Flight to Liquidity and Systemic Bank Runs

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Previously circulated as “Financial Crises and Systemic Bank Runs in a Dynamic Model of Banking”
Banking crises (U.S. 2008, Great Depression)

1. Many financial intermediaries insolvent, and subject to runs
   - This paper: panics, multiple equilibria

2. Flight to liquidity
   - Friedman-Schwartz hypothesis:
     Fed did not increase money supply in the ’30s ⇒ great depression
   - 2008: Fed injected liquidity ⇒ mitigated the crisis
   - What are the effects of monetary injections?
     Can the central bank rule out self-fulfilling panics?
A new macroeconomic model of banking

- General equilibrium, monetary model
- 3-period (motivated by infinite horizon, different from typical bank run model)

Two main contributions:

- New channel, multiplicity of equilibria
  - Good equilibrium
  - Bad equilibrium:
    1. many banks insolvent and subject to runs (*systemic crisis*)
    2. flight to liquidity, deflation

- Monetary policy analysis:
  - How to eliminate the bad equilibrium
  - Effects of monetary injections that do not eliminate bad equilibrium
The model in one slide

- Two assets in fixed supply: money, capital
- Consumption ≤ money in wallet + money withdrawn from bank
- Good equilibrium: all banks are solvent, no runs
- Bad equilibrium:
  - Fear of runs: precautionary money hoarding (flight to liquidity)
    - money demand ↑, demand for capital ↓ ⇒ price of capital ↓
  - Some money held for precautionary reasons
    → unspent money → deflation

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<tr>
<th>Assets</th>
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<th>Weakest banks: insolvent</th>
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<td>Value of capital ↓</td>
<td>Deposits (nominal)</td>
<td>run on insolvent banks</td>
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<td>Net worth ↓</td>
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5 / 29
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- Two assets in fixed supply: money, capital
- Consumption $\leq$ money in wallet + money withdrawn from bank
- Good equilibrium: all banks are solvent, no runs
- Bad equilibrium:
  - **Fear of runs**: precautionary money hoarding (*flight to liquidity*)
    - money demand ↑, demand for capital ↓ $\Rightarrow$ price of capital ↓

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- Weakest banks: *insolvent* $\Rightarrow$ *run* on insolvent banks
- Some money held for precautionary reasons
  - $\Rightarrow$ unspent money $\Rightarrow$ deflation
Preview of the results

• Model: flight to liquidity
  - $\downarrow$ $M1 = \text{money held by households} + \text{deposits}$
  - $\downarrow$ Deposits
  - $\uparrow$ Money held by households

• Monetary policy
  1. What policy commitment eliminates the bad equilibrium?
     “Large” monetary injections
  2. Effects of ”small” monetary injections. In some cases:
     nominal prices $\downarrow$, flight to liquidity $\uparrow$, losses of depositors $\uparrow$
Comparison with literature

- Panic, multiplicity of equilibria: Diamond and Dybvig (1983)
  Not applicable to monetary injections; exogenous returns

  *This paper: Money, interactions between runs and monetary policy*

    *Endogenous drop in asset prices (systemic crises)*

    *Multiplicity of equilibria: new channel*

- Bank runs with money: Diamond and Rajan (2006), Allen et al. (2013)
  Exogenous shocks to money demand (policy-invariant)

  *This paper: Flight to liquidity is endogenous*

    *Monetary injections amplify flight to liquidity (in some cases)*
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• Deflation and banking crises: Carapella (2012), Brunnermeier and Sannikov (2015) “I-Theory”: banks are intermediaries, no runs

  *Companion paper: Real model (no deflation), multiple equilibria*

  *Source of multiplicity: flight to liquidity*
Outline

• Model without policy intervention

• Equilibria:
  • Good equilibrium
  • Bad equilibria (up to two bad equilibria)

• Monetary policy

• Robustness
• Model without policy intervention

• Equilibria:
  • good equilibrium
  • bad equilibria (up to two bad equilibria)

• Monetary Policy
Timing, agents, assets

- 3 periods: $t = 0, 1, 2$

- Agents:
  - Banks: continuum $b \in [0, 1]$, perfect competition
  - Households: double continuum $h \in [0, 1] \times [0, 1]$
    (continuum $[0, 1]$ of households per bank)
  - Central bank

- Assets:
  - money (numeraire): fixed supply $M$
  - capital: fixed supply $K$
  - deposits
Preferences

- Banks: linear utility from consumption $C_{2}^{h}$

- Households: consumption $C_{1}^{h}$ and $C_{2}^{h}$:

$$
\mathbb{E} [\tilde{u} (C_{1}^{h})] + \beta C_{2}^{h}
$$

where:

$$
\tilde{u} (\cdot) = \begin{cases} 
\bar{u} (\cdot) & \text{(impatient) probability } \kappa \\
0 & \text{(patient) probability } 1 - \kappa 
\end{cases}
$$

- private information

- realized at $t = 1$

- i.i.d. across agents, LLN at each bank
Utility of impatient households, \( t = 1 \)

- Concavity: risk aversion
- Linearity (almost everywhere): tractability
Technology and markets

- **t = 0**: Walrasian market, price of capital $Q_0$

- **t = 1**:
  - 1 unit of capital $\to A$ units consumption good
  - Purchases of $C^h$ at price $P_1$, s.t. cash-in-advance
  - Capital cannot be traded at $t = 1$ (illiquid)

- **t = 2**:
  - 1 unit money $\to \frac{1}{P_2}$ units of consumption good
  - 1 unit capital $\to \frac{Q_2}{P_2}$ units of consumption good

$P_2$ and $Q_2$ exogenous, motivated by infinite horizon
Technology and markets

Production: $A \overline{K}$

$t = 0$

Walrasian market

$Q_0$: price of capital

Preference shocks realized

$t = 1$

Market for consumption good

(cash-in-advance constraint)

$P_1$: price of consumption

$t = 2$

1 unit money $\rightarrow \frac{1}{P_2}$ units consumption good

1 unit capital $\rightarrow \frac{Q_2}{P_2}$ units consumption good

• $P_2$ and $Q_2$ exogenous, motivated by infinite horizon
Technology and markets

Production: $A \overline{K}$

$t = 0$
- Walrasian market
- $Q_0$: price of capital
- Preference shocks realized

$t = 1$
- Market for consumption good (cash-in-advance constraint)
- $P_1$: price of consumption

$t = 2$
- 1 unit money $\rightarrow \frac{1}{\overline{P}_2}$ units consumption good
- 1 unit capital $\rightarrow \frac{\overline{Q}_2}{\overline{P}_2}$ units consumption good

$\overline{P}_2$ and $\overline{Q}_2$ exogenous, motivated by infinite horizon

Trade-off:
- Preference shock known at $t = 1$ & no trading of capital at $t = 1$
- Opportunity cost of holding money (return from holding capital)
Endowments at $t = 0$

Bank $b$

\[
\begin{pmatrix}
M_{-1}^b, K_{-1}^b, D_{-1}^b
\end{pmatrix}
\]

money, capital, deposits

Household $h$

\[
\begin{pmatrix}
M_{-1}^h, K_{-1}^h, D_{-1}^h
\end{pmatrix}
\]

money, capital, deposits

- Deposits
  - Bank: obligation to pay money, on demand
  - Household: claim redeemable for money (on demand)

- Banks are heterogeneous in $K_{-1}^b \in \{K^H, K^L\}$, $K^H > K^L$

**Private information at $t = 0$, common knowledge at $t = 1$**

- All households are alike
Net worth (beginning of $t = 0$)

- Household $h$: $N^h_0 \equiv K^h_{-1} Q_0 + M^h_{-1} + D^h_{-1}$
  - value of capital
  - money
  - deposits

- Bank $b$, balance sheet:

<table>
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<th>Assets</th>
<th>Liabilities</th>
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<tr>
<td>Capital $K^b_{-1} Q_0$</td>
<td>Deposits $D^b_{-1}$</td>
</tr>
<tr>
<td>Money $M^b_{-1}$</td>
<td>Net worth $N^b_0$</td>
</tr>
</tbody>
</table>

Net worth of bank $b$: $N^b_0 \equiv K^b_{-1} Q_0 + M^b_{-1} - D^b_{-1}$

- $Q_0$ high $\Rightarrow N^b_0 \geq 0$ for all $b$ (all banks are solvent)

- $Q_0$ low $\Rightarrow \begin{cases} N^b_0 \geq 0 & \text{for banks with large endowment, } K^b_{-1} = K^H \\ N^b_0 < 0 & \text{for banks with low endowment, } K^b_{-1} = K^L \end{cases}$
Banking

- **Households**
  - $t = 0$, portfolio choice:
    \[
    M^h_0 + D^h_0 + Q_0 K^h_0 \leq M^h_{-1} + D^h_{-1} + Q_0 K^h_{-1}
    \]
    
    - $t = 1$, withdrawal:
      \[
      W^h_1 \leq \begin{cases} 0 & \text{if run & last in line} \\ D^h_0 & \text{otherwise} \end{cases}
      \]
    
    and consumption $C^h_1$ s.t. $P_1 C^h_1 \leq M^h_0 + W^h_1$

- $t = 2$: get return on deposits not withdrawn at $t = 1$

- **Banks, $t = 0$**:
  - hold money $= \kappa \times \text{deposits}$
    (to pay withdrawals by fraction $\kappa$ of impatient households, at $t = 1$)
  - other resources invested capital
    (return on capital utilized to pay return on deposits at $t = 2$)
One good equilibrium, up to 2 bad equilibria

- **Good equilibrium:** high price capital $Q_0$
  - banks: all solvent; households: money $M_h^0 = 0$, deposits $D_h^0 = D^*$
  - impatient households, $t = 1$: withdraw, consume
    patient households, $t = 2$: deposits + return

- **Bad equilibrium:** low price of capital $Q_0$
  - some banks are insolvent, nobody knows which one
    (asymmetric information about banks’ balance sheet at $t = 0$)
  - $t = 0$: fear of runs $\rightarrow$ flight to liquidity, $M_h^0 > 0$, $D_h^0 < D^*$
  - $t = 1$: run on insolvent banks
  - households last in line in a run:
    consumption expenditure $= M_h^0$ (less than households who withdraw)
    losses on deposits
Outline

Introduction

Model

Monetary policy

Conclusions
Monetary policy during the recent crisis

- Temporary increase in money supply
  
  \[ M_0 = \bar{M} (1 + \mu) \]
  
  \[ M_1 = \bar{M} (1 + \mu) \]
  
  \[ M_2 = \bar{M} \]

- Increase money supply in two ways

  - Asset purchases
    
    - Central bank buys capital at \( t = 0 \)
    
    - Capital “sold” at \( t = 2 \)

  - Loans to banks
    
    - Central bank offers loans to private banks at \( t = 0 \)
    
    - Loan + return: repaid at \( t = 2 \)

- Profits/losses of central bank: lump-sum transfers to households
The effects of monetary injections

- **Small monetary injection:**
  
<table>
<thead>
<tr>
<th></th>
<th>in one bad equilibrium</th>
<th>in the other bad equilibrium</th>
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<tbody>
<tr>
<td>price capital</td>
<td>( \frac{dQ_0}{d\mu} )</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>price level</td>
<td>( \frac{dP_1}{d\mu} )</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>deposits</td>
<td>( \frac{dD_0^h}{d\mu} )</td>
<td>( \leq 0 )</td>
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- **Large monetary injection:** eliminate the bad equilibria
The effects of “small” monetary injections

Two forces:

1. Direct effect pushes endogenous variables closer to good equilibrium
   
   More money $\Rightarrow$ nominal prices $\uparrow$

2. Indirect effect exacerbates strategic complementarity and flight to liquidity

\[
\left( \frac{\text{nominal price of capital, } Q_0}{\text{capital}} \right) \uparrow \Rightarrow \left( \text{return on capital} \right) \downarrow \Rightarrow \left( \text{return on deposits, } r^D_2 \right) \downarrow \Rightarrow \left( \text{deposits, } D^h_0 \right) \downarrow
\]

Banks use return on capital to pay $r^D_2$
Conclusions

- Monetary, general equilibrium model of banking:
  - New channel produces bank runs, flight to liquidity, deflation
  - Systemic crisis

- Policy:
  - Monetary injection $\rightarrow$ interest rates $\downarrow$ $\rightarrow$ may worsen crisis
    Contrary to conventional wisdom (reducing rates, or spreads, is good)
  - How to eliminate bad equilibria: large monetary injections
  - Same forces robust to (slightly) richer specification

- Future work:
  - Companion paper: real model; equity injections
  - Take a dynamic version of the model to the data
Comparison with literature

- Panic, multiplicity of equilibria: Diamond-Dybvig (1983)
  Not applicable to monetary injections; exogenous returns

  *This paper:* Money (monetary policy)

  *Endogenous drop in asset prices (systemic crises)*

  Exogenous shocks to money demand (policy-invariant)

  *This paper:* Flight to liquidity is endogenous;

  *Monetary injections amplify flight to liquidity (in some cases)*

- Deflation and banking crises: Carapella (2012),
  Brunnermeier-Sannikov (2015) “I-Theory”: banks are intermediaries

  *This paper:* Banks insure against preference shocks, runs;

  *Deflation is not necessary to have bad equilibrium, companion paper: fire-sales, no deflation (multiple equilibria)*
Outline

Appendix
Monetary injections

- Money supply $\overline{M} (1 + \mu)$, $t = 0$:
  
  $\mu \overline{M} = B_0^{CB} + Q_0 K_0^{CB}$

  - monetary injection
  - loans to banks
  - asset purchases

- Budget constraint of banks at $t = 0$:
  
  $M^b_0 + Q_0 K^b_0 \leq D^b_0 + B^b_0 (M^b_{-1} + Q_0 K^b_{-1} - D^b_{-1})$

  - money
  - capital
  - deposits
  - loans from central bank
  - net worth

- Real profits (or losses) at $t = 2$:
  
  $T_2 = \frac{1}{P_2} \left[ Q_0 K_0^{CB} (1 + r^K_2) + \int B^b_0 (1 + \max \{ r^{CB}_2, r^b_2 \}) \, db - \mu \overline{M} \right]$

  (lump-sum transfers to households)
Bank problem

\[ \max C_2^b \quad \text{subject to:} \]

\[ t = 0: \quad \begin{array}{l}
M_0^b + Q_0 K_0^b \\
\text{money}\quad \text{capital (private info)}
\end{array} \leq \begin{array}{l}
D_0^b + \left( M_{-1}^b + Q_0 K_{-1}^b - D_{-1}^b \right) \\
\text{money}\quad \text{capital}\quad \text{deposits}
\end{array} \]

\[ \text{net worth (endowment and price } Q_0) \]

\[ t = 1: \quad \begin{array}{l}
W_1^b \leq M_0^b \\
\text{withdrawals}
\end{array} \quad \text{(feasibility, with "=" in equilibrium)} \]

\[ t = 2: \quad C_2^b = \max \left\{ 0; \frac{1}{P_2} \left[ Q_0 K_0^b (1 + r_2^K) - (D_0^b - W_1^b) (1 + r_2^D) \right] \right\} \]

\[ 1 + r_2^D: \text{promised return on deposits (taken as given by banks)} \]
Bank problem: solution

- \( t = 0 \): for every $ of deposits
  - fraction \( \kappa \) invested in money (to serve withdrawals at \( t = 1 \))
  - fraction \( 1 - \kappa \) invested in capital

- \( t = 2 \):
  - return = \( (1 - \kappa) \left(1 + r_2^K\right) \)
    return on capital
  - repayment to depositors = \( (1 - \kappa) \left(1 + r_2^D\right) \)
    promised return on deposits
  - profit per $ of deposits = \( (1 - \kappa) \left[\left(1 + r_2^K\right) - \left(1 + r_2^D\right)\right] \)

- In equilibrium: zero profits \( \Rightarrow r_2^D = r_2^K \)
banks indifferent among any amount of deposits
Actual return on deposits

\[ C_2^b = \max \left\{ 0; \frac{1}{P_2} \left[ Q_0 K_0^b (1 + r_2^K) - (D_0^b - W_1^b) (1 + r_2^D) \right] \right\} \]

Define actual return on deposits \( 1 + r_2^b \)

\[
1 + r_2^b = \begin{cases} 
1 + r_2^D & \text{if } C_2^b > 0 \quad (\text{assets} > \text{liabilities}) \\
\frac{Q_0 K_0^b (1 + r_2^K)}{D_0^b - W_1^b} & \leq 1 + r_2^D \quad \text{if } C_2^b = 0 \quad (\text{assets} \leq \text{liabilities})
\end{cases}
\]

Bad equilibrium: \( r_2^b < r_2^D \) if \( b = L \) (bank with endowment \( K^L \))
Household problem

\[
\max \ E \left[ \tilde{u} \left( C_1^h \right) \right] + \beta C_2^h \quad \text{subject to:}
\]

\[ t = 0 : \quad \begin{aligned}
M_0^h + & \quad D_0^h + Q_0 K_0^h \\
\text{money} & \quad \text{deposits,} \\
\text{at one bank} & \quad \text{capital}
\end{aligned} \leq \left( M_{-1}^h + D_{-1}^h + Q_0 K_{-1}^h \right) \quad \text{wealth (endowment and price } Q_0) \]

\[ t = 1 : \quad W_1^h \leq \begin{cases} 
D_0^h \\
0
\end{cases} \quad \text{if run & last in line} \]

\[ P_1 C_1^h \leq M_0^h + W_1^h \quad \text{(cash-in-advance)} \]

\[ t = 2 : \quad C_2^h = \frac{1}{P_2} \left[ \begin{aligned}
Q_0 K_0^h (1 + r_2^K) + (D_0^h - W_1^h) (1 + r_2^b) \\
\text{capital + return } r_2^K & \quad \text{deposits not withdrawn} \\
\text{+ actual return } r_2^b & \quad \text{unspent money}
\end{aligned} \right]
\]

\[ + \left( M_0^h + W_1^h - P_1 C_1^h \right) \]

\[ \#35 / 29 \]
Household problem

$$\max \mathbb{E} [\tilde{u}(C_1^h)] + \beta C_2^h$$

subject to:

$$t = 0 : \quad \underbrace{M_0^h}_{\text{money}} + \underbrace{D_0^h}_{\text{deposits, at one bank}} + Q_0 K_0^h \leq \underbrace{(M_{-1}^h + D_{-1}^h + Q_0 K_{-1}^h)}_{\text{wealth (endowment and price } Q_0)}$$

$$t = 1 : \quad \underbrace{W_1^h}_{\text{withdrawals}} \leq \begin{cases} D_0^h \\ 0 \text{ if run & last in line} \end{cases}$$

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$$t = 2 : \quad C_2^h = \frac{1}{P_2} \left[ \underbrace{Q_0 K_0^h (1 + r_2^K)}_{\text{capital + return } r_2^K} + \underbrace{(D_0^h - W_1^h) (1 + r_2^b)}_{\text{deposits not withdrawn} + \text{actual return } r_2^b} \leq \underbrace{\text{promised return } r_2^D}_{\text{unspent money}} \right]$$
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\max \ E \left[ \tilde{u} \left( C_1^h \right) \right] + \beta C_2^h \\
\text{subject to:}
\]

\[ t = 0 : \quad \begin{aligned}
& \underbrace{M_0^h} \quad + \underbrace{D_0^h} + \underbrace{Q_0 K_0^h} \\
& \quad \text{money} \quad \text{deposits, at one bank} \quad \text{capital} \\
& \leq \underbrace{M_{-1}^h + D_{-1}^h + Q_0 K_{-1}^h} \quad \text{wealth (endowment and price } Q_0) \\
\end{aligned} \]

\[ t = 1 : \quad \begin{aligned}
& \underbrace{W_1^h} \\
& \quad \text{withdrawals} \\
& \leq \begin{cases} 
D_0^h \\
0 & \text{if run & last in line}
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& \underbrace{Q_0 K_0^h (1 + r_2^K)} + \underbrace{(D_0^h - W_1^h) \left(1 + r_2^b\right)} \\
& \quad \text{capital + return } r_2^K \quad \text{deposits not withdrawn} \\
& \quad + \text{actual return } r_2^b \leq \text{promised return } r_2^d \\
& \underbrace{M_0^h + W_1^h - P_1 C_1^h} \quad \text{unspent money}
\end{aligned} \right] \]
Market clearing conditions

\[ t = 0 \]

Capital: \[ \int K_0^b \, db + \int K_0^h \, dh = \overline{K} \]

Money: \[ \int M_0^b \, db + \int M_0^h \, dh = \overline{M} \]

Deposits: \[ \int D_0^b \, db = \int D_0^h \, dh \]

\[ t = 1 \]

Goods: \[ \int C_1^h \, dh = A\overline{K} \]
Equilibrium: definition

- Prices $Q_0, P_1$ and promised return on deposits $r_2^D$
- Actual return on deposits $r_t^b$ for each $b$
- Households:
  - beliefs $(t = 0)$ about $r_2^b$ and probability of “run & last in line”
  - choices: $M_0^h, D_0^h, K_0^h$ $(t = 0)$ and $W_1^h, C_1^h$ $(t = 1)$
- Banks’ choices: $M_0^b, D_0^b, K_0^b$
- Set of households “last in line” during a run

Such that:

- **households have rational beliefs** and maximize utility
- banks maximize utility
- markets clear
Good equilibrium

- A good equilibrium exists (under some restrictions on banks’ endowment)

- Each bank pools the liquidity risk of its depositors
  - households: money $M^h_0 = 0$, deposits $D^h_0 = D^*$, capital $K^h_0 > 0$
  - banks: money $M^b_0 = \kappa D^*$
    (hold money to finance withdrawals by impatient households)
    all banks are solvent, pay promised return $(r^D_2)^* $; no runs
  - prices $Q_0 = Q^* , P_1 = P^*$

*The good equilibrium achieves the first-best*
Bad equilibria with deflation

PROPOSITION. If:

- utility patient households: \( u \left( C^h_1 \right) = 0 \)
- some other parameter restrictions hold

there exists either one or two bad equilibria, characterized by:

- \( t = 0 \):
  - households: money \( M^h_0 > 0 \), deposits \( D^h_0 < D^* \) (flight to liquidity)
  - price of capital: \( Q_0 < Q^* \) (good equilibrium)

- banks:
  - if \( K_{-1}^b = K^L \): insolvent (net worth \( t=0 < 0 \))
  - if \( K_{-1}^b = K^H \): solvent (net worth \( t=0 \geq 0 \))
Bad equilibria with deflation

- \( t = 1 \):
  - \( K^b_- \) becomes common knowledge, run on insolvent banks
  - withdrawals:

\[
W^h_1 = \begin{cases} 
D^h_0 & \text{if patient} \\
0 & \text{if impatient} 
\end{cases}
\]

- insolvent bank, run

\[
W^h_1 = \begin{cases} 
D^h_0 & \text{if first in line} \\
0 & \text{if last in line} 
\end{cases}
\]

- deflation: \( P_1 < \underbrace{P^*}_{\text{good equilibrium}} \) (money held by impatient is not spent)

- \( t = 2 \): actual return on deposits of insolvent banks

\[
r^b_2 < 0 \quad (\text{loose money if you don’t run})
\]

Two bad equilibria: qualitatively identical, but fundamentally different
PROPOSITION. Under some restrictions on endowment:

1. good equilibrium:

\[ K_0^b = K_{-1}^b \text{ for all } b \in \mathbb{B} , \quad K_0^h = K_{-1}^h \text{ for all } h \in \mathbb{H}. \]

2. bad equilibrium (with deflation or fire-sales):

\[ K_0^b < K_{-1}^b \text{ for all } b \in \mathbb{B} , \quad K_0^h > K_{-1}^h \text{ for all } h \in \mathbb{H}. \]

- Bad equilibrium with deflation: debt-deflation
  - *nominal* endowment of deposits \( D_{-1}^h \) and \( D_{-1}^b \)
  - wealth transferred from debtor (banks) to creditors (households)
Multiplicity of equilibria

Strategic complementarity in households’ deposits holdings

1. Fix deposits \( D_0 \)

2. Solve for other equilibrium objects (dropping household’s deposit FOC)

3. Using prices computed at (2): allow one agent to choose \( D_0^h \)
Multiplicity of bad equilibria

1. Fix actual return on deposits of insolvent bank $= \bar{r}_{2L}$ (Low endowment)

2. Solve for other equilibrium objects (dropping definition of $r_{2L}^L$)

3. Use objects from (2) and definition of $r_{2L}^L$: $r_{2L} = \text{function} (\bar{r}_{2L})$
The effects of monetary injections

1. Fix actual return on deposits of insolvent bank $= \bar{r}_2^L$

2. Solve for other equilibrium objects (dropping definition of $r_2^L$)

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The effects of monetary injections

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The effects of monetary injections

Fix actual return on deposits of insolvent bank $r^L_2$

- Money supply $\mu \uparrow$

- FOCs depend only on $r^L_2$ (fixed), and prices $Q_0$, $P_1$
  \[ \rightarrow Q_0 \text{ and } P_1 \text{ unchanged} \]

- $P_1$ unchanged and $\mu \uparrow \Rightarrow M_0^h \uparrow$, $D_0^h \downarrow$ (due to deposit multiplier)

- $\frac{\partial r^L_2}{\partial D_0^h} > 0$ (strategic complementarity)

  $\Rightarrow$ equilibrium best response $r^L_2 \downarrow$
Asset Purchases vs Loans to Banks

Asset purchases

Loans to banks

- Loans to banks
  - central bank bears some of the losses of insolvent banks
  - best response $\mu \uparrow$ (partially)
- Large monetary injections: eliminate bad equilibrium
Monetary policy and multiple bad equilibria

- Assume only one bad equilibrium exists without monetary injection
- With monetary injection: a second bad equilibrium might arise
How general is the result?

- Utility of impatient: piecewise-log-linear

\[ \bar{u}(C) = \begin{cases} 
\theta \log C & \text{if } C < \bar{C} \\
\theta \log \bar{C} + (C - \bar{C}) & \text{if } C \geq \bar{C}
\end{cases} \]

Utility of patients: \( u(C) = 0 \)

- No policy intervention: one or two bad equilibria (with deflation)

- Policy affects marginal utility (numerical simulations)
  - equilibrium best response “bent” in more complicated fashion
  - \( \frac{dD_0^h}{d\mu} < 0 \) quite robust (in some cases, for both bad equilibria)
  - \( \frac{dQ_0}{d\mu}, \frac{dP_1}{d\mu}, \frac{dr_2^L}{d\mu} < 0 \) (only for few parameterization)
  - even if prices \( \uparrow \): quantitative difference between equilibria
Monetary injections: bad equilibrium with fire-sale

PROPOSITION. If:

- a bad equilibrium with fire-sales exists
- central bank changes \( \overline{M} (1 + \mu) \) (either asset purchases or loans to banks)

\[
\frac{dQ_0}{d\mu} > 0 \quad \frac{dP_1}{d\mu} > 0
\]

\[
\frac{dr^L_2}{dK^L} < 0 \quad \frac{dD^h_0}{d\mu} < 0
\]

- Large monetary injection: eliminate bad equilibrium
Price of capital $Q_0$ and banks’ net worth

Net worth $N_0^b$

Bank $b$ with endowment $K^H$

Bank $b$ with endowment $K^L$

High $Q_0$, $N_0^b \geq 0$ for all $b$ (all banks are solvent)

Low $Q_0$, $N_0^b$

$$N_0^b \equiv \underbrace{K_{-1}^b Q_0}_{\text{value of capital}} + \underbrace{M_{-1}^b}_{\text{money}} - \underbrace{D_{-1}^b}_{\text{deposits}}$$