Global Price Shocks and International Monetary Coordination*

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When the central bank of an individual country is trying to contain domestic inflation it mostly takes as given global supply conditions—which determine oil prices, shipping costs, the prices of tradable intermediate goods, bottlenecks in global supply chains, and so on. However, the combined actions of all the world central banks matter for global demand, which matters for how global supply conditions translate into price and non-price pressures on global inflation. This suggests the possibility that, following a negative world supply shock, uncoordinated monetary policy may deliver excessively loose policy from a global perspective: an expansionary bias. We investigate this possibility in some simple models of a world economy with an integrated global commodity market and derive sufficient conditions for an expansionary bias to emerge.

1 Introduction

The inflation surge of 2021-2022 had two visible features: it had a global nature, as it played out in fairly similar way in many advanced and emerging economies, and it was strongly associated to symptoms of scarcity in the global supply of tradable inputs, as captured, for example, by measures of supply chain pressures, increases in shipping costs, and spikes in energy prices.

These two observations have led many economists to view inflation as driven by global forces, mostly outside the control of individual central banks. However, if we think of the world as a whole, we need to recognize that global scarcity is always relative, in other words, it is always a symptom that supply is insufficient given the level of global aggregate demand. And global aggregate demand is eventually the outcome of the combined choices of individual countries' fiscal and monetary authorities. Each of these national authorities may take as given the level of world commodity prices and bottlenecks in the global supply of tradable intermediates. But their aggregate decisions contribute to determine these prices and the tightness of these supply constraints.

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A natural question is whether, in these circumstances, uncoordinated policy responses around the world will produce an optimal global response, or whether lack of coordination will produce a sub-optimal outcome.

A widely held view, that goes back to the classic work of Canzoneri and Henderson (1991), is that in the fight against global inflation, uncoordinated policy choices lead to a contractionary bias, that is, to monetary policy that is excessively tight. The argument is that each domestic central bank has a stronger incentive to tighten because of international economic forces. Namely, monetary tightening in an open economy with a flexible exchange rate leads to a domestic currency appreciation, which reduces the domestic currency price of imports, and thus contributes to disinflation. However, when we look at the world as a whole, all currencies cannot appreciate at the same time, so, this is a zero-sum game that only leads to excessive tightening.

In this paper we revisit this conventional wisdom and show that, once we take into account the role of global commodity markets and other globally determined input prices, the conclusion above can be turned around, and lack of coordination can lead to excessively loose policy at the global level: an expansionary bias.

The first crucial ingredient in our exercise is an explicit model of a global commodity market in which the global price of the scarce commodity is determined. For ease of exposition we focus on a model with two inputs, a local input, labor, and a global input, oil. Labor is immobile and the labor market clears inside each country. Oil is traded in a frictionless world market and the price of oil is perfectly flexible.

The second crucial ingredient in our model is the assumption of some form of wage rigidity. Wage rigidity matters because it introduces a trade-off between inflation stabilization and output gap stabilization following a supply shock.

Once these two ingredients are present in a relatively standard new Keynesian framework, the possibility of excessively loose uncoordinated policy arises.

A simple intuition for our result can be derived from analyzing country-level and world-level Phillips curves. Because the world oil market is completely integrated, the relative scarcity of oil is only determined at the world level. Each local central bank faces a country Phillips curve which takes the world price of oil as given. The world Phillips curve can be steeper than the country-level Phillips curve, because at the world-level the oil price is endogenous and a higher level of world activity pushes up the equilibrium oil price. Local central banks perceive their choice set to be the flatter, country Phillips curve. A world planner, coordinating the actions of all central banks, would instead perceive the choice set to be the steeper world Phillips curve. In other words, the planner would internalize the effects of world demand on oil prices. This will lead the world planner to choose lower demand and lower inflation.

There are two loose ends in the intuition just provided. First, we need to show under what conditions the world Phillips curve is indeed steeper than the country one. Second, it is not obvious that the local central bank and the world planner have the same objective functions. In the paper, we provide a detailed analysis of both issues.

First, we show that the relative slope of the Phillips curves depends crucially on the elasticity of substitution between the local input (labor) and the global input (oil). When the elasticity of substitution is sufficiently the world Phillips curve is steeper than the country Phillips curve.

Second, we show that indeed the objective functions of the local and world planner are not the same. In particular, each local economy perceives that it can adjust its oil imports and perceives the cost of oil imports in terms of running a larger trade deficit. At the world level, however, oil trade must be balanced in equilibrium. This observation means that the conditions for an expansionary bias do not depend only on the slopes of the Phillips curves, but also on the country-level perceived cost of oil imports. In the paper, we provide both numerical examples and analytical sufficient conditions for an expansionary bias to emerge in the uncoordinated equilibrium, taking into account all motives for the country and world planner. A low elasticity of substitution between labor and oil is still the crucial parameter that generates an expansionary bias following a negative supply shock.

Our paper is related to a recent wave of papers that have revived attention to the classic problem of coordination in international monetary policy. In particular, in a paper strongly complementary to ours, Fornaro and Romei (2022) focus on the problem of insufficient global supply of tradable goods and identify conditions for a contractionary bias. Their emphasis is on the fact that an expansionary policy at the country level increases the supply of traded goods, easing inflation in other countries. Our emphasis is on the mirror problem arising on the input side, an expansionary policy at the country level makes global inputs scarcer, exacerbating global inflation. It is an open empirical question what is the relative force of these opposite channels and how it depends on the nature of the shocks affecting the world economy.

Another recent paper on coordination is Bianchi and Coulibaly (2024), who emphasize a different channel by which countries are tightly integrated, the financial markets. Their analysis shows how policy aimed at containing demand in one country have spillovers on other countries, by tightening world financial conditions. Our focus here is less on demand management and more on the characterization of the inflation-output tradeoff and we describe domestic policy directly in terms of the level of aggregate demand in the country, leaving aside how that is achieved by the country policy makers.

More generally, our paper is related to recent analyses of optimal monetary policy in open economies, including Itskhoki and Mukhin (2023). Their paper is focused on identifying the minimal number of instruments needed to achieve first-best efficiency and shows general conditions for so called "divine coincidence" results to hold in an international context. Here, we instead we focus on an environment in which the planner does not have sufficient tools to reach divine coincidence and in which, due to wage rigidity, central banks face a trade-off between inflation stabilization and output stabilization. We think this approach is useful to capture in a stylized way trade-offs faced by central bankers in the inflation fight of the last few years.

Paper organization In Sections 2 to 5 we develop our argument in the context of a simple model with tradable and non-tradable goods, and many small identical countries. This model help us identify our externality in a transparent way because the tradable good is homogenous and we can abstract from term-of-trade manipulation motives.

In Section 6, we consider a model where terms of trade considerations play a more prominent role and we show that the logic of our baseline model is still at work.

In Section 7 we explore models with asymmetries between countries (large and small, energy exporters and energy importers), to evaluate how these asymmetries affect the direction and size of the externality.

2 The World Economy

The world is made of a continuum of small, identical countries. Time is discrete and infinite. In each country, there is a representative consumer who consumes tradable goods and non-tradable goods, C_{Tt} and C_{Nt}, and supplies labor services L_t. The preferences of the consumer are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_{Tt}, C_{Nt}, L_t),$$

where

$$U(C_{Tt}, C_{Nt}, L_t) = C_{Tt} + u(C_{Nt}) - v(L_t).$$

The assumption of linear utility in tradable consumption simplifies the analysis but is not needed for our argument. For ease of notation, we omit adding a subscript to denote each individual country. We focus on symmetric equilibria, so this will not cause confusion.

The representative consumer in each country receives an endowment $\bar{Y}_T$ of tradable goods and an endowment $\bar{X}_t$ of a commodity needed for production. To be concrete, we refer to this commodity as "oil", but we have in mind a broader set of global inputs into domestic production.

The model features no explicit treatment of uncertainty. We study a deterministic economy, we characterize the steady state, and then we introduce a one-time, unexpected, temporary shock to the oil supply $\bar{X}_t$. The shock affects symmetrically all countries, so in equilibrium countries never trade oil in the world market. But the world market is open and competitive, so each country will take the oil price as given and, in contemplating deviations from equilibrium behavior, each country's central bank will assume that the oil price is unaffected by policy.

Non-tradable goods come in a continuum of varieties $[0, 1]$ that are aggregated according to a constant elasticity of substitution function with elasticity $\varepsilon > 1$,

$$C_{Nt} = \left(\int_0^1 \left(C_{Nt}\left(i\right)\right)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Each variety i is produced by a firm with production function

$$Y_t\left(i\right) = F\left(L_t\left(i\right), X_t\left(i\right)\right),$$

where $L_t\left(i\right)$ is the labor input and $X_t\left(i\right)$ the oil input. The fact that oil enters the production of non-tradables and not of tradables is a simplification due to the fact that we assumed a tradable endowment, but is not essential for our results. The function F has constant returns to scale. In the general analysis, we do not make specific functional form assumptions on F, but in numerical examples we use a simple CES function.

The government of each country pays firms a proportional subsidy $\frac{\varepsilon}{\varepsilon-1}$ that exactly offsets the monopolistic distortion, financed with a lump-sum tax on the representative consumer. Given that there are no terms-of-trade effects, this subsidy is set at the level that is optimal in steady state from the point of view of a benevolent fiscal authority in each country.

Each country has its own currency, there are floating exchange rates, and each country runs monetary policy independently. Prices in local currency for the tradable good and for oil are perfectly flexible and are denoted by P_{Tt} and P_{Xt}.

Non-tradable goods prices are subject to a nominal rigidity which works as follows. In each period, a fraction λ of firms in the non tradable sector set their price at the beginning of the period, before the aggregate shock is realized, and $1 - \lambda$ set their price after the aggregate shock is realized.

The nominal wage rate is W_t. For wages, we consider two extreme cases: flexible wages and perfectly rigid nominal wages.

The budget constraint of the representative agent in each country is

$$P_{Tt}C_{Tt} + P_{Nt}C_{Nt} + P_{Xt}X_t = P_{Tt}\bar{Y}_T + W_tL_t + P_{Xt}\bar{X}_t + T_t,$$

where T_t includes any profits from monopolistic firms and transfers from the government sector.

The law of one price applies to tradable consumption goods and oil, so that

$$P_{Tt} = E_tP^*_{Tt},$$
$$P_{Xt} = E_tP^*_{Xt},$$

where P^*_{Tt} and P^*_{Xt} are world prices of tradable goods and oil, denominated in the world unit of account, and E_t is the nominal exchange rate, the price of the world unit of account in terms of local currency. The world unit of account is a basket made of all countries' currencies, with equal weights.

We focus on symmetric equilibria where all currencies trade at the same exchange rate $E_t = 1$. But when domestic central banks choose policy, they consider off-equilibrium policy deviations that would cause the exchange rate of their individual country to be different from 1.

3 Equilibrium and Policy Regimes

Despite the dynamic setting, we can study the equilibrium period by period, so whenever there is no risk of confusion we drop time subscripts.

Suppose the world economy starts with all countries at a steady state with zero inflation, which, given our assumptions reaches the first best allocation. We normalize units so that in this steady state all prices, including the wage rate, are equal to 1.

Our objective is to study what happens when the world is hit by an unexpected, aggregate, temporary shock to the oil endowment $\bar{X}$.

Price setting Consider a monopolistic firm in the non-tradable sector, who is allowed to reset its price. The firm will set the price $\tilde{P}_N$ equal to its marginal cost, due to the presence of the subsidy, so

$$\tilde{P}_N = \mathcal{M}(W, P_X), \tag{1}$$

where $\mathcal{M}$ is the marginal cost function

$$\mathcal{M}(W, P_X) = \min \{WL + P_X X : F(L, X) = 1\}.$$

The firms who cannot adjust keep their price at the level set at the beginning of the period, when they were assigning zero probability to the aggregate shock. Therefore, their price is at the steady state value of 1.

Aggregating the prices of adjusters and non-adjusters, the price index of non-tradable goods is

$$P_N = \left(\lambda + (1-\lambda)\tilde{P}_N^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{2}$$

All firms use the same oil-to-labor ratio $L_t(i) / X_t(i)$ in equilibrium. Therefore, we can aggregate the input demands of all firms, and obtain the aggregate resource constraint

$$\Delta \cdot Y = F(L, X), \tag{3}$$

where the quantity index of non-tradable output Y is equal to real consumption

$$Y = C_N, \tag{4}$$

and the factor

$$\Delta = \lambda \left(\frac{1}{P_N}\right)^{-\varepsilon} + (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{-\varepsilon}.$$

reflects the distortionary effects of price dispersion.[1] When $\tilde{P}_N = P_N = 1$ there is zero inflation, all firms sell at the same price, and $\Delta = 1$. Whenever $\tilde{P}_N \neq 1$ we have $\Delta > 1$, as inputs are misallocated across firms selling at different prices. This is the traditional distortionary effect of inflation in sticky price models.

Notice that X, oil consumption, is in general different from $\bar{X}$. In a symmetric equilibrium they will be equal, but an individual country's central bank needs to evaluate what happens in equilibrium when X deviates from $\bar{X}$.

Equilibrium Conditions To complete our characterization of an equilibrium we need four more conditions.

[1]The demand for sticky goods is $(1/P_N)^{-\varepsilon} Y$ and the demand for flexible goods is $\left(\tilde{P}_N/P_N\right)^{\varepsilon} Y$, so total demand is

$$\left[\lambda \left(\frac{1}{P_N}\right)^{-\varepsilon} + (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{-\varepsilon}\right] Y.$$

The consumer demand for non-tradable goods is given by the optimality condition

$$u'(C_N) = \frac{P_N}{P_T}. \tag{5}$$

The firms' choice of the optimal factor mix of labor and oil gives

$$\frac{F_X(L,X)}{F_L(L,X)} = \frac{P_X}{W}. \tag{6}$$

The country consolidated budget constraint is

$$C_T = \bar{Y} - Q \cdot (X - \bar{X}), \tag{7}$$

where Q is the world relative price of oil, expressed in tradable goods

$$Q \equiv \frac{P_X^*}{P_T^*}.$$

Here, we are assuming that whenever a country buys oil in excess of its endowment, $X - \bar{X} > 0$, it pays for it by reducing the consumption of tradable goods in the same period. In general a country could borrow and pay partly in the future, but given linear preferences the timing of oil payments does not matter. It is easy to extend the model to the case of non-linear, separable utility in tradable goods, in which countries will optimally spread payments over all future periods. All our results below go through in that environment.

The law of one price implies that the relative price of oil in tradable goods is equal in all countries and equal to Q,

$$\frac{P_X}{P_T} = Q. \tag{8}$$

We look at two cases for wages. For most of the analysis, we assume rigid nominal wages

$$W = 1.$$

But we will also consider the case of flexible wages, satisfying the optimal labor supply condition

$$u'(Y)\frac{W}{P_N} = v'(L).$$

Equilibrium Definitions Let us focus on the rigid wage case. We have eight equilibrium conditions (1)-(8) and nine endogenous variables

$$(P_T, \tilde{P}_N, P_N, P_X, C_N, C_T, Y, L, X),$$

so the central bank has one degree of freedom. We describe the central bank's problem in terms of choosing Y and letting equations (1)-(8) determine all other variables. In most of the paper, we take this approach and let the central bank directly pick Y. The advantage of this approach is that the central bank is explicitly choosing a point on the Phillips curve, as we shall see shortly. However, it is perfectly equivalent to let the central bank choose the nominal interest rate and use the consumer Euler equation and an uncovered interest parity condition to derive endogenously the values of the exchange rate E and of output Y. In the text of the paper, sometimes we use this second approach to build intuition about the exchange rate channel.

Assume that the domestic central bank in each country is benevolent and maximizes the utility of the representative consumer.

We compare two regimes for international monetary policy coordination.

In the "no coordination" regime, each central banks acts independently and we study the Nash equilibrium of the game in which each central bank chooses Y, taking as given the choice of all other central banks.

In the "coordination" regime, a world planner chooses the same level of Y for all countries, to maximize the utility of the representative world consumer.

In the case of no coordination, since the countries are all identical, we focus on symmetric Nash equilibria.

Definition 1. *(No coordination equilibrium)* A symmetric Nash equilibrium is a vector

$$\left(P_T, \tilde{P}_N, P_N, P_X, C_N, C_T, Y, L, X\right)$$

that satisfies two conditions:

1. *Country's optimization.* It satisfies conditions (1)-(8) and, among all the vectors that satisfy these conditions, it maximizes the utility of the representative household, taking Q as given.

2. *Market clearing in the world oil market.* The price Q is such that $X = \bar{X}$.

In the case of coordination, we look at prices and quantities chosen by a social planner that internalizes the market clearing condition in the world oil market. Since all countries are identical, the global social planner chooses identical prices and quantities in all countries.

Definition 2. *(Coordination equilibrium)* An equilibrium with coordination is a vector

$$\left(Q, P_T, \tilde{P}_N, P_N, P_X, C_N, C_T, Y, L, X\right)$$

that maximizes the utility of the representative household, taking as given conditions (1)-(8), plus the condition $X = \bar{X}$.

The crucial difference is that Q is taken as given in the no coordination case, while it is a choice variable in the coordination case.

4 Phillips Curves

In this section we derive Phillips curves, i.e., equilibrium relations between real activity and inflation. First we do it at the country level, taking the world oil price as given. Next, we do it at the world level, taking the world oil supply as given and taking into account the endogenous determination of the world oil price. This allows us to show in what cases the Phillips curve perceived by an individual country is flatter than the world Phillips curve, a condition that will play a crucial role in the normative analysis.

4.1 The country Phillips curve

The world economy is in steady state, with constant endowments of tradable goods and oil and constant nominal prices (zero inflation). Consider the effect of a temporary negative shock to oil supply that reduces oil supply, in logs, by $\bar{x} < 0$. All lowercase variables denote log deviations from the initial steady state. Suppose for now that the effect of the shock is to increase the world oil price by $q > 0$. We linearize the model near its steady state and use π_N to denote non-tradable inflation in the period in which the shock occurs.[2] Derivations in Appendix A.1 show that the central bank of any individual country faces the country-level Phillips curve

$$\pi_N = \frac{(1-\lambda)\, s_X}{1-(1-\lambda)\, s_X} \left(\frac{1}{\epsilon} y + q \right), \tag{9}$$

where ϵ is the steady-state elasticity of substitution between tradable and non-tradable goods, and s_X is the steady-state share of oil in total costs. Consumption in the non-tradable sector is our measure of aggregate activity because tradable output is a fixed endowment. We focus on inflation in the non-tradable sector because tradable inflation does not have distortionary effects in our environment. It is easy to obtain a similar relation between y and inflation in the consumer price index (CPI), which includes both

[2] Given our assumptions and notation $\pi_N = p_N$.

tradable and non-tradable goods.[3]

From the point of view of the individual country, the oil supply shock $q > 0$ causes an upward shift of the Phillips curve, outside the control of the domestic central bank. The central bank can partly or completely offset the effect of the oil shock on inflation, by accepting a contraction in domestic activity $y < 0$.

Why does a contraction in domestic activity help contain inflation? Since nominal wages are fixed, the labor market channel is muted here. The only reason why monetary policy is effective is through the exchange rate channel. Contractionary policy leads to an appreciation, i.e., to lower E which makes oil cheaper in domestic currency, since $P_X = EP_X^*$. This reduces marginal costs and cools non-tradable inflation. The price of traded goods P_T also falls, and it falls in proportion to the nominal appreciation, due to $P_T = EP_T^*$. Because of wage rigidities and price stickiness, non-tradable prices fall by less than tradable prices, so non-traded goods become relatively more expensive. In other words, the nominal appreciation causes a real appreciation. This relative price effect reduces non-tradable output. Therefore, the currency appreciation achieves lower inflation at the expense of real activity.[4]

4.2 The world Phillips curve

Turning to the world Phillips curve, it is useful to first take an intermediate step and ask how does contractionary monetary policy in a given economy, all else equal, affect that economy's oil consumption. There are two opposing effects. On the one hand, by reducing domestic output, contractionary policy moves oil demand to the left. On the other hand, by making oil cheaper in domestic currency it increases the quantity of oil demanded, for a given oil demand curve. The first effect dominates if the following inequality holds

$$\eta s_L < \epsilon \left[1 - (1 - \lambda) s_X\right], \tag{10}$$

[3]Just combine (9) and the following relation between CPI inflation π and non-tradable inflation π_N:

$$\pi = \omega \frac{1}{\epsilon} y + \pi_N,$$

where ω is the steady state share of spending in tradable goods.

[4]The exchange rate elasticity of P_T is 1, while the exchange rate elasticity of P_N is $(1 - \lambda) s_X < 1$. The elasticity of the relative price P_N/P_T is then $(1 - \lambda) s_X - 1 < 0$, and that of non-tradable consumption $\epsilon (1 - (1 - \lambda) s_X) > 0$. Therefore, a 1% nominal appreciation gives a $(1 - \lambda) s_X \cdot 1\%$ reduction in non-tradable inflation at the cost of a $(1 - (1 - \lambda) s_X) \epsilon \cdot 1\%$ reduction in non-tradable consumption. This gives an exact intuition for the slope of the Phillips curve 9.

An equivalent (but algebraically more cumbersome) way of reaching this result is to derive the domestic interest rate increase necessary to obtain the same exchange rate appreciation and to use the consumer Euler equation to derive the effect on consumer spending.

where η is the steady-state elasticity of substitution between labor and oil and $s_L = 1 - s_X$ is the steady-state labor share in total costs. The inequality holds if the elasticity of substitution η is low enough, which seems the empirically relevant configuration.[5]

Let $X = \mathcal{X}(Y, Q)$ denote the country's oil consumption as a function of domestic output and the oil price. This mapping is derived from equations (1)-(8). The following result formalizes the result in the previous paragraph. Throughout the paper we use a subscript to denote the partial derivative of a function, e.g., $\mathcal{X}_Y$ is the same as $\partial \mathcal{X} / \partial Y$.

Proposition 1. *The elasticity of domestic oil consumption to domestic output Y, at the steady state allocation, is*

$$\frac{Y \mathcal{X}_Y}{\mathcal{X}} = 1 - \frac{\eta}{\epsilon} \frac{s_L}{1 - (1 - \lambda) s_X},$$

and is positive if and only if inequality (10) *holds.*

From an individual country perspective, under assumption (10), a contraction in Y leads to a contraction in the local oil demand X. If a single country chooses a lower Y, since all countries are small, the effect on world oil demand is negligible. From a world general equilibrium perspective though, if all countries contract spending at the same time, the price of oil needs to decrease to keep the oil market in equilibrium. Therefore, if (10) holds, there is an increasing equilibrium relation between world output and world oil prices. We formalize this observation in next proposition, which is proved in Appendix A.1.

Let $Q = \mathcal{Q}(\bar{X}, Y)$ denote the general equilibrium mapping that gives the world oil price as a function of oil supply $\bar{X}$ and world output Y, assuming all countries choose the same Y.

Proposition 2. *At the zero-inflation steady state, the function $\mathcal{Q}$ satisfies*

$$\frac{\bar{X} \mathcal{Q}_{\bar{X}}}{\mathcal{Q}} = \zeta_{\bar{X}} = -\frac{1}{\eta} \frac{1 - (1 - \lambda) s_X}{s_L},$$
$$\frac{Y \mathcal{Q}_Y}{\mathcal{Q}} = \zeta_Y = \frac{1}{\eta} \frac{1 - (1 - \lambda) s_X}{s_L} - \frac{1}{\epsilon}.$$

The coefficient $\zeta_{\bar{X}}$ is negative. The coefficient ζ_Y is positive iff inequality (10) *holds.*

It follows that when a global supply shock hits, the oil price response, conditional on a given response of world spending y, can be written as follows, in log-deviations from

[5] Following up on the intuition in footnote 4, the reduction in consumption caused by a 1% appreciation is $(1 - (1 - \lambda) s_X) \epsilon \cdot 1\%$, and it causes a proportional reduction in oil demand. The price effect of the appreciation, due to substitution between oil and labor, is $\eta s_L \cdot 1\%$. Therefore, the two terms on the right-hand side of the inequality capture exactly the two effects described in the text.

the initial steady state:

$$q = \zeta_{\bar{X}} \bar{x} + \zeta_Y y.$$

Substituting the last equation in the local Phillips curve (9), we obtain the world Phillips curve:

$$\pi_N = \frac{(1-\lambda)\, s_X}{1-(1-\lambda)\, s_X} \left[\left(\frac{1}{\epsilon} + \zeta_Y \right) y + \zeta_{\bar{X}} \bar{x} \right], \tag{11}$$

and we obtain the following result, which, once again, depends on our inequality (10).

Proposition 3. *The world Phillips curve is steeper than the local Phillips curve iff $\zeta_Y > 0$, that is, if inequality* (10) *holds.*

In other words, whenever the world oil price is an increasing function of real activity, the global Phillips curve is steeper. The result is intuitive: when (10) holds, by running a coordinated world monetary expansion, world oil demand increases, and we end up with a higher oil price in general equilibrium, which causes higher world inflation.

The results so far are illustrated in Figure 1, where we plot local and global Phillips curves in an example with $\zeta_Y > 0$. The figure shows two local Phillips curves, one (in blue) corresponding to a higher level of world demand, and thus a higher oil price, one (in yellow) corresponding to a lower level of world demand and a lower oil price. A higher oil price shows up like an exogenous shift in the country-level curve. When countries act in coordination, however, local and world demand move together, the oil price is endogenous, and we move along the steeper world Phillips curve shown in the figure (in red).

It is useful to clarify the role of the exchange rate in the comparison between local and global Phillips curves. The exchange rate channel was crucial in our intuition for the slope of the local Phillips curve. Since all countries cannot appreciate at once, one could ask: why the absence of the exchange rate channel at the world level is not making the global Phillips curve flatter than the local one? Recall that the exchange rate channel was captured by the term $\frac{1}{\epsilon} y$ in equation (9). Why is that term still present in equation (11)? The answer is that the term $\frac{1}{\epsilon} y$ in equation (11) captures a different but equivalent effect: it captures the fact that a global planner controls the world prices P_T^* and P_X^*. These prices are taken as given by the individual country, but not by a global planner. When we look at the price of oil in domestic currency

$$P_X = E \cdot P_X^*,$$

a local planner thinks it can change it by levering E, taking P_X^* as given, the global planner thinks it can change it by levering P_X^*, with $E = 1$. Given that E and P_X^* enter perfectly

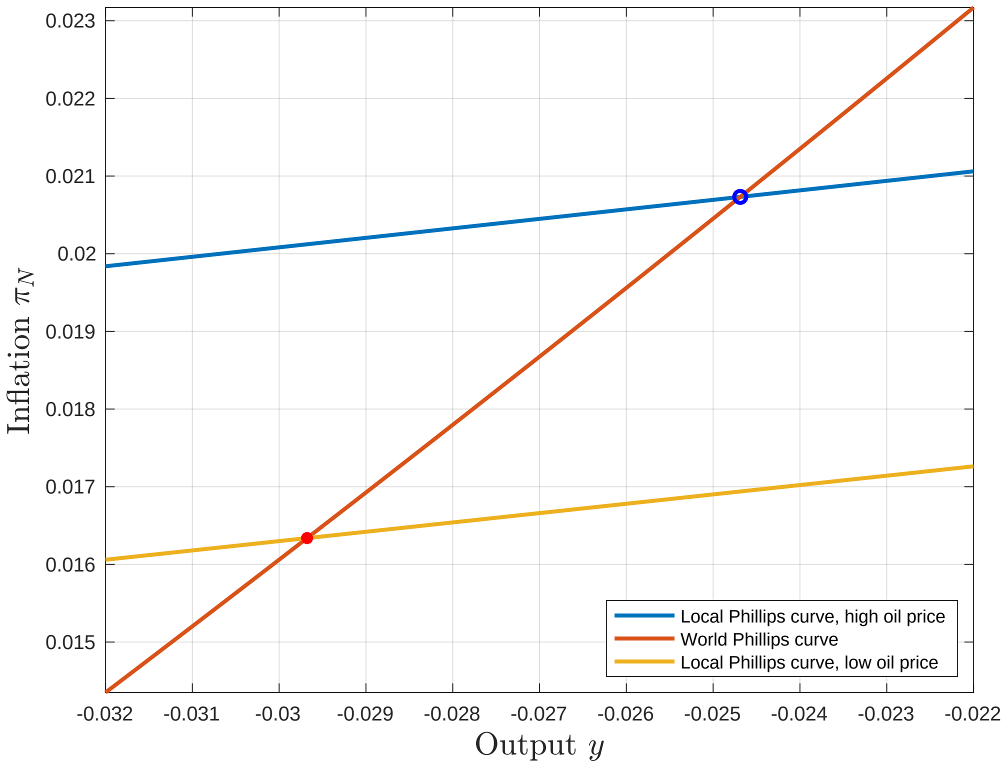

Figure 1: Phillips curves

symmetrically in this equation, the two effects are identical. When we go from the local Phillips curve to the global Phillips curve the term $\frac{1}{\epsilon}y$ that was capturing the exchange rate channel, now captures the nominal world oil price channel.

Inspecting Figure 1, one could be tempted to immediately conclude that in the uncoordinated Nash equilibrium countries tend to underestimate the disinflationary benefit of a contraction in spending, leading to excessive inflation from a world planner point of view. However, as we will see in the next section, welfare does not necessarily depend only on output and inflation, so it will take a bit more work to figure out under what conditions we get an expansionary bias.

5 Expansionary Bias and Gains from Coordination

In this section, we analyze the decision problem of the central bank of a given country and characterize equilibria with and without coordination. This will give us conditions for the uncoordinated equilibrium to display an excessive level of real activity and inflation.

5.1 No coordination

Using the equilibrium conditions (1)-(8), we can express the levels of domestic equilibrium variables as functions of the level of local activity Y and of the world oil price Q. We already introduced the mapping $X = \mathcal{X}(Y, Q)$ constructed in this way for oil consumption. In this section, we also use the following mappings for employment and the price of non-tradable goods:

$$L = \mathcal{L}(Y, Q), \qquad P_N = \mathcal{P}(Y, Q).$$

In a world with no coordination, the domestic central bank's best response, that is, the level of activity Y chosen for a given world oil price Q, is characterized compactly as the solution to the following problem:

$$\mathcal{Y}(Q) = \arg\max_{Y} U\left(\bar{Y}_T - Q \cdot \left(\mathcal{X}(Y, Q) - \bar{X}\right), Y, \mathcal{L}(Y, Q)\right). \tag{12}$$

Substituting this best response in the oil demand function, and imposing market clearing in the world oil market, gives

$$\mathcal{X}(\mathcal{Y}(Q), Q) = \bar{X}.$$

These two conditions fully characterize a Nash equilibrium.

To further analyze the tradeoff faced by the individual central bank, let us derive the first order condition of the maximization problem in (12). Omitting the functions' arguments for ease of notation, and, again, using subscripts to denote partial derivatives, we have

$$U_{C_N} + U_L \cdot \mathcal{L}_Y - Q \cdot U_{C_T} \cdot \mathcal{X}_Y = 0.$$

To interpret this expression and similar expressions to come, it is useful to recall that $U_L < 0$, capturing the marginal disutility of labor.

If we substitute for $\mathcal{L}_Y$ in the expression above, we obtain the following decomposition that is easier to interpret (the necessary steps are in the appendix):

$$\underbrace{\left(U_{C_N} + \frac{\Delta}{F_L} U_L\right)}_{\text{Labor wedge}} + \underbrace{U_L \frac{\Delta'(P_N) C_N}{F_L}}_{\text{Inflation distortion}} \cdot \mathcal{P}_Y + \underbrace{\left(-U_L \frac{F_X}{F_L} - Q \cdot U_{C_T}\right)}_{\text{Oil wedge}} \cdot \mathcal{X}_Y = 0. \tag{13}$$

The first two terms on the right-hand side of this equation are standard.

The first captures a notion of output gap. When the expression is positive, there is a negative output gap: increasing output is welfare improving as the marginal utility of consumption exceeds the marginal labor cost.

The second term captures the welfare cost of inflation, due to the distortionary effect

Δ. Notice that the factor Δ can be written solely in terms of P_N. With a slight abuse of notation, we define the function,[6]

$$\Delta(P_N) \equiv P_N^{\varepsilon}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right]. \tag{14}$$

The derivative of this function appears in (13). When $P_N > 1$ and inflation is positive we have $\Delta'(P_N) > 0$. Increasing output increases inflation distortions, given that the partial derivative $\mathcal{P}_Y$ is positive. This lowers welfare because $U_L < 0$.

The third term in (13) is novel and captures the gap between the domestic social benefits of higher oil consumption and the cost of importing oil. Suppose we are in the case discussed in Proposition (1), where an expansion causes oil imports to increase, $\mathcal{X}_Y > 0$. Oil imports are socially beneficial, as more oil reduces the need for labor in production in proportion to the marginal rate of substitution between oil and labor F_X/F_L. The cost of oil is given by the loss of tradable consumption, as the country needs to pay for the oil. The difference between these two expressions captures the net social benefit of stimulating oil demand through expansionary policy. We call this term the "oil wedge". The sign of this wedge can in general be positive or negative. In our examples below it is typically negative, so the central bank perceives an incentive to reduce domestic oil consumption.

Wage rigidity plays an important role in our analysis, as we shall see shortly. We already see its importance here: with flexible wages the oil wedge is always zero. With flexible wages the real wage in tradable goods W/P_T is equalized to the ratio $-U_L/U_{C_T}$ by workers' optimality. Since the ratio of the factor prices QP_T/W is equal to the factors' marginal rate of substitution, by firms' optimality, we get

$$-\frac{U_L}{U_{C_T}} = \frac{W}{P_T} = \frac{F_L}{F_X}Q,$$

which immediately implies a zero oil wedge.

By a similar line of reasoning, when wages are rigid, the oil wedge always has the opposite sign of the expression:

$$\frac{W}{P_T} + \frac{U_L}{U_{C_T}},$$

which is the gap between the real wage and its' flexible price counterpart. When this expression is positive firms perceive an over-priced labor input and tend to use too much

[6] Inverting condition (2) gives

$$\tilde{P}_N = \left(\frac{P_N^{1-\varepsilon} - \lambda}{1-\lambda}\right)^{\frac{1}{1-\varepsilon}}.$$

Substituting in the expression for Δ gives the function $\Delta(P_N)$.

oil from a social perspective. This makes the oil wedge negative.

5.2 Coordination and inefficiency

Let's turn to the case of coordinated macroeconomic policies.

The world social planner solves the same problem of an individual country's central bank, but with two differences: the quantity of oil used in the economy cannot be changed through imports, and the oil price is endogenously determined. This leads to the following problem

$$\max_{Y,Q} \left\{ U\left(\bar{Y}_T, Y, \mathcal{L}\left(Y, Q\right)\right) | Q = \mathcal{Q}\left(\bar{X}, Y\right) \right\}.$$

The fact that we have $C_T = \bar{Y}_T$ instead of $C_T = \bar{Y}_T - Q \cdot (X - \bar{X})$ captures the fact that the world as a whole cannot import oil.

Proceeding as in the case of the individual country, the planner first order condition is

$$U_{C_N} + U_L \cdot \mathcal{L}_Y + U_L \cdot \mathcal{L}_Q \cdot \mathcal{Q}_Y = 0$$

which, after substituting for $\mathcal{L}_Y$ and $\mathcal{L}_Q$ can be rewritten as follows (steps in the appendix):

$$\underbrace{\left(U_{C_N} + \frac{\Delta}{F_L} U_L\right)}_{\text{Labor wedge}} + \underbrace{U_L \frac{\Delta'\left(P_N\right) C_N}{F_L}}_{\text{Inflation distortion}} \cdot \left(\mathcal{P}_Y + \mathcal{P}_Q \mathcal{Q}_Y\right) = 0. \tag{15}$$

Comparing the optimality conditions (13) and (15) shows two differences between the world planner and the individual central bank.

First, individual countries fail to internalize the benefit of reducing world inflation through lower oil prices, as they ignore the expression

$$\mathcal{P}_Q \mathcal{Q}_Y$$

in the last term in parenthesis in (15). This expression captures exactly the difference between the slopes of the local and global Phillips curves, so the discussion in the previous section is immediately relevant. As shown in Proposition 3, condition (10) implies $\mathcal{P}_Q \mathcal{Q}_Y > 0$ and countries underestimate the social benefit of world-wide restrictive policy.

However, there is a second difference between the world planner and country optimization, captured in the third term in (13), the oil wedge, which is absent in (15). Individual countries can increase oil imports, and due to price rigidities, the domestic planner can perceive a net positive or net negative benefit of importing more oil.

In the examples below there is a negative oil wedge in equilibrium, which pushes the domestic central bank to be more contractionary in reaction to the oil shock. The logic of this negative wedge was discussed above: the real wage is above its natural level, so firms tend to import too much oil from a social welfare point of view, to substitute for labor. Contracting the economy is a way for the domestic central bank to partly correct for this bias, reducing oil imports.

Whether the Nash equilibrium displays excessively expansionary policy or the opposite, depends on the sign and magnitude of the two differences just discussed, as shown in the following proposition.

Proposition 4. *(Expansionary bias) In the no coordination equilibrium real output and inflation are inefficiently high iff*

$$\underbrace{-U_L \frac{\Delta'(P_N)\,Y}{F_L}}_{\text{Inflation distortion}} \cdot \underbrace{\mathcal{P}_Q \mathcal{Q}_Y}_{\text{Extra slope of the world P.C.}} > -\underbrace{\left(-U_L \frac{F_X}{F_L} - Q \cdot U_{C_T}\right)}_{\text{Oil wedge}} \cdot \mathcal{X}_Y. \tag{16}$$

When the inequality holds, the marginal welfare benefit of reducing inflation distortions at the world level is stronger in absolute value than the negative oil wedge perceived by each country central bank.[7] A global planner would recommend all countries to reduce output below the Nash equilibrium level.

The result above is stated in terms of equilibrium allocations and is derived without making specific assumptions on the functional forms of $u(C)$, $v(N)$ and $F(L, X)$. We will shortly make functional form assumptions to construct examples that satisfy condition (16) and derive sufficient conditions on the model parameters that imply it. Before we do so, there is one more general result that can be derived, showing special cases in which divine coincidence holds and the Nash equilibrium is efficient.

5.3 Flexible prices and flexible wages

The combination of price and wage rigidities is crucial to produce a need for international coordination in our model. If we allow for either flexible prices or flexible wages the following proposition shows that in equilibrium condition (16) becomes an equality and the Nash equilibrium is efficient.

Proposition 5. *(Divine coincidence, no coordination benefits.) With either flexible prices or flexible wages, the Nash equilibrium yields the first best allocation and coincides with the world planner optimum.*

[7] An immediate corollary is that if the oil wedge positive we always have an expansionary bias. However, in our examples, we typically find a negative oil wedge in equilibrium.

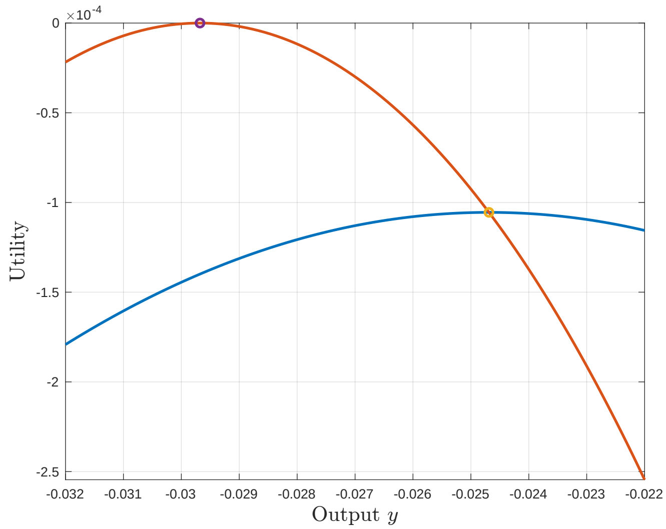

Figure 2: Nash equilibrium and world optimum

This result is independent of the slopes of the world and country Phillips curves. When divine coincidence holds, both Phillips curves intersect at the natural allocation with zero inflation, so the slope of the trade off is irrelevant.[8]

5.4 Examples and sufficient conditions

We now parametrize preferences and technology to construct examples of inefficiency and to derive sufficient conditions for expansionary bias.

The utility function is

$$U\left(C_T, C_N, L\right) = C_T + \ln C_N - \frac{\Psi}{1+\phi} L^{1+\phi},$$

and the production function is

$$F\left(L, X\right) = \left(a_L^{\frac{1}{\eta}} L^{1-\frac{1}{\eta}} + a_X^{\frac{1}{\eta}} X^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

with $a_L + a_X = 1$. The parameter Ψ in the utility function is set to $\Psi = a_L^{-\phi}$, and the endowment in steady state is $\bar{X} = a_X$. These are normalizations that ensure that all steady state equilibrium prices are 1.

[8]Notice also that this result is only meaningful if some nominal rigidity is present in the model, either on the price or wage side. If both prices and wages are perfectly flexible, monetary policy no longer has the power to affect the real allocation, so the problem of coordination is trivial as both planners' choice set is a point.

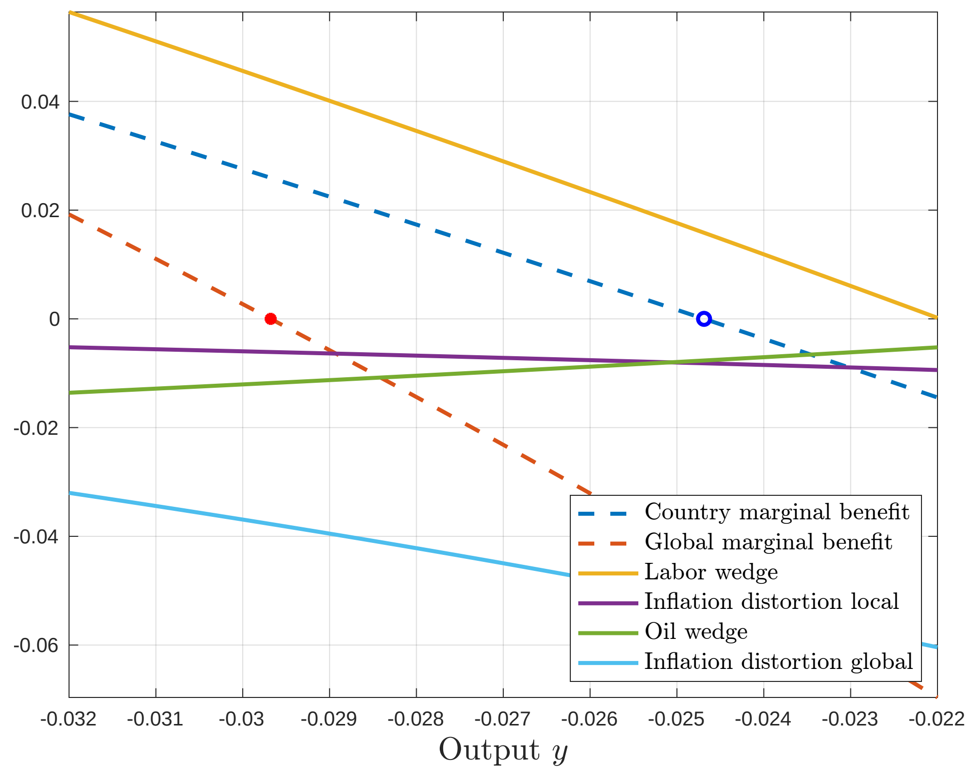

Figure 3: Decomposition of net marginal benefits for the country and world planner

First, consider a simple example with the following parameters:

$$a_X = 0.2, \quad \eta = 1/6, \quad \phi = 2, \quad \lambda = 1/2, \quad \varepsilon = 3.$$

In Figure (2) we show the objective function of the domestic central bank, given Q at its Nash equilibrium level (blue line), and social welfare at a symmetric allocation (red line), as functions of the output level y. The point where the two curves intersect gives the Nash equilibrium payoff. As the figure shows, social welfare is higher when output is lower. The world planner would prefer a lower level of aggregate activity. This is an example where condition 16 is satisfied.

To investigate what is behind this result, in Figure (3) we plot the first derivative of the two objective functions, and decompose them in the different components identified in equations (13) and (15). At the Nash equilibrium (the small blue circle), the country's central bank balances the cost of keeping output below its natural level (positive labor wedge, in yellow), with the benefit of reducing inflation (in purple), and of reducing an inefficiently high level of oil consumption (in green). The world planner chooses instead the allocation at the small red circle, with lower output y. The crucial difference is that the world planner internalizes a much larger cost of inflation distortions (in light blue).

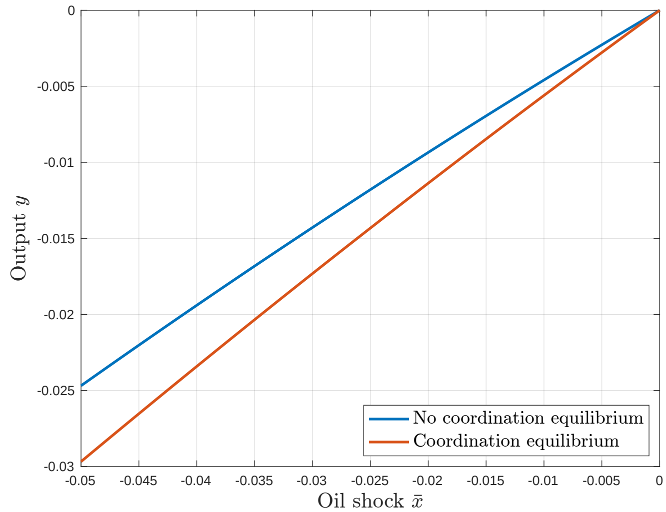

Figure 4: Comparative statics

Even though the planner does not take into account the oil distortion, the larger cost of inflation distortions is sufficient to make optimal output lower.

The outcomes in terms of inflation are marked in the Phillips curve in Figure (1), which is drawn for the same numerical example analyzed here. The two intersections of the world Phillips curve with the country Phillips curves correspond to the Nash equilibrium (on the right) and the world planner optimum (on the left). Therefore, in this example, world coordination delivers lower world inflation.

The example above is for a given oil shock $\bar{x}$. Figure 4 shows comparatice statics for different values of $\bar{x}$. With no shock ($\bar{x} = 0$) there is no inefficiency, that is, no gain from coordination. As the size of the shock grows (in absolute value) the distance between the two equilibria grows as well. The figure only looks at negative shocks, but the model behavior is symmetric, so following a positive oil shock we get a contractionary bias, i.e., countries expand too little and there is a too large deflation.

The following result offers sufficient conditions on the parameters that deliver an expansionary bias in equilibrium. A low enough elasticity of substitution η between labor and oil ensures that these conditions are satisfied. There are two inequalities involving η in the proposition, the first one is just a restatement of our familiar condition (10), which ensures the world Phillips curve is steeper than the country Phillips curve (with $\epsilon = 1$, due to log preferences). The second inequality is new and is needed to ensure that the oil wedge motive is weaker than the planner disinflation motive. Since inflationary distortions are larger when the goods are more substitutable, this second inequality is more easily satisfied when ε is large.

Proposition 6. *(Sufficient conditions for expansionary bias) Following a small negative supply* *shock $\bar{x} < 0$, output and inflation are inefficiently high in the no coordination equilibrium if the following conditions hold:*

$$\lambda > 0, a_X > 0,$$
$$\eta < 1 + \lambda \frac{a_X}{a_L}, \quad \eta < \varepsilon (1 - \lambda).$$

The proof is based on linearizing the Nash equilibrium conditions near the steady state and checking if the condition in Proposition 4 holds. The steps are in Appendix.

We conclude this section by showing by counterexample that the second condition in Proposition 6 is indeed needed. When the condition is not satisfied, even if the economy displays a steeper world Phillips curve, the planner may prefer a more expansionary stance. In Figure 5, we show an example where the oil wedge is large enough that countries, in Nash equilibrium, have a strong incentive to contract, in fact, too strong from the point of view of a global planner, who would select a larger level of output. The parameters needed to produce a result like this are equal to the ones above except for

$$\eta = 1.2, \quad , a_X = 2/3, \quad \varepsilon = 2.$$

In particular, we need to assume an unrealistically large oil share (which is equal to a_X in steady state), to get at the same time a steeper world Phillips curve and a contractionary bias. Of course, the model is too simplified to offer a full quantification. But our impression is that the result in Figure 5 is mostly a theoretical curiosum.[9]

6 A Model with Terms of Trade Manipulation

In this section, we study a variant of the model in which we have countries producing differentiated tradable goods. The model in this section is similar to the canonical model of Gali and Monacelli (2005) and introduces term-of-trade effects of monetary policy. For convenience, we will make assumptions of unit elasticity of substitution in most places, so this economy has many convenient properties of a Cole and Obstfeld economy.

Term-of-trade effects can in general impart a contractionary bias to monetary policy, as reducing domestic production has the beneficial side effect of making domestically produced goods more expensive in the world. However, as we will show, this is a bias that is present independently of shocks. The question we are asking in this paper is not whether on average monetary policy is too contractionary or too expansionary, but how

[9]This does not mean that other relevant forces may lead overall to a contractionary bias in a quantified model. Our statement here is just that this specific way of obtaining a contractionary bias seems quantitatively far fetched.

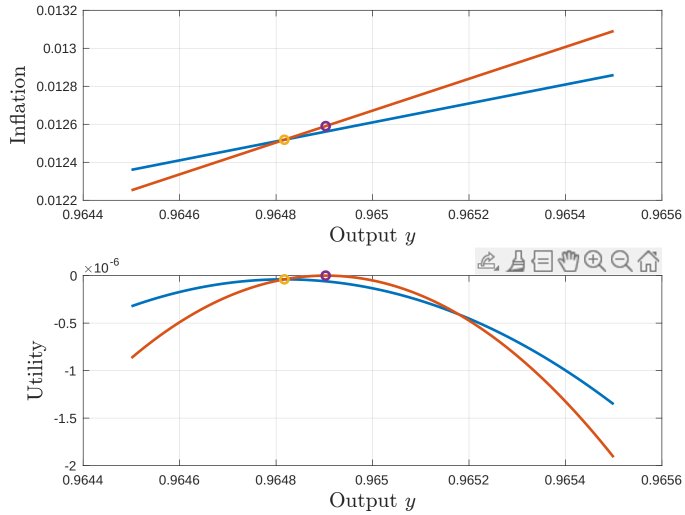

Figure 5: Nash equilibrium and world optimum in an example with a relatively large oil wedge

supply shocks affect this bias. We will show that by allowing for a sufficient set of policy instruments, we can separate these two questions and derive results that are parallel to the ones derived in the economy of Section 2. Namely, we show that it is possible to have an expansionary bias in response to a world supply shock, which comes from the same mechanism identified in the model of the previous sections: a failure to internalize the effects of world demand on the demand for scarce traded inputs.

6.1 Environment

To simplify, we consider a two-period version of the model.

The representative consumer in a typical country has preferences represented by the utility function

$$\ln C - \frac{\Psi}{1+\phi} L^{1+\phi} + Z$$

where C and L are consumption and labor supply in the first period, and Z is consumption of a uniform tradable good in the second period. The consumption index C is a unit-elasticity composite of the home good H and of imported foreign goods,

$$C = \left(\frac{C_H}{\omega}\right)^{\omega} \left(\frac{C_F}{1-\omega}\right)^{1-\omega},$$

where C_H is a constant-elasticity index of consumption of a continuum of varieties $j \in$

$[0,1]$ produced in the home economy,

$$C_H = \left(\int_0^1 \left(C_H \left(j \right) \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

C_F is a unit-elasticity index of consumption of the goods produced in all other countries $i \in [0,1]$

$$C_F = \exp \int_0^1 \ln C_{Fi} di,$$

and C_{Fi} is a consumption index of the varieties produced in country i imported by the domestic economy

$$C_{Fi} = \left(\int_0^1 \left(C_{Fi} \left(j \right) \right)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Each variety j of home good is produced according to the production function

$$Y\left(j\right) = F\left(L\left(j\right), X\left(j\right)\right)$$

where

$$F\left(L, X\right) \equiv \left(a_L^{\frac{1}{\eta}} L^{1-\frac{1}{\eta}} + a_X^{\frac{1}{\eta}} X^{1-\frac{1}{\eta}} \right)^{\frac{1}{1-\frac{1}{\eta}}},$$

and $a_L + a_X = 1$. Price setting is as in Section 2, with λ firms setting prices before shocks are realized, and $1-\lambda$ after. As in Section 2 we have fully sticky wages: the nominal wage $\bar{W}$ is set before shocks are realized at its steady-state flexible wage level but cannot adjust when shocks are realized.

The following assumptions are all as in Section 2. Each country has a fixed supply of oil, all countries have the same endowment $\bar{X}$,the world oil market is perfectly integrated, and the oil price is perfectly flexible. There are flexible exchange rates, the world unit of account is an equally-weighted basket of all currencies, and the exchange rate is the price of the world unit of account in domestic currency denoted by E.

Let R^* denote the world real interest rate in tradable goods, defined as the price at which one unit of the composite good F (which is the same for all countries) can be converted into a unit of good Z in the second period. The real rate R^* is determined by the collective choices of all central banks, as it depends on nominal interest rates in each currency and by the nominal prices of each local good in terms of the world unit of account. Since each country is small, each country takes R^* as given.

From expenditure minimization, the price index of the composite good F, expressed in the world unit of account, is

$$P^{*}=\exp\int_{0}^{1}\ln\left(\frac{P_{i}}{E_{i}}\right)di,$$

(just for this expression we index the exchange rate E_i explicitly with the country index i). The price index of good H and the domestic CPI are then respectively

$$P_{H}=\left(\int_{0}^{1}P_{H}\left(j\right)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}},$$
$$P=P_{H}^{\omega}\left(EP^{*}\right)^{1-\omega}.$$

The country's real output is defined as

$$Y=\frac{1}{P_{H}}\int_{0}^{1}P_{H}\left(j\right)Y\left(j\right)dj,$$

and the country-level intertemporal budget constraint is

$$Z=\frac{R^{*}}{EP^{*}}\left[P_{H}Y-PC-EP_{X}^{*}\left(X-\bar{X}\right)\right],$$

where P_X^* is the oil price expressed in the world unit of account and X is the country's oil consumption $X=\int_0^1 X\left(j\right)dj$.

6.2 Equilibrium

From intertemporal optimization, we have the Euler equation for consumption of the foreign good

$$\frac{1-\omega}{C_{F}}=R^{*}. \tag{17}$$

Optimal relative demand for home and foreign goods gives

$$\frac{C_{H}}{C_{F}}=\frac{\omega}{1-\omega}\left(\frac{P_{H}}{EP^{*}}\right)^{-1}. \tag{18}$$

Demand for the home good comes from home and foreign consumers and is equal to

$$Y=C_{H}+\left(\frac{P_{H}}{EP^{*}}\right)^{-1}C_{F}^{*}, \tag{19}$$

where the second term on the right-hand side is derived from the demand of all other countries, and C_F^* denotes the average demand for imported goods in all other countries.

The price setting condition for firms who set their price after the shock is

$$\tilde{P}_H = (1+\mu)\left(a_L \bar{W}^{1-\eta} + a_X \left(EP_X^*\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{20}$$

where $\bar{W}$ is the steady state wage rate, μ is the markup that satisfies

$$1+\mu = \frac{1}{1+\sigma}\frac{\varepsilon}{\varepsilon-1},$$

and σ is a proportional subsidy to producers set by the domestic fiscal authority.

The price of good H is then

$$P_H = \left(\lambda \bar{P}_H^{1-\varepsilon} + (1-\lambda)\tilde{P}_H^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \tag{21}$$

where $\bar{P}_H$ is the initial price level, set at its steady state level by firms who expect no shocks.

The distortion function $\Delta\left(P_H\right)$ has the same form as (14), except with P_H in place of P_N. The resource constraint is then

$$\Delta\left(P_H\right) \cdot Y = F\left(L, X\right). \tag{22}$$

Cost minimization by each domestic firm gives the relative demand for oil and labor

$$\frac{X}{L} = \frac{a_X}{a_L}\left(\frac{EP_X^*}{\bar{W}}\right)^{-\eta}. \tag{23}$$

Conditions (17)-(23) give seven equilibrium condition, which determine, up to one degree of freedom, the vector of eight equilibrium variables:

$$\left(E, \tilde{P}_H, P_H, C_H, C_F, Y, L, X\right). \tag{24}$$

As in the previous section, there is one degree of freedom and we assume that a benevolent domestic central bank selects the equilibrium vector.

We can now define a Nash equilibrium. The definition is similar to 1, but now four global variables need to be specified: R^*, P^*, P_X^*, C_F^*.

Definition 3. *(No coordination equilibrium)* A symmetric Nash equilibrium is given by a vector

$$\left(R^*, P^*, P_X^*, C_F^*, E, \tilde{P}_H, P_H, C_H, C_F, Y, L, X\right)$$

that satisfies two conditions:

1. *Country's optimization.* The vector of local prices and quantities (24) satisfies equilib-

rium conditions (1)-(8) and, among all vectors of prices and quantities that satisfy those conditions, it maximizes the utility of the representative household in country H, taking (R^*, P^*, P_X^*, C_F^*) as given.

2. *World equilibrium.* The exchange rate satisfies $E = 1$, the price of imports satisfies $P^* = P_H$, foreign demand satisfies $C_F^* = C_F$, and oil consumption satisfies $X = \bar{X}$.

The definition of an equilibrium with coordination is analogous to Definition 2, and we omit it.

Dealing with steady state bias

Take a steady state allocation and suppose that, as we did in Section 3, we set the subsidy σ to achieve a first-best steady-state allocation. Then the following first order condition is satisfied in equilibrium

$$\frac{1}{C} - \frac{1}{F_L}\Psi L^{\phi} = 0. \tag{25}$$

Consider now the marginal benefit of increasing consumption of the home good C_H for a domestic central bank, satisfying the local equilibrium conditions (1)-(8). The marginal benefit for the representative local consumer is now

$$\frac{\omega}{C_H} - \frac{1}{\omega}\frac{1}{F_L}\Psi L^{\phi} = \frac{1}{C} - \frac{1}{\omega}\frac{1}{F_L}\Psi L^{\phi} < 0,$$

where the equality follows because in equilibrium $C = C_H/\omega$, and the inequality follows from (25) and $\omega < 1$. The presence of $1/\omega$ in the last term has a simple interpretation: to stimulate domestic consumption the central bank needs to depreciate the real exchange rate $\frac{P_H}{EP^*}$, but this stimulates foreign spending on domestic goods, so the increase in labor effort needed to satisfy total demand is magnified relative to the increase in domestic consumption. In our simple Cole-Obstfeld economy the magnification is simply given by the factor $1/\omega$.[10] Therefore, if we start at a steady state that is first-best optimal for the global planner, increasing consumption is welfare reducing for a local planner. A domestic central bank will have an incentive to contract the domestic economy.

In other words, the local monetary authority has a deflationary bias in steady state. This bias arises from a terms-of-trade manipulation incentive: reducing domestic activity

[10] Combining equations (18) and (19) we have

$$\frac{Y}{C_H} = \frac{1}{\omega}.$$

improves the domestic terms of trade $\frac{P_H}{EP^*}$. This is a well-known source of deflationary bias in open economy new Keynesian models, but it is a bit of a nuisance in the present context, since we are trying to identify the effects of supply shocks, not the steady state inflationary bias of the two regimes.

Fortunately, there is a simple solution to avoid this tension. In the analysis that follows, when we compare the two regimes, no coordination and coordination, we assume that in the uncoordinated case local authorities choose *both* the response of monetary policy to shocks and the level of the fiscal subsidy σ in an uncoordinated way, while in the coordinated case, both are chosen by the world planner. This implies that the steady state is different in the two cases. In the coordination equilibrium the steady state satisfies (25), in the no coordination equilibrium the steady state satisfies

$$\frac{1}{C} - \frac{1}{\omega}\frac{1}{F_L}\Psi L^{\phi} = 0. \tag{26}$$

This condition comes from letting countries choose σ optimally ex ante, taking the σ chosen by all other countries as given, and assigning zero probability to the oil shock.

An immediate implication of this approach is that the benevolent central bank will choose zero inflation in steady state in both no coordination and coordination equilibrium.

6.3 Supply shocks and expansionary bias

We can now turn to the central question of our paper: is there an expansionary bias following a supply shock?

Let $P_H = \mathcal{P}(Y)$ and $\mathcal{X}(Y)$ denote the equilibrium mappings that give home consumption, the price of the home price good, and oil consumption as functions of home output Y. For economy of notation, we leave implicit the global variables R^*, P^*, P^*_X, C^* and assume they are at their equilibrium level.

The optimality condition for the domestic planner can then be written as follows, as shown in the appendix,

$$\underbrace{\left(\frac{\omega}{C_H} - \frac{1}{\omega}\frac{\Delta}{F_L}\Psi L^{\phi}\right)}_{\text{Labor wedge (local)}} - \underbrace{\Psi L^{\phi} \cdot \frac{\Delta'(P_H)\,Y}{F_L}}_{\text{Inflation distortion}}\mathcal{P}'(Y) + \underbrace{\left(\Psi L^{\phi}\frac{F_X}{F_L} - \frac{R^* P^*_X}{P^*}\right)}_{\text{Oil wedge}}\mathcal{X}'(Y) = 0.$$

Notice the similarity with the decomposition (13) in Section 5.

From the optimality of the world planner we get the optimality conditions

$$\underbrace{\left(\frac{1}{C}-\frac{\Delta}{F_L}\Psi L^{\phi}\right)}_{\text{Labor wedge (world)}}-\underbrace{\Psi L^{\phi}\frac{\Delta'\left(P\right)C}{F_L}}_{\text{Inflation distortion}}\cdot\mathcal{P}^{*\prime}\left(Y\right)=0$$

where the mapping $\mathcal{P}^{*}\left(Y\right)$ captures the relation between aggregate activity and the world CPI in a symmetric world equilibrium, and it includes by definition the endogenous effects of oil prices on inflation. Notice again the similarity with 15 in Section 5.

Both the domestic central bank and the world planner are trading off distortions in the aggregate labor allocation against inflation distortions. However, the domestic central bank takes into account the additional effects of its choices on domestic oil imports (see the presence of the oil wedge), while the world central bank internalizes the equilibrium in world oil markets and thus uses a different relation between aggregate activity and inflation (see the different inflationary effects in the second element of our decompositions).

We now present a numerical example based on the following parameters:

$$a_X=0.2,\quad \eta=1/6,\quad \omega=0.8,\quad \phi=2,\qquad \lambda=1/2,\quad \varepsilon=3.$$

The parameter Ψ is set equal to $a_L^{-\phi}$ and the oil supply in steady state is set to $\bar{X}=a_X$, so that in the first-best steady state all prices and consumption levels are equal to 1, while $L=a_L$ and $X=a_X$.

Figure 6 illustrates what happens to the oil price P_X^*, to CPI inflation P, and to employment in the two regimes considered, as a function of the oil price shock $\bar{x}$. All variables are expressed in log deviations from the steady state.[11]

By construction, in steady state both the local planner and the world planner choose to implement the zero inflation allocation. However, for negative oil shocks, both the local and world planner face a trade off between reducing employment below its efficient level and containing inflation. In the example, the inflation distortion perceived by the world planner is large enough, due to the endogeneity of the oil price, that the world planner chooses a larger contraction and lower world inflation.

Overall, the example shows a result that is qualitatively similar to the one analyzed in Section 5 for the model with tradable and non-tradable goods. In response to negative oil shocks a world planner would recommend a tighter monetary stance to contain inflation.

[11] The equilibrium allocations are computed numerically using the equilibrium relations and the planner's optimality conditions in their non-linear form.

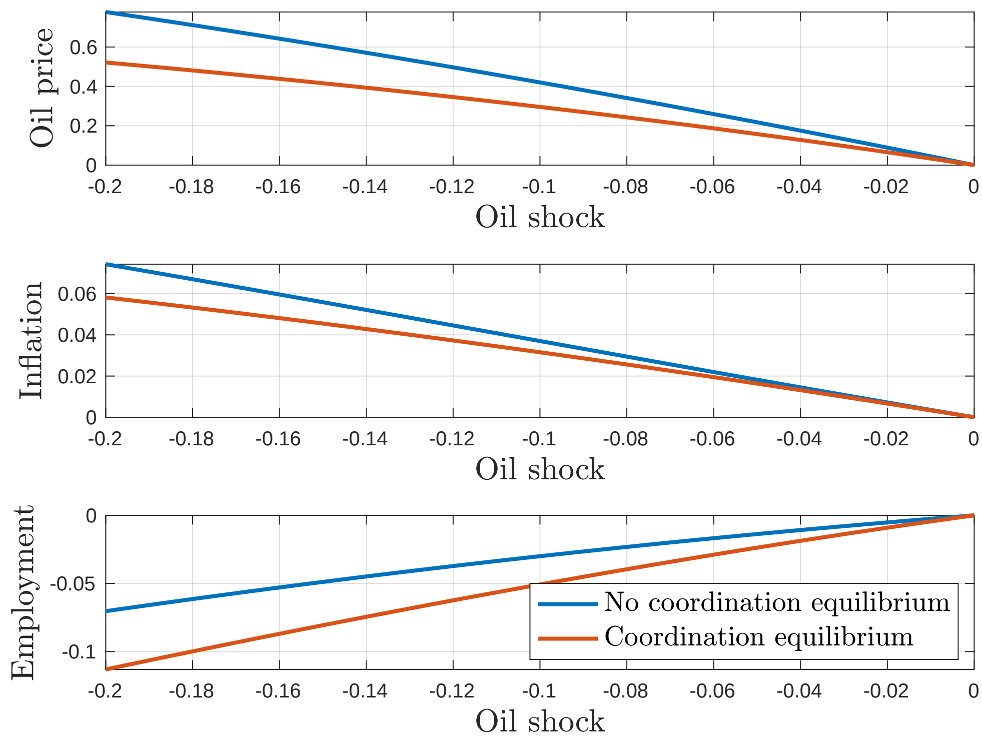

Figure 6: Comparative statics: no coordination and coordination for different oil shocks

7 Asymmetries Across Countries

[TBC]

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A Appendix

A.1 First-order properties of the equilibrium

The system of equations (1)-(8) characterizes an equilibrium for given Y and Q. In this section, we use differential notation to derive first-order properties of this system of equations. These properties will be used for two purposes:

1. Derive a linear approximation of the equilibrium near the initial steady state;
2. Derive the derivatives of the mappings $\mathcal{L}(Y, Q)$ and $\mathcal{X}(Y, Q)$ in general (away from the initial steady state) to analyze the efficiency properties of the equilibrium.

The Phillips curves in Section 4 are a side product of step 1.

For a generic variable X, we use the following notation

$$X = X_o e^x,$$

so X_o denotes the steady state level and x log deviations from the steady state.

The labor share and the oil share are

$$s_L = \frac{LF_L(L, X)}{F(L, X)}, \quad s_X = \frac{XF_X(L, X)}{F(L, X)}. \tag{27}$$

Combining (1) and (2) gives

$$P_N^{1-\varepsilon} = \lambda + (1-\lambda)(\mathcal{M}(1, P_X))^{1-\varepsilon}.$$

Differentiating both sides and rearranging we obtain

$$dp_N = h \cdot dp_X \tag{28}$$

where

$$h = (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{1-\varepsilon} \frac{P_X \mathcal{M}_{P_X}}{\mathcal{M}}.$$

Rearranging (2) we have

$$1 - \lambda P_N^{\varepsilon-1} = (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{1-\varepsilon},$$

and standard properties of the cost function imply

$$\frac{P_X \mathcal{M}_{P_X}}{\mathcal{M}} = s_X.$$

We can then rewrite h as a function of P_N and s_X:

$$h = \left(1 - \lambda P_N^{\varepsilon-1}\right) s_X. \tag{29}$$

Differentiating the resource constraint (3) gives

$$Y\Delta'\left(P_N\right)dP_N+\Delta\left(P_N\right)dY=F_L(L,X)dL+F_X(L,X)dX,$$

dividing, side by side, by (3) and rearranging gives

$$\delta\left(P_N\right)dp_N+dy=s_Ldl+s_Xdx, \tag{30}$$

where the function δ is the elasticity of the distortion function

$$\delta\left(P_N\right)=\frac{P_N\Delta'\left(P_N\right)}{\Delta\left(P_N\right)}. \tag{31}$$

Differentiating (5)-(8) gives

$$dy=-\epsilon\left(dp_N-dp_T\right), \tag{32}$$

$$dx-dl=-\eta\cdot dp_X, \tag{33}$$

$$dy=-\frac{Q\left(X-\bar{X}\right)}{C_T}dq-\frac{QX}{C_T}dx, \tag{34}$$

$$dp_X=dq+dp_T, \tag{35}$$

where ϵ is the elasticity of substitution between tradable and non-tradable goods in preferences

$$\epsilon=-\frac{C_Nu''(C_N)}{u'\left(C_N\right)},$$

and η is the elasticity of substitution between labor and oil in the production function

$$\eta=\frac{F_L\cdot F_X}{F_{XL}\cdot F}.$$

Substituting (32) and (35) in (28) gives

$$dp_N=h\left(dq+dp_N+\frac{1}{\epsilon}dy\right)$$

and solving for the non-tradable price

$$dp_N=\frac{h}{1-h}\frac{1}{\epsilon}dy+\frac{h}{1-h}dq. \tag{36}$$

Computing this expression at the steady state gives the country Phillips curve (9) in the text. Substituting back in (32) and (35) we get the other two prices,

$$\begin{aligned} dp_T &= \frac{1}{1-h}\frac{1}{\epsilon}dy + \frac{h}{1-h}dq, \\ dp_X &= \frac{1}{1-h}\frac{1}{\epsilon}dy + \frac{1}{1-h}dq. \end{aligned} \tag{37}$$

Rewrite (30) as

$$dl = \delta \cdot dp_N + dy - s_X (dx - dl),$$

and, using (33) and (37), we obtain

$$dx - dl = -\eta \left(\frac{1}{1-h}dq + \frac{1}{1-h}\frac{1}{\epsilon}dy \right)$$

and

$$dl = \delta \left(\frac{h}{1-h}dq + \frac{h}{1-h}\frac{1}{\epsilon}dy \right) + dy + \eta s_X \left(\frac{1}{1-h}dq + \frac{1}{1-h}\frac{1}{\epsilon}dc_N \right),$$

which, grouping terms, give

$$dl = \left(1 + \frac{1}{\epsilon}\frac{\eta s_X + \delta h}{1-h} \right) dy + \frac{\eta s_X + \delta h}{1-h}dq, \tag{38}$$

$$dx = \left(1 - \frac{1}{\epsilon}\frac{\eta s_L - \delta h}{1-h} \right) dy - \frac{\eta s_L - \delta h}{1-h}dq. \tag{39}$$

A.2 Proof of Proposition (1)

From the conditions (38)-(39) derived above, we immediately derive the following properties of the mappings $\mathcal{L}$ and $\mathcal{X}$:

$$\begin{aligned} \mathcal{L}_Y &= \left(1 + \frac{1}{\epsilon}\frac{\eta s_X + \delta h}{1-h} \right) \frac{L}{Y}, \\ \mathcal{L}_Q &= \frac{\eta s_X + \delta h}{1-h}\frac{L}{Q}, \\ \mathcal{X}_Y &= \left(1 - \frac{1}{\epsilon}\frac{\eta s_L - \delta h}{1-h} \right) \frac{X}{Y}, \\ \mathcal{X}_Q &= -\frac{\eta s_L - \delta h}{1-h}\frac{X}{Q}. \end{aligned}$$

A.3 Derivation of equations (13) and (15)

Totally differentiating the resource constraint

$$\Delta (P_N) Y = F (L, X),$$

and using the notation $P_N = \mathcal{P}(Y, Q)$ to denote the equilibrium mapping for the non-tradable price, yields

$$\Delta'(P_N) Y \mathcal{P}_Y + \Delta = F_L(L, X) \mathcal{L}_Y + F_X \mathcal{X}_Y,$$

which can be rearranged to give

$$\mathcal{L}_Y = \frac{1}{F_L} \Delta'(P_N) Y \mathcal{P}_Y + \frac{1}{F_L} \Delta - \frac{F_X}{F_L} \mathcal{X}_Y.$$

Similar steps, differentiating with respect to Q, give

$$\mathcal{L}_Q = \frac{1}{F_L} \Delta'(P_N) C_N \mathcal{P}_Q - \frac{F_X}{F_L} \mathcal{X}_Q.$$

Substituting for $\mathcal{L}_Y$ in the country optimality condition yields

$$U_{C_N} + \frac{1}{F_L} U_L \left(\Delta'(P_N) Y \mathcal{P}_Y + \Delta - F_X \mathcal{X}_Y\right) - Q \cdot U_{C_T} \cdot \mathcal{X}_Y = 0$$

which, after rearranging, yields (13) in the text.

Substituting for $\mathcal{L}_Y$ and $\mathcal{L}_Q$ in the planner optimality condition, yields

$$\begin{aligned} U_{C_N} + \frac{1}{F_L} U_L \left(\Delta'(P_N) Y \mathcal{P}_Y + \Delta - F_X \mathcal{X}_Y\right) + \\ + U_L \cdot \left(\frac{1}{F_L} \Delta'(P_N) C_N \mathcal{P}_Q - \frac{F_X}{F_L} \mathcal{X}_Q\right) \cdot \mathcal{Q}_Y = 0. \end{aligned}$$

Differentiating the market clearing condition in the oil market gives $\mathcal{X}_Y + \mathcal{X}_Q \mathcal{Q}_Y = 0$, and we obtain expression (15).

A.4 Proof of Proposition 2

The equilibrium oil price function $\mathcal{Q}(\bar{X}, Y)$ satisfies, by definition,

$$\mathcal{X}(Y, \mathcal{Q}(\bar{X}, Y)) = \bar{X}.$$

Differentiating both sides with respect to $\bar{X}$, using the expressions for $\mathcal{X}_Y$ and $\mathcal{X}_Q$ derived in A.1, and rearranging gives

$$\frac{\bar{X} \mathcal{Q}_X}{\mathcal{Q}} = -\frac{1-h}{\eta s_L - \delta h}.$$

Similarly, differentiating w.r.t. Y, and rearranging gives

$$\frac{Y \mathcal{Q}_Y}{\mathcal{Q}} = \frac{1-h}{\eta s_L - \delta h} - \frac{1}{\epsilon}.$$

At the steady state with zero inflation, we have $\delta = 0$ and $h = (1-\lambda)\, s_X$. Substituting in the expressions above we obtain the expressions for $\zeta_{\bar{X}}$ and ζ_Y in the proposition.

A.5 General equilibrium linearized

The following are linearized relations that hold in the global equilibrium, that is, taking into account the endogenous determination of Q. Differentiating the equilibrium condition

$$\mathcal{X}\,(Y, Q) = \bar{X}$$

yields

$$\left(1 - \frac{1}{\epsilon}\frac{\eta s_L - \delta h}{1-h}\right) dy - \frac{\eta s_L - \delta h}{1-h} dq = d\bar{x}$$

and rearranging we obtain

$$dq = \left(\frac{1-h}{\eta s_L - \delta h} - \frac{1}{\epsilon}\right) dy - \frac{1-h}{\eta s_L - \delta h} d\bar{x}.$$

At the zero inflation steady state this becomes

$$dq = \left(\frac{1}{\eta}\frac{s_L + \lambda s_X}{s_L} - \frac{1}{\epsilon}\right) dy - \frac{1}{\eta}\frac{s_L + \lambda s_X}{s_L} d\bar{x} \tag{40}$$

Also at the zero inflation steady state, equilibrium conditions imply:

$$dl = \frac{1}{s_L}\,(dy - s_X d\bar{x})\,, \tag{41}$$

$$dp_X = \frac{1}{\eta}\,(dl - d\bar{x}) = \frac{1}{\eta}\frac{1}{s_L}\,(dy - d\bar{x})\,,$$

$$dp_N = (1-\lambda)\, s_X dp_X = \frac{1}{\eta}\frac{s_X}{s_L}\,(dy - d\bar{x})\,. \tag{42}$$

A.6 Steady state, natural allocation, divine coincidence

We show that the first-best efficient allocation is a no coordination equilibrium, if nominal wages in the initial steady state are at their flexible wage level.

We assume that the economy starts in steady state, at the first best efficient allocation that satisfies

$$U_{C_N} F_L + U_L = 0, \tag{43}$$

and the world price of oil is

$$Q = \frac{U_{C_N}}{U_{C_T}} F_X. \tag{44}$$

Totally differentiating the resource constraint gives

$$Y\Delta'(P_N)\,dP_N + \Delta(P_N)\,dY = F_L(L,X)dL + F_X(L,X)\,dX.$$

Therefore, at an allocation with $P_N = 1$, $\Delta(P_N) = 1$, and $\Delta'(P_N) = 0$ we have

$$1 = F_L(L,X)\mathcal{L}_Y + F_X(L,X)\,\mathcal{X}_Y \tag{45}$$

and

$$0 = F_L(L,X)\mathcal{L}_Q + F_X(L,X)\,\mathcal{X}_Q. \tag{46}$$

Computing the optimality condition for the Nash equilibrium and using (43) and (44) gives

$$U_{C_N} + U_L\mathcal{L}_Y - QU_{C_T}\mathcal{X}_Y = U_{C_N} - U_{C_N}F_L\mathcal{L}_Y - U_{C_N}F_X\mathcal{X}_Y = 0,$$

where the last equality follows from (45).

B Iso-elastic preferences and technology

Preferences areThe cost function is

$$\left(a_L W^{1-\eta} + a_X P_X^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$

Marginal productivities are

$$F_L = a_L^{\frac{1}{\eta}} C_N^{\frac{1}{\eta}} L^{-\frac{1}{\eta}},$$
$$F_X = a_X^{\frac{1}{\eta}} C_N^{\frac{1}{\eta}} X^{-\frac{1}{\eta}}.$$

Marginal utilities

$$U_{C_N} = C_N^{-1}, \quad U_L = -a_L^{-\phi} L^{\phi}.$$

B.1 Steady state

In steady state we have quantities

$$L_o = a_L, \bar{X} = a_X, C_{No} = 1,$$

marginal utilities

$$U_{C_T o} = 1, U_{C_N o} = 1, U_{Lo} = -1,$$

and prices

$$P_{To} = 1, P_{No} = 1, W_o = 1, P_{Xo} = 1, Q = 1.$$

B.2 Algorithms

To compute Nash equilibrium. Given Δ, Y:

1. Solve for L from
$$\Delta \cdot Y = \left(a_L L^{1-\frac{1}{\eta}} + a_X X^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

2. Solve for $P_X, \tilde{P}_N, P_N, P_T, Q$ iteratively from
$$\begin{aligned} P_X &= \left(\frac{a_X}{a_L}\frac{L}{X}\right)^{\frac{1}{\eta}}, \\ \tilde{P}_N &= \left(a_L + a_X P_X^{1-\eta}\right)^{\frac{1}{1-\eta}}, \\ P_N &= \left(\lambda + (1-\lambda)\tilde{P}_N^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \\ P_T &= P_N C_N, \\ Q &= \frac{P_X}{P_T}. \end{aligned}$$

3. Solve for
$$\Delta^{+1} = P_N^{\varepsilon}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right]$$
check if Δ^{+1} close to Δ, set $\Delta = \Delta^{+1}$ and iterate steps 1-3 until convergence.

4. Compute s_L, s_X, h from
$$\begin{aligned} \frac{s_L}{1-s_L} &= \left(\frac{a_L}{a_X}\right)^{\frac{1}{\eta}}\left(\frac{L}{X}\right)^{1-\frac{1}{\eta}}, \\ s_X &= 1 - s_L, \\ h &= \left(1 - \lambda P_N^{\varepsilon-1}\right) s_X. \end{aligned}$$

5. Substitute in FOC of Nash and coordinated equilibrium, find zeros.

To compute allocation for single country given Y and Q:

1. Define a function that given P_N computes
$$\begin{aligned} P_T &= C_N P_N, \\ \tilde{P}_N &= \left(a_L + a_X (Q P_T)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \\ f &= \left(\lambda + (1-\lambda)\tilde{P}_N^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} - P_N, \end{aligned}$$
and find p_N such that $f = 0$.

2. Compute

$$\Delta = P_N^{\varepsilon}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon}-\lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right],$$

compute X/L ratio ξ

$$\xi = \frac{a_X}{a_L}\left(P_T Q\right)^{-\eta},$$

and find L from

$$\Delta \cdot Y = \left(a_L + a_X \xi^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}} L$$

and $X = \xi L$.

3. Compute utility

B.3 Conditions for inefficiency

The country optimality condition can be written as

$$1 - a_L^{-\phi} L^{1+\phi}\left(1 + \frac{\eta s_X + \delta h}{1-h}\right) - \left(1 - \frac{\eta s_L - \delta h}{1-h}\right) QX = 0,$$

or

$$\left(1 - a_L^{-\phi} L^{1+\phi} - QX\right) - \frac{\eta s_L - \delta h}{1-h}\left(\frac{\eta s_X + \delta h}{\eta s_L - \delta h} a_L^{-\phi} L^{1+\phi} - QX\right) = 0. \tag{47}$$

The planner optimality condition can be written as

$$1 - a_L^{-\phi} L^{1+\phi}\left(1 + \frac{\eta s_X + \delta h}{1-h}\right) - a_L^{-\phi} L^{1+\phi} \frac{\eta s_X + \delta h}{\eta s_L - \delta h}\left(1 - \frac{\eta s_L - \delta h}{1-h}\right) = 0.$$

The expression

$$\mathcal{X}_Y\left(U_L \frac{\mathcal{L}_Q}{\mathcal{X}_Q} - U_{C_T} Q\right)$$

has the same sign as

$$\left(1 - \frac{\eta s_L - \delta h}{1-h}\right)\left(\frac{\eta s_X + \delta h}{\eta s_L - \delta h} a_L^{-\phi} L^{1+\phi} - QX\right)$$

and is positive (excess inflation) if

$$1 - \frac{\eta s_L - \delta h}{1-h} > 0$$

and

$$\frac{\eta s_X + \delta h}{\eta s_L - \delta h} a_L^{-\phi} L^{1+\phi} > QX. \tag{48}$$

Our objective is to linearize (47) w.r.t. Y and $\bar{X}$, at the steady state, using equilibrium conditions and conditions (40)-(42), and to check when (48) holds. We proceed in steps.

Linearizing the expression

$$1 - a_L^{-\phi} L^{1+\phi} - QX$$

gives

$$-\left(1+\phi\right) a_L l - a_X \left(q + \bar{x}\right)$$

which becomes

$$-\left(1+\phi\right)\left(y - a_X \bar{x}\right) - a_X \left[\left(\frac{1}{\eta}\frac{a_L + \lambda a_X}{a_L} - 1\right) y - \frac{1}{\eta}\frac{a_L + \lambda a_X}{a_L}\bar{x} + \bar{x}\right]$$

or

$$-\left(1+\phi\right)\left(y - a_X \bar{x}\right) - a_X \left(\frac{1}{\eta} - 1 + \frac{\lambda}{\eta}\frac{a_X}{a_L}\right)\left(y - \bar{x}\right).$$

The next step is to linearize the expression

$$\frac{\eta s_L - \delta h}{1-h}\left(a_L^{-\phi} L^{1+\phi}\frac{\eta s_X + \delta h}{\eta s_L - \delta h} - Q\bar{X}\right).$$

Since the term in parenthesis is zero at the steady state, this is equivalent to linearizing

$$\eta \frac{a_L}{a_L + \lambda a_X}\left(a_L^{-\phi} L^{1+\phi}\frac{\eta s_X + \delta h}{\eta s_L - \delta h} - Q\bar{X}\right).$$

To do so we derive some preliminary results. From

$$\frac{s_L}{s_X} = \frac{F_L L}{F_X X} = \frac{a_L}{a_X}\left(\frac{L}{X}\right)^{1-\frac{1}{\eta}}$$

we get

$$\frac{ds_L}{s_L} - \frac{ds_X}{s_X} = \left(1 - \frac{1}{\eta}\right)\left(l - x\right)$$

which combined with

$$y - x = s_L \left(l - x\right)$$

gives

$$\frac{ds_L}{s_L} - \frac{ds_X}{s_X} = \left(1 - \frac{1}{\eta}\right)\frac{1}{a_L}\left(y - \bar{x}\right)$$

Differentiating twice the distortion function (14) gives

$$\Delta'\left(P_N\right) = \varepsilon P_N^{\varepsilon-1}\left[\lambda + \left(1-\lambda\right)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right] +$$
$$-\varepsilon\left(1-\lambda\right)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{1}{1-\varepsilon}}$$

and

$$\Delta''(P_N) = \varepsilon(\varepsilon-1) P_N^{\varepsilon-2}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon}-\lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right] +$$
$$-\varepsilon^2 P_N^{\varepsilon-1}(1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon}-\lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}-1} P_N^{-\varepsilon} +$$
$$+\varepsilon(1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon}-\lambda\right)^{-\frac{1}{1-\varepsilon}-1} P_N^{-\varepsilon}$$

which computed at the steady state give

$$\Delta'(1) = 0, \quad \Delta''(1) = \varepsilon\frac{\lambda}{1-\lambda}.$$

From the definition of δ and $\Delta'(P_N) = 0$ we have

$$\left[\frac{d\delta}{dP_N}\right]_{P_N=1} = \left[P_N\frac{\Delta''(P_N)}{\Delta(P_N)} + \frac{\Delta'(P_N)}{\Delta(P_N)} - P_N\left(\frac{\Delta'(P_N)}{\Delta(P_N)}\right)^2\right]_{P_N=1} = \varepsilon\frac{\lambda}{1-\lambda}$$

and, using (42), we can write

$$d\delta = \varepsilon\frac{\lambda}{1-\lambda}p_N = \varepsilon\frac{\lambda}{\eta}\frac{a_X}{a_L}(y-\bar{x}). \tag{49}$$

Using the results just derived, we can linearize the expression

$$\frac{\eta s_X + \delta h}{\eta s_L - \delta h}$$

as follows

$$\frac{a_X}{a_L}\left(\frac{ds_X}{s_X} + \frac{1-\lambda}{\eta}d\delta - \frac{ds_L}{s_L} + \frac{1-\lambda}{\eta}\frac{a_X}{a_L}d\delta\right) = \frac{a_X}{a_L}\left(\frac{ds_X}{s_X} - \frac{ds_L}{s_L} + \frac{1-\lambda}{\eta}\frac{1}{a_L}d\delta\right) =$$
$$= \frac{a_X}{a_L^2}\left(\frac{1}{\eta} - 1 + \varepsilon\frac{\lambda(1-\lambda)}{\eta^2}\frac{a_X}{a_L}\right)(y-\bar{x}). \tag{50}$$

Linearizing $Q\bar{X}$ gives

$$a_X(q+\bar{x}) = a_X\left(\frac{1}{\eta} - 1 + \frac{\lambda}{\eta}\frac{a_X}{a_L}\right)(y-\bar{x}).$$

Combining all previous results, the linearized first order condition of the individual country is

$$
\begin{aligned}
-\left(1+\phi\right)\left(y-a_X\bar{x}\right)-a_X\left(\frac{1}{\eta}-1+\frac{\lambda}{\eta}\frac{a_X}{a_L}\right)\left(y-\bar{x}\right)+ \\
-\frac{\eta a_X}{a_L+\lambda a_X}\left(1+\phi\right)\left(y-a_X\bar{x}\right)+ \\
-\frac{\eta a_X}{a_L+\lambda a_X}\left(\frac{1}{\eta}-1+\varepsilon\frac{\lambda\left(1-\lambda\right)}{\eta^2}\frac{a_X}{a_L}\right)\left(y-\bar{x}\right)+ \\
+\frac{\eta a_X}{a_L+\lambda a_X}a_L\left(\frac{1}{\eta}\frac{a_L+\lambda a_X}{a_L}-1\right)\left(y-\bar{x}\right)=&0,
\end{aligned}
$$

or

$$
\begin{aligned}
\left(1+\phi\right)\left(c_N-a_X\bar{x}\right)+a_X\left(\frac{1}{\eta}\frac{a_L+\lambda a_X}{a_L}-1\right)\left(y-\bar{x}\right)+ \\
+\frac{\eta a_X}{a_L+\lambda a_X}\left\{\left(1+\phi\right)\left(y-a_X\bar{x}\right)+\left[\left(\frac{1}{\eta}-1\right)a_X-\frac{\lambda}{\eta}a_X+\varepsilon\frac{\lambda\left(1-\lambda\right)}{\eta^2}\frac{a_X}{a_L}\right]\left(y-\bar{x}\right)\right\}=&0.
\end{aligned}
$$

Write this equation compactly as

$$
B_1\left(y-a_X\bar{x}\right)+B_2\left(y-\bar{x}\right)+\xi\left(B_1\left(y-a_X\bar{x}\right)+B_3\left(y-\bar{x}\right)\right)=0 \tag{51}
$$

where

$$
\xi=\frac{\eta a_X}{a_L+\lambda a_X}
$$

and

$$
\begin{aligned}
B_1 &= 1+\phi, \\
B_2 &= a_X\left(\frac{1}{\eta}-1\right)+\frac{\lambda}{\eta}\frac{a_X^2}{a_L}, \\
B_3 &= a_X\left(\frac{1}{\eta}-1\right)+\varepsilon\frac{\lambda\left(1-\lambda\right)}{\eta^2}\frac{a_X}{a_L}-\frac{\lambda}{\eta}a_X.
\end{aligned}
$$

We now analyze under what conditions $\bar{x}<0$ implies

$$
B_1\left(y-a_X\bar{x}\right)+B_2\left(y-\bar{x}\right)<0, \tag{52}
$$

since this inequality implies a Nash equilibrium in which

$$
1-a_L^{-\phi}L^{1+\phi}-QX>0
$$

which, from 47, implies

$$
\frac{\eta s_X+\delta h}{\eta s_L-\delta h}a_L^{-\phi}L^{1+\phi}>QX.
$$

Proof. Solving (51) gives

$$c_N = \frac{a_X\left(1+\xi\right)B_1 + B_2 + \xi B_3}{\left(1+\xi\right)B_1 + B_2 + \xi B_3}\bar{x}.$$

Condition (52) becomes

$$\left(\left(B_1+B_2\right)\frac{a_X\left(1+\xi\right)B_1 + B_2 + \xi B_3}{\left(1+\xi\right)B_1 + B_2 + \xi B_3} - \left(a_X B_1 + B_2\right)\right)\bar{x} < 0$$

and, with $\bar{x} < 0$, we need

$$\left(B_1+B_2\right)\frac{a_X\left(1+\xi\right)B_1 + B_2 + \xi B_3}{\left(1+\xi\right)B_1 + B_2 + \xi B_3} > a_X B_1 + B_2. \tag{53}$$

The denominator on the left-hand side is positive because it can be written as

$$\left(1+\frac{\eta a_X}{a_L+\lambda a_X}\right)\left(1+\phi+a_X\frac{1-\eta}{\eta}\right) + \frac{\lambda}{\eta}\frac{a_X}{a_L} + \frac{\eta a_X}{a_L+\lambda a_X}\left(\varepsilon\lambda\left(1-\lambda\right)\frac{1}{\eta^2}\frac{a_X}{a_L} - \frac{\lambda}{\eta}a_X\right),$$

and

$$1+\phi+a_X\frac{1-\eta}{\eta} = a_L+\phi+a_X\frac{1}{\eta} > 0$$

and the last term is parenthesis is positive if $\eta < \varepsilon\left(1-\lambda\right)$. Therefore, we can rearrange (53) and obtain

$$\left(a_X B_1 + B_3\right)\left(B_1+B_2\right) > \left(a_X B_1 + B_2\right)\left(B_1+B_3\right)$$

which is equivalent to

$$B_3 > B_2,$$

or

$$\varepsilon\lambda\left(1-\lambda\right)\frac{1}{\eta^2}\frac{a_X}{a_L} - \frac{\lambda}{\eta}a_X > \frac{\lambda}{\eta}\frac{a_X^2}{a_L},$$

As long as $\lambda > 0, a_X > 0$ this is equivalent towhich is equivalent to

$$\varepsilon\frac{1-\lambda}{\eta} > 1.$$

□

B.4 Numerical example with inefficiently low inflation

Set parameter values to

$$\eta = 1.2, \varepsilon = 2$$
$$\frac{a_X}{a_L} = 2, \lambda = 0.5$$

so that world Phillips curve is steeper:

$$\eta = 1.2 < 2 = 1 + \lambda \frac{a_X}{a_L}$$

but

$$\varepsilon \frac{1-\lambda}{\eta} = 2\frac{0.5}{1.2} < 1.$$

B.5 Decomposition in example

From

$$\underbrace{\left(U_{C_N} + \frac{\Delta}{F_L} U_L\right)}_{\text{Labor wedge}} + \underbrace{U_L \frac{\Delta'(P_N)\, C_N}{F_L}}_{\text{Inflation distortion}} \cdot \frac{\partial \mathcal{P}_N}{\partial C_N} + \underbrace{\left(-U_L \frac{F_X}{F_L} - Q \cdot U_{C_T}\right)}_{\text{Oil imports wedge}} \cdot \frac{\partial \mathcal{X}}{\partial C_N} = 0$$

The first term is

$$U_{C_N} + \frac{\Delta}{F_L} U_L = \frac{1}{C_N} + \frac{U_L L}{C_N} \frac{1}{s_L}.$$

Using

$$\frac{\partial \mathcal{P}_N}{\partial C_N} = \frac{P_N}{C_N} \frac{h}{1-h}$$

the second term is

$$U_L \frac{\Delta'(P_N)\, C_N}{F_L} \cdot \frac{\partial \mathcal{P}_N}{\partial C_N} = \frac{U_L L}{C_N} \frac{1}{s_L} \frac{\delta h}{1-h}.$$

Using

$$\frac{\partial \mathcal{X}}{\partial C_N} = \frac{X}{C_N} \left(1 - \frac{\eta s_L - \delta h}{1-h}\right)$$

the third term is

$$-\left(U_L \frac{F_X}{F_L} + Q \cdot U_{C_T}\right) \cdot \frac{\partial \mathcal{X}}{\partial C_N} = -\left(\frac{U_L L}{C_N} \frac{s_X}{s_L} + \frac{QX}{C_N}\right) \left(1 - \frac{\eta s_L - \delta h}{1-h}\right).$$

The planner's extra term is

$$U_L \frac{\Delta'(P_N)\, C_N}{F_L} \cdot \mathcal{P}_{N,Q} \mathcal{Q}_{C_N},$$

where

$$\mathcal{P}_{N,Q} = \frac{P_N}{Q} \frac{h}{1-h}, \quad \mathcal{Q}_{C_N} = \frac{Q}{C_N} \left(\frac{1-h}{\eta s_L - \delta h} - 1\right),$$

so

$$U_L \frac{\Delta'(P_N)\, C_N}{F_L} \cdot \mathcal{P}_{N,Q} \mathcal{Q}_{C_N} = \frac{U_L L}{C_N} \frac{1}{s_L} \frac{\delta h}{1-h} \left(\frac{1-h}{\eta s_L - \delta h} - 1\right).$$

Is there a connection between labor wedge and oil wedge?

Labor wedge can be written as

$$\left(U_{C_N}+\frac{\Delta}{F_L}U_L\right)=U_L\left(\frac{U_{C_N}}{U_L}+\frac{\Delta}{F_L}\right)=U_L\frac{\Delta}{F_L}\left(\frac{F_L}{\Delta}\frac{U_{C_N}}{U_L}+1\right),$$

the oil wedge as

$$-\left(U_L\frac{F_X}{F_L}+Q\cdot U_{C_T}\right)=-\left(U_L\frac{Q}{W/P_T}+Q\cdot U_{C_T}\right)=-U_LQ\frac{P_T}{W}\left(1+\frac{WU_{C_T}}{U_LP_T}\right)$$
$$=-U_LQ\frac{P_T}{W}\left(1+\frac{W}{P_N}\frac{U_{C_N}}{U_L}\right).$$

With sticky prices and fixed nominal wages we tend to have after supply shock a positive labor wedge

$$1<\frac{F_L}{\Delta}\frac{U_{C_N}}{-U_L}$$

and a negative oil wedge because

$$\frac{W}{P_N}>\frac{F_L}{\Delta}$$

implies

$$\frac{W}{P_N}\frac{U_{C_N}}{-U_L}>\frac{F_L}{\Delta}\frac{U_{C_N}}{-U_L}>1$$

Natural output

$$(1+\phi)\left(c_N-a_X\bar{x}\right)+a_X\frac{1-\eta}{\eta}\left(c_N-\bar{x}\right)=0$$

$$c_N=a_X\frac{\phi+\frac{1}{\eta}}{\phi+a_L+a_X\frac{1}{\eta}}\bar{x}$$