

# Bond Market Liquidity and the Role of Repo

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Board of Governors of the Federal Reserve System

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<sup>1</sup>The views of this presentation are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System

# Motivation

- ▶ Market claim U.S. Treasury liquidity is low and blame regulation:  
*“...many market participants also expect ongoing changes in regulation to raise the cost of providing immediacy services during normal times...” — CGFS Report, November 2014*
- ▶ Literature takes as given the link between the cash and repo markets  
*“Repo market liquidity is an important ingredient in the general liquidity of bond markets, and especially cash treasury market.” — Duffie (2016)*

## Goal

- ▶ Model how repo markets affect dealer liquidity provision in the cash market
- ▶ How does regulation—specifically the leverage ratio (SLR)—affect bond market liquidity?

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# Perspective

- ▶ Two main roles of repo:
  - ▶ Collateralized lending and borrowing: **general collateral repo (GC repo)**
  - ▶ Source assets without taking a position: **specific issue repo (SI repo)**
    - ▶ establish short position
    - ▶ deliver to counterparty
- ▶ Box constraint: Dealers must have assets in order to deliver
  - ▶ Cannot use GC market to source assets
- ▶ High *specialness*  $\implies$  sourcing assets through repo is expensive
- ▶ Using repo markets to intermediate cash markets increases dealer balance sheets
  - Leverage restrictions limit dealers' market making abilities

# Main Results

1. Repos allow dealers to intermediate without compromising their optimal portfolio, but increases balance sheet size  $\rightarrow$  *repo footprint*
2. When specialness is high, dealers demand higher compensation from customers for intermediation
3. Use of repo is balance sheet intensive  $\rightarrow$  size restrictions increase customers' intermediation cost

# Literature Review

- ▶ Treasury Market Liquidity:  
Fleming, Remolona (1999), Fleming (1997, 2003), Mizrach, Neely (2006)
- ▶ On-the-run/Off-the-run Pricing:  
Goldreich et. al (2005), Krishnamurthy (2002)
- ▶ Inventory Management:  
Amihud, Mendelson (1986), Stoll (1978), Ho, Stoll (1983)
- ▶ Trading Frictions in Repo Markets:  
Duffie (1996), Bottazzi et. al (2012), Foley-Fisher et. al. (2016)

# Model Setup

# Dealers, Clients, and Asset Market

- ▶ One risky asset with price  $p$ , one riskless asset numeraire (cash)
  - ▶ Risky asset payoff:  $\tilde{v} \sim \mathcal{N}(\mu, \sigma^2)$
- ▶ Continuum of *Dealers* with CARA utility and risk aversion  $\gamma$ 
  - ▶ Symmetric dealer's initial position: asset  $D$ , no cash, no repo position
- ▶ Dealers are approached by *Clients* to intermediate “random” order flow.
- ▶ Each client uses one dealer  $\implies$  segmented markets
  - ▶ Dealers post bid  $p - b$  and ask  $p + a$  to clients (dealer fees)
- ▶ Frictionless interdealer markets (U.S. Treasury Market)
- ▶ Simplified client orders: each dealer receives either short or levered long order  
 $\implies$  Repo and Cash trade!



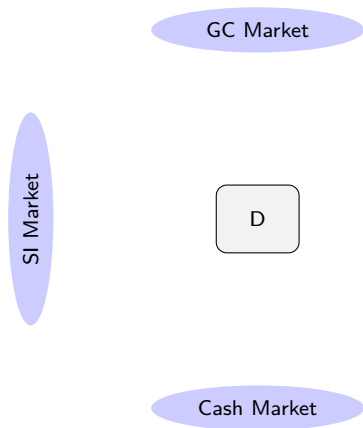
# GC and SI Repo Markets

- ▶ Dealers can access GC and SI repo markets
- ▶ Haircuts are zero and repos are risk free
- ▶ GC repo market
  - ▶ Exogenous GC repo rate  $R$
  - ▶ Assets supplied through GC market cannot be used further
- ▶ SI repo market
  - ▶ Endogenous SI repo rate  $R^S$
  - ▶ Focus on  $R - R^S > 0$ , i.e., when SI is on special
  - ▶ Additional participant: *Securities Lenders*:
    - ▶ Securities lenders exogenously supply  $\mathcal{S}\mathcal{L}(R - R^S; \eta)$  assets in the SI repo market

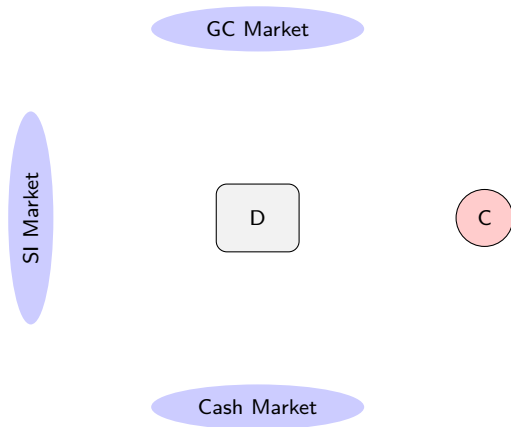
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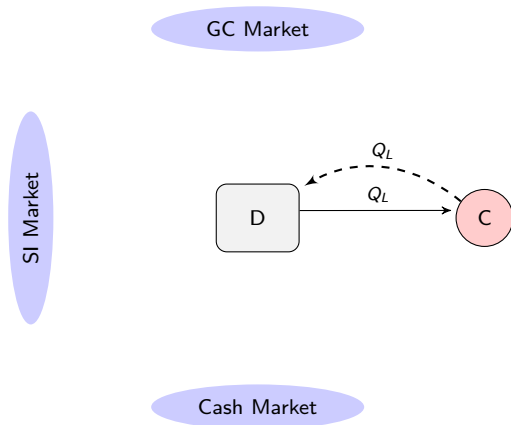
# Model Market Structure: tracking collateral



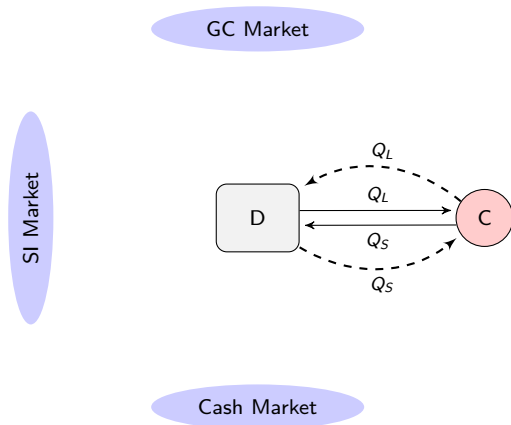
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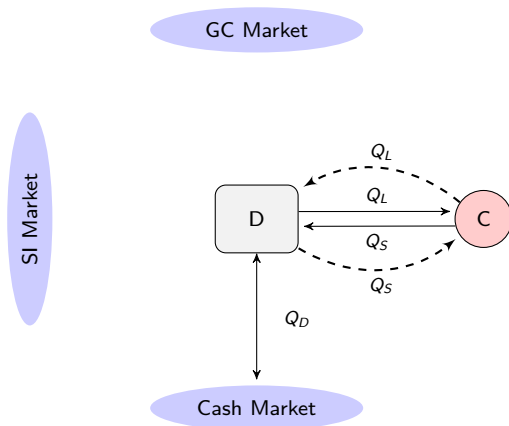
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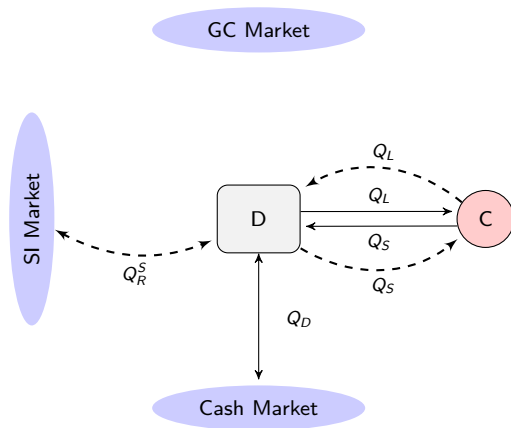
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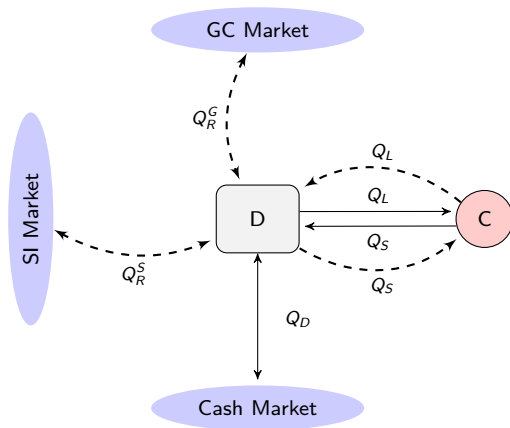
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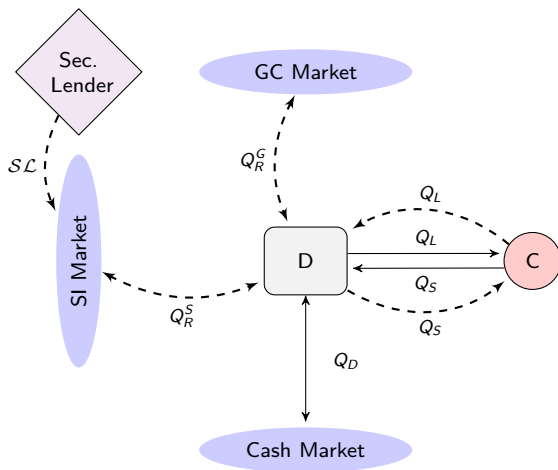


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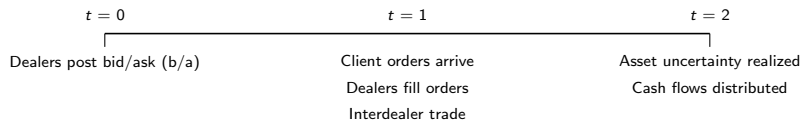




## Model Market Structure: tracking collateral



# Timeline



SI Market

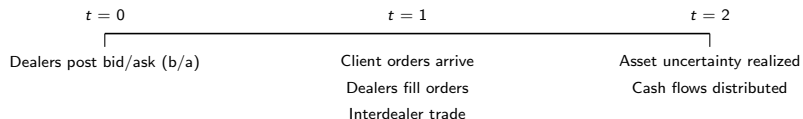
GC Market

D

C

Cash Market

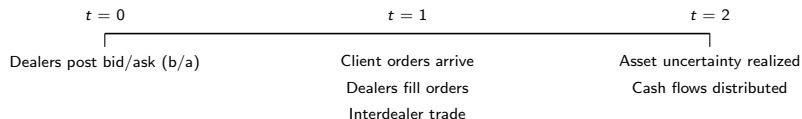
# Timeline



- ▶ Client order size depends on bid and ask
  - ▶  $t = 0$ :  $\tilde{Q}_L \sim \text{Exp}(\lambda(a))$ ,  $\tilde{Q}_S \sim \text{Exp}(\lambda(b))$

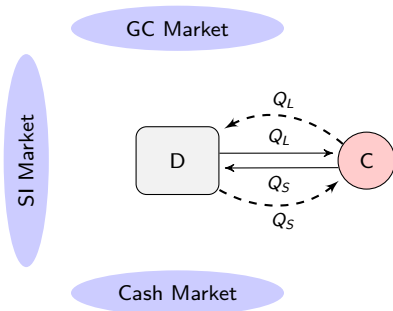


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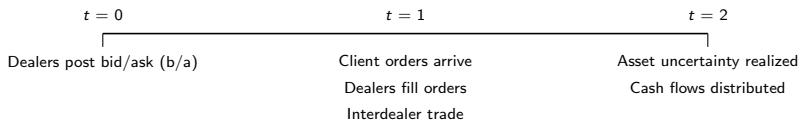


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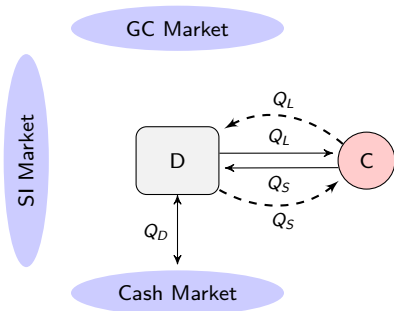
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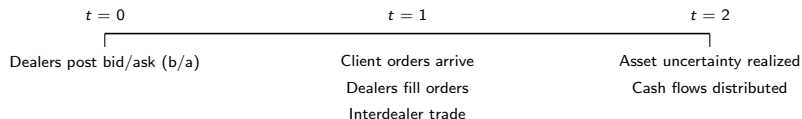
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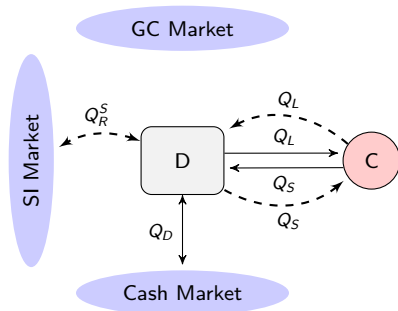
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- ▶  $Q_D$ : Asset rebalancing



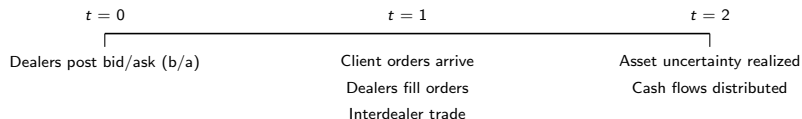
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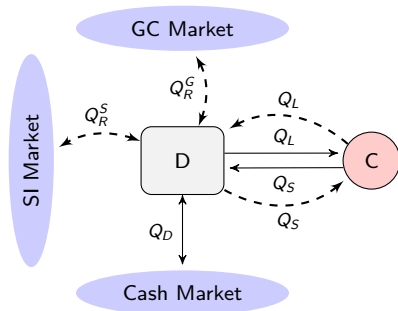
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- ▶  $Q_R^S$ : Asset sourced in from SI repo



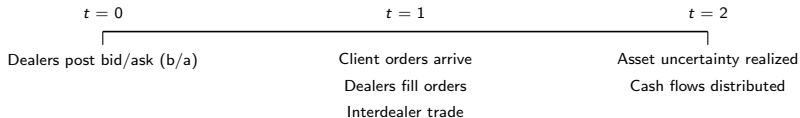
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- ▶  $Q_R^G$ : Asset brought in from GC repo



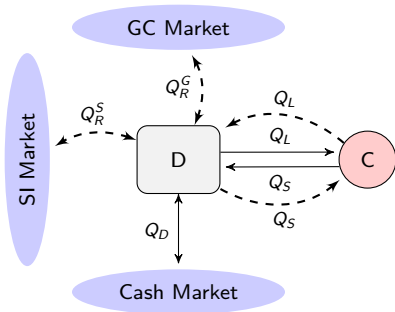
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SI box constraint:

$$D + Q_D + Q_R^S \geq g(Q_S, Q_L)$$



▶ Evidence of  $g$

▶ Dealer Optimization



# Unrestricted Balance Sheet

## Optimal rebalancing strategy: $t = 1$

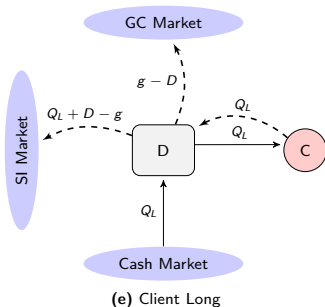
- ▶ In this CARA-Normal setting  $\implies$  optimal portfolio =  $\frac{\mu - pR^S}{\gamma\sigma^2}$
- ▶ Case with:  $D = \frac{\mu - pR^S}{\gamma\sigma^2}$ ,  $g(Q_S, Q_L) = g > D$ , and “large” order size:

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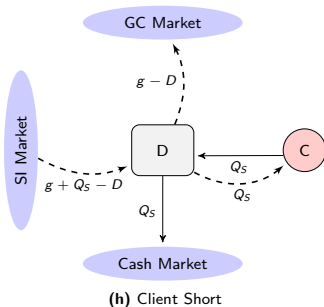
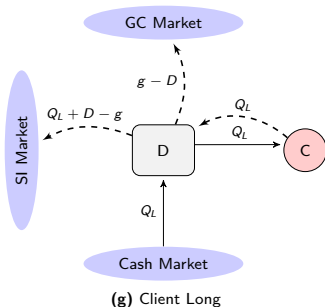
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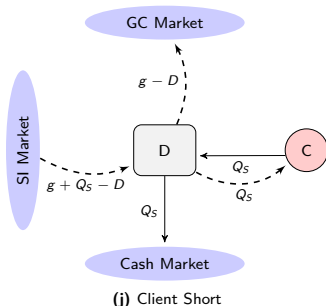
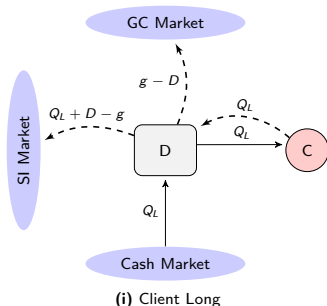
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Asset position will be

$$D + Q_D^* - \tilde{Q} = \frac{\mu - pR^S}{\gamma\sigma^2}$$

▶ Trade Vol vs. Repo

# “Interdealer” Equilibrium: $t = 1$

► Market clearing

$$\int_i Q_{Di} di = 0, \quad \text{Asset mkt clearing}$$

$$\int_i Q_{Ri}^S di = S\mathcal{L}(R - R^S; \eta), \quad SI \text{ mkt clearing}$$

► “Interdealer” equilibrium is given by

$$\frac{\mu - R^S p}{\gamma \sigma^2} = D - \frac{\mathbb{P}(CL)}{\lambda(a)} + \frac{\mathbb{P}(CS)}{\lambda(b)}$$
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## Optimal Bid/Ask: $t = 0$

- ▶ Expected payoff to service a levered long or short position ,

$$\mathbb{E}(u(W^*)) \propto \mathbb{E} \left( \exp \left\{ \gamma p [a \tilde{Q}_L + b \tilde{Q}_S - (R - R^S)g(\tilde{Q}_L, \tilde{Q}_S)] \right\} \right)$$

Note: if  $g$  symmetric then  $b^* = a^*$ .

- ▶ Simplified case:  $g(Q, 0) = g(0, Q) = g_0 \times Q$ , with  $g_0 \geq 0$
- ▶ Optimal ask  $a^*$  solves,

$$\lambda'(a^*)[a^* - g_0(R - R^S)] - \lambda(a^*) = 0$$

- ▶ Comparative statics to sec. lenders willingness to lend

$$\frac{\partial(R - R^S)}{\partial \eta} < 0, \quad \frac{\partial a^*}{\partial \eta} < 0$$

More assets available to lend, lower specialness & more liquid markets!

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# Constrained Balance Sheet

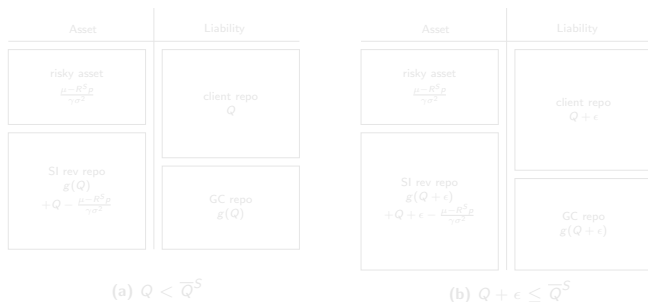
## Leverage Restriction (SLR)

- ▶ With fixed equity, leverage restriction  $\iff$  balance sheet size restriction:  $C$

$$\text{Assets} + \text{Liabilities} \leq 2C \times p$$

- ▶ Case with:  $D = 0, W = 0$  and  $\mu > R^S p$
- ▶ Dealers intermediate as unrestricted case when size restriction is slack, but...
- ▶ a binding balance sheet size restriction  $\implies$  Cap intermediation:  $(\bar{Q}^L(C), \bar{Q}^S(C))$

Figure: Client Short Order  $Q$



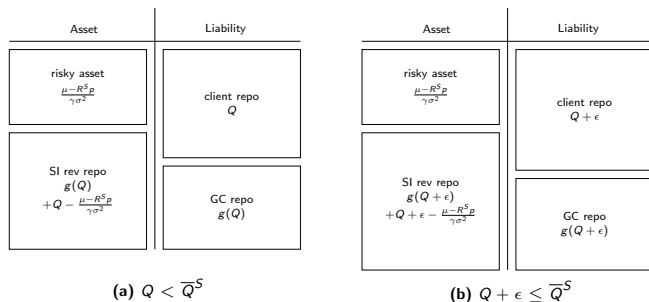
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Figure: Client Short Order  $Q$



# Optimal Bid with Leverage Restriction for Individual Firm (SLR)

- ▶  $b_{\infty}^*$ : optimal bid with unrestricted balance sheet
- ▶  $b_C^*$ : optimal bid with restricted balance sheet with size bound  $\bar{Q}^S(C)$

## Optimal bid

$$b_C^* > b_{\infty}^*$$

- ▶ Changes in **individual** balance sheet constraint:

$$\frac{\partial b_C^*}{\partial C} < 0$$

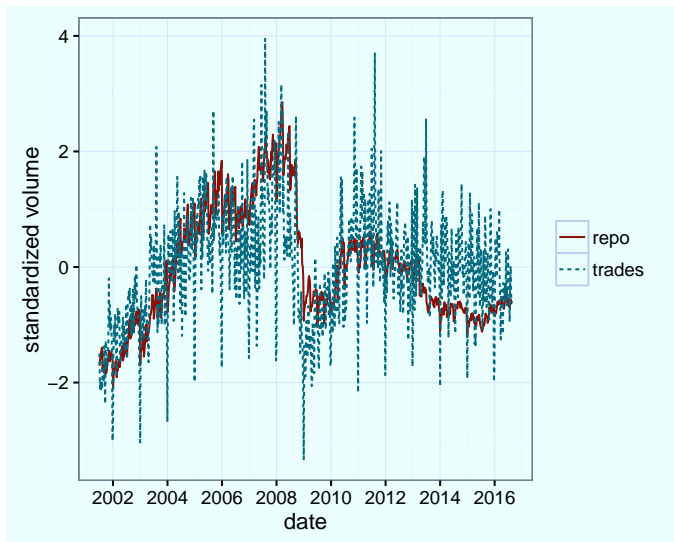
- ▶ Intuition:  
Dealers cannot intermediate large trades  $\implies$   
choose to lower the probability of attracting large trades and reap more profits from smaller ones

## Concluding Remarks

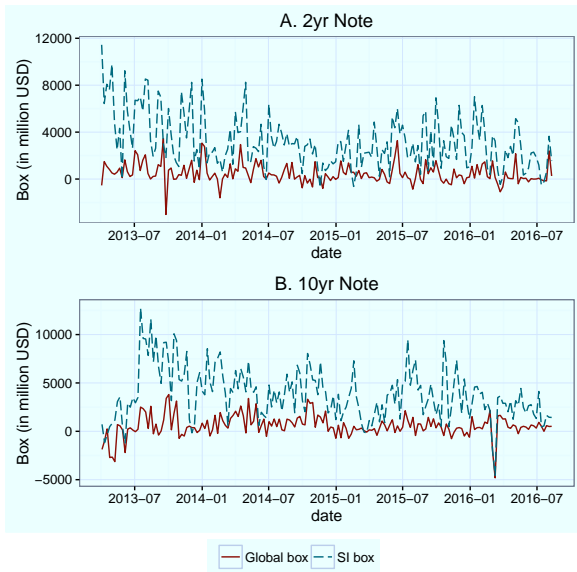
- ▶ Flexibility of repo book is important for Treasury market liquidity.
- ▶ Frictions in settlement and delivery ( $g$ ) imply a cost to intermediation due to repo specialness, affecting bond market liquidity.
- ▶ Restrictions on the size of repo book restricts dealer capacity to intermediate, reducing market depth and liquidity.



# Repo and trading volume in Treasury markets



# Evidence of $g$



# Dealer Optimization in $t = 1$

$$\max_{\{Q_D, Q_R^S, Q_R^G\}} \mathbb{E}(u(\tilde{W}) | \tilde{Q}_L = Q_L, \tilde{Q}_S = Q_S),$$

subject to

$$\begin{aligned} pQ_D + pQ_R^S + pQ_R^G &\leq 0 \\ D + Q_D + Q_R^S &\geq g(Q_L, Q_S) \end{aligned}$$

where

$$\begin{aligned} \tilde{W} = & (\tilde{v} - p)Q_D + (R^S - 1)pQ_R^S + (R - 1)pQ_R^G \\ & - (\tilde{v} - R^S p)\tilde{Q}_L + (\tilde{v} - R^S p)\tilde{Q}_S + ap\tilde{Q}_L + bp\tilde{Q}_S \\ & + \tilde{v}D \end{aligned}$$

► Dealer Strat