is strictly higher in the case with third-party servicer than in the case with in-house servicer. That is,*

$$\gamma U_H(\lambda_H^{tp}, \lambda_L^{tp}) + (1-\gamma)U_L(\lambda_L^{tp}) > \gamma U_H^{ih} + (1-\gamma)U_L^{ih} \tag{15}$$

This result highlights the role played by the third-party servicer in the securitisation process. With in-house servicing, the foreclosure policy is determined ex post by the securitiser given his private information and given that he retains a residual levered equity claim from the mortgage pool. With third-party servicing, the separation of cash flow rights and servicing rights allows the securitiser to effectively commit to a set of *ex post* inefficient foreclosure policy, that maximises the *ex ante* expected value of the mortgage pool.

6 Empirical implications

This section summarises the empirical implications of our model related to foreclosure policy and characteristics of mortgage servicers' compensation contracts. Here we emphasise the information required in these predictions: Statements 1–2 apply to ex ante average-quality mortgage pools, whereas 3–5 depend on the (unobservable) quality of the mortgage pools (ex post observable given the securities issued in separating equilibria). Moreover, statements 2–4 apply to securitised mortgages both with in-house servicing and with third-party servicing.

1. *For a given quality of mortgage pool, third-party servicers foreclose more delinquent mortgages than in-house servicers do.* In our model, third-party servicers—who act according to their incentive contracts—implement the ex ante optimal foreclosure policy, thus foreclosing more (fewer) delinquent mortgages than the ex post efficient rates when their mortgage pools are of low (high) quality. In contrast, in-house servicers foreclose to maximise the value of their retained securities. Proposition 4 shows that in-house servicers foreclose few mortgages when their mortgage pools are of high quality because they retain levered equity claims, and foreclose at the ex post efficient rate when their mortgage pools are of low quality. Therefore, regardless of the mortgage pools' quality, third-party servicers foreclose more mortgages than in-house servicers do.

2. *Securitised mortgages on average feature larger variations in the amount of foreclosures than comparable bank-held mortgages.*

3. *Low-quality (high-quality) securitised mortgage pools have a weakly higher (lower) foreclosure rate than comparable bank-held mortgages.* This and the previous prediction follow from the main result of our model, which shows that the information friction in the process of securitisation will distort foreclosures

of delinquent mortgages towards the extremes in a bad state (Proposition 3). That is, securitstaion leads to more (fewer) foreclosures for low-quality (high-quality) mortgage pools.[15]

4. *Foreclosing the marginal delinquent mortgage in a low-quality (high-quality) securitised pool returns weakly less (more) than the mortgage's expected recovery value.* Proposition 3 shows that foreclosures in a low-quality (high-quality) mortgage pool are excessive (insufficient) comparing to the ex post efficient foreclosure rates, at which the marginal proceeds from foreclosing an additional mortgage equals the mortgage's expected recovery value. In other words, at the point when servicers implement the ex ante optimal foreclosure policy, foreclosing an additional delinquent mortgage in a low-quality (high-quality) securitised pool will decrease (increase) the pool's expected value.

5. *Third-party servicers of low-quality (high-quality) mortgage pools have compensation contracts that are biased weakly towards (against) foreclosure.* In our model, securitisers offer optimal incentive contracts to third-party servicers in order to implement the ex ante optimal foreclosure policy (Corollary 1). Since the optimal policy is biased comparing to the ex post efficient benchmark, the servicers' incentives have to be biased accordingly.

7 Conclusion

This paper formally studies the relationship between the foreclosure decision of delinquent mortgages and the securitisation of mortgages, and examines the role of mortgage servicers in this process.

We investigate the optimal servicing arrangement and foreclosure decision in a model of mortgage-backed securitisation under asymmetric information. A securitiser with a pool of mortgages has private information regarding the mortgages' recovery rate when they become delinquent. The securitiser initially designs and sells mortgage-backed securities, and decides what fraction of the delinquent mortgages in the

[15]Existing empirical finding of Piskorski et al. (2010), Agarwal et al. (2011a) and Krueger (2014) show that on average, securitisation leads to more foreclosures during the recent subprime mortgage crisis. This is consistent with our model's prediction if the mortgage pools' average quality is sufficiently low.

mortgage pool to foreclose or modify.

Relative to the case with full information, we show that the optimal foreclosure policy under asymmetric information involves excessive foreclosure if the mortgage pool is of low quality, and insufficient foreclosure if the mortgage pool is of high quality. This is because by distorting the ex post foreclosure policy , the securitiser reduces the ex ante signalling cost incurred at the securitisation stage. Our model thus predicts that securitisation distorts foreclosure policy due to information friction.

Our paper also addresses the role played by third-party servicers in the mortgage backed securitisation industry and their optimal compensation contracts. By contracting with third-party servicers, the securitisers can effectively commit to the ex ante optimal foreclosure policy. As the optimal policy is biased (from an ex post perspective), the servicers' compensation contracts are hence *endogenously* biased.

We close by listing some predictions coming from our model for future empirical works. Different from the setting of most existing empirical tests, some of our predictions depend on the heterogeneous, unobservable quality of delinquent mortgages, namely the recovery probability. We believe, in additional to the proposed economic mechanism, this difference in the requirement of the econometrician's information set makes our predictions novel.[16]

We conclude with some conjecture of directions for future works and extensions. First, this framework can be extended to a setting with multiple securitisers to study the spillover effects of foreclosures. For instance, it will be interesting to study the interaction between the distorting effect of securitisation and the fire-sale externality in the distressed property market. It could also be fruitful to analyse, in a general equilibrium, the potential impact of securitisation on the quantity, quality, and the prices of mortgages originated. Finally, a dynamic framework could shed light on how the distorting effect of securitisation interacts with property prices across business cycles.

[16] Begley and Purnanandam (2013) finds evidence that retained equity trench as proxies of unobservable quality of RMBS. However, they do not study the foreclosure likelihood *conditional on delinquency*.

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Appendices

A Proofs

A.1 Proof of Proposition 1

The first-best foreclosure level λ_i^{FB} for type i maximises the expected payoff of the mortgage pool $V_B + \mathcal{L}(\lambda_i) + (1-\lambda_i)\theta_i X$. Thus the first order condition is $\frac{\partial \mathcal{L}(\lambda_i^{FB})}{\partial \lambda_i} = \theta_i X$ for $i \in \{H, L\}$. Using the functional form of $\mathcal{L}(\lambda) = aX \ln(1 + \frac{\lambda}{a})$, the results follow immediately.

A.2 Proof of Proposition 2

Before proceeding to the proof, let's recall the possible realisation of cash flows of the mortgage pool and the security payoff issued by the high type.

Table 3: Payoffs of the security issued by the high type

Realisation of cash flow	Security payoff $\mathcal{F}$
$c_1 \equiv V_G$	f_1
$c_2 \equiv V_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X$	f_2
$c_3 \equiv V_B + \mathcal{L}(\lambda_L) + (1-\lambda_L)X$	f_3
$c_4 \equiv V_B + \mathcal{L}(\lambda_L)$	f_4
$c_5 \equiv V_B + \mathcal{L}(\lambda_H)$	f_5

We focus on the case with the following ranking of cash flow: $c_1 > c_2 > c_3 > c_4 > c_5$, which will be the case if the equilibrium foreclosure rate satisfies $\lambda_L > \lambda_H$.

In a separating equilibrium, the optimal strategy for the low type is to issue all of the equity to outside investors in order to minimise the retention cost. And the optimal (monotone) security for the high type has to satisfy incentive compatibility (IC) constraint to prevent the low type from mimicking, the set of limited liability constraints and the monotonicity constraints for both securities hold by outside investors (MNO) and retained by insiders (MNI) securities. Combining with limited liability constraints, the constraints can be re-written as:

$$(IC) \qquad U_L \geq \pi f_1 + (1-\pi)[\theta_H f_2 + (1-\theta_H) f_5] + \delta\{\pi(c_1 - f_1) + (1-\pi)[\theta_L(c_3 - f_3) + (1-\theta_L)(c_4 - f_4)]\} \tag{16}$$

$$(MNO) \quad f_1 \geq f_2 \geq f_3 \geq f_4 \geq f_5 \geq 0 \tag{17}$$

$$(MNI) \quad c_1 - f_1 \geq c_2 - f_2 \geq c_3 - f_3 \geq c_4 - f_4 \geq c_5 - f_5 \geq 0 \tag{18}$$

And the objective is to design an security $\mathcal{F} = \{f_1, f_2, f_3, f_4, f_5\}$ in order to maximise the high type's securities selling price

$$p(\mathcal{F}) = \pi f_1 + (1-\pi)[\theta_H f_2 + (1-\theta_H) f_5] \tag{19}$$

The proof is constructed by establishing several claims in succession. Let us call a security $\mathcal{F}$ *permissible* if it satisfies (IC), (MNO), and (MNI). An optimal security is a permissible security that maximises the payoff $p(\mathcal{F})$.

Claim 1. *For any optimal security $\mathcal{F}^* = \{f_1^*, f_2^*, f_3^*, f_4^*, f_5^*\}$, $f_1^* < c_1$.*

Proof. If $f_1^* = c_1$, by (MCI), $f_j^* = c_j$ for $j = \{2, 3, 4, 5\}$. This security (full equity) violates (IC). □

Claim 2. *For any optimal security $\mathcal{F}^*$, the (IC) must bind.*

Proof. Suppose instead the (IC) is slack for some optimal security $\mathcal{F}^* = \{f_1^*, f_2^*, f_3^*, f_4^*, f_5^*\}$. By Claim 1, $f_1^* < c_1$. Unless $c_1 - f_1^* = c_2 - f_2^*$, there exists another permissible security $\hat{\mathcal{F}} = \{\hat{f}_1, f_2^*, f_3^*, f_4^*, f_5^*\}$ with $\hat{f}_1 > f_1^*$ that satisfies (IC). As $p(\mathcal{F})$ strictly increases with f_1, $p(\hat{\mathcal{F}}) > p(\mathcal{F}^*)$, contradicting the assumption that $\mathcal{F}^*$ is optimal.

If $f_1^* < c_1$ and $c_1 - f_1^* = c_2 - f_2^*$, one can increase the objective function $p(\mathcal{F}^*)$ by increasing both f_1^* and f_2^* by some $\epsilon > 0$ without violating (IC), unless $f_2^* = c_2$ or $c_2 - f_2^* = c_3 - f_3^*$. Note that $f_2^* = c_2$ is not possible as it implies $f_1^* = c_1$ violating Claim 1.

Suppose now $f_1^* < c_1$ and $c_1 - f_1^* = c_2 - f_2^* = c_3 - f_3^*$, similarly one can increase all f_1^*, f_2^*, f_3^* without violating (IC) to strictly increase $p(\mathcal{F}^*)$, unless $c_3 - f_3^* = c_4 - f_4^*$.

By similar argument, we reach the last possible case with $f_1^* < c_1$ and $c_1 - f_1^* = c_2 - f_2^* = c_3 - f_3^* = c_4 - f_4^* = c_5 - f_5^*$. One can increase all f_i^* by a small amount without violating (IC) unless $f_5^* = c_5$ which also leads to a contradiction that

$f_1^* = c_1$. Since we have shown any optimal security with a slacking (IC) can be improved upon, all optimal securities must have the (IC) binding. □

Claim 3. *For any optimal security $\mathcal{F}^*$, either $f_{j-1}^* = f_j^*$ or $c_j - f_j^* = c_{j+1} - f_{j+1}^*$ (or both) for $j = \{3, 4\}$.*

Proof. Let's start with the case with $j = 3$. The proof proceeds by contradiction. Suppose there is an optimal security $\mathcal{F}^* = \{f_1^*, f_2^*, f_3^*, f_4^*, f_5^*\}$ with $f_2^* > f_3^*$ and $c_3 - f_3^* > c_4 - f_4^*$. Since the (IC) is relaxed by increasing f_3, one can construct another permissible security $\hat{\mathcal{F}} = \{f_1^*, f_2^*, \hat{f}_3, f_4^*, f_5^*\}$ with some $\hat{f}_3 > f_3^*$. Notice that this security $\hat{\mathcal{F}}$ has the same price as $\mathcal{F}^*$, i.e. $p(\hat{\mathcal{F}}) = p(\mathcal{F}^*)$, as $p(\mathcal{F})$ does not depend on f_3. However, by Claim 2 and the fact that the (IC) is slack under security $\hat{\mathcal{F}}$, $\hat{\mathcal{F}}$ is not an optimal security. In other words, there exists another permissible security $\hat{\hat{\mathcal{F}}}$ such that $p(\hat{\hat{\mathcal{F}}}) > p(\hat{\mathcal{F}}) = p(\mathcal{F}^*)$. As a result, the assumption that $\mathcal{F}^*$ is optimal is contradicted.

The proof is identical for the case with $j = 4$. □

Claim 4. *For any optimal securities $\mathcal{F}^*$, $f_3^* > f_4^*$.*

Proof. Suppose that $f_3^* = f_4^*$, which implies $c_3 - f_3^* > c_4 - f_4^*$. By Claim 3, $f_2^* = f_3^* = f_4^*$. But (IC) is slack under this $\mathcal{F}^*$, violating its optimality assumption by Claim 2. To see this, note that the (IC) can be written as:

$$(1-\delta)U_L \geq \pi(1-\delta)f_1 + (1-\pi)[\theta_H f_2 + (1-\theta_H)f_5] - \delta(1-\pi)[\theta_L f_3 + (1-\theta_L)f_4] \tag{20}$$

For security $\mathcal{F}^*$ with $f_2^* = f_3^* = f_4^*$, the right-hand side of the IC in Eq. (20) is strictly less than its left-hand side:

$$\begin{aligned} &\pi(1-\delta)f_1^* + [(1-\pi)\theta_H - \delta(1-\pi)]f_4^* + (1-\pi)(1-\delta_H)f_5^* \\ \leq&\pi(1-\delta)f_1^* + (1-\pi)(1-\delta)f_4^* \\ <&(1-\delta)\{\pi c_1 + (1-\pi)[\theta_L c_3 + (1-\delta_L)c_4]\} = (1-\delta)U_L \end{aligned}$$

The first weak inequality follows from $f_4^* \geq f_5^*$. The second strict inequality follows from $f_1^* < c_1$ (Claim 1), $f_4^* \geq f_3^*$, $c_3^* \geq f_3^*$, and $c_4^* \geq f_4^*$. □

Claim 5. *For an optimal security $\mathcal{F}^*$, $f_4^* = c_4$ and $f_5^* = c_5$.*

Proof. To prove Claim 5, we will show that for any optimal security $\mathcal{F}^*$ with $f_4^* < c_4$, there exists another permissible, payoff-equivalent security with which the (IC) is slack. By Claim 2, therefore, $\mathcal{F}^*$ cannot be an optimal security.

By Claim 3 and 4, we know $f_3^* > f_4^*$ and $c_4 - f_4^* = c_5 - f_5^*$. Suppose that $f_4^* < c_4$, implying $c_4 - f_4^* = c_5 - f_5^* > 0$, we will show that one can simultaneously (i) increase both f_4^* and f_5^* by a small positive ϵ and (ii) decrease some $\{f_i^*\}$ to keep the payoff unchanged while relaxing the (IC). Consider the follow two (exhaustive) cases for any optimal security $\mathcal{F}^*$:

(I). $c_1 - f_1^* > c_2 - f_2^*$ and $f_2^* \geq f_3^*$:

Pick an arbitrarily small, positive ϵ. Construct a new security $\hat{\mathcal{F}}$ from $\mathcal{F}^*$ by increasing f_4^* and f_5^* by $\frac{1}{1-\theta_H}\epsilon$ and decreasing f_2^* and f_3^* by $\frac{1}{\theta_H}\epsilon$. By construction, $p(\hat{\mathcal{F}}) = p(\mathcal{F}^*)$ whereas the (IC) is slack under $\hat{\mathcal{F}}$. To see this, the RHS of the (IC) shown in eq. (20) is reduced by $\delta(1-\pi)\frac{\theta_H - \theta_L}{\theta_H(1-\theta_H)} > 0$. Therefore, by Claim 2, $\hat{\mathcal{F}}$ is not optimal and there exists another permissible $\hat{\hat{\mathcal{F}}}$ such that $p(\hat{\hat{\mathcal{F}}}) > p(\hat{\mathcal{F}}) = p(\mathcal{F}^*)$, contradicted the optimality assumption of $\mathcal{F}^*$.

(II). $c_1 - f_1^* = c_2 - f_2^*$ and $f_2^* \geq f_3^*$:

Pick an arbitrarily small, positive ϵ. Construct a new security $\hat{\mathcal{F}}$ from $\mathcal{F}^*$ by increasing f_4^* and f_5^* by $\frac{1}{(1-\pi)(1-\theta_H)}\epsilon$ and decreasing f_1^* by $\frac{1}{\pi}\epsilon$ (Note that $c_1 - f_1^* = c_2 - f_2^*$ implies $f_1^* > f_2^*$). It is immediate to check that $p(\hat{\mathcal{F}}) = p(\mathcal{F}^*)$ and the (IC) is relaxed by $\delta(\frac{\theta_H - \theta_L}{1-\theta_H})\epsilon > 0$. By similar reasons stated in case (I), $\mathcal{F}^*$ is not optimal.

As all the possible cases of a potential optimal security with $f_4^* < c_4$ lead to contradictions, we have shown that (with limited liability constraint) for any optimal security, $f_4^* = c_4$.

By (MCI), $0 = c_4 - f_4^* \geq c_5 - f_5^*$, thus $f_5^* = c_5$. □

Claim 6. *For any optimal security $\mathcal{F}^*$, $c_1 - f_1^* = c_2 - f_2^*$.*

Proof. Suppose contrary that $c_1 - f_1^* > c_2 - f_2^*$, pick an arbitrarily small, positive ϵ and construct a new security $\hat{\mathcal{F}}$ from $\mathcal{F}^*$ by increasing f_1^* by $\frac{1}{\pi}\epsilon$ and decreasing f_2^* and f_3^* by $\frac{1}{(1-\pi)\theta_H}\epsilon$. It is immediate to check that $p(\hat{\mathcal{F}}) = p(\mathcal{F}^*)$ and the (IC) is relaxed by $\delta(1 - \frac{\theta_L}{\theta_H})\epsilon > 0$. □

To sum up, Claim 1–6 establish that the optimal security $\mathcal{F}^*$ will be one of the following two cases:

1. $c_1 > f_1^* = (c_1 - c_2) + f_2^* > f_2^* > f_3^* = c_3 > f_4^* = c_4 > f_5^* = c_5$
2. $c_1 > f_1^* = (c_1 - c_2) + f_2^* > f_2^* = f_3^* > f_4^* = c_4 > f_5^* = c_5$

The only differences between the two cases is whether $f_2^* > f_3^*$ or $f_2^* = f_3^*$.[17] In both cases, there remains only one free parameter f_2^* in the optimal security. And since we know the (IC) binds for any optimal security, f_2^* will be pinned down by the (IC).

For case 1, substitute $f_1^* = c_1 - c_2 + f_2^*, f_3^* = c_3, f_4^* = c_4$, $f_5^* = c_5$ into the (IC). Then the f_2^* satisfies (IC)

$$U_L = \pi(c_1 - c_2 + f_2^*) + (1-\pi)[\theta_H f_2^* + (1-\theta_H)c_5] + \delta\pi(c_1 - c_1 + c_2 - f_2^*)$$

Hence

$$f_2^* = \frac{U_L - \pi c_1 + \pi(1-\delta)c_2 - (1-\pi)(1-\theta_H)c_5}{\pi(1-\delta) + (1-\pi)\theta_H} \tag{21}$$

$$= \frac{\pi(1-\delta)c_2 + (1-\pi)[\theta_L c_3 + (1-\theta_L)c_4 - (1-\theta_H)c_5]}{\pi(1-\delta) + (1-\pi)\theta_H} \tag{22}$$

Similarly, for case 2, substitute $f_1^* = c_1 - c_2 + f_2^*, f_3^* = f_2^*, f_4^* = c_4$, $f_5^* = c_5$ into the (IC). Then

$$f_2^* = \frac{U_L - \pi c_1 + \pi(1-\delta)c_2 - \delta(1-\pi)\theta_L c_3 - (1-\pi)(1-\theta_H)c_5}{\pi(1-\delta) + (1-\pi)(\theta_H - \delta\theta_L)} \tag{23}$$

$$= \frac{\pi(1-\delta)c_2 + (1-\pi)[\theta_L(1-\delta)c_3 + (1-\theta_L)c_4 - (1-\theta_H)c_5]}{\pi(1-\delta) + (1-\pi)(\theta_H - \delta\theta_L)} \tag{24}$$

Finally, we need to check whether $f_2^* > f_3^* = c_3$ in case (1) and $f_2^* = f_3^* \le c_3$ in case (2). Define $G(\theta_H, \theta_L; \pi; \delta)$ as

$$G(\theta_H, \theta_L; \pi; \delta) \equiv \theta_H(c_3 - c_5) - \theta_L(c_3 - c_4) - (c_4 - c_5) - \frac{\pi}{1-\pi}(1-\delta)(c_2 - c_3) \tag{25}$$

For $G(\theta_H, \theta_L; \pi; \delta) < 0$, we have the $f_2^* > f_3^* = c_3$ as in case (1). Similarly for

[17] Note that $f_2^* > f_3^*$ implies $f_3^* = c_3$ because of Claim 3 and 5.

$G(\theta_H, \theta_L; \pi; \delta) \geq 0$, the optimal security is described as in case (2) where $f_2^* = f_3^* \leq c_3$.

A.3 Proof of Lemma 1

This result follows immediately by implicitly differentiating the first order condition $\beta \frac{\partial \mathcal{L}(\lambda_i)}{\partial \lambda_i} = \theta_i X$ with regard to β.

A.4 Proof of Proposition 3

To show this proposition, we first construct an equilibrium with foreclosure policy $(\tilde{\lambda}_H, \tilde{\lambda}_L)$. We then characterise the properties of the equilibrium foreclosure policy $(\tilde{\lambda}_H, \tilde{\lambda}_L)$.

To construct the equilibrium, we first conjecture that the equilibrium foreclosure policy is given by $(\tilde{\lambda}_H, \tilde{\lambda}_L)$, where $\tilde{\lambda}_H < \tilde{\lambda}_L$. We then show that there exists a menu of servicer contracts that implements such foreclosure policy and satisfies (IC_i) and binds (PC), given in the optimisation programme Eq. 7. It then follows that such contracts indeed maximises the expected payoff to the securitiser as specified by Eq. 7.

In order to implement an equilibrium foreclosure policy equal to $(\tilde{\lambda}_H, \tilde{\lambda}_L)$, the equilibrium servicer contracts must have unique β_i^{tp} such that $\beta_i^{tp} \frac{\partial \mathcal{L}(\lambda_i^{tp})}{\partial \lambda_i} = \theta_i X$. The uniqueness follows from Lemma 1.

We then construct a menu of servicer contracts such that (PC) binds. The binding PC implies that $w_H(\alpha_H, \beta_H^{tp}, \tau_H) = U_H^{ih} + \frac{1-\gamma}{\gamma}[U_L^{ih} - w_L(\alpha_L, \beta_L^{tp}, \tau_L)]$. Suppose further that $w_L(\alpha_L, \beta_L^{tp}, \tau_L) = U_L^{ih}$. We can then write the flat transfer τ_i in the contracts as

$$\begin{aligned} \tau_L^{tp} &= U_L^{ih} - \delta \alpha_L K(\theta_L, \beta_L^{tp}) \\ \tau_H^{tp} &= U_H^{ih} - \delta \alpha_H K(\theta_H, \beta_H^{tp}) \end{aligned} \tag{26}$$

where $K(\theta, \beta) \equiv \beta \, \mathcal{L}(\lambda^s(\beta)) + (1 - \lambda^s(\beta))\theta X$. This implies that

$$\tau_H^{tp} - \tau_L^{tp} = U_H^{ih} - U_L^{ih} - \delta[\alpha_H K(\theta_H, \beta_H^{tp}) - \alpha_L K(\theta_L, \beta_L^{tp})] \tag{27}$$

Rearranging (IC_i) yields the following

$$\alpha_L K(\theta_L, \theta_L^{tp}) - \alpha_H K(\theta_L, \theta_H^{tp}) \geq \tau_H^{tp} - \tau_L^{tp} \geq \alpha_L K(\theta_H, \beta_L^{tp}) - \alpha_H K(\theta_H, \beta_H^{tp}) \quad (28)$$

Substituting Eq. 27 into 28 and rearranging produces

$$\alpha_L[K(\theta_H, \beta_L^{tp}) - K(\theta_L, \beta_L^{tp})] < \frac{U_H^{ih} - U_L^{ih}}{\delta} < \alpha_H[K(\theta_H, \beta_H^{tp}) - K(\theta_L, \beta_H^{tp})] \quad (29)$$

Claim 7 in Appendix A.6 shows that $U_H^{ih} > U_L^{ih}$. By the Envelope Theorem, we have $\frac{\partial K(\theta,\beta)}{\partial \beta} > 0$, $\frac{\partial K(\theta,\beta)}{\partial \theta} > 0$ and $\frac{\partial^2 K(\theta,\beta)}{\partial \beta \partial \theta} < 0$. This implies $0 < K(\theta_H, \beta_L^{tp}) - K(\theta_L, \beta_L^{tp}) < K(\theta_H, \beta_H^{tp}) - K(\theta_L, \beta_H^{tp})$. Therefore there exist $\alpha_H^{tp}, \alpha_L^{tp} \geq 0$ such that the equilibrium contracts satisfy (IC_i), bind (PC), and implement the foreclosure policy $\lambda_i^{tp} = \tilde{\lambda}_i$.

Having established that the solution to the optimisation programme given by Eq. 8 coincides with the solution to the optimisation programme given by Eq. 9, we next characterise the equilibrium foreclosure policy by examining the solution to the latter optimisation programme, to show that $\lambda_H^{tp} \leq \lambda_H^{FB} < \lambda_L^{FB} \leq \lambda_L^{tp}$. Because the optimisation programme has a continuous objective function over a compact set, an maximum always exists.

There are two cases given by Appendix A.2, depending on the specific feature of the security issued in equilibrium. Which case is in equilibrium depends on the value of the $G(\cdot)$ function (Eq. 25). In order to see the effect of the foreclosure policy (λ_H, λ_L) on this condition, we rewrite the function $G(\cdot)$ as follows

$$\begin{aligned} G(\lambda_H, \lambda_L) &= \theta_H[\mathcal{L}(\lambda_L) + (1-\lambda_L)X - \mathcal{L}(\lambda_H)] - \theta_L(1-\lambda_L)X - [\mathcal{L}(\lambda_L) - \mathcal{L}(\lambda_H)] \\ &\quad - \frac{\pi}{1-\pi}(1-\delta)[\mathcal{L}(\lambda_H) - \mathcal{L}(\lambda_L) + (\lambda_L - \lambda_H)X] \quad (30) \end{aligned}$$

We can then consider the two cases separately, by studying the optimisation program (Eq. 9) subject to the constraints $G(\lambda_H, \lambda_L) \leq 0$ and $G(\lambda_H, \lambda_L) > 0$ respectively. Afterwards we finally summarise the conditions for the equilibrium, considering both cases.

Consider first the case (1). Substituting in the optimal security characterised in

Appendix A.2, the optimisation programme 9 within this case can be written as

$$\begin{aligned}
\max_{(\lambda_H,\lambda_L)} \quad & \gamma\delta\left(\pi V_G + (1-\pi)[V_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)\theta_H X]\right) \\
& +\gamma(1-\delta)\pi[f_2(\lambda_H,\lambda_L) + V_G - V_B - \mathcal{L}_H(\lambda_H) - (1-\lambda_H)X] \\
& +\gamma(1-\delta)(1-\pi)\theta_H f_2(\lambda_H,\lambda_L) \\
& +\gamma(1-\delta)(1-\pi)(1-\theta_H)[V_B + \mathcal{L}(\lambda_H)] \\
& +(1-\gamma)U_L(\lambda_L) \\
s.t. \quad & G(\lambda_H,\lambda_L) \leq 0
\end{aligned} \tag{31}$$

$$\begin{aligned}
\text{where} \quad f_2(\lambda_H,\lambda_L) = & \frac{1}{\pi(1-\delta)+(1-\pi)\theta_H}\left(\pi(1-\delta)[V_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X]\right. \\
& \left. +(1-\pi)[V_B + \mathcal{L}(\lambda_L) + (1-\lambda_L)\theta_L X - (1-\theta_H)(V_B + \mathcal{L}(\lambda_H))]\right)
\end{aligned}$$

We confirm that the second order conditions are satisfied given that $\mathcal{L}(\lambda)$ is concave by Assumption 1.

$$\begin{aligned}
(SOC_H) \quad & [\delta(1-\pi) - (1-\delta)\pi + (1-\delta)(1-\pi)(1-\theta_H)]\frac{\partial^2 \mathcal{L}(\lambda_H)}{\partial \lambda_H^2} \\
& \qquad +(1-\delta)[\pi + (1-\pi)\theta_H]\frac{\partial^2 f_2(\lambda_H,\lambda_L)}{\partial \lambda_H^2} \\
& < \delta[1-\pi-(1-\delta)\pi]\frac{\partial^2 \mathcal{L}(\lambda_H)}{\partial \lambda_H^2} < 0 \\
(SOC_L) \quad & \gamma(1-\delta)[\pi + (1-\pi)\theta_H]\frac{\partial^2 f_2(\lambda_H,\lambda_L)}{\partial \lambda_L^2} + (1-\gamma)(1-\pi)\frac{\partial^2 \mathcal{L}(\lambda_L)}{\partial \lambda_L^2} \\
& < (1-\pi)[\gamma(1-\delta) + (1-\gamma)]\frac{\partial^2 \mathcal{L}(\lambda_H)}{\partial \lambda_L^2} < 0
\end{aligned}$$

If the constraint $G(\lambda_H,\lambda_L) \leq 0$ is slack at the optimum, the interior solution to the above programme is thus given by the following first order conditions

$$
\begin{aligned}
(FOC_H) \qquad h_H(\lambda_H) \equiv\ & \delta(1-\pi)(\frac{\partial \mathcal{L}(\lambda_H)}{\partial \lambda} - \theta_H X) - (1-\delta)\pi(\frac{\partial \mathcal{L}(\lambda_H)}{\partial \lambda} - X) \\
& +(1-\delta)[\pi + (1-\pi)\theta_H]\frac{\partial f_2(\lambda_H, \lambda_L)}{\partial \lambda_H} \\
& +(1-\delta)(1-\pi)(1-\theta_H)\frac{\partial \mathcal{L}(\lambda_H)}{\partial \lambda} = 0 \\
(FOC_L) \qquad h_L(\lambda_L) \equiv\ & \gamma(1-\delta)[\pi + (1-\pi)\theta_H]\frac{\partial f_2(\lambda_H, \lambda_L)}{\partial \lambda_L} \\
& +(1-\gamma)(\frac{\partial \mathcal{L}(\lambda_L)}{\partial \lambda} - \theta_L X) = 0
\end{aligned}
$$

Recall that the first-best λ_i^{FB} is given by $\frac{\partial \mathcal{L}(\lambda_i^{FB})}{\partial \lambda} = \theta_i X$. Substituting this into the (FOC_i) we have the $h_i(\lambda_i^{FB}) = 0$ for $i \in \{H, L\}$. It follows that $\lambda_H^{tp} = \lambda_H^{FB}$ and $\lambda_L^{tp} = \lambda_L^{FB}$ for interior solution, which is the case if $G(\lambda_H^{FB}, \lambda_L^{FB}) \leq 0$.

If $G(\lambda_H^{FB}, \lambda_L^{FB}) > 0$, we can have a boundary solution for case 1, where the constraint $G(\lambda_H, \lambda_L) = 0$ binds. Substitute this binding constraint into the objective function to eliminate λ_L. The first derivative regard to λ_H can thus be expressed as the following, where the superscript B stands for the boundary case in Case (1).

$$
h^B(\lambda_H) \;\; = \;\; h_H(\lambda_H) + h_L(\lambda_L^B(\lambda_H))\frac{\partial \lambda_L^B(\lambda_H)}{\partial \lambda_H} \tag{32}
$$

where $\lambda_L^B(\lambda_H)$ is defined implicitly by $G(\lambda_H, \lambda_L^B(\lambda_H)) = 0$, and $\frac{\partial \lambda_L^B(\lambda_H)}{\partial \lambda_H}$ is given by implicitly differentiating $G(\lambda_H, \lambda_L) = 0$ with regard to λ_H.

$$
\frac{\partial \lambda_L^B(\lambda_H)}{\partial \lambda_H} \equiv -\frac{\partial G(\lambda_H, \lambda_L)/\partial \lambda_H}{\partial G(\lambda_H, \lambda_L)/\partial \lambda_L} > 0 \tag{33}
$$

Again the solution can be either interior, as given by $h^B(\lambda_H) = 0$, or boundary, with $h^B(\lambda_H) \neq 0$. We first examine the interior solutions. Because $\frac{\partial \lambda_L^B(\lambda_H)}{\partial \lambda_H} > 0$, the first order condition $h_B(\lambda_H) = 0$ can only be satisfied if (i) $\lambda_H < \lambda_H^{FB}$ and $\lambda_L^B(\lambda_H) > \lambda_L^{FB}$, (ii) $\lambda_H > \lambda_H^{FB}$ and $\lambda_L^B(\lambda_H) < \lambda_L^{FB}$, or (iii) $\lambda_H = \lambda_H^{FB}$ and $\lambda_L^B(\lambda_H) = \lambda_L^{FB}$, because the $h_i(\lambda_i)$ is decreasing in λ_i by the (SOC_i). However, notice that $\frac{\partial G(\lambda_H, \lambda_L)}{\partial \lambda_H} > 0$ and $\frac{\partial G(\lambda_H, \lambda_L)}{\partial \lambda_L} < 0$. Therefore $G(\lambda_H^{FB}, \lambda_L^{FB}) > 0$ implies that only the first scenario, $\lambda_H < \lambda_H^{FB}$ and $\lambda_L^B(\lambda_H) > \lambda_L^{FB}$, satisfies $G(\lambda_H, \lambda_L) = 0$.

We next consider potential boundary solutions with $h^B(\lambda_H) \neq 0$. This can be (i) $\lambda_H = 0$ and $h^B(0) < 0$, or (ii) for some $\lambda_H \geq \lambda_H^{FB}$ and $h^B > 0$. The first scenario exists only if $h_L(\lambda_L(0)) < 0$, which implies that $\lambda_L^{FB} < \lambda_L^{tp}$ given $G(\lambda_H^{FB}, \lambda_L^{FB}) > 0$. The second scenario can be ruled out, since for all $\lambda_H \geq \lambda_H^{FB}$, $\lambda_L(\lambda_H) > \lambda_L^{FB}$ by $G(\lambda_H^{FB}, \lambda_L^{FB}) > 0$, this contradicts with $h^B > 0$.

The analysis of Case 1 therefore lead to two scenarios. If $G(\lambda_H, \lambda_L) \leq 0$, the solution is $(\lambda_H^{tp}, \lambda_L^{tp}) = (\lambda_H^{FB}, \lambda_L^{FB})$. Otherwise, the solution is such that $\lambda_H < \lambda_H^{FB}$ and $\lambda_L(\lambda_H) > \lambda_L^{FB}$.

We next turn to consider case (2) and follow similar line of reasoning. The optimisation programme is similar to above, but with the constraint that $G(\lambda_H, \lambda_L) > 0$, and

$$\begin{aligned} f_2(\lambda_H, \lambda_L) \quad = \quad & \frac{1}{\pi(1-\delta) + (1-\pi)(\theta_H - \delta\theta_L)} (\pi(1-\delta)[V_B + \mathcal{L}(\lambda_H) + (1-\lambda_H)X] \\ & +(1-\pi)\theta_L(1-\delta)[V_B + \mathcal{L}(\lambda_L) + (1-\lambda_L)X] \\ & +(1-\pi)[(1-\theta_L)[V_B + \mathcal{L}(\lambda_L)] - (1-\theta_H)[V_B + \mathcal{L}(\lambda_H)]]) \end{aligned} \tag{34}$$

We confirm that the second order conditions are satisfied given that $\mathcal{L}(\lambda)$ is concave by Assumption 1.

$$\begin{aligned} (SOC_H) \quad & [\delta(1-\pi) - (1-\delta)\pi + (1-\delta)(1-\pi)(1-\theta_H)] \frac{\partial^2 \mathcal{L}(\lambda_H)}{\partial \lambda_H^2} \\ & \qquad + (1-\delta)[\pi + (1-\pi)\theta_H] \frac{\partial^2 f_2(\lambda_H, \lambda_L)}{\partial \lambda_H^2} \\ & < \delta[1 - \pi - (1-\delta)\pi] \frac{\partial^2 \mathcal{L}(\lambda_H)}{\partial \lambda_H^2} < 0 \\ (SOC_L) \quad & \gamma(1-\delta)[\pi + (1-\pi)\theta_H] \frac{\partial^2 f_2(\lambda_H, \lambda_L)}{\partial \lambda_L^2} + (1-\gamma)(1-\pi) \frac{\partial^2 \mathcal{L}(\lambda_L)}{\partial \lambda_L^2} \\ & < (1-\pi)[\gamma(1-\delta)^2\theta_L + \gamma(1-\delta)(1-\theta_L) + (1-\gamma)] \frac{\partial^2 \mathcal{L}(\lambda_H)}{\partial \lambda_L^2} < 0 \end{aligned}$$

Suppose that the constraint $G(\lambda_H, \lambda_L) > 0$ is slack at the optimum. Substituting the equilibrium $f_2(\lambda_H, \lambda_L)$ into the first order conditions have that $h_H(\lambda_H^{FB}) < 0$ and $h_L(\lambda_L^{FB}) > 0$. It then follows that $\lambda_H^{tp} < \lambda_H^{FB}$ and $\lambda_L^{tp} > \lambda_L^{FB}$ for a solution in Case 2, if $G(\lambda_H^{tp}, \lambda_L^{tp}) > 0$ is satisfied.

$$\begin{aligned} h_H(\lambda_H^{FB}) &= \delta\theta_L(1-\pi) \frac{-[\pi + (1-\pi)\theta_H]}{\pi(1-\delta) + (1-\pi)(\theta_H - \delta\theta_L)} X < 0 \\ h_L(\lambda_L^{FB}) &= \delta\theta_L(1-\pi) \frac{\delta(1-\theta_L)\gamma(1-\gamma)[\pi + (1-\pi)\theta_H]}{\pi(1-\delta) + (1-\pi)(\theta_H - \delta\theta_L)} X > 0 \end{aligned}$$

To summarise, an the equilibrium exists and must be one of the following three types.

(a) An interior solution in Case 1 with $(\lambda_H^{tp}, \lambda_L^{tp}) = (\lambda_H^{FB}, \lambda_L^{FB})$, which exists if $G(\lambda_H^{FB}, \lambda_L^{FB}) \leq 0$.

(b) A boundary solution in Case 1 with $\lambda_H^{tp} < \lambda_H^{FB} < \lambda_L^{FB} < \lambda_L^{tp}$, which exists if $G(\lambda_H^{FB}, \lambda_L^{FB}) > 0$.

(c) An interior solution in Case 2 with $\lambda_H^{tp} < \lambda_H^{FB} < \lambda_L^{FB} < \lambda_L^{tp}$, which exists if $G(\lambda_H^{tp}, \lambda_L^{tp}) > 0$.

It is clear that the solutions (a) and (b) are mutually exclusive. Moreover, since $G(\lambda_H^{FB}, \lambda_L^{FB}) \leq 0$ implies that $G(\lambda_H^{tp}, \lambda_L^{tp}) < 0$, solutions (a) and (c) are also mutually exclusive. Therefore for any equilibrium, the equilibrium satisfies that

$$\lambda_H^{tp} \leq \lambda_H^{FB} < \lambda_L^{FB} \leq \lambda_L^{tp}$$

where the inequalities are strict if and only if $G(\lambda_H^{FB}, \lambda_L^{FB}) > 0$, which can be rearranged into the condition provided by Eq. 11 in Proposition 3.

A.5 Proof of Corollary 1

The result follows immediately from Lemma 1 and Proposition 3.

A.6 Proof of Proposition 4

To establish this proposition, we approach the optimisation problem in Eq. 13 in two steps. First, we solve for the relaxed optimisation problem for any given $\hat{\lambda}_H$ and $\hat{\lambda}'_L$, $\hat{\lambda}_H \leq \hat{\lambda}'_L$, without (IH_H) and (IH_L). This coincides with the optimisation problem in Eq. 4, whose solution is given in Appendix A.2. Next, we find λ_H and λ'_L that satisfy (IH_H) and (IH_L) respectively, for an optimal security given by the previous step. The solution to the optimisation programme in Eq. 13 is then given by $\mathcal{F}$ such that $\hat{\lambda}_H = \lambda_H$ and $\hat{\lambda}'_L = \lambda'_L$.

For any given $\hat{\lambda}_H$ and $\hat{\lambda}'_L$, $\hat{\lambda}_H \leq \hat{\lambda}'_L$, there are two cases for the solution to the relaxed optimisation problem, as given in Appendix A.2. In both cases, $f_2 < V_B + \mathcal{L}(\hat{\lambda}_H) + (1 - \hat{\lambda}_H)X$ and $f_5 = V_B + \mathcal{L}(\hat{\lambda}_H)$. This suggests that, for any given security with such properties, (IH_H) implies that $\lambda_H = \arg\max_\lambda \mathcal{L}(\lambda) + (1-\lambda_H)X = 0$. In order to analyse λ_L, we consider the two cases separately below.

In case 1, $f_3 = V_B + \mathcal{L}(\hat{\lambda'}_L) + (1-\hat{\lambda'}_L)X$ and $f_4 = V_B + \mathcal{L}(\hat{\lambda'}_L)$. For a given security with such property, (IH_L) becomes independent of λ'_L. We continue to assume that in this case, the low type's security chooses the fist best level of foreclosure $\lambda'_L = \lambda_L^{FB}$. Such an equilibrium exists if and only if $G(\cdot) < 0$, where $G(\cdot)$ is given by Eq. 25 for cash flows c_j, $j \in \{1, 2, 3, 4, 5\}$, determined by $\lambda_H = 0$ and $\lambda_L = \lambda_L^{FB}$. That is,

$$\frac{\theta_H - \theta_L}{1-\theta_H}(1-\lambda_L^{FB})X < \mathcal{L}(\lambda_L^{FB}) - \frac{\pi}{1-\pi}\frac{1-\delta}{1-\theta_H}[\lambda_L^{FB}X - \mathcal{L}(\lambda_L^{FB})] \tag{35}$$

In case 2, $f_3 < V_B + \mathcal{L}(\hat{\lambda'}_L) + (1-\hat{\lambda'}_L)X$ and $f_4 = V_B + \mathcal{L}(\hat{\lambda'}_L)$. For a given security with such property, (IH_L) implies that $\lambda'_L = \arg\max_\lambda \mathcal{L}(\lambda) + (1-\lambda'_L)X = 0$. Such an equilibrium exists if and only if $G(\cdot) > 0$ for cash flows determined by $\lambda_H = \lambda_L = 0$. This can be expressed as the following, which holds always true.

$$(\theta_H - \theta_L)X > 0 \tag{36}$$

To summarise, for both types of equilibria, the equilibrium foreclosure policy for the high-type securitiser is $\lambda_H = 0$, following an issue of a debt like security. The equilibrium foreclosure policy for the low-type securitiser is λ_L^{FB}, following an issue of a full pass through security.

Finally, as a corollary to this proposition, we have the following claim.

Claim 7. *Given in-house servicing, the expected of payoff to a high-type securitiser is higher than the expected payoff to a low-type securitiser. That is,* $U_H^{ih} > U_L^{ih}$.

Proof. This claim is proved by examining the (IC) constraint in the optimisation programme given by Eq. 13. Given that the equilibrium security satisfies $c_4 - f_4^* = 0$ as shown in Appendix A.2, the (IC) constraint in equilibrium is satisfied with

$$U_L^{ih} = p(\mathcal{F}^{ih}) + \delta\pi(c_1 - f_1^*) + \delta(1-\pi)\theta_L(c_3 - f_3^*) \tag{37}$$

Appendix A.2 also establishes that the equilibrium security also satisfies $c_5 - f_5^* = 0$ and $c_2 - f_2^* \geq c_3 - f_3^*$. This then implies that the equilibrium payoff to the high-type securitiser is higher than the equilibrium payoff to the low-type securitiser.

$$\begin{aligned} U_H^{ih} &= p(\mathcal{F}^{ih}) + \delta\pi(c_1 - f_1^*) + \delta(1-\pi)\theta_H(c_2 - f_2^*) \\ &> p(\mathcal{F}^{ih}) + \delta\pi(c_1 - f_1^*) + \delta(1-\pi)\theta_L(c_3 - f_3^*) = U_L^{ih} \end{aligned} \tag{38}$$

□

A.7 Proof of Proposition 5

Notice that $(\lambda_H^{ih}, \lambda_L^{ih})$ and its corresponding equilibrium security also satisfies the constraint in the optimisation programme Eq. 9. The results of this corollary thus follows from the fact that $(\lambda_H^{tp}, \lambda_L^{tp})$ is the optimiser of said programme.