Funding Liquidity Risk From a Regulatory Perspective

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Outline

- In the Basel 2 regulation, the computation of required capital is based on a measure of risk of a future portfolio value.
- The portfolio is crystallized: the volumes are fixed. Therefore the uncertainty concerns only the market prices (market risk) and the counterparty risk (credit risk).
- Thus the risk is mainly on the asset component of the balance sheet, with a constant liability component.

- The crystallization has to be considered more carefully.
- Volume changes on the asset and liability sides are very different.
- On the asset side, they correspond to portfolio reallocation according to price movements, or needs to answer funding liquidity shocks. They are **endogenous**.
- On the liability side, they result from changes in the behaviour of customers and investors. These volume changes are **exogenous**. *Examples:* lapses of life insurance contracts, bank runs, withdrawals by hedge fund investors...
- Basel 3 and Solvency 2 introduce new features to avoid relying too much on the assumption of portfolio crystallization.

- In the spirit of new regulations, a more symmetric analysis of the balance sheets takes into account both the uncertainty on the asset and liability components.
- On the asset side, the uncertainty on the asset value, i.e. **market risk** (and credit risk).
- On the liability side, the uncertainty on the volume of debt, i.e. **funding liquidity risk**.
- Unfortunately, these two fundamental risks are connected. We need to also consider **market liquidity risk** that combines risks on the asset side and on the liability side.

- The two fundamental risks leads to the existence of two types of default of a financial institution.
    - i) A financial institution may be in default due to **liquidity shortage** stemming mostly from funding liquidity risk. An institution in liquidity default may be structurally in good financial health.
    - ii) A financial institution may be in default due to a **lack of solvency** resulting mainly from market risk.
- With two types of default to disentangle, the regulation need to define and set two reserve accounts: a reserve account for funding liquidity risk and a reserve account for solvency risk.

$\Longrightarrow$ **The objective of the paper is two explain how to set and manage jointly these two types of reserve.**

Outline

We consider a simplified balance sheet:

Asset	Liability
$x_{1,t}p_{1,t}$	$L_{1,t}$
	$L_{0,t}$
$x_{0,t}$	0
	Y_t

Table : Stylized balance sheet at initial date t

On the asset side:

- $x_{1,t}p_{1,t}$ are illiquid assets (volume $\times$ price).
- $x_{0,t}$ is cash.

On the liability side:

- $L_{1,t}$ is long-term debt (without possibility of prepayment).
- $L_{0,t}$ is short-term debt or long term debt with possible prepayment.
- 0 is the value of a (currently unused) credit line with interest rate γ and limit M.
- Y_t is the equity (shareholders's value).

- The exogenous shocks affect the asset and liability sides.
- On the asset side, prices are shocked:

$$p_{1,t} \longrightarrow p_{1,t+1}, \tag{1}$$

- On the liability side, the (volumes of) long term and short term debts are shocked:

$$L_{1,t} \longrightarrow L_{1,t+1} \quad (L_{1,t+1} > L_{1,t}), \tag{2}$$

$$L_{0,t} \longrightarrow L_{0,t+1} \quad (L_{0,t+1} \geq 0). \tag{3}$$

- We denote these shocks:

$$\delta p_{1,t+1} \equiv p_{1,t+1} - p_{1,t} \geq -p_{1,t}, \tag{4}$$

$$\delta L_{1,t+1} \equiv L_{1,t+1} - L_{1,t} \geq 0, \tag{5}$$

$$\delta L_{0,t+1} \equiv L_{0,t+1} - L_{0,t} \in [-L_{0,t}, \infty). \tag{6}$$

- For simplicity, we aggregate shock on the liability side:

$$\delta L_{t+1} \equiv \delta L_{1,t+1} + \delta L_{0,t+1}. \tag{7}$$

- The liability shocks and the asset shocks are **simultaneously**.
- The exogenous shocks on the liability side can create a need for cash. This need will be fulfilled recursively:
  - by using the existing cash $x_{0,t}$,
  - by using the credit line,
  - by selling the illiquid assets with an haircut (H).
- If these successive operations are not sufficient, the institution will be in default for funding liquidity risk.
- Even if the institution is not in default for liquidity shortage, it can still become in default for solvency risk.

- After the reaction of the firm to liquidity need, the new equity value is:

$$Y^*_{t+1|t} = \tilde{Y}^*_{t+1|t} \mathbb{1}_{\left(-x_{0,t}-\delta L_{t+1}-\tilde{M}\right)^+ < x_{1,t}(p_{1,t}+\delta p_{1,t+1})H} \tag{8}$$

where:

$$\begin{aligned} \tilde{Y}^*_{t+1|t} &= Y_t + x_{1,t}\delta p_{1,t+1} - \gamma min\left[\tilde{M}; (-x_{0,t} - \delta L_{t+1}))^+\right] \\ &- \left(\frac{1}{H} - 1\right)\left(-x_{0,t} - \delta L_{t+1} - \tilde{M}\right)^+, \end{aligned} \tag{9}$$

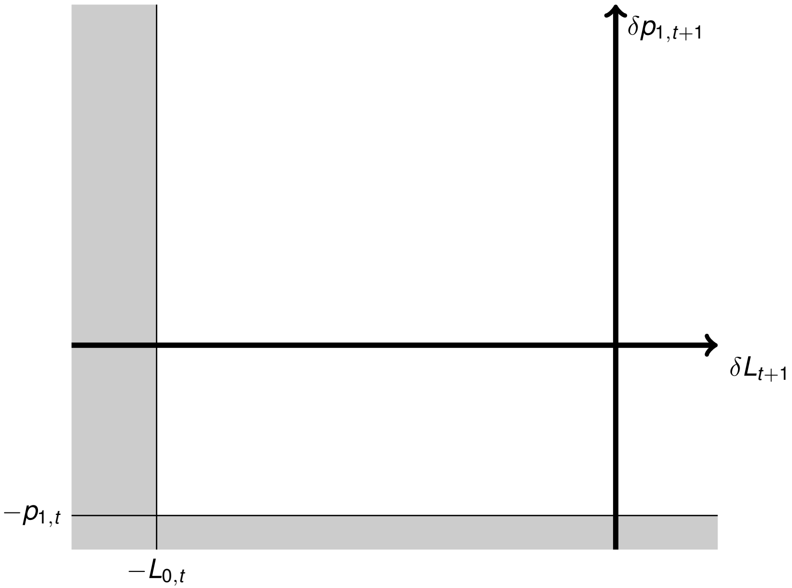
and

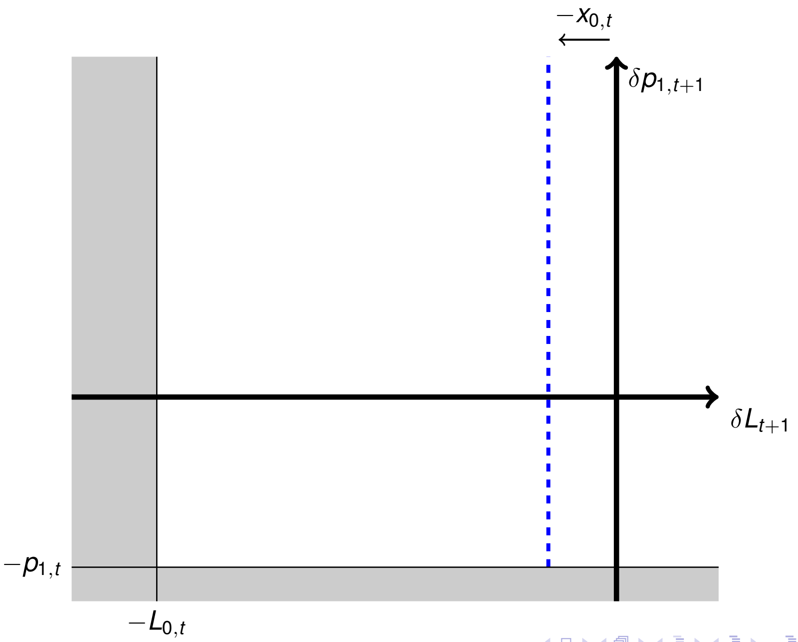
$$\tilde{M} = M/(1+\gamma). \tag{10}$$

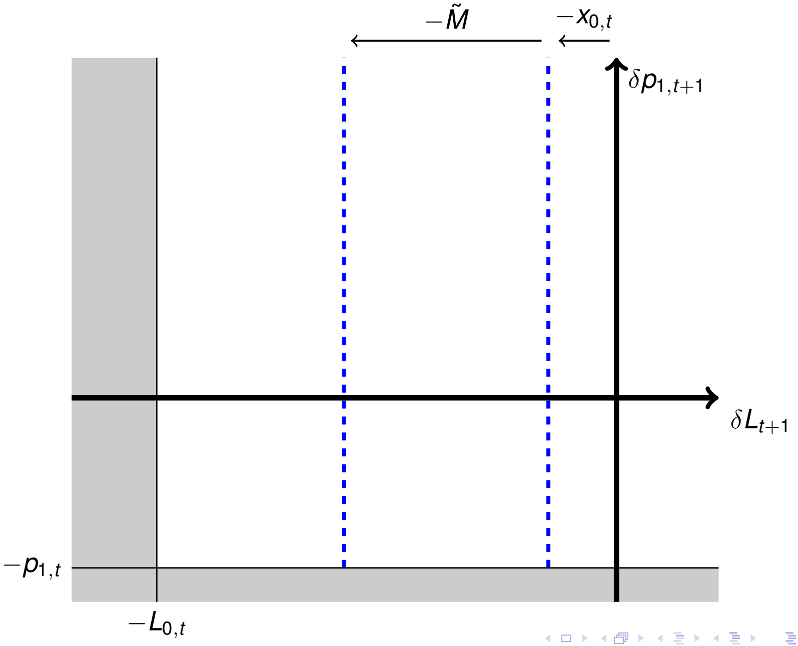
- $\tilde{Y}^*_{t+1|t}$ is the standard formula of Basel 2 ("$Y_t + x_{1,t}\delta p_{1,t+1}$"), adjusted for tranches written on the liquidity shortage $-x_{0,t} - \delta L_{t+1}$.
- The dummy in the expression $Y^*_{t+1|t}$ indicates that the need for cash exceeds the maximum of cash the institution can get.

Outline

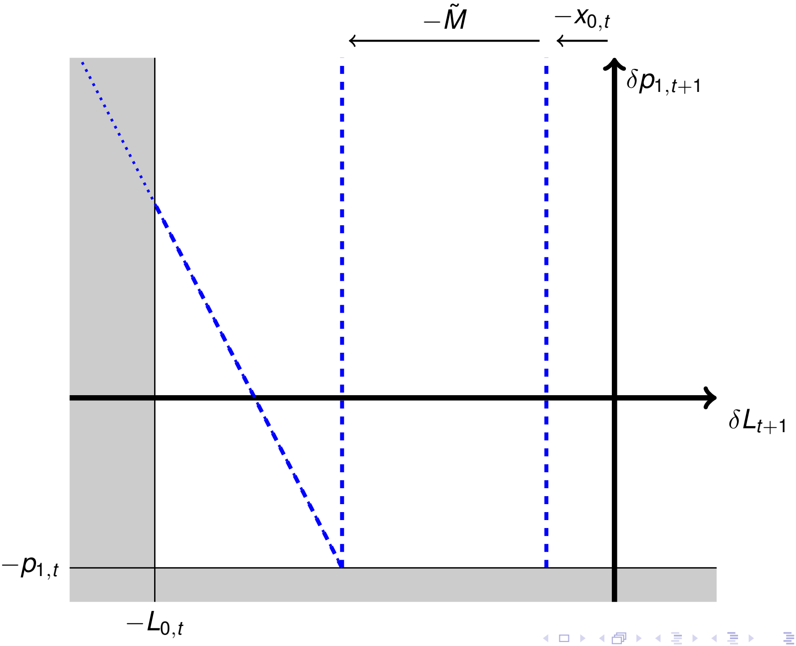
- The expression of the P&L in tranches leads us to identify different regimes. We denote them using rating grades to indicate the severity of the potential distress.
- 4 liquidity regimes are identified:
  - $\mathcal{R}^{\ell}(AA)$: there is enough cash,
  - $\mathcal{R}^{\ell}(A)$: the credit line is activated,
  - $\mathcal{R}^{\ell}(B)$: some illiquid asset are sold,
  - $\mathcal{R}^{\ell}(D)$: liquidity needs cannot be met.
- 2 solvency regimes are identified:
  - $\mathcal{R}^{\mathcal{S}}(A)$: the financial institution is solvent $\tilde{Y}^{*}_{t+1|t} > 0$,
  - $\mathcal{R}^{\mathcal{S}}(D)$: the financial institution is insolvent, otherwise.
- The observed regimes depends on:
  - The initial balance sheet structure and market conditions (γ, M, H),
  - the magnitude of the two types of shocks: $\delta p_{1,t+1}$ and δL_{t+1}.
- In general, there are 7 regimes.

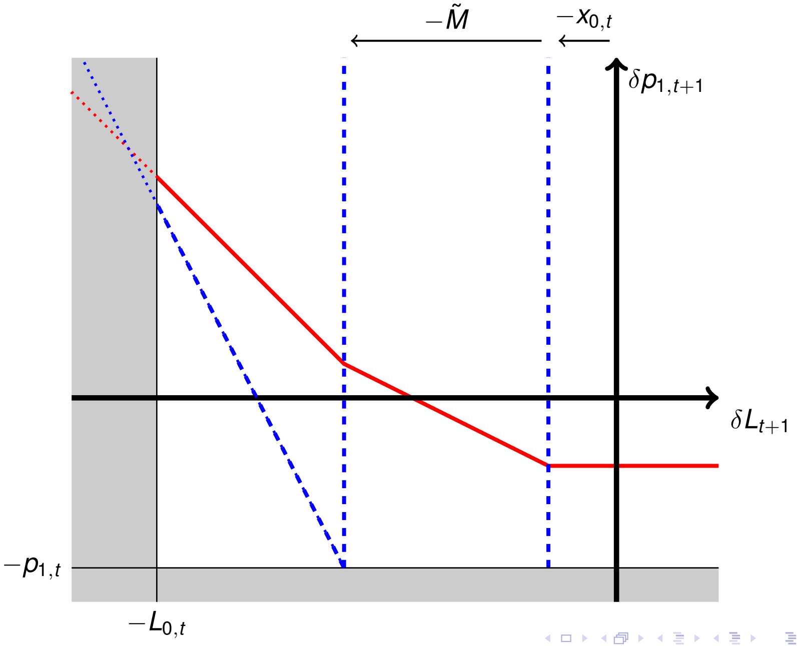

$\delta p_{1,t+1}$
δL_{t+1}
$-p_{1,t}$
$-L_{0,t}$

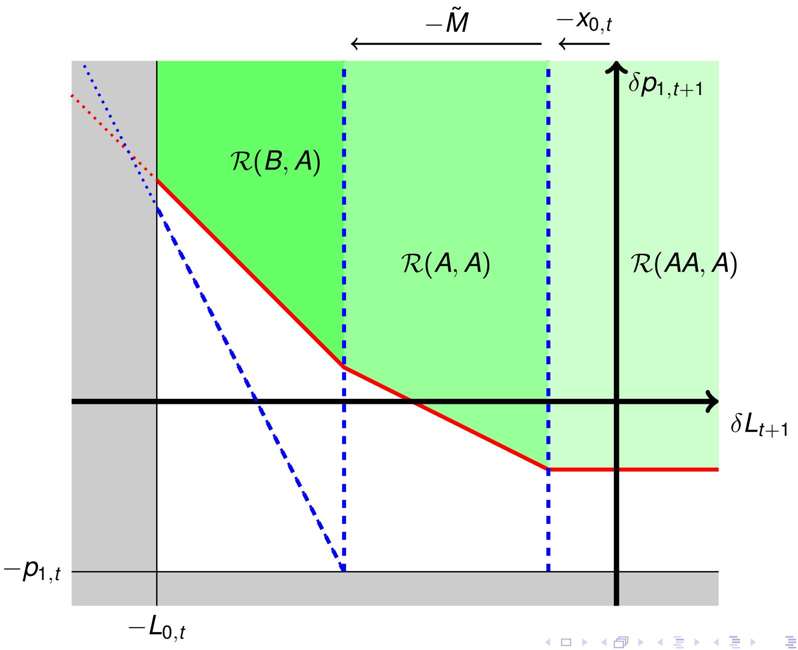

$-x_{0,t}$
$\delta p_{1,t+1}$
δL_{t+1}
$-p_{1,t}$
$-L_{0,t}$

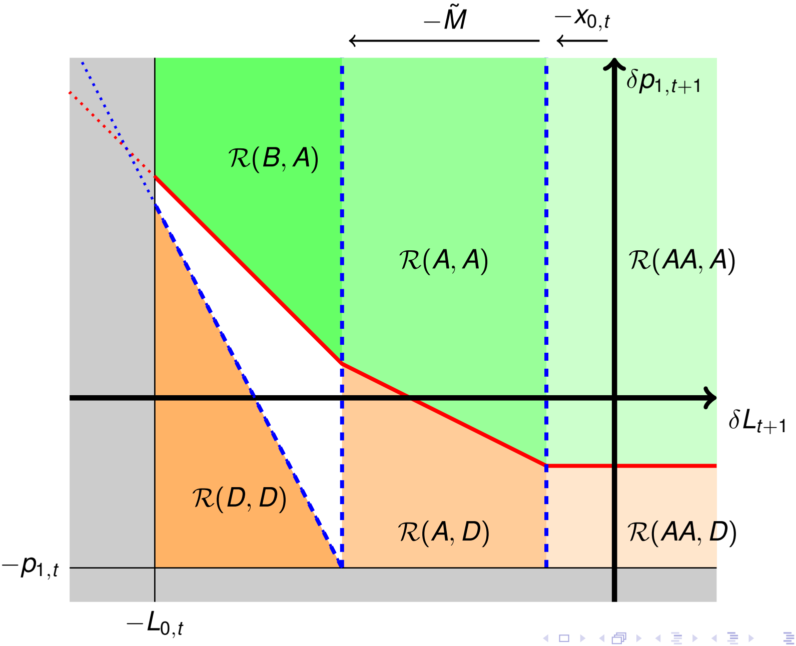


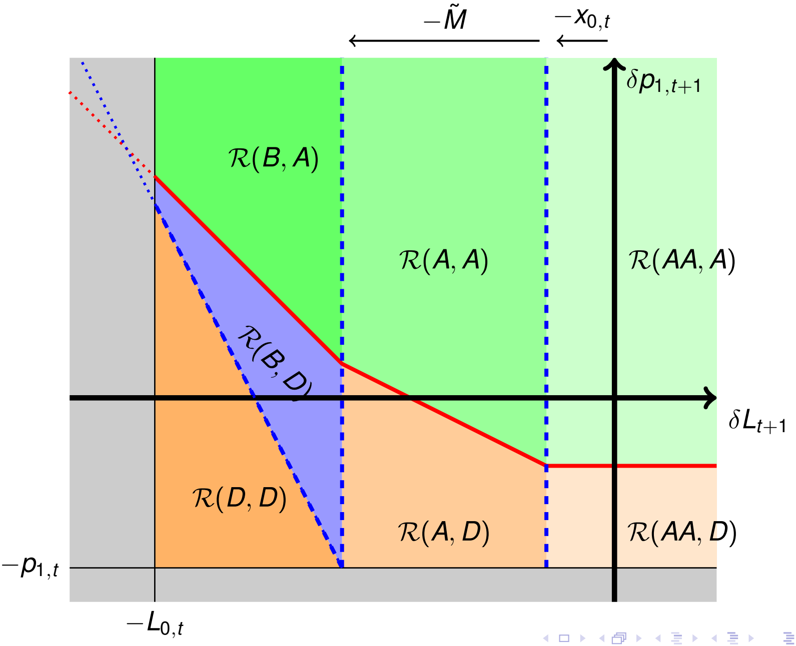
$-\tilde{M}$
$-x_{0,t}$
$\delta p_{1,t+1}$
δL_{t+1}
$-p_{1,t}$
$-L_{0,t}$


$-\tilde{M}$
$-x_{0,t}$
$\delta p_{1,t+1}$
δL_{t+1}
$-p_{1,t}$
$-L_{0,t}$


$-\tilde{M}$
$-x_{0,t}$
$\delta p_{1,t+1}$
δL_{t+1}
$-p_{1,t}$
$-L_{0,t}$


$-\tilde{M}$
$-x_{0,t}$
$\delta p_{1,t+1}$
$\mathcal{R}(B, A)$
$\mathcal{R}(A, A)$
$\mathcal{R}(AA, A)$
δL_{t+1}
$-p_{1,t}$
$-L_{0,t}$


$-\tilde{M}$
$-x_{0,t}$
$\delta p_{1,t+1}$
$\mathcal{R}(B, A)$
$\mathcal{R}(A, A)$
$\mathcal{R}(AA, A)$
δL_{t+1}
$\mathcal{R}(D, D)$
$\mathcal{R}(A, D)$
$\mathcal{R}(AA, D)$
$-p_{1,t}$
$-L_{0,t}$



$-\tilde{M}$
$-x_{0,t}$
$\delta p_{1,t+1}$
$\mathcal{R}(B, A)$
$\mathcal{R}(A, A)$
$\mathcal{R}(AA, A)$
$\mathcal{R}(B, D)$
δL_{t+1}
$\mathcal{R}(D, D)$
$\mathcal{R}(A, D)$
$\mathcal{R}(AA, D)$
$-p_{1,t}$
$-L_{0,t}$

- The partition in regimes allows to disentangle liquidity risk from solvency risk. With little adaptation, usual statistics can be used to do so in our framework.

- **i) The Value-at-Risk:**
The *VaR* is written on the equity and depends on γ, *M* and *H*, and a critical level α.
The $VaR^{\alpha}(\gamma, M, H)$ can be decomposed to identify the impact of the different risks:

$$\begin{aligned} VaR^{\alpha}(\gamma, M, H) &= VaR^{\alpha}(0, \infty, H) \\ &\qquad \textit{[standard VaR under Basel 2]} \\ &+ (VaR^{\alpha}(\gamma, \infty, H) - VaR^{\alpha}(0, \infty, H)) \\ &\qquad \textit{[additional cost for a credit line]} \\ &+ (VaR^{\alpha}(\gamma, M, H) - VaR^{\alpha}(\gamma, \infty, H)) \\ &\qquad \textit{[additional cost of market liquidity risk]} \end{aligned}$$

- **ii) Measures of funding liquidity risk:**
  - The probability of using the credit line ($\approx PD$):

$$PL = 1 - \mathbb{P}\left[\mathcal{R}(AA, A) \cup \mathcal{R}(AA, D)\right],$$

  - The expected use of the credit line ($\approx ELGD$),
  - The probability of selling illiquid assets:

$$PS = \mathbb{P}\left[\mathcal{R}(B, A) \cup \mathcal{R}(B, D) \cup \mathcal{R}(D, D)\right],$$

  - ...
- **iii) Probabilities of default:**
  - Probability of Default due to funding liquidity risk:

$$PD^F = \mathbb{P}\left[\mathcal{R}(B, D) \cup \mathcal{R}(D, D)\right],$$

  - Probability of Default due to a lack of solvency:

$$PD^S = \mathbb{P}\left[\mathcal{R}(A, D) \cup \mathcal{R}(AA, D)\right],$$

  - ...

- Funding liquidity risk is salient when there are:
  - an intrinsic maturity mismatch (very little cash on the asset side and mainly short term debt on the liability side),
  - inappropriate liquidity hedging (a small credit line with respect to the short term debt),
  - and a strong exposure to market liquidity risk (a large haircut H for selling illiquid assets).
- For banks, deposits are actually short-term debt while life-insurance policies are exposed to lapses.
- The dependence between shocks is crucial since liquidity funding risk is amplified by market/credit risk (Morris and Shin, 2010).

- We propose an illustration using the average of the top 5 US banks at 2014Q1 (JPM, BoA, Citi, WF and GS).
- Based on their Consolidated Financial Statements for Holding Companies (FR Y-9C reports), we calibrate (in % of total assets):

Asset	Liability
$x_{1,t} \approx 85\%$	$L_{1,t} \approx 71\%$
	$L_{0,t} \approx 19\%$
$x_{0,t} \approx 15\%$	0
	$Y_{0,t} \approx 10\%$

NB: Short-term/cash $\equiv$ maturity up to 1 year. Prices are normalized: $p_{1,t} = 1$.

Table : Calibrated initial balance sheet

- We set the credit line parameters: $\gamma = 1\%$, $M = 0.1 \times x_{0,t}$.

- We consider Gaussian shocks:

$$\begin{pmatrix} \delta p_{1,t+1} \\ \delta L_{t+1} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_p \\ \mu_L \end{pmatrix}; \begin{pmatrix} \sigma_p^2 & \rho\sigma_p\sigma_L \\ \rho\sigma_p\sigma_L & \sigma_L^2 \end{pmatrix} \right).$$

- We calibrate shocks assuming no interaction between them. Calibration relies on the following assumptions:
  - Return-on-asset based on FR Y-9C net income: $ROA^{|\rho=0,\delta L=0} = 0.19\%$,
  - Given probability of default: $PD^{|\rho=0,\delta L=0} = 1\%$,
  - No issuing long-term debt: $\delta L_{1,t+1} = 0$,
  - No expected change in depositor: $\mathbb{E}(\delta L|\rho = 0, \delta p = 0) = 0$,
  - Given stressed runoff rate: $\mathbb{P}\left[L_{0,t+1} \leq 0.28 L_{0,t}|\rho = 0, \delta p = 0\right] = 1\%$ (crudely derived from Basel 3's liquidity coverage ratio[1]).
- Hair cut on illiquid asset: $H = 50\%$ (haircut on level 2B asset in Basel 3's Liquidity Coverage Ratio).
- Results are based on Monte Carlo approach (1,000,000 simulations).

[1] $0.28 \approx 0.90^{12}$

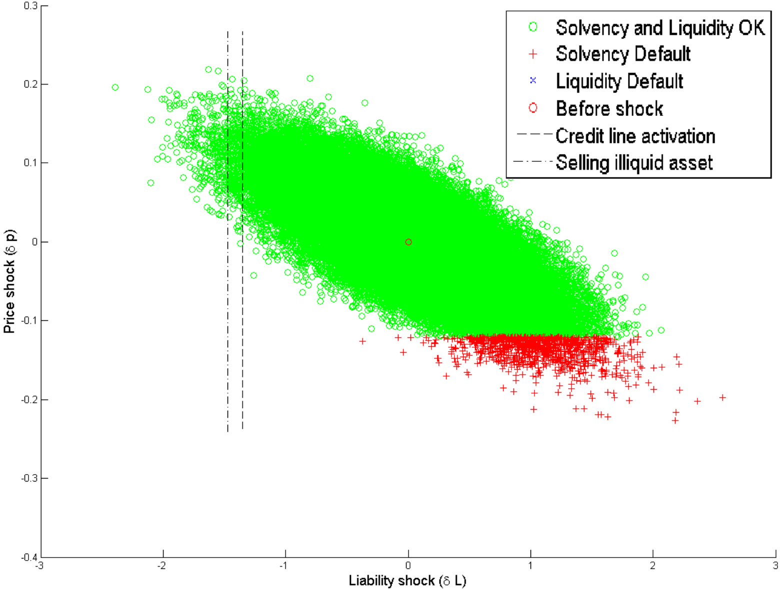


Figure : Observed regimes when $\rho = -0.75$.

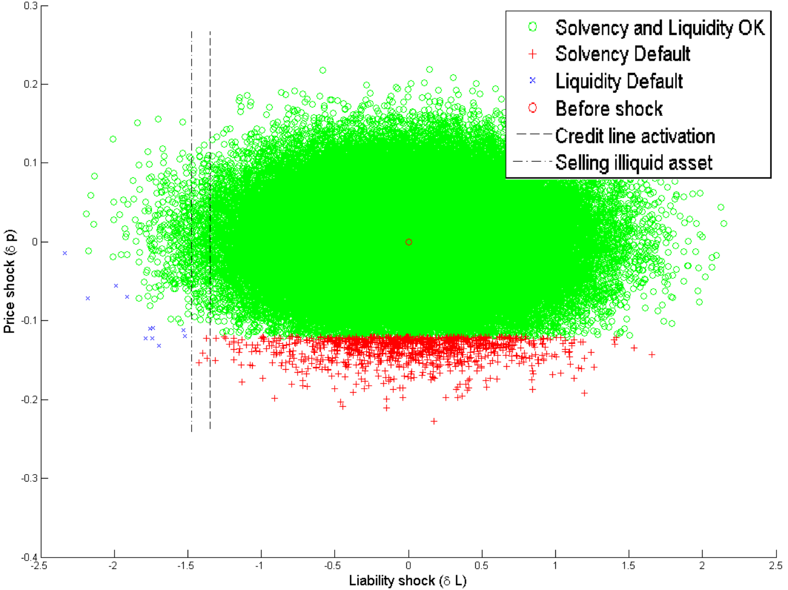


Figure : Observed regimes when $\rho = 0$.

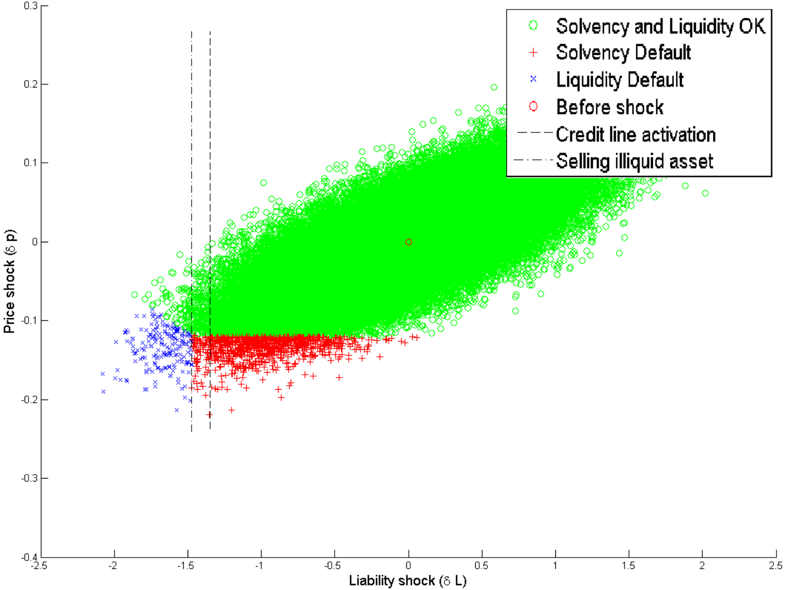


Figure : Observed regimes when $\rho = +0.75$.

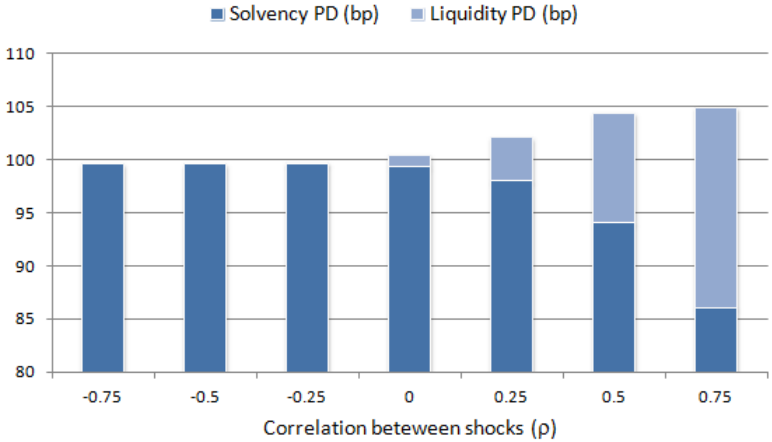


Figure : Probabilities of Default.

The probability of default increases with ρ, as liquidity distress become salient.

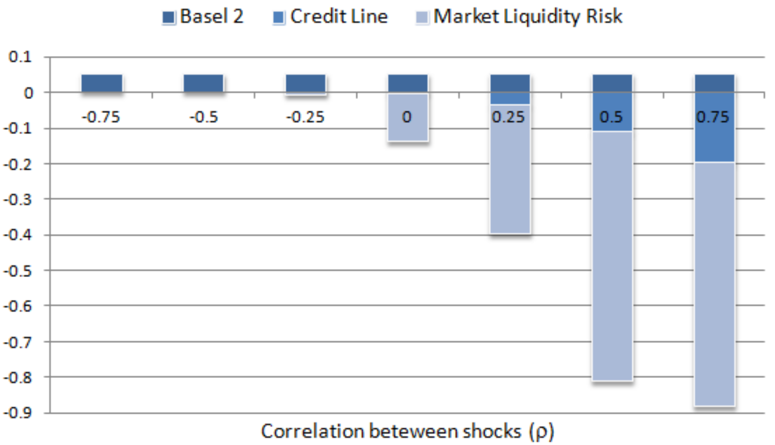


Figure : Value-at-Risk Composition (in % of initial equity).

Solvency part (Basel 2) is constant. Additional terms increase with ρ up to about 1% of initial equity.

Outline

- We introduce two reserve accounts in the stylized balance sheet:

Asset	Liability
$x_{1,t}p_{1,t}$	$L_{1,t}$
	$L_{0,t}$
$x_{0,t}$	0
$R_{1,t}$	
$R_{2,t}$	
	Y_t

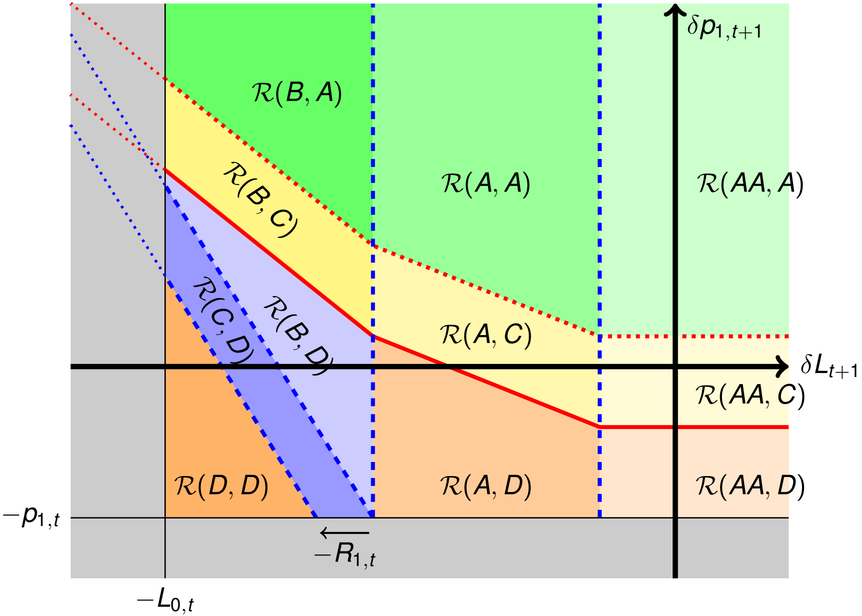
Table : Initial balance sheet with reserve accounts

- The two reserve accounts $R_{1,t}$ and $R_{2,t}$ are composed of cash.
- They differ from the cash account $x_{0,t}$, since they can only be used with the authorization of the supervisor.
- To satisfy the regulatory solvency constraint, we assume that

$$Y_t > R_{1,t} + R_{2,t}. \tag{11}$$

- Of course, the portfolio has been **previously readjusted** to satisfy the regulation.

- We analyze the following supervisor's intervention scheme (intervention "in last resort"):
- Rule 1: The reserve for liquidity risk $R_{1,t}$ can be unlocked to avoid a default due to funding liquidity risk, once the other solutions have been used.
- Rule 2: If there is a problem of solvency, the total reserve $R_{2,t}$ plus the residual $R_{1,t}$ can be unlocked to avoid default.
- Rule 1 introduces a fifth regime for liquidity corresponding to the use of the reserve account to pay short-term debt holders.
- Rule 2 introduces a third regime for solvency corresponding to a solvent institution (positive equity), but with a capital lower than the required one.


$\delta p_{1,t+1}$
$\mathcal{R}(B, A)$
$\mathcal{R}(A, A)$
$\mathcal{R}(AA, A)$
$\mathcal{R}(B, C)$
$\mathcal{R}(A, C)$
$\mathcal{R}(C, D)$
$\mathcal{R}(B, D)$
δL_{t+1}
$\mathcal{R}(AA, C)$
$\mathcal{R}(D, D)$
$\mathcal{R}(A, D)$
$\mathcal{R}(AA, D)$
$-p_{1,t}$
$-R_{1,t}$
$-L_{0,t}$

$\Longrightarrow$ **How to set the reserves?**

- By controlling the two probabilities of default:

$$\left\{ \begin{array}{r} \mathbb{P}_t\,[\text{default due to funding liquidity}] = PD_t^F(R_{1,t}, R_{2,t}), \\ \mathbb{P}_t\,[\text{default due to lack of solvency}] = PD_t^S(R_{1,t}, R_{2,t}). \end{array} \right. \quad (12)$$

$$\Leftrightarrow \left\{ \begin{array}{r} \mathbb{P}_t\,[\mathcal{R}(D,D)] = PD_t^F(R_{1,t}, R_{2,t}), \\ \mathbb{P}_t\,[\mathcal{R}(B,D) \cup \mathcal{R}(C,D) \cup \mathcal{R}(A,D) \cup \mathcal{R}(AA,D)] = PD_t^S(R_{1,t}, R_{2,t}). \end{array} \right.$$

- Then we can set two risk levels α_1 and α_2 and solve the bivariate system:

$$\left\{ \begin{array}{rcl} PD_t^F(R_{1,t}, R_{2,t}) & = & \alpha_1, \\ PD_t^S(R_{1,t}, R_{2,t}) & = & \alpha_2. \end{array} \right. \quad (13)$$

- There is no close form formulas, but simulation techniques are feasible.

- Let us use a numerical illustration with the following balance sheet:
  - Total debt: $L_{1,t} + L_{0,t} = 100$,
  - Illiquid assets: $x_{1,t}p_{1,t} = 100$ (with $p_{1,t} = 1$),
  - Cash: $x_{0,t} \in [1; 5]$,
  - Equity: $Y_{0,t} = x_{0,t} \in [1; 5]$.
- Gaussian shock for $\delta p_{1,t+1}$ and δL_{t+1}:
  - Correlation between shocks: $\rho \in [-0.9; +0.9]$,
  - No drift: $\mu_L = \mu_p = 0$,
  - Liability shock magnitude: σ_L such that the probability to use the credit line is $0.6bp$,
  - Asset shock magnitude: $\sigma_p = \sigma_L/(L_{1,t} + L_{0,t})$ to have similar risk on the asset side and the liability side.

	$R_2 = 0$	$R_2 = 1$	$R_2 = 2$	$R_2 = 3$	$R_2 = 4$	$R_2 = 5$
$R_1 = 0$	2.59%	2.59%	2.59%	2.59%	2.59%	2.59%
$R_1 = 1$	2.05%	2.05%	2.05%	2.05%	2.05%	2.05%
$R_1 = 2$	1.60%	1.60%	1.60%	1.60%	1.60%	1.60%
$R_1 = 3$	1.23%	1.23%	1.23%	1.23%	1.23%	1.23%
$R_1 = 4$	0.94%	0.94%	0.94%	0.94%	0.94%	0.94%
$R_1 = 5$	0.72%	0.72%	0.72%	0.72%	0.72%	0.72%

Table : Probability of default due to funding liquidity PD_t^F

	$R_2 = 0$	$R_2 = 1$	$R_2 = 2$	$R_2 = 3$	$R_2 = 4$	$R_2 = 5$
$R_1 = 0$	38.36%	35.36%	32.48%	29.8%	27.31%	24.97%
$R_1 = 1$	35.89%	33.02%	30.34%	27.85%	25.51%	23.40%
$R_1 = 2$	33.48%	30.8%	28.3%	25.96%	23.86%	21.91%
$R_1 = 3$	31.16%	28.67%	26.33%	24.23%	22.27%	20.53%
$R_1 = 4$	28.96%	26.62%	24.51%	22.56%	20.81%	19.26%
$R_1 = 5$	26.84%	24.74%	22.79%	21.04%	19.49%	18.12%

Table : Probability of default due to lack of solvency PD_t^S

Outline

1. Introduction
2. The balance sheets and their responses to exogenous shocks on liability
3. The regimes and the profit and loss distributions
4. Definition of reserves
5. Concluding remarks

We have seen in a simplified framework how to extend the standard computation of reserve based on the *VaR* of the asset portfolio to disentangle the liquidity and the solvency risk in the reserve account(s).
Different extensions can be considered to be closer to either the reality, or the practice:

- The haircut has been assumed fixed (no fire-sale mechanism),
- The analysis has been presented for an isolated financial institution. The distribution between an account for specific risk and for systemic risk would be relevant for liquidity risk as well as solvency risk.
- The assumption on the intervention of the supervisor can be changed. For instance, the reserve account R_1 can be unlocked before the institution sold all its illiquid assets to breakdown the possible liquidity spiral.

NB: in our framework, hundreds of lines of usual balance sheet have been aggregated into "illiquid asset"; therefore managing few reserve accounts is not too complex.