Bond Market Liquidity and the Role of Repo

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September 2016

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Motivation

- Market claim U.S. Treasury liquidity is low and blame regulation:
  
  "...many market participants also expect ongoing changes in regulation to raise the cost of providing immediacy services during normal times..." — CGFS Report, November 2014

- Literature takes as given the link between the cash and repo markets
  
  "Repo market liquidity is an important ingredient in the general liquidity of bond markets, and especially cash treasury market." — Duffie (2016)

Goal

- Model how repo markets affect dealer liquidity provision in the cash market
- How does regulation—specifically the leverage ratio (SLR)—affect bond market liquidity?
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How does regulation—specifically the leverage ratio (SLR)—affect bond market liquidity?
Two main roles of repo:
  ▶ Collateralized lending and borrowing: general collateral repo \((GC\ repo)\)
  ▶ Source assets without taking a position: specific issue repo \((SI\ repo)\)
    ▶ establish short position
    ▶ deliver to counterparty

Box constraint: Dealers must have assets in order to deliver
  ▶ Cannot use \(GC\) market to source assets

High \textit{specialness} \(\implies\) sourcing assets through repo is expensive

Using repo markets to intermediate cash markets increases dealer balance sheets
  \(\rightarrow\) Leverage restrictions limit dealers' market making abilities
Main Results

1. Repos allow dealers to intermediate without compromising their optimal portfolio, but increases balance sheet size \( \rightarrow \) repo footprint

2. When specialness is high, dealers demand higher compensation from customers for intermediation

3. Use of repo is balance sheet intensive \( \rightarrow \) size restrictions increase customers’ intermediation cost
Literature Review

- Treasury Market Liquidity:

- On-the-run/Off-the-run Pricing:
  Goldreich et. al (2005), Krishnamurhty (2002)

- Inventory Management:

- Trading Frictions in Repo Markets:
  Duffie (1996), Bottazzi et. al (2012), Foley-Fisher et. al. (2016)
Model Setup
Dealers, Clients, and Asset Market

- One risky asset with price \( p \), one riskless asset numeraire (cash)
  - Risky asset payoff: \( \tilde{v} \sim \mathcal{N}(\mu, \sigma^2) \)

- Continuum of Dealers with CARA utility and risk aversion \( \gamma \)
  - Symmetric dealer’s initial position: asset \( D \), no cash, no repo position

- Dealers are approached by Clients to intermediate “‘random” order flow.

- Each client uses one dealer \( \implies \) segmented markets
  - Dealers post bid \( p - b \) and ask \( p + a \) to clients (dealer fees)

- Frictionless interdealer markets (U.S. Treasury Market)

- Simplified client orders: each dealer receives either short or levered long order
  \( \implies \) Repo and Cash trade!
GC and SI Repo Markets

- Dealers can access GC and SI repo markets
- Haircuts are zero and repos are risk free

**GC repo market**
- Exogenous GC repo rate $R$
- Assets supplied through GC market cannot be used further

**SI repo market**
- Endogenous SI repo rate $R^S$
- Focus on $R - R^S > 0$, i.e., when SI is on special
- Additional participant: Securities Lenders:
  - Securities lenders exogenously supply $SL(R - R^S; \eta)$ assets in the SI repo market

\[
\frac{\partial SL(R - R^S; \eta)}{\partial (R - R^S)} > 0
\]
\[
\frac{\partial SL(R - R^S; \eta)}{\partial \eta} > 0
\]
Model Market Structure: tracking collateral

- Cash Market
- GC Market
- SI Market

D
Model Market Structure: tracking collateral
Model Market Structure: tracking collateral
Model Market Structure: tracking collateral

Cash Market

GC Market

SI Market

D

C

QL

QS

QL

QS
Model Market Structure: tracking collateral
Model Market Structure: tracking collateral

\[ Q_D \]

\[ Q_{SR} \]

\[ Q_L \]

\[ Q_L \]

\[ Q_S \]

\[ Q_S \]
Model Market Structure: tracking collateral

SI Market

GC Market

D

QL

QS

Q_D

QL

QS

Q_R^G

Q_R^S

C

Cash Market

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FRB
Model Market Structure: tracking collateral

- Sec. Lender
- SI Market
- GC Market
- D
- Cash Market

$Q^S_R$, $Q^G_R$, $Q_L$, $Q_S$, $Q_D$
Timeline

$t = 0$
- Dealers post bid/ask (b/a)

$t = 1$
- Client orders arrive
- Dealers fill orders
- Interdealer trade

$t = 2$
- Asset uncertainty realized
- Cash flows distributed

Cash Market
GC Market
SI Market

D
C
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- Client order size depends on bid and ask
  - \( t = 0: \tilde{Q}_L \sim \text{Exp}(\lambda(a)), \tilde{Q}_S \sim \text{Exp}(\lambda(b)) \)
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- \( Q_D \): Asset rebalancing

Diagram:
- GC Market
- SI Market
- Cash Market
- D
- C
- Arrows indicating flow between markets and asset rebalancing.
Timeline

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- $Q_D$: Asset rebalancing

- $Q^S_R$: Asset sourced in from SI repo
Timeline

- $t = 0$: Dealers post bid/ask (b/a)
- $t = 1$: Client orders arrive
- $t = 1$: Dealers fill orders
- $t = 2$: Interdealer trade
- $t = 2$: Asset uncertainty realized
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Timeline

- **t = 0**: Dealers post bid/ask (b/a)
- **t = 1**: Client orders arrive, Dealers fill orders, Interdealer trade
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**SI box constraint:**

\[
D + Q_D + Q^S_R \geq g(Q_S, Q_L)
\]

- Evidence of \( g \)
- Dealer Optimization
Unrestricted Balance Sheet
Optimal rebalancing strategy: $t = 1$

- In this CARA-Normal setting $\implies$ optimal portfolio $= \frac{\mu - \rho R^S}{\gamma \sigma^2}$

- Case with: $D = \frac{\mu - \rho R^S}{\gamma \sigma^2}$, $g(Q_S, Q_L) = g > D$, and “large” order size:
Optimal rebalancing strategy: $t = 1$

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- Case with: $D = \frac{\mu - pR_S}{\gamma \sigma^2}, g(Q_S, Q_L) = g > D$, and “large” order size:

Asset position will be

$$D + Q^*_D - \tilde{Q} = \frac{\mu - pR_S}{\gamma \sigma^2}$$
"Interdealer" Equilibrium: $t = 1$

- Market clearing

$$
\int_i Q_{Di} di = 0, \quad \text{Asset mkt clearing}
$$

$$
\int_i Q_{Si} di = SL(R - R^S; \eta), \quad SI \text{ mkt clearing}
$$

- "Interdealer" equilibrium is given by

$$
\frac{\mu - R^S p}{\gamma \sigma^2} = D - \frac{\mathbb{P}(CL)}{\lambda(a)} + \frac{\mathbb{P}(CS)}{\lambda(b)}
$$

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SL(R - R^S; \eta) = \mathbb{E}(g(\tilde{Q}_L, \tilde{Q}_S)) - D
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“Interdealer” Equilibrium: \( t = 1 \)

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\[
SL(R - R^S; \eta) = E(g(\tilde{Q}_L, \tilde{Q}_S)) - D
\]
Optimal Bid/Ask: $t = 0$

- Expected payoff to service a levered long or short position,

\[ \mathbb{E}(u(W^*)) \propto \mathbb{E}\left( \exp \left\{ \gamma p [a\tilde{Q}_L + b\tilde{Q}_S - (R - R^S)g(\tilde{Q}_L, \tilde{Q}_S)] \right\} \right) \]

Note: if $g$ symmetric then $b^* = a^*$.

- Simplified case: $g(Q, 0) = g(0, Q) = g_0 \times Q$, with $g_0 \geq 0$

- Optimal ask $a^*$ solves,

\[ \lambda'(a^*)[a^* - g_0(R - R^S)] - \lambda(a^*) = 0 \]

- Comparative statics to sec. lenders willingness to lend

\[ \frac{\partial(R - R^S)}{\partial \eta} < 0, \quad \frac{\partial a^*}{\partial \eta} < 0 \]

More assets available to lend, lower specialness & more liquid markets!
Optimal Bid/Ask: $t = 0$

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Constrained Balance Sheet
Leverage Restriction (SLR)

- With fixed equity, leverage restriction \(\iff\) balance sheet size restriction: \(C\)

\[
\text{Assets} + \text{Liabilities} \leq 2C \times p
\]

- Case with: \(D = 0, W = 0\) and \(\mu > R^S p\)

- Dealers intermediate as unrestricted case when size restriction is slack, but...

- a binding balance sheet size restriction \(\implies\) Cap intermediation: \((\overline{Q^L}(C), \overline{Q^S}(C))\)

Figure: Client Short Order \(Q\)
Leverage Restriction (SLR)

- With fixed equity, leverage restriction ⇐⇒ balance sheet size restriction: 
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**Figure:** Client Short Order \( Q \)

### (a) \( Q < \bar{Q}^S \)

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>risky asset</td>
<td>client repo ( Q )</td>
</tr>
<tr>
<td>( \frac{\mu - R^S p}{\gamma \sigma^2} )</td>
<td></td>
</tr>
<tr>
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<td>GC repo ( g(Q) )</td>
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### (b) \( Q + \epsilon \leq \bar{Q}^S \)

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Optimal Bid with Leverage Restriction for Individual Firm (SLR)

- $b^*_\infty$: optimal bid with unrestricted balance sheet
- $b^*_C$: optimal bid with restricted balance sheet with size bound $\overline{Q}^S(C)$

Optimal bid

$$b^*_C > b^*_\infty$$

- Changes in *individual* balance sheet constraint:
  $$\frac{\partial b^*_C}{\partial C} < 0$$

- Intuition:
  Dealers cannot intermediate large trades $\implies$
  choose to lower the probability of attracting large trades and reap more profits
  from smaller ones
Concluding Remarks

- Flexibility of repo book is important for Treasury market liquidity.

- Frictions in settlement and delivery \((g)\) imply a cost to intermediation due to repo specialness, affecting bond market liquidity.

- Restrictions on the size of repo book restricts dealer capacity to intermediate, reducing market depth and liquidity.
Repo and trading volume in Treasury markets

![Graph showing repo and trading volume trends over time.](image-url)
Evidence of $g$

A. 2yr Note

B. 10yr Note
Dealer Optimization in $t = 1$

$$\max_{\{Q_D, Q^S_R, Q^G_R\}} \mathbb{E}(u(\tilde{W})|\tilde{Q}_L = Q_L, \tilde{Q}_S = Q_S),$$

subject to

$$pQ_D + pQ^S_R + pQ^G_R \leq 0$$
$$D + Q_D + Q^S_R \geq g(Q_L, Q_S)$$

where

$$\tilde{W} = (\tilde{v} - p)Q_D + (R^S - 1)pQ^S_R + (R - 1)pQ^G_R$$
$$- (\tilde{v} - R^S p)\tilde{Q}_L + (\tilde{v} - R^S p)\tilde{Q}_S + ap\tilde{Q}_L + bp\tilde{Q}_S + \tilde{v}D$$