Macro factors and sovereign bond spreads: a quadratic no-arbitrage model*

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*All opinions are personal and should not be attributed to the BIS or the ECB.
Motivation

10-year yield spreads vs Germany

- Greece
- Portugal
- Spain
- France
- Italy

Euro sovereign bond spreads
Motivation

- Peripheral euro area sovereign bond yields have risen sharply above comparable yields on German government bonds.

- These spread increases have coincided with rapidly deteriorating fiscal conditions and eroding market confidence.

- Can country-specific fundamentals explain the rise in spreads? Or do factors unrelated to fundamentals play an important role (e.g. self-fulfilling dynamics)?

- Attempts to model the dynamics of sovereign yields/spreads as functions of observable fundamentals have been challenging.
We examine the drivers of euro area sovereign bond spreads, including observable macro fundamentals

- key features:
  
  (i) allow sovereign spreads to depend on observable fundamentals (debt/GDP, economic growth)

  (ii) allow for non-linear effects of fiscal fundamentals on spreads

  (iii) include a common credit factor

  (iv) let all factors affect debt/GDP to allow us to capture feedback of higher spreads on debt (to a first approximation)
Outline

- Methodology / Literature / Model
- Data and estimation method
- Results: model fit, estimated common factor, quantification of nonlinear effects, risk premia estimates, spread decompositions
- Conclusions
Starting point: reduced-form credit models (Lando (1994), Duffie & Singleton (1998), Duffee (1999), ...), in which assumptions are made about the process for default intensity.

Default is doubly stochastic: default arrives randomly, and the arrival intensity process $\Lambda_t$ varies randomly over time.

Advantage: tractable bond pricing; for zero recovery & discrete time:

$$P_t(n) = E_t^Q \left[ \exp \left( - \sum_{j=1}^{n} (r_{t+j-1} + \Lambda_{t+j}) \right) \right]$$

The intensity $\Lambda_t$ may be a function of a set of factors.
We apply this framework to euro sovereign bonds. Much of the available literature on sovereign credit has focused on emerging markets (Duffie, Pedersen & Singleton (2003), Pan & Singleton (2008), Longstaff et al. (2011)); Dai and Philippon (2006) focused on US, but assumed no credit risk.

More recent interest in advanced economies, in particular the euro area: Laubach (2011), Monfort & Renne (2011) Borgy et al. (2011), Ang & Longstaff (2011), ...

These studies typically model default intensities and sovereign spreads as affine functions of some unobservable factors (Borgy et al. use observable macro factors)
Methodology

- We want to allow macro fundamentals to play an important role for
  spreads; in particular (expected) debt/GDP.
- A linear relationship may not be optimal: theoretical results suggest a
  non-linear relationship (e.g. Bi, 2011, Juessen et al., 2011, Corsetti et
  al., 2012)
- As economies approach the fiscal limit — the point where a
government no longer has the ability to finance its debt by raising
taxes — bond yields become steeply non-linear.
- There is also empirical evidence of a non-linear relationship between
  bond yields and fiscal fundamentals (Alesina et al., 1992; Ardagna et
  al. 2007; Bernoth et al., 2004).
Looking at recent euro area developments: challenging to assume a linear relationship between spreads and fiscal fundamentals — a quadratic specification seems more promising:
We start with the result of Duffie and Singleton (1999): under RMV, credit risky bonds can be priced as default-free bonds using a default-adjusted discount rate \( r^*_t = r_t + \Lambda_t \).

\( \Lambda_t \) is a “recovery-adjusted default intensity”

We assume that \( \Lambda_t \) depends on:

- country-specific expected GDP growth: \( g_t \)
- country-specific expected debt/GDP: \( d_t \)
- debt-to-GDP squared: \( d_t^2 \)
- a common latent factor: \( C_t \)

We assume VAR factor dynamics for \( g, d \) and \( C \).
Model (cont.)
Default intensities and state variables

- Default intensity: $\Lambda_t^i = \lambda_0^i + \lambda_t^i X_t^i + (X_t^i)' \Xi_t X_t^i$

- State vector, country $j$: $X_t = [C_t, g_t^j, d_t^j]'$

- State dynamics: $X_t = \Phi X_{t-1} + \varepsilon_t$ with restrictions:
  - $C_t$: AR(1); independent of macro fundamentals
  - $g_t$: AR(1) [interactions not significant]
  - $d_t$: allowed to depend on all factors
Assuming a standard SDF, and affine market prices of risk (Duffee, 2002), $\psi_t = \psi_0 + \psi_1 X_t$, we can price risky bonds using exponential quadratic bond prices of Ahn, Dittmar & Gallant (2002) and Leippold & Wu (2002) (Realdon (2006) in discrete time):

$$
\begin{align*}
    P^n_t &= \exp (A_n + B_n X_t + X_t' C_n X_t) \\
    A_n &= A_{n-1} + \ldots \\
    B_n &= B_{n-1} \Phi + \ldots \\
    C_n &= \Phi' C_{n-1} \Phi + \ldots
\end{align*}
$$
Data

- 2, 3, 4, 5, 7, 10-year zero-coupon yields on Greek, Portuguese, Spanish, Italian and French government bonds (NS estimates), minus corresponding German yields.
- \( g^j_t \): one-year ahead expected GDP growth; constructed from semi-annual European Commission forecasts; monthly data obtained using the Kalman filter (VAR)
- \( d^j_t \): one-year ahead expected debt-to-GDP ratio (constructed in the same way).
- \( g \) and \( d \) are in deviation from sample mean (pre-crisis mean for \( d \))
Joint ML estimation assuming Gaussian shocks.
Allow all spreads and macro factors to be imperfectly observed: Gaussian measurement errors
Spreads are not linear in the factors: we can’t use a regular Kalman filter
We use the unscented Kalman filter (Julier & Uhlmann (1997, 2004)):
- deterministic sampling of ‘sigma points’ around the mean of the state variables
- propagate points through the non-linear function
- recover first two moments of the non-linear system
- use this to update the filter
Parameter estimates: default intensities

\[ \Lambda_t = \lambda_0 + \lambda X_t + (X_t)' \Xi X_t; \quad \lambda = (\lambda_C, \lambda_g, \lambda_d) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Greece</th>
<th>Portugal</th>
<th>Spain</th>
<th>France</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda_0 \times 10^2)</td>
<td>0.036</td>
<td>0.021**</td>
<td>0.039**</td>
<td>0.003**</td>
<td>0.026**</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.007)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>(\lambda_C \times 10^2)</td>
<td>(-0.220)</td>
<td>(-0.074)</td>
<td>(-0.017**)</td>
<td>(-0.008)</td>
<td>(-0.024**)</td>
</tr>
<tr>
<td></td>
<td>(1.739)</td>
<td>(0.220)</td>
<td>(0.005)</td>
<td>(0.013)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(\lambda_g \times 10^2)</td>
<td>(-0.031**)</td>
<td>(-0.001**)</td>
<td>(-0.015**)</td>
<td>(-0.004**)</td>
<td>(-0.003)</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>(\lambda_d \times 10^2)</td>
<td>(-0.020)</td>
<td>(-0.019)</td>
<td>0.441**</td>
<td>0.019**</td>
<td>0.410**</td>
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<tr>
<td></td>
<td>(0.059)</td>
<td>(0.027)</td>
<td>(0.117)</td>
<td>(0.006)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>(\Xi_{d,d})</td>
<td>0.116**</td>
<td>0.041**</td>
<td>0.021**</td>
<td>0.003**</td>
<td>0.094**</td>
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<tr>
<td></td>
<td>(0.019)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.000)</td>
<td>(0.009)</td>
</tr>
</tbody>
</table>
Results: model validation

Estimated and observed spreads

Greece

Portugal

Spain

P. Hördač (BIS)

Euro sovereign bond spreads
Results: model validation
Estimated and observed spreads (cont.)

France

Italy


0

0

0.5

0.5

1

1

1.5

1.5

2

2

4

4

8

8

2y fitted
2y actual
5y fitted
5y actual
10y fitted
10y actual
Results: model validation

One-year ahead forecasts of 10-year spreads

Spain

France

Italy

realized
model
Consensus forecast
Results: model validation

Conditional spread volatilities

Greece spread vols

Greece, 5y vol

Portugal spread vols

Portugal, 5y vol

model-implied

GARCH
Results

Estimated common factor

P. Hördahl (BIS)
Results

Is allowing for nonlinearities important?

Greece

France

Italy

linearized model
full non-linear model
observed
Results

The role of the distress risk premium: 10-year spreads

Greece

France

Italy

- total spread
- spread net of risk premium
Results

One-year (risk-neutral) default probabilities (50% loss assumed)
Results

Decomposition of default risk component (10y spread less premium)

![Graphs showing Euro sovereign bond spreads for Greece, France, and Italy from 2004 to 2012.]

- Greece
- France
- Italy

Legend:
- Blue: common
- Green: growth
- Red: debt
- Cyan: constant

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Results

Decomposition of distress risk premium (10y)

Greece

France

Italy

common
growth
debt
constant
Results

10-year spread responses to adverse shocks
Conclusions

- We estimate a non-linear reduced-form credit model for sovereign spreads of five euro area countries.
- Using country-specific debt-to-GDP and GDP growth along with a common factor, our model can capture well features of sovereign spreads along several dimension.
- Macro factors are important drivers of sovereign spreads, mainly through the default risk component.
- In particular, spreads depend non-linearly on debt-to-GDP.
- For all countries, except Greece, the surge in spreads has mainly been due to rising distress risk premia. For this component, the common factor plays a central role.
German zero-coupon yields vs. OIS zero rates
Extra slides
Macroeconomic data: observed and filtered (KF)

- Expected GDP growth, France
- Expected debt/GDP, France
- Expected GDP growth, Italy
- Expected debt/GDP, Italy
Extra slides
Filtered macro variables (UKF) and data

- Expected GDP growth, Greece
- Expected debt/GDP, Greece
- Expected GDP growth, Portugal
- Expected debt/GDP, Portugal
- Expected GDP growth, Spain
- Expected debt/GDP, Spain

filtered data
Extra slides
Filtered macro variables (UKF) and data

- Expected GDP growth, France
- Expected debt/GDP, France

- Expected GDP growth, Italy
- Expected debt/GDP, Italy