DOCUMENT

DE TRAVAIL

N° 260

IDENTIFICATION OF LAGGED DURATION DEPENDENCE IN MULTIPLE-SPELL COMPETING RISKS MODELS

Guillaume Horny and Matteo Picchio

December 2009

BANQUE DE FRANCE

EUROSYSTÈME

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

DIRECTION GÉNÉRALE DES ÉTUDES ET DES RELATIONS INTERNATIONALES

IDENTIFICATION OF LAGGED DURATION DEPENDENCE IN MULTIPLE-SPELL COMPETING RISKS MODELS

Guillaume Horny and Matteo Picchio

December 2009

Identification of lagged duration dependence in multiple-spell competing risks models*

Guillaume Horny[†] and Matteo Picchio[‡]

December 11, 2009

*The authors acknowledge financial support from the Belgian Federal Science Policy Office (PAI P6/07). In addition, Matteo Picchio is grateful for the research grant he received from the Special Research Fund (FSR) when he was at the Université catholique de Louvain. The authors also wish to thank Bart Cockx, Michel Mouchart, Sébastien Van Bellegem, and Bas van der Klaauw for their comments and suggestions.

[†]Banque de France, DGEI-DEMS-SAMIC, 31 rue Croix des Petits Champs, 75 049 Paris Cedex 01, France and ECON, Université Catholique de Louvain, Belgium. E-mail: guillaume.horny@banque-france.fr

[‡]CentER, ReflecT, and Department of Economics, Tilburg University, PO BOX 90153, 5000 LE Tilburg, The Netherlands. E-mail: m.picchio@uvt.nl.

Abstract

We show that lagged duration dependence is non-parametrically identified in mixed proportional hazard models for duration data, in the presence of competing risks and consecutive spells.

Keywords: lagged duration dependence, competing risks, mixed proportional hazard models, identification.
JEL classification codes: C14, C41

Résumé

Nous montrons l'identification non-paramétrique de la dépendance aux durées écoulées dans les modèles de mélange de hasards proportionnels, en présence de risques concurrents et d'épisodes multiples.

Mots-clés: dépendance aux durées écoulées, risques concurrents, modèles de mélange de hasards propportionnels, identification.
Classification JEL: C14, C41

1 Introduction

Time spent in a previous state can affect the duration of sojourn in the current state. Multiple destinations are also possible at the end of each sojourn. Appropriate modelling of this situation can be found in a competing risks framework, where random variables measure the duration until a risk materialization, and only the smallest of all these durations is observed along with the corresponding exit destination. Applications involve, for example, investigations into the way in which the layoff-rehire process affects unemployment outcomes (Katz and Meyer, 1990), repeated temporary jobs (Gagliarducci, 2005), and youth job stability after early unemployment events (Doiron and Gørgens, 2008; Gaure et al., 2008; Cockx and Picchio, 2009).

The joint distribution of all the durations, observed and censored, is not nonparametrically identified in a single-spell competing risks framework (Cox, 1962; Tsiatis, 1975). With no assumptions, bounds on the unidentified marginal distributions can be generated (Peterson, 1976). The way in which these bounds can be tightened by imposing parametric assumptions is studied in Honoré and Lleras-Muney (2006).

Identification requires structure, such as independent risks, parametric failure times joint distribution (see van den Berg, 2001, for a survey), or variation in the explanatory variables (Heckman and Honoré, 1989; Abbring and van den Berg, 2003; Lee, 2006). We show the identification of mixed proportional hazard (MPH) models with lagged duration dependence in a multiple-spell competing risks framework. We consider the simplest case where two consecutive spells are observed per unit, and each spell can terminate because of two competing risks. Our identification result can be easily extended to more than two spells or two destination states. We thus generalize the single-risk result in Honoré (1993).

2 The MPH multiple-spell competing risks model

Let $t = 0$ be the start of the process and $\{Z(t), t \in \Re_+\}$ be a finite-state point process. $Z(t)$ indicates the state occupied by each unit at time t and takes values in $\{o, a, b, c, d, e, f\}$. It is generated by the following sequence:

(i) The state space is $\{o, a, b\}$. State o is the origin state of the first spell, for all the units under study. Every unit can experience at most a unique transition to a state in $\{a, b\}$. The observed outcome of the first spell is:

$$\begin{aligned} T^1 &= \min(T^*_{oa}, T^*_{ob}), \\ \Delta^1 &= \arg\min_{\{a,b\}}(T^*_{oa}, T^*_{ob}), \\ T^*_{ok} &= \inf\{t|Z(t) = k\},\ \forall\, k \in \{a, b\}. \end{aligned}$$

The T_{ok}^*'s are latent origin-destination-specific durations. We only observe their minimum and the destination state of the first spell. Assume that tied observations have zero probability and define the latent duration distributions by the following mixed proportional hazard rates:

$$\theta_{ok}(t|x, v_{ok}) = \lambda_{ok}(t)\phi_{ok}(x)v_{ok}, \ \forall \, k \in \{a, b\},$$

where the functions $\lambda_{ok}(\cdot)$ are the baseline hazards, $\phi_{ok}(\cdot)$ the systematic parts, x a vector of regressors and v_{ok}, for all $k \in \{a, b\}$, a vector of unobserved non-negative specific random variables. Dependence between T_{oa}^* and T_{ob}^* is assumed to be captured by observed and unobserved characteristics.

(ii) After a random time, the unit moves to either a or b. For all $k \in \{a, b\}$, let us denote by t_{ok} the observed duration of a first spell ending up in k; that is, $t_{ok} = T_{ok}^*$ when $T^1 = T_{ok}^*$. New state spaces are available: a transition to a leads to a new state space where the only possible further transitions are toward $\{c, d\}$, whereas a first transition to b leads to a new state space with transitions toward $\{e, f\}$. Consider a first transition toward a. We have:

$$\begin{aligned} T^2 &= \min(T_{ac}^*, T_{ad}^*), \\ \Delta^2 &= \arg\min_{\{c,d\}}(T_{ac}^*, T_{ad}^*), \\ T_{ak}^* &= \inf\{t - t_{oa}|Z(t) = k\}, \ \forall \, k \in \{c, d\}. \end{aligned}$$

Distributions of the T_{ak}^* are characterized by the origin-destination-specific hazard functions:

$$\theta_{ak}(t|x, t_{oa}, v_{ak}) = \lambda_{ak}(t)\phi_{ak}(x)h_{ak}(t_{oa})v_{ak}, \ \forall \, k \in \{c, d\},$$

where $h_{ak}(\cdot)$ captures the effect of the duration in the previous state on the current transition intensity. Dependence between T_{ac}^* and T_{ad}^* is assumed to be captured by observed characteristics, unobservables, and lagged duration t_{oa}. Duration of a sojourn in b is defined in a symmetric way. Vector $v \equiv (v_{oa}, v_{ob}, v_{ac}, v_{ad}, v_{be}, v_{bf})$ has distribution G, which is allowed to have a mass point at 0.

At the end of the second spell, we observe $(T^1, \Delta^1, T^2, \Delta^2)$, and possible trajectories are in $\{oac, oad, obe, obf\}$. Denote by $\mathcal{D}^1 = \{oa, ob\}$ the set of the possible transitions from the first spell, and by $\mathcal{D}^2 = \{ac, ad, be, bf\}$ the set of the possible transitions from the second spell. The joint survival function is :

$$\Pr\{\cap_{j\in(\mathcal{D}^1\cup\mathcal{D}^2)}(T_j^*>t_j)|x\} = \int_{\Re_+^6} \exp\Big[-\sum_{k\in\mathcal{D}^1}\Lambda_k(t_k)\phi_k(x)v_k - \sum_{\substack{l\in\mathcal{D}^2\\ m(l)\in\mathcal{D}^1}}\Lambda_l(t_l)\phi_l(x)h_l(t_{m(l)})v_l\Big]dG(v), \tag{1}$$

where $\Lambda_k(t_k)=\int_0^{t_k}\lambda_k(u)du$, for $k\in(\mathcal{D}^1\cup\mathcal{D}^2)$. It is equal to:

$$\mathscr{L}_G\big\{\Lambda_{oa}(t_{oa})\phi_{oa}(x), \Lambda_{ob}(t_{ob})\phi_{ob}(x), \Lambda_{ac}(t_{ac})\phi_{ac}(x)h_{ac}(t_{oa}), \\ \Lambda_{ad}(t_{ad})\phi_{ad}(x)h_{ad}(t_{oa}), \Lambda_{be}(t_{be})\phi_{be}(x)h_{be}(t_{ob}), \Lambda_{bf}(t_{bf})\phi_{bf}(x)h_{bf}(t_{ob})\big\},$$

where $\mathscr{L}_G$ is the Laplace transform of G.[1] This is not observable, as we observe only $(T^1,\Delta^1,T^2,\Delta^2)$.

The subsurvival probability functions (Tsiatis, 1975) provide the probability to survive t_1 time periods in the origin state and t_2 time periods in the subsequent state. These are observed and taken to be known for all $x\in\mathcal{X}$. Let us denote the subsurvivors by $Q_l(t_1,t_2|x)$, for $l\in\mathcal{D}^2$. Focusing on Q_{ac}, we have:

$$Q_{ac}(t_1,t_2|x)=\Pr(T_{oa}^*>t_1, T_{ob}^*>T_{oa}^*, T_{ac}^*>t_2, T_{ad}^*>T_{ac}^*|x).$$

This subsurvival can be related to a subdensity $f(T_{oa}^*=t_1, T_{ob}^*>t_1, T_{ac}^*=t_2, T_{ad}^*>t_2|x)$, also known from the data. Integrating the subdensity over the duration in the first state from t_1 to infinity, we obtain the following observed function in terms of subsurvivals:

$$Q_l'(t_1,t_2|x)=\frac{\partial}{\partial T_l^*}\Pr\{\cap_{j\in\mathcal{D}^1}(T_j^*>t_1), \cap_{k\in\mathcal{D}^2}(T_k^*>t_2)|x\}, \forall l\in\mathcal{D}^2.$$

More specifically, for a transition to state c at the end of the second spell:

$$\begin{aligned} Q_{ac}'(t_1,t_2|x) =& -\lambda_{ac}(t_2)\phi_{ac}(x)h_{ac}(t_1)\int_{\Re_+^4} v_{ac}\exp\Big[-\sum_{k\in\{oa,ob\}}\Lambda_k(t_1)\phi_k(x)v_k \\ & -\sum_{l\in\{ac,ad\}}\Lambda_l(t_2)\phi_l(x)h_l(t_1)v_l\Big]dG(v) \\ =& -\lambda_{ac}(t_2)\phi_{ac}(x)h_{ac}(t_1) \qquad (2) \\ & D_{ac}\mathscr{L}_G^a\left[\Lambda_{oa}(t_1)\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), \Lambda_{ac}(t_2)\phi_{ac}(x), \Lambda_{ad}(t_2)\phi_{ad}(x)\right], \end{aligned}$$

[1] See, e.g., Lancaster (1990), appendix 2, for an overview of the Laplace transform and its properties.

where $D_{ac}\mathscr{L}_G^a() \equiv \partial \mathscr{L}_G^a\{s_{oa}, s_{ob}, s_{ac}, s_{ad}\}/\partial s_{ac}$, and $\mathscr{L}_G^a$ is the Laplace transform of G defined for first spells ending in state a. We similarly define $\mathscr{L}_G^b$.

Applications of a competing risks model with lagged duration dependence may involve assessment of the participation in the labour market. Suppose state o denotes unemployment, a employment, and b inactivity. An employment spell can be terminated by a transition either to a second unemployment event c or to inactivity $(d = b)$. Inactivity may end because of a transition either to a second unemployment event $(e = c)$ or to employment $(f = a)$. Similarly, Doiron and Gørgens (2008), Gaure et al. (2008), and Cockx and Picchio (2009) estimated models to study youth job stability after early unemployment events. Other examples involve the analysis of types of training programs on job stability. Our theoretical framework is more general than what is required in these examples.

3 Identification result

Theorem 1 *Assume that the joint survivor function of* $(T_{oa}^*, T_{ob}^*, T_{ac}^*, T_{ad}^*, T_{be}^*, T_{bf}^*)$ *conditional on* x *is given by (1). Functions* $\mathscr{L}_G$*,* (Λ_j, ϕ_j)*,* $\forall j \in \mathcal{D}_1 \cup \mathcal{D}_2$*, and* h_l*,* $\forall l \in \mathcal{D}_2$*, are identified from the distribution of* $(T_1, \Delta_1, T_2, \Delta_2)$ *conditional on* x *under the following assumptions:*

- *A1* $\Lambda_j(t) < \infty$ *is non-negative, differentiable, and strictly increasing* $\forall j \in (\mathcal{D}_1 \cup \mathcal{D}_2)$ *and* $\forall t \in \Re_+$.
- *A2 The support* χ *of* x *is an open set in* $\Re^n$*. For all* $j \in \mathcal{D}_1 \cup \mathcal{D}_2$*,* ϕ_j *is a continuous function such that* $\{\phi_{oa}(x), \phi_{ob}(x), \phi_{ac}(x), \phi_{ad}(x), \phi_{be}(x), \phi_{bf}(x)\}$ *contains a non-empty open set in* $\Re_+^6$.
- *A3* h_l *is non-negative on* $\Re_+$*,* $\forall l \in \mathcal{D}_2$.
- *A4 Vector* v *has non-negative components with distribution function* G *independent of* x*, and* $E[v] < \infty$.
- *A5 For all* $l \in \mathcal{D}_2$*,* $h_l(t^0) = 1$ *for some fixed* $t^0 \in \Re_+$*. For all* $j \in (\mathcal{D}_1 \cup \mathcal{D}_2)$*,* $\Lambda_j(t^{00}) = 1$ *for some fixed* $t^{00} \in \Re_+$*. For all* $j \in (\mathcal{D}_1 \cup \mathcal{D}_2)$*,* $\phi_j(x^{00})$*=1 for some fixed* $x^{00} \in \chi$.

Proof. Conditional on x, the duration in the second spell depends on the duration in the first spell, and we can not iteratively apply a single-spell identification result. Our proof successively establishes the identification of **(a)** the functions involved in the distribution of the first-spell durations, **(b)** functions $(\phi_{ac}, \phi_{ad}, \phi_{be}, \phi_{bf})$, **(c)** the unobserved heterogeneity distribution G, **(d)** functions $(h_{ac}, h_{ad}, h_{be}, h_{bf})$, **(e)** the baseline hazards $(\lambda_{ac}, \lambda_{ad}, \lambda_{be}, \lambda_{be})$.

(a) From the marginal distribution of $(T_1, \Delta_1)|x$ we can identify $(\Lambda_k, \phi_k), \forall k \in \mathcal{D}^1$ (Heckman and Honoré, 1989; Abbring and van den Berg, 2003).

(b) As $t_2 \to 0$:

$$\frac{Q'_{ac}(t_1, t_2|x)}{Q'_{ac}(t_1, t_2|x^{00})} \to \frac{\phi_{ac}(x)}{\phi_{ac}(x^{00})} \frac{D_{ac}\mathscr{L}^a_G\left[\Lambda_{oa}(t_1)\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), 0, 0\right]}{D_{ac}\mathscr{L}^a_G\left[\Lambda_{oa}(t_1)\phi_{oa}(x^{00}), \Lambda_{ob}(t_1)\phi_{ob}(x^{00}), 0, 0\right]}.$$

As $t_1 \to 0$, $D_{ac}\mathscr{L}^a_G(.) \to \mathrm{E}(v_{ac}) < \infty$ and identification of ϕ_{ac} is obtained. Analogously working on Q'_l, $\forall l \in \{ad, be, bf\}$, yields the identification of ϕ_{ad}, ϕ_{be}, and ϕ_{bf}.

(c) Evaluate the joint survivor function (1) at $t_{oa} = t_{ob} = t^0$ and $t_l = t^{00}, \forall l \in \mathcal{D}^2$. We obtain:

$$\mathscr{L}_G\big[\Lambda_{oa}(t^0)\phi_{oa}(x), \Lambda_{ob}(t^0)\phi_{ob}(x), \phi_{ac}(x), \phi_{ad}(x), \phi_{be}(x), \phi_{bf}(x)\big].$$

This is observed, as t^0 is unique for all the first spells, as well as t^{00} for the second spells. From assumption A2, we can vary x and trace $\mathscr{L}_G$ on a non-empty open set. The Laplace transform $\mathscr{L}_G$ is completely monotone, and is thus identified on a non-empty open set from Proposition 1 in Abbring and van den Berg (2003). As $\mathscr{L}_G$ is real analytic, its identification on an open set can be extended to $\Re^6_+$. Uniqueness of the Laplace transform concludes the identification of G. Identification of $\mathscr{L}^a_G$ and $\mathscr{L}^b_G$ can be shown along the same lines.

(d) As $t_2 \to 0$:

$$\frac{Q'_{ac}(t_1, t_2|x)}{Q'_{ac}(t^0, t_2|x)} \to \frac{h_{ac}(t_1)}{h_{ac}(t^{00})} \frac{D_{ac}\mathscr{L}^a_G\left[\Lambda_{oa}(t_1)\phi_{oa}(\hat{x}), \Lambda_{ob}(t_1)\phi_{ob}(x), 0, 0\right]}{D_{ac}\mathscr{L}^a_G\left[\Lambda_{oa}(t^{00})\phi_{oa}(x), \Lambda_{ob}(t^{00})\phi_{ob}(x), 0, 0\right]}. \quad (3)$$

Since the hazards of the first spell and G have been identified, we identify h_{ac} by varying t_1. Identification of h_{ad}, h_{be}, and h_{bf} is analogous.

(e) To identify Λ_{ac}, compute Q'_{ac} and solve in λ_{ac}. This yields a differential equation:

$$\lambda_{ac}\Big(t_2, \Lambda_{ac}(t_2), \Lambda_{ad}(t_2)\Big) = \frac{Q'_{ac}(t_1, t_2|x)}{\phi_{ac}(x)h_{ac}(t_1)M_{ac}}, \quad (4)$$

where

$$M_{ac} = D_{ac}\mathscr{L}^a_G\big[\Lambda_{oa}(t_1)\phi_{oa}(x), \Lambda_{ob}(t_1)\phi_{ob}(x), \Lambda_{ac}(t_2)\phi_{ac}(x)h_{ac}(t_1), \Lambda_{ad}(t_2)\phi_{ad}(x)h_{ad}(t_1)\big]$$

and with initial condition $\Lambda_{ac}(t^{00}) = 1$. Set $t_2 = t^{00}$ and fix x. The numerator of (4) is observed and Λ_{ac}, ϕ_{ac}, h_{ac} and $\mathscr{L}_G^a$ have already been identified. We can compute $\lambda_{ac}(t^{00})$ using the normalization in assumption A5. We can also compute $\Lambda_{ac}(t^0 + \varepsilon)$ for a sufficiently small ε, and deduce the marginal changes λ_{ac}. Plugging it in the differential equation (4) and solving iteratively, we can trace out Λ_{ac} on $\Re_+$. A generalized smoothness Lipschitz continuity is satisfied, ensuring the uniqueness of the traced-out Λ_{ac} (Abbring and van den Berg, 2003). Identification of λ_{ad}, λ_{be} and λ_{be} proceeds in the same way. This completes the proof.||

A consequence of the result is an informal test for lagged duration dependence. In applications, a quick plot of relation (3) can display a discernible non-constant pattern.

References

Abbring, J. H., van den Berg, G. J., 2003. The identifiability of the mixed proportional hazards competing risks model. Journal of the Royal Statistical Society Series B 65 (3), 701–710.

Cockx, B., Picchio, M., 2009. Are short-lived jobs stepping stones to long-lasting jobs?, IZA Discussion Paper No. 4007, Bonn.

Cox, D. R., 1962. Renewal Theory. Methuen, London.

Doiron, D., Gørgens, T., 2008. State dependence in youth labor market experiences, and the evaluation of policy interventions. Journal of Econometrics 145 (1), 81–97.

Gagliarducci, S., 2005. The dynamics of repeated temporary jobs. Labour Economics 12 (4), 429–448.

Gaure, S., Røed, K., Westlie, L., 2008. The impacts of labour market policies on job search behavior and post-unemployment job quality, IZA discussion paper No. 3802, Bonn.

Heckman, J. J., Honoré, B. E., 1989. The identifiability of the competing risks model. Biometrika 76 (2), 325–330.

Honoré, B. E., 1993. Identification results for duration models with multiple spells. Review of Economic Studies 60 (1), 241–246.

Honoré, B. E., Lleras-Muney, A., 2006. Bounds in competing risks models and the war on cancer. Econometrica 74 (6), 1675–1698.

Katz, L. F., Meyer, B. D., 1990. Unemployment insurance, recall expectations, and unemployment outcomes. Quarterly Journal of Economics 105 (4), 973–1002.

Lancaster, T., 1990. The Econometric Analysis of Transition Data. Cambridge University Press, Cambridge.

Lee, S., 2006. Identification of a competing risks model with unknown transformations of latent failure times. Biometrika 93 (4), 996–1002.

Peterson, A. V., 1976. Bounds for a joint distribution with fixed sub-distribution functions: Application to competing risks. Proceedings of the National Academy of Science 73 (1), 11–13.

Tsiatis, A., 1975. A nonidentifiability aspect of the problem of competing risks. Proceedings of the National Academy of Sciences of the United States of America 72 (1), 20–22.

van den Berg, G. J., 2001. Duration models: Specification, identification, and multiple durations. In: Heckman, J. J., Leamer, E. (Eds.), Handbook of Econometrics, Volume V. North-Holland, Amsterdam, Ch. 55, pp. 3381–3460.

Documents de Travail

250. A. Monfort, «Une modélisation séquentielle de la VaR,» Septembre 2009

251. A. Monfort, "Optimal Portfolio Allocation under Asset and Surplus VaR Constraints," September 2009

252. G. Cette and J. Lopez, "ICT Demand Behavior: An International Comparison," September 2009

253. H. Pagès, "Bank Incentives and Optimal CDOs," September 2009

254. S. Dubecq, B. Mojon and X. Ragot, "Fuzzy Capital Requirements, Risk-Shifting and the Risk Taking Channel of Monetary Policy," October 2009

255. S. Frappa and J-S. Mésonnier, "The Housing Price Boom of the Late '90s: Did Inflation Targeting Matter?" October 2009

256. H. Fraisse, F. Kramarz and C. Prost, "Labor Court Inputs, Judicial Cases Outcomes and Labor Flows: Identifying Real EPL," November 2009

257. H. Dixon, "A unified framework for understanding and comparing dynamic wage and price-setting models," November 2009

258. J. Barthélemy, M. Marx and A. Poisssonnier, "Trends and Cycles: an Historical Review of the Euro Area," November 2009

259. C. Bellégo and L. Ferrara, "Forecasting Euro-area recessions using time-varying binary response models for financial variables," November 2009

260. Guillaume Horny and Matteo Picchio, "Identification of lagged duration dependence in multiple-spell competing risks models," December 2009

Pour accéder à la liste complète des Documents de Travail publiés par la Banque de France veuillez consulter le site :
http://www.banque-france.fr/fr/publications/documents_de_travail/documents_de_travail_09.htm

For a complete list of Working Papers published by the Banque de France, please visit the website:
http://www.banque-france.fr/fr/publications/documents_de_travail/documents_de_travail_09.htm

Pour tous commentaires ou demandes sur les Documents de Travail, contacter la bibliothèque de la Direction Générale des Études et des Relations Internationales à l'adresse suivante :

For any comment or enquiries on the Working Papers, contact the library of the Directorate General Economics and International Relations at the following address :

BANQUE DE FRANCE
49- 1404 Labolog
75049 Paris Cedex 01
tél : 0033 (0)1 42 92 49 55 ou 62 65 ou 48 90 ou 69 81
email : thierry.demoulin@banque-france.fr
jeannine.agoutin@banque-france.fr
veronique.jan-antuoro@banque-france.fr
nathalie.bataille-salle@banque-france.f