WORKING paper

BANQUE DE FRANCE
EUROSYSTÈME

Evaluating the Financial Instability Hypothesis: a Positive and Normative Analysis of Leveraged Risk-Taking and Extrapolative Expectations

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September 2025, WP #1009

Abstract

Historical accounts of financial crises emphasize the joint contribution of extrapolative beliefs and leveraged risk-taking to financial instability. This paper proposes a simple macro-finance framework to evaluate these views. We find a novel interplay between non-rational extrapolation and investment risk-taking that amplifies financial instability relative to a rational expectation benchmark. Furthermore, the analysis provides guidance on the design of cyclical policy interventions. Specifically, relative to a rational expectations benchmark, extrapolative expectations command tighter financial regulation, irrespective of whether the regulator shares these expectations.

Keywords: Non-Rational Expectations, Financial Stability and Regulation

JEL classification: E44, E71, G01

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NON-TECHNICAL SUMMARY

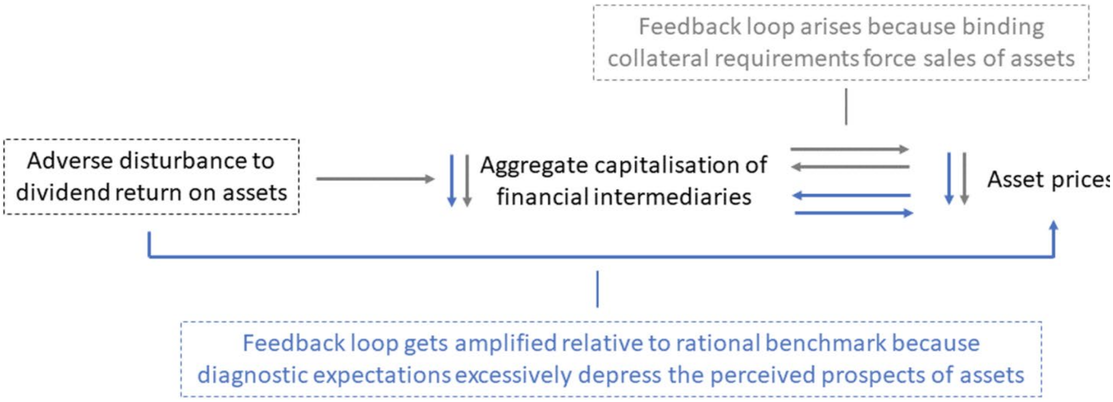
The classical writings of Hyman Minsky and Charles P. Kindleberger popularized the Financial Instability Hypothesis (FIH), which emphasizes the vulnerability of financial markets to boom-bust cycles – specifically, robust expansions that eventually lead to asset price collapses and economic contractions. Their analysis identifies a cyclical pattern where periods of prosperity and unfounded optimism fuel excessive risk-taking, particularly through leveraged investments. As optimism drives higher borrowing, even minor asset price declines can trigger "fire sales", leading to a severe market downturn accompanied by economic contraction. This research formally evaluates the FIH within a standard macroeconomic framework that incorporates constraints on leveraged investment and non-rational extrapolative expectations.

To evaluate the FIH appropriately, the analysis includes three critical components. First, credit markets are subject to collateral constraints, which lead to episodes of "fire sales" that amplify the effects of economic shocks (Figure 1). Second, the model includes non-rational, extrapolative expectations, which can cause significant deviations in asset valuations from economic fundamentals. Finally, the framework allows for the allocation of resources to productive technologies with different risk-return profiles. These elements provide a basis for understanding how the interplay between financial frictions and non-rational expectations generates financial instability and economic fluctuations.

The study's first key finding is that diagnostic (or extrapolative) expectations - whereby economic agents form (incorrect) expectations about future economic conditions based on recent movements in asset prices and macroeconomic outcomes- exacerbate financial instability relative to a benchmark economy in which agents form rational expectations that are consistent with economic fundamentals. This result rests on the interaction between the allocation of resources across productive technologies and extrapolative expectations. Consider a financial cycle that begins with a sequence of favorable economic shocks. These increase financiers' wealth and borrowing capacity but also lead to excessive optimism regarding asset returns, encouraging risk taking and driving up asset prices. At the peak of the financial cycle, the economy becomes fragile to minor disturbances, which trigger a sharp correction of the appreciation of economic fundamentals. This event sets up a loop of asset price declines, tightening borrowing constraints and reallocation of assets to less productive but less risky technologies. During the bust, the effects of extrapolative beliefs on asset price adjustments are milder, because aggregate risk decreases. This asymmetric effect of diagnostic beliefs along the financial cycle leads to overall higher financial instability, thereby destabilizing both the financial system and the real economy, consistent with the FIH.

The study argues that financial regulation plays a critical role in limiting these risks. Specifically, the analysis emphasizes that restrictions on financial leverage and risk-taking are socially desirable, regardless of the regulator's expectations. However, the cyclical nature of these measures depends on the expectation process of the policymaker. A benevolent policymaker, whose expectations align with those of private agents, would impose leverage restrictions during post-crisis recoveries to support the restoration of balance sheets and the orderly recapitalization of the financial sector. By contrast, a paternalistic planner, with rational expectations in a world of private agents holding diagnostic expectations, would prefer to act pre-emptively, imposing leverage restrictions, like a countercyclical capital buffer, during expansions to reduce vulnerability to future financial downturns and fire sales episodes. Overall, the results underscore the importance of macroprudential policy throughout the financial cycle, with the design of measures shaped by the expectations of both market participants and regulators.

Figure 1. Financial implications of financial frictions and diagnostic expectations



Note: the figure illustrates interactions between fluctuations in financial conditions (i.e. aggregate capitalisation of financial intermediaries) and fluctuations in asset prices. Blue lines indicate additional effects that stem from non-rational extrapolative expectations. These additional effects are larger during booms, when risk-taking in financial markets and in real investments is more aggressive.

Une évaluation de l'hypothèse de l'instabilité financière

RÉSUMÉ

Les écrits classiques de Hyman Minsky et Charles P. Kindleberger ont popularisé l'Hypothèse de l'Instabilité Financière (HIF), soulignant la vulnérabilité des marchés financiers aux cycles d'expansion et de contraction. Ces cycles sont alimentés par un optimisme et une prise de risque excessive, notamment par des investissements financés par emprunt. Ce document de travail propose un cadre macroéconomique simple pour évaluer cette hypothèse, en étudiant les interactions entre les contraintes sur les investissements à levier et les anticipations extrapolatives non rationnelles. L'analyse identifie une interaction nouvelle entre extrapolation non rationnelle et prise de risque économique, qui amplifie l'instabilité financière par rapport à une économie ou les agents forment des anticipations rationnelles. Ces éléments analytiques sont alignés avec les postulats et conclusions de l'HIF.

L'analyse met également en évidence l'importance de la régulation financière pour atténuer ces effets. En particulier, les anticipations non rationnelles justifient une régulation financière plus stricte, indépendamment du degré des anticipations non rationnelles du régulateur. En revanche, les caractéristiques de ces restrictions dépendent des anticipations du régulateur. Ainsi, la régulation macroprudentielle, adaptée aux différentes phases du cycle financier, apparaît essentielle pour minimiser les effets déstabilisateurs des cycles économiques sur les marchés financiers et l'économie réelle.

Mots-clés : anticipations non rationnelles, stabilité financière, politique macroprudentielle.

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1 Introduction

Hyman Minsky (1977, 1986) and Charles P. Kindleberger (1978) have popularized the view that financial markets are prone to robust expansion leading to asset price collapses and economic contractions. Kindleberger's "Anatomy of a Typical Crisis" (1978, Chapter 2) identifies a sequence starting with extended periods of prosperity and investment gains, which propel a surge in leveraged risk-taking fueled by investors' optimism:

> *"During the expansion phase, investors become more optimistic about the future, revising upward their estimates of the profitability of a wide range of investments, and thus, they become more eager to borrow."*

The seeds of financial instability lie in highly leveraged investors exposed at the cycle's peak to minor declines in asset prices. Domino effects unfold as losses on asset values trigger credit restrictions and force investors to sell assets at discount prices – a phenomenon called "fire-sales" – rapidly transforming a minor downturn into a collapse of asset prices, intensified by investors' pessimism:

> *"Soon, some of the investors who had financed most of their purchases with borrowed money become distress sellers (. . .) lead to sharp declines in the prices of the assets, and a crash and panic may follow."*

These narratives, often referred to as the Financial Instability Hypothesis (FIH), highlight three critical elements: investment decisions are undertaken by leveraged investors, even when investment projects are inherently risky; the reallocation of assets across investment projects is difficult, even when these projects present similar characteristics; and investors' appreciations of future economic developments are tied to recent economic events, even in the presence of incompatible statistical evidence.

The objective of this study is to assess these narratives and their policy implications. We propose a macroeconomic framework with a financial sector whose capitalization directly influences the supply of credit and the resilience of business fluctuations and financial stability to different economic disturbances. In addition, the environment features the critical components necessary to evaluate the FIH. First, credit market frictions provoke episodes of fire sales that amplify the economic effects of the disturbances. Second, non-rational, extrapolative (or diagnostic) expectations shape asset valuations that may significantly deviate from economic fundamentals.[1] Finally, the possibility of allocating productive resources to businesses with different risk-return profiles enables an endogenous risk-taking channel in real investments.[2] We find that this real risk-taking channel creates a novel mechanism that ties extrapolative expectations and endogenous aggregate risk, leading to an amplification of financial and economic instability. In addition, the analysis prescribes the design of tighter financial regulations, regardless of the regulator's degree of diagnostic expectation, relative to the optimal design derived assuming agents form rational expectations.

Our modeling environment features a single productive asset and two production technologies. One of the technologies is more productive on average, but it is also riskier. Because agents are risk neutral, if expectations were rational, real risk-taking would always be perceived as privately optimal and, in the absence of financial frictions, it would never be financially constrained. However, because agents form diagnostic expectations, real risk-taking can temporarily be perceived as privately suboptimal when the technologies have been impacted by a sequence of adverse disturbances. Moreover, because only some expert investors subject to leverage restrictions can channel the asset to the productive technology, real risk-taking may occasionally be financially constrained when the experts' aggregate capitalization is sufficiently low. The interplay between the agents' perceptions and the experts' financial capacity determines the allocation of the asset between the technologies (i.e., the degree of real risk-taking) and the asset price in equilibrium. Real risk-taking falls and a fire sales episode occurs when adverse disturbances erode the experts' wealth share sufficiently to force them to sell assets to non-experts.

Our first main finding is that diagnostic expectations intensify financial instability relative to a benchmark economy in which agents form rational expectations. The mechanism behind this result is as follows. A favorable sequence of technological disturbances increases the wealth share of experts, thereby enhancing financial stability, whereas an unfavorable

[1] The concept of diagnostic expectations is grounded on the psychological theory of "the representativeness heuristic" (Tversky and Kahneman 1983). Under diagnostic expectations, agents tend to assign excessive likelihood to possible future events that are reminiscent of those realized in the recent past. As shown by Bordalo, Gennaioli and Shleifer 2018 and Bordalo et al. 2019, these expectations generate systematic forecast errors on asset returns that are consistent with those empirically measured.

[2] Models of financial intermediation with multiple technologies are standard (e.g., Brunnermeier and Sannikov (2014)). Heterogeneity in risk–return profiles across assets or technologies is likewise well established and traces back, notably, to the classic portfolio-selection problems studied by Markowitz (1952) and Sharpe (1964), among others. Our framework generates a pro-cyclical risk profile for real investment, consistent with the empirical evidence discussed in the literature review.

sequence deteriorates it. Moreover, a favorable sequence renders the agents overly optimistic about the prospects of the technologies, whereas an unfavorable sequence makes them overly pessimistic. Together, these two elements generate a negative co-movement between the experts' wealth share and the systematic forecasts errors resulting from extrapolative expectations. In addition, excessive extrapolation encourages real risk-taking when the wealth share is high (and the forecast errors are negative), whereas it discourages it when the wealth share is low, strengthening the economic impact of forecast errors during upturn while weakening it during downturns. This asymmetry, in general, is sufficiently strong to tilt leftward the ergodic (i.e., long-run) distribution of the expert's wealth share relative to the rational expectations benchmark, thus deteriorating stability in both the financial system and the real economy, in line with the FIH.

Extensions of our baseline environment demonstrate that our results remain robust to alternative modeling specifications. In particular, the amplification of financial instability is quantitatively similar when agents form diagnostic expectations tied to the endogenous return on capital investment, rather than being associated to exogenous disturbances as in the baseline model. Similarly, our findings of increased financial instability under diagnostic beliefs persist when experts' leverage constraints are endogenously linked to the present discounted value of future investment profits rather than solely to the contemporaneous wealth share.[3]

Kindleberger and Minsky devote discussions to cyclical policy interventions designed to temper financial instability and associated undesirable economic volatility:

> *"The appearance of a mania or a bubble raises the policy issue of whether governments should seek to moderate the surge in asset prices to reduce the likelihood or the severity of the ensuing financial crisis or to ease the economic hardship that occurs when asset prices begin to decline."* (Ibid.)

Our analysis allows us to determine whether policy intervention is required at each stage of the financial cycle. Our framework with a limited commitment friction and incomplete markets features a standard pecuniary externality, reflected in financial amplification of fundamental shocks, depressed asset prices and episodes of fire sales.[4] To isolate these

[3] Under this specification, non-rational extrapolation directly influences the perception of future profits—and thus the leverage limit as well—but these effects are generally of second-order importance, because the financing constraint becomes slack when the experts are well capitalized and optimism is strong.

[4] As mentioned, aggregate shocks are amplified when experts lose net worth and become less willing to hold assets, further depressing asset prices and growth. The mechanism is reminiscent of the seminal

effects and study policy implications, we characterize a socially optimal allocation and compare its properties with the competitive equilibrium.[5] Specifically, we assess the magnitude of the depressing effect of the pecuniary externality on asset prices, reinvestment rates, and the allocation of the asset.

The pecuniary externality motivates an active role for financial regulation, even under rational expectations. Generally, reinvestment is structurally depressed in the competitive equilibrium, because it is tied to the price of the asset. As for restrictions on financial leverage and risk-taking, these are socially desirable only in the economy with diagnostic expectations, regardless of the regulator's degree of diagnostic expectations. Importantly, the cyclical nature of the restrictions depends on the degree of diagnosticity of the social planner. Notably, a benevolent planner (i.e., a planner whose expectations are equally diagnostic to those of private agents) restricts the allocation of the asset to the productive technology during recoveries from busts to promote the recapitalization of the financial sector. By contrast, a paternalistic planner (i.e., a planner with rational expectations in an environment in which private agents have diagnostic expectations) restricts it during economic expansions, to mitigate the possible adverse effects of a reversal and associated fire sales.[6] Both types of interventions mitigate financial instability and enhance long-run welfare. Relative to the benevolent interventions, the paternalistic interventions yields a higher mean consumption level with a substantially lower level of consumption variability. Overall, our analysis identifies how macro-prudential regulations are required at different phases of the financial cycle depending on the expectation processes of regulators.

Related literature. A wealth of literature has studied the implications of financial and behavioral frictions on financial markets and macroeconomic outcomes.[7] Among these studies, ours characterizes the nonlinear stochastic global dynamics in a continuous time

analysis of Lorenzoni (2008). See Dávila and Korinek (2017) for a recent attempt at structuring formally the literature on credit market frictions and allocative inefficiencies.

[5]Using constrained efficiency rather than parametrized instruments means that there is no need to commit to an arbitrary set of policy instruments, but rather let the model guide the choice of instruments, as in Di Tella (2019).

[6]Note that these restrictions have benefits that are realized throughout the financial cycle, not only in periods when they are actively enforced, due to the general equilibrium effect on asset prices.

[7]Recent studies that incorporate diagnostic expectations into dynamic general equilibrium models include Krishnamurthy and Li (2025); L'Huillier, Singh and Yoo (2021); L'Huillier, Phelan and Wieman (2022); Bianchi, Ilut and Saijo (2023). Also, this paper fits into the literature on financial amplification mechanisms going back to Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke, Gertler and Gilchrist (1999).

general equilibrium framework, and hence is closest to Brunnermeier and Sannikov (2014) and Maxted (2023). As previously noted, Brunnermeier and Sannikov (2014) highlights the role of fire sales to financial instability in an economy with rational expectations, whereas Maxted (2023) emphasizes the absence of contribution of diagnostic expectations to financial instability in an economy without fire sales or real risk-taking. To the best of our knowledge, this is the first study to incorporate extrapolative expectations, financial frictions, and risk-taking in the composition of real investment to appropriately evaluate the FIH.

Importantly, our analysis establishes that extrapolative expectations amplify financial instability when one accounts for the real risk taking channel. This contrasts with results presented in Maxted (2023), where the effects of extrapolative beliefs on asset prices and the dynamics of net worth tend to be symmetric along the financial cycle, so that positive forecasts errors during busts compensate negative forecasts errors during booms. Instead, our environment with endogenous procyclical aggregate risk highlights an amplification of financial instability under diagnostic expectations relative to rational expectations.[8] Procyclical real risk-taking is documented in formal empirical studies, notably during the boom-bust cycle that culminated in the Global Financial Crisis (e.g., Mian and Sufi 2011, Dell'Ariccia, Igan and Laeven 2012). Once we account for this endogenous risk taking channel, the effects of extrapolative beliefs are asymmetric along the financial cycle. This feature contributes to foster financial instability, a view that aligns closely with the historical views of financial crises.

A central piece of our analysis derives the implications of financial instability and extrapolative beliefs for macro-prudential regulation. As noted in Phelan (2016), Brunnermeier and Sannikov (2014) is a critical contribution to the literature, but the model is not designed to study leverage regulation: "limiting leverage can improve stability, but it is very difficult to improve the welfare of either households or banks." In contrast, our analysis points to beneficial leverage restrictions when agents form extrapolative expectations. Overall, our study completes the review of the FIH and identifies conditions that motivate beneficial policy interventions and leverage restrictions at each phase of the financial cycle.

Finally, our study differs from other analyses that focus on the normative implications

[8] Our framework departs critically from the one presented in Maxted (2023) by allowing productive assets to be allocated to different production technologies that differ in risk-return profiles. This is the key feature that accounts for the risk taking channel. In Section 4.1, we investigate the contribution of technology heterogeneity to our results and highlight the central role of endogenous cyclical aggregate risk.

of non-rational expectations for financial regulation. Fontanier (2022) presents a stylized environment, in which externalities may arise when the non-rational component of expectations is tied to asset prices. To conduct the welfare analysis, Fontanier (2022) restricts attention to the socially optimal allocation derived by a paternalistic planner. Instead, our study considers socially optimal allocations for both paternalistic and benevolent planners in a nonlinear stochastic environment. Dávila and Walther (2021) also study the optimal design of financial regulation when private agents have distorted beliefs relative to a planner, but they focus on implications for the optimal regulation of differences in beliefs between investors and creditors. Finally, Farhi and Werning (2020) study an economy with diagnostic expectations in which social inefficiencies may arise from aggregate demand externalities. Their study focuses on the implications of extrapolative expectations for the coordination of monetary policy and macro-prudential policy.

Layout. The paper is structured as follows. Section 2 describes the model and characterizes its equilibrium. Section 3 conducts the positive analysis, and section 4 establishes robustness to alternative model specifications. Section 5 presents the normative analysis. Section 6 concludes. An appendix to the paper provides the proofs of all lemmas, propositions and corollaries stated in the sections.

2 The Model

Consider a production economy with financial frictions, in which agents form diagnostic expectations over future events and allocate productive resources to technologies with different risk-return profiles.

2.1 Environment

Time $t \in \mathbb{R}_+$ is continuous and unbounded. There is a single real asset $k_t \geq 0$ and a single output good $y_t \geq 0$. The asset can be allocated between two production technologies.

Technologies. The technologies produce the output good using the asset according to

$$y_{j,t} = A_j k_{j,t} \geq 0 \ , \tag{1}$$

where $A_j > 0$ is the productivity of technology $j \in \{1,2\}$ and $k_{j,t} \geq 0$ are the units of the asset allocated to that technology. In addition, the technologies allow for internal reinvestment of the asset at a standard rate of return, $\mathcal{I}_j(\iota_{j,t})k_{j,t}dt$, which satisfies $\mathcal{I}_j(0) = 0$, $\mathcal{I}'_j(\cdot) > 0$, and $\mathcal{I}''_j(\cdot) < 0$, where $\iota_{j,t} \in [0, A_j]$ is the reinvestment rate per unit of the asset. One of the technologies (i.e., $j = 1$) is more productive but it is also riskier. Formally, $A_1 \geq A_2$, $\mathcal{I}_1(\cdot) \geq \mathcal{I}_2(\cdot)$, and $\sigma_1 > \sigma_2 \geq 0$, with

$$\frac{dk_{j,t}}{k_{j,t}} = \mathcal{I}_j(\iota_{j,t})dt + \sigma_j dZ_t \ , \tag{2}$$

where $dZ_t \sim_{i.i.d.} \mathcal{N}(0, dt)$ is a Brownian disturbance common to the technologies. The disturbance can be interpreted as an aggregate shock to the productive quality of the asset, or in other words, as a *quality shock*.[9]

Preferences. The economy is populated by a continuum of agents with linear preferences over the output good. Thus, agents only derive utility from the present value of consumption flows, discounted at subjective time discount rate, $r > 0$. All agents share common expectations but these feature non-rational (or diagnostic) extrapolation.

Non-rational Extrapolation. If expectations were rational, agents would not extrapolate from past disturbances the likelihood of future disturbances, since disturbances are serially uncorrelated. Additionally, agents would correctly forecast $\hat{E}_t[dZ_t] = E_t[dZ_t] = 0$, where hat variables indicate perceptions. By contrast, under extrapolative expectations, agents rely on recently realized disturbances to estimate the future average disturbance, a process that leads to systematic forecast errors.

Formally, we assume agents synthesize information about past disturbances as

$$\omega_t \equiv \int_0^t e^{-\delta(t-s)} dZ_s \ , \tag{3}$$

[9]The existence of a more productive yet riskier technology gives rise to the possibility of a risk-return trade-off. The possibility of that trade-off is standard in the literature, and its incorporation in portfolio selection problems or in problems concerning real investment decisions can be traced back at least to Markowitz (1952) and Sharpe (1964) for the former application and to Debreu (1959) and Arrow (1964) for the latter. One could think that technology $j = 1$ represents a relatively new sector of the economy, with high prospects but also with high uncertainty on its viability, whereas technology $j = 2$ represents a more traditional sector, with lower opportunities for growth but also with lower exposure to operational or profitability risks.

where $\delta > 0$ indicates the memory rate of decay at which past realizations are discounted.[10] Agents then use recent information $\omega_t \in \mathbb{R}$ to forecast future disturbances according to

$$\hat{E}_t[dZ_t] \equiv E_t[d\hat{Z}_t], \text{ with } d\hat{Z}_t \equiv \hat{\mu}\omega_t dt + dZ_t, \tag{4}$$

where parameter $\hat{\mu} > 0$ is the extrapolative (or diagnostic) weight of information on expectation formation. Thus, forecast $\hat{E}_t[dZ_t] = \hat{\mu}\omega_t dt$ positively depends on information ω_t, and forecast errors $\hat{E}_t[dZ_t] \neq 0$ are possible if $\hat{\mu} > 0$. Provided that information $\omega_t < 0$ is sufficiently negative, it could well be the case that estimates about the growth rate of the asset under the technologies satisfy $\hat{E}_t[dk_2/k_2] > \hat{E}_t[dk_1/k_1]$, which as detailed next can induce agents to perceive the *unproductive* technology (i.e., $j = 2$) as the most profitable. Based on these results—and because past disturbances do not have predictive power over future disturbances—in what follows, we interpret ω_t as *sentiment*.[11]

Perceived Returns on Technologies. The asset can be traded in spot markets at a price $q_t > 0$. We postulate that the price evolves according to an Ito process

$$\frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{q,t} dZ_t \ , \ \text{ with } \ \sigma_{q,t} \geq 0 \ , \tag{5}$$

where $\mu_{q,t} \in \mathbb{R}$ and $\sigma_{q,t} \geq 0$ are endogenous drift and diffusion processes to be determined in equilibrium and dZ_t is the aggregate disturbance introduced in (2). Let $dR_{j,t} \in \mathbb{R}$ denote the rate of return to allocate the asset to technology j, defined as

$$dR_{j,t} \equiv \frac{A_j - \iota_{j,t}}{q_t} dt + \frac{d\left(q_t k_{j,t}\right)}{q_t k_{j,t}} \ , \tag{6}$$

where the first term on the RHS is the dividend yield from operating the technology and the second term is the percentage change of the market value of asset holdings. Applying

[10] Other papers in the literature (e.g., Maxted (2023)) assume instead that agents infer past information from the historical rates of change of physical capital. Subsection 4.2 shows that these two specifications deliver quantitatively similar results in our environment with a risk-taking channel.

[11] As shown in (3), the information operator ω_t is a weighted sum of past shocks, weighing the shocks with a constant time discount rate. This functional form implies a single time-consistent subjective probability measure. Consequently, the law of iterated expectations (LOIE) holds for subjective beliefs, as in Bordalo, Gennaioli and Shleifer (2018) and Maxted (2023). Intuitively, agents' forecasts are dynamically consistent: the expectation they hold today for a future variable coincides with their expectation of how they will forecast it in the future. For an analysis of failure of LOIE under diagnostic expectations, see Bianchi, Ilut and Saijo (2023).

Ito's product rule, one gets

$$dR_{j,t} = \Big[\frac{A_j - \iota_{j,t}}{q_t} + \mu_{q,t} + \mathcal{I}_j(\iota_{j,t}) + \sigma_{q,t}\sigma_j\Big]dt + (\sigma_{q,t} + \sigma_j)dZ_t \ , \tag{7}$$

which implies that agents' forecasts are

$$\hat{E}_t[dR_{j,t}] = \left[\frac{A_j - \iota_{j,t}}{q_t} + \mu_{q,t} + \mathcal{I}_j(\iota_{j,t}) + \sigma_{q,t}\sigma_j + (\sigma_{q,t} + \sigma_j)\,\hat{\mu}\omega_t\right] dt \ . \tag{8}$$

The last term on the RHS of (7) reflects the influence of diagnostic expectations over the perceptions of price risk $\sigma_{q,t}dZ_t$ and quality risk $\sigma_j dZ_t$. Notably, if sentiment ω_t is sufficiently low, diagnostic perceptions of risks are sufficiently strong to render $\hat{E}_t[dR_{2,t}] > \hat{E}_t[dR_{1,t}]$. Consequently, when sentiment is relatively high, agents correctly perceive the *productive* technology (i.e., $j = 1$) as the most profitable investment, but when sentiment is sufficiently low, they incorrectly perceive the *unproductive* technology (i.e., $j = 2$) as the most profitable.[12]

Frictions. There are two types of agents: households and financiers. Households can only operate the unproductive technology whereas financiers can only operate the productive technology.[13] Both types of agents can issue debt, but only financiers are subject to a financing constraint. This constraint is motivated by a standard agency problem in credit markets that allows financiers to walk away with a fraction of their assets immediately after issuing debt. The constraint restricts asset holdings of financiers to satisfy $q_t k_{1,t} \leq \lambda n_t$, where $n_t \geq 0$ is their net worth and parameter $\lambda > 1$ is the upper limit on their debt-to-net-worth ratio. For simplicity, we assume debt is short term and non-contingent, meaning that debt issued at time t matures at time $t + dt$ and promises a fixed rate of return

[12]Our environment's preferences and technologies give rise to an endogenous risk–return trade-off across technologies, which hinges on whether agents form rational (subsection 3.1) or diagnostic expectations (subsection 3.2). Importantly, if the technologies were identical, no trade-off could arise. Subsection 4.1 evaluates how our three modeled sources of heterogeneity—productivity A_j, reinvestment opportunity $\mathcal{I}(\iota_{j,t})$, and exposure to capital-quality risk σ_j—influence the trade-off and the key results of the model.

[13]This assumption is made for expositional clarity, one could allow financiers to operate both technologies without changing allocation decisions and equilibrium outcomes. Differences in access to markets or technologies across agents is a standard assumption in the literature. For instance, Brunnermeier and Sannikov (2014) consider an environment with two types of agents and two technologies similar to ours, in which each type operates a different technology. Another example is He and Krishnamurthy (2013), who consider an environment with many types of agents but a single technology, in which only some types can operate the technology.

regardless of realization dZ_t. Given linearity in preferences over consumption of the output good, the interest rate on debt is given by the agents' subjective time discount rate, rdt.

Portfolio Problems. Agents are competitive. Households maximize the present discounted value of consumption subject to the law of motion of their wealth. Formally, they solve

$$\max_{c_t,\iota_{2,t},k_{2,t}\geq 0} \hat{E}_t \int_t^{+\infty} e^{-r(s-t)} c_s ds \ , \tag{9}$$

subject to

$$dw_s = dR_{2,s} q_s k_{2,s} + r(w_s - q_s k_{2,s})ds - c_s ds + \tau_s ds \ , \tag{10}$$

where $c_t \geq 0$ is consumption, $w_t \in \mathbb{R}$ is wealth, and $\tau_t \in \mathbb{R}$ are net transfers from financiers. As in Gertler and Kiyotaki (2010), Gertler and Karadi (2011), and Maggiori (2017), financiers do not consume; instead, each pays dividends to a unique household, according to an exogenous Poisson process with an arrival rate $\theta > 0$. When they pay out, financiers transfer their entire net worth to their associated household, and immediately afterwards, they are replaced by an identical newcomer whose starting net worth is specified below. Financiers maximize the present discounted value of dividend payouts

$$\max_{\iota_{1,s},k_{1,s}\geq 0} \hat{E}_t \int_t^{+\infty} \theta e^{-(r+\theta)(s-t)} n_s ds \ , \tag{11}$$

subject to the law of motion of net worth

$$dn_s = dR_{1,s} q_s k_{1,s} - r(q_s k_{1,s} - n_s)ds \ , \tag{12}$$

and the collateral constraint

$$q_s k_{1,s} \leq \lambda n_s \ . \tag{13}$$

Equilibrium. A competitive equilibrium is an allocation $\{\iota_{1,t}, k_{1,t}, \iota_{2,t}, k_{2,t}, c_t\}$ and an asset price process $\{q_t, \mu_{q,t}, \sigma_{q,t}\}$ such that (i) the allocation solves portfolio problems (9)-(10) and (11)-(13) given the price process; (ii) the markets for the good, the asset, and debt clear.

2.2 Solving the Equilibrium

To solve the equilibrium we proceed as follows. First, we derive the optimal choices of households and financiers, which combined with market clearing conditions deliver analytical equilibrium conditions. Then, we restrict attention to a Markov equilibrium, which allows to characterize equilibrium as a tractable system of second-order partial differential equations (PDEs).

2.2.1 Households' Problem

The lemma below characterizes the optimal choices of households.

Lemma 1. *At any given time t, households are indifferent among all possible consumption rate c_t. Moreover, they choose reinvestment rate $\iota_{2,t}$ and asset holding $k_{2,t}$ as follows:*

$$\mathcal{I}_2'(\iota_{2,t}) = \frac{1}{q_t} \ , \tag{14}$$

and

$$q_t k_{2,t} \begin{bmatrix} = 0 & \text{if } \alpha_{2,t} < 0 \\ \in [0, +\infty) & \text{if } \alpha_{2,t} = 0 \end{bmatrix} \ , \tag{15}$$

where the estimated risk-adjusted excess return to allocate the asset to the unproductive technology over holding debt, that is, $\alpha_{2,t} \leq 0$, is given by

$$\alpha_{2,t} \equiv \frac{1}{dt} \hat{E}_t \left[dR_{2,t} \right] - r \leq 0 \ . \tag{16}$$

The intuition behind this result is as follows. Households are indifferent to any consumption rate because the interest rate on debt equals their subjective time discount rate. When $\alpha_{2,t} < 0$, households strictly prefer holding debt securities to allocating the asset to the unproductive technology. Thus, $k_{2,t} = 0$ is optimal. By contrast, when $\alpha_{2,t} = 0$, households are indifferent between the two investment opportunities, and therefore, any $k_{2,t} \geq 0$ is optimal. Excess return $\alpha_{2,t} \leq 0$ cannot be positive in equilibrium. Otherwise, households would take unbounded leveraged positions on the asset, since they are not subject to financing constraints. Reinvestment rule (14) indicates that reinvestment positively depends on asset price q_t.

2.2.2 Financiers' Problem

Let $V_t \geq 0$ be the value function associated with problem (11)-(13). We postulate that the value is linear in net worth. Formally, $V_t \equiv v_t n_t$, where marginal value of net worth $v_t \geq 1$ is endogenous but independent of individual choices. In addition, we postulate that marginal value v_t evolves stochastically over time, according to an Ito process with disturbance dZ_t and endogenous drift and diffusion $\mu_{v,t} \in \mathbb{R}$ and $\sigma_{v,t} \leq 0$, respectively. The following lemma characterizes the optimal choices of financiers.

Lemma 2. *At any given time t, financiers choose reinvestment rate $\iota_{1,t}$ and asset holding $k_{1,t}$ as follows:*

$$\mathcal{I}_1'(\iota_{1,t}) = \frac{1}{q_t} \ , \tag{17}$$

and

$$\frac{q_t k_{1,t}}{n_t} \begin{bmatrix} = 0 & \textit{if } \alpha_{1,t} < 0 \\ \in [0, \lambda] & \textit{if } \alpha_{1,t} = 0 \\ = \lambda & \textit{if } \alpha_{1,t} > 0 \end{bmatrix} , \tag{18}$$

where the estimated risk-adjusted excess return to allocate the asset to the productive technology over holding debt, namely, $\alpha_{1,t} \in \mathbb{R}$, is given by

$$\alpha_{1,t} \equiv \frac{1}{dt} \hat{E}_t \left[dR_{1,t} \right] - r + \left(\sigma_{q,t} + \sigma_1 \right) \sigma_{v,t} \ . \tag{19}$$

The marginal value of net worth, v_t, satisfies

$$0 = \alpha_{1,t} \frac{q_t k_{1,t}}{n_t} + \mu_{v,t} + \hat{\mu} \omega_t \sigma_{v,t} + \frac{\theta}{v_t} - \theta \ . \tag{20}$$

When $\alpha_{1,t} > 0$, financiers expect a positive excess return to allocating the asset to the productive technology. Thus, they take leveraged positions on the asset until they hit their limit on debt. When $\alpha_{1,t} = 0$, financiers are willing to take any position on the asset, because they are indifferent between the two investment alternatives. Lastly, when $\alpha_{1,t} < 0$, financiers do not acquire the asset, because they expect a higher return from holding debt. The last term in (19) is a risk premium, compensating agents for holding quality risk $\sigma_1 dZ_t$ and price risk $\sigma_{q,t} dZ_t$. It arises from the risk of facing a binding collateral constraint, which makes financiers concerned with the co-movement between the rate of change in the marginal value of net worth and the return on investments. Reinvestment rule (17) is analogous to reinvestment rule (14).

Condition (20) expresses marginal value v_t as a present discounted value of expected rents $\alpha_{1,t} q_t k_{1,t} / n_t \geq 0$. These rents are the profits earned by financiers from operating the productive technology.[14]

2.2.3 Equilibrium Characterization

We postulate that in equilibrium households and financiers cannot simultaneously be marginal buyers of the asset. Put formally, excess returns $\alpha_{1,t}$ and $\alpha_{2,t}$ cannot simultaneously be null "almost surely." The following proposition characterizes the equilibrium.

Proposition 1. *Let $\eta_t \equiv n_t / q_t k_t \in [0,1]$ be the aggregate net worth of financiers as a share of total wealth and let $\kappa_t \equiv k_{1,t}/k_t \in [0,1]$ be the aggregate share of the asset allocated to the productive technology. Then, the equilibrium outcome is partitioned into the following three regimes,*

$$\begin{array}{llll} \text{1. Financially unconstrained regime:} & \kappa_t = 1 \leq \lambda \eta_t \,, & \alpha_{1,t} = 0 \,, & \alpha_{2,t} < 0 \,; \\ \text{2. Financially constrained regime:} & \kappa_t = \lambda \eta_t \in [0,1] \,, & \alpha_{1,t} > 0 \,, & \alpha_{2,t} = 0 \,; \\ \text{3. Precautionary regime:} & \kappa_t = 0 \,, & \alpha_{1,t} < 0 \,, & \alpha_{2,t} = 0 \,; \end{array} \tag{21}$$

The equilibrium allocation is summarized as $\{\iota_{1,t}, \iota_{2,t}, \kappa_t\}$, and the equilibrium is characterized by $\{(14), (16), (17), (19), (20), (21)\}$. The equilibrium utility of households per unit of the asset, noted $u_t > 0$, satisfies

$$\begin{aligned} 0 \;=\; & \kappa_t \left\{ A_1 - \iota_{1,t} + \left[\mathcal{I}_1(\iota_{1,t}) + \sigma_1 \hat{\mu} \omega_t \right] u_t \right\} + \\ & + (1-\kappa_t) \left\{ A_2 - \iota_{2,t} + \left[\mathcal{I}_2(\iota_{2,t}) + \sigma_2 \hat{\mu} \omega_t \right] u_t \right\} + \hat{E}_t \left[du_t \right] - r u_t . \end{aligned} \tag{22}$$

Notations do not distinguish between individual and aggregate variables because, in equilibrium, a representative household and a representative financiers exist.[15] The equilibrium regimes directly follow from combining the optimality conditions from lemmas 1

[14] If financiers never earn any rent—and thus the collateral constraint is always slack—then $v_t = 1$. By contrast, if $\alpha_{1,t} > 0$ at least occasionally, then $v_t \geq 1$.

[15] There is a representative household because individual households are identical. A representative financier exists because the behavior of individual financiers is linear in net worth.

and 2 together with market clearing for the asset. Consumption per unit of the asset c_t/k_t is given by net output flows $y_t/k_t = (A_1 - \iota_{1,t})\,\kappa_t + (A_2 - \iota_{2,t})\,(1-\kappa_t)$. Utility u_t is the present discounted value of consumption per unit of the asset. Thus, it is interpreted as the *social value* of the asset.

2.2.4 Markov Equilibrium

For tractability, we restrict attention to a Markov equilibrium, which allows to reduce the equilibrium conditions to a system of second-order PDEs. As is common practice, we no longer report the time subscript.

Definition 1. *A Markov equilibrium is a set of state variables* $\{\eta, \omega, k\}$ *and a set of mappings* $\{q, v\}$ *defined over states* $\{\eta, \omega\}$*, such that* (i) *the mappings satisfy conditions* $\{(14), (16), (17), (19), (20), (21)\}$ *and* (ii) *the states evolve according to laws of motion consistent with these conditions.*

Wealth share η relates to the tightness of the collateral constraint while sentiment ω indicates the degree of extrapolation relative to the rational expectations benchmark. The aggregate quantity of the asset is also a state variable, but it is not relevant for the derivations, because the equilibrium is scale invariant with respect to k. Mappings $\{q, v\}$ alone are sufficient to characterize the equilibrium because any other endogenous variable can be expressed as a function of those mappings or of their partial derivatives with respect to the states.

Regarding the laws of motion, the aggregate quantity of the asset evolves endogenously, according to

$$\frac{dk}{k} = \mu_k dt + \sigma_k dZ \ , \tag{23}$$

with

$$\mu_k = \kappa \mathcal{I}_1(\iota_1) + (1-\kappa)\,\mathcal{I}_2(\iota_2) \ , \tag{24}$$

$$\sigma_k = \kappa \sigma_1 + (1-\kappa)\,\sigma_2 \ . \tag{25}$$

The wealth share of financiers evolves endogenously as well, according to

$$\frac{d\eta}{\eta} = \mu_\eta dt + \sigma_\eta dZ \ , \tag{26}$$

with

$$\mu_\eta = \left[\frac{A_1 - \iota_1}{q} + \mathcal{I}_1(\iota_1) + \sigma_q \sigma_1\right]\phi + (\mu_q - r)(\phi - 1) - \mu_k \tag{27}$$

$$-\sigma_q \sigma_k + (\sigma_q + \sigma_k)\left[(\sigma_q + \sigma_k) - \phi(\sigma_q + \sigma_1)\right] - \left(\theta - \frac{\gamma}{\eta}\right),$$

$$\sigma_\eta = \phi(\sigma_q + \sigma_1) - (\sigma_q + \sigma_k), \tag{28}$$

where $\phi \equiv qk_1/n \geq 0$ is the leverage multiple of financiers and the last term in μ_η is the net transfers from financiers to households.[16] The first term of the transfers is the aggregate dividend payout and the second term is the starting endowment of newcomers. As in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), each newcomer receives $\gamma/\theta > 0$ units of the asset from a unique household.

Lastly, sentiment evolves exogenously, according to

$$d\omega = -\delta\omega dt + dZ. \tag{29}$$

Proposition 2. *The Markov equilibrium can be analytically characterized as the solution to a system of second-order PDEs for the mappings $\{q, v\}$ in the states $\{\eta, \omega\}$.*

To conduct the positive and the normative analysis, we solve the PDEs numerically using spectral methods, when necessary. To do so, we parametrize return functions $\mathcal{I}_j(\cdot)$ and assign numerical values to the parameters, as detailed in the next subsection.

2.3 Parametrization and Parameter Values

In our baseline specification, we consider a risk-less unproductive technology without reinvestment opportunities. Formally, $\mathcal{I}_2(\cdot) = 0$ and $\sigma_2 = 0$. Throughout the analysis, as is common in the literature (e.g., Brunnermeier and Sannikov 2014; Phelan 2016; He and Krishnamurthy 2019), we consider quadratic costs for reinvestment. That is, $\mathcal{I}_1(\iota_1) = \chi_1\sqrt{\iota_1}$, where $\chi_1 > 0$ is a parameter.

Table 1 presents the baseline parameter values. Our calibration follows standard principles in the literature: technology and preference parameters are either drawn from existing studies or chosen to match unconditional averages in an economy with diagnostic expecta-

[16] This law of motion follows from applying Ito's quotient rule to $\eta = n/qk$ and then subtracting from the resulting expression the net transfers from financiers to households, $\theta - \gamma/\eta$. Note then that $\tau = (\theta\eta - \gamma)qk$.

tions and financial frictions, thereby facilitating direct comparisons with the literature.[17] To compute unconditional averages, we use the limiting probability density function of the state, which measures the share of time the economy spends on average at each state point over a sufficiently long (i.e., infinite) time horizon.

Table 1: Parameter Values

Description	Parameter	Value	Target / Source
Panel A. Technologies			
Productivity gap	$A_1 - A_2$	0.34	Av. credit spread (1%)
Quality risk	σ_1	2.9%	Av. volatility of output (4%)
Return on reinvestment	χ_1	1.93%	Av. investment-output ratio (20%)
Panel B. Agents			
Subjective time discount rate	r	2%	Interest rate
Limit on debt	$\lambda - 1$	2.90	Av. leverage multiple (3.7)
Frequency of dividend payouts	θ	10%	Av. life span of financiers (10 years)
Starting endowment of financiers	γ/θ	14%	Av. wealth share of financiers (25%)
Panel C. Expectation formation			
Memory decay rate	δ	0.86	Corr. sentiment-wealth share (0.71)
Extrapolation weight	$\hat{\mu}$	0.20	Output bias (0.75%)

Notes. The table reports the parameter values in the baseline specification of the model. The time frequency is annual.

The time frequency is annual. The productivity of the productive technology, $A_1 = 1$, is normalized to 1. The productivity gap $A_1 - A_2 = 0.34$ targets an excess return to operating the productive technology of $E\left[dR_1\right] - r = 1\%$. The investment return $\chi_1 = 1.93\%$ targets an average ratio of reinvestment to output of $E[\iota/[A_1\kappa + A_2(1-\kappa)]] = 20\%$, whereas volatility $\sigma_1 = 2.9\%$ targets an average volatility of detrended output of $Var[A_1\kappa + A_2(1-\kappa)] = 4\%$. The subjective time discount rate is consistent with a standard value for the real interest rate of $r = 2\%$.[18] The debt limit of financiers, $\lambda - 1 = 2.9$, targets an average leverage multiple of $E[\phi] = 3.7$, which is consistent with Gertler and Kiyotaki (2010) and

[17] A previous version of the analysis used a calibration targeting moments in an economy with rational expectations. The results are robust to either approach.

[18] In the economy with rational expectations and without financial frictions (presented in section 3.1), the growth rate of the economy is constant over time and under the parameter values presented in Table 1, it takes value $E\left[\kappa\mathcal{I}_1(\iota_1)\right] = 1.48\% < r = 2\%$. This guarantees that relevant present discounted values, such as those for consumption or output, are bounded and dynamics are not explosive.

Gertler and Karadi (2011).[19] The average frequency of dividend payouts is $\theta = 10\%$, and the endowment of financiers satisfies $\gamma/\theta = 14\%$. The former value implies an average lifespan of financial firms of 10 years, whereas the latter value targets an average share of wealth of financiers of $E[\eta] = 25\%$. These values are consistent with Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) as well.

Finally, the extrapolation weight $\hat{\mu} = 0.2$ targets an output bias of 0.75% for a standard deviation in sentiment, as reported by Bordalo et al. (2020). The persistence parameter $\delta = 0.86$ targets a correlation between sentiment and wealth share of $Corr[\omega, \eta] = 71\%$, as in Maxted (2023).[20]

We perform robustness analyses in section 4, where we consider alternative specifications for the properties of production technologies, the formation of diagnostic expectations, and the collateral constraint.

3 Positive Analysis

We now derive the equilibrium outcome and investigate its positive properties. To clarify exposition, first, we examine three simplified versions of the model, and then we consider the original model economy with diagnostic expectations and financial frictions.

3.1 Rational Expectations and No Financial Frictions

Consider first an economy with rational expectations and without financial frictions. Formally, extrapolation weight $\hat{\mu} = 0$ is null, financiers' net worth $n \in \mathbb{R}$ can be negative, and leverage limit $\lambda = +\infty$ is unbounded.[21] The following corollary derives from proposition 1 and describes the equilibrium outcome in this economy.

Corollary 1. *In the economy with rational expectations and without financial frictions, neither sentiment ω nor wealth share η influences the equilibrium outcome. The asset price*

[19] The leverage constraint is not systematically binding in equilibrium, hence the possible discrepancy between debt limit and average leverage multiple.

[20] The precautionary regime does not occur under rational expectations. Under diagnostic expectations, it occurs approximately once every 500 years.

[21] If the net worth of financiers could not be negative, boundary condition $(\sigma_q + \sigma_1) \to 0$ as $\eta \to 0$ would be required to ensure $n \geq 0$—as in Maggiori (2017). The reason is that external financing is limited to non-contingent debt and the asset is risky. The possibility of $n < 0$ can then be interpreted as the possibility of issuing risky debt (i.e., debt whose rate of return depends on shock dZ) or of issuing equity.

is a constant that satisfies

$$\alpha_1 = 0 \Leftrightarrow \frac{A_1 - \iota_1}{q} + \mathcal{I}_1(\iota_1) - r = 0 \; , \; \textit{with} \;\; \mathcal{I}_1'(\iota_1) = \frac{1}{q} \; . \tag{30}$$

Value $v = 1$ is also a constant. The aggregate quantity of the asset is allocated to the productive technology, that is, $\kappa = 1$. The social value of the asset equals the asset price, that is, $u = q$.

In this economy, out of the three regimes presented in proposition 1, only the *financially unconstrained* occurs. The economy fluctuates—because of variations in the aggregate quantity of the asset k—but no deviation from linear trend k occurs. Thus, there is no notion of an economic or of a financial cycle.

3.2 Diagnostic Expectations but No Financial Frictions

Consider next an economy with diagnostic expectations and without financial frictions. That is, the extrapolation weight $\hat{\mu} > 0$ is positive, net worth $n \in \mathbb{R}$ can be negative, and leverage limit $\lambda = +\infty$ is unbounded.

Corollary 2. *In the economy with diagnostic expectations and without financial frictions, sentiment ω is the only relevant state that influences the equilibrium outcome. A threshold state $\bar{\omega} < 0$ exists such that*

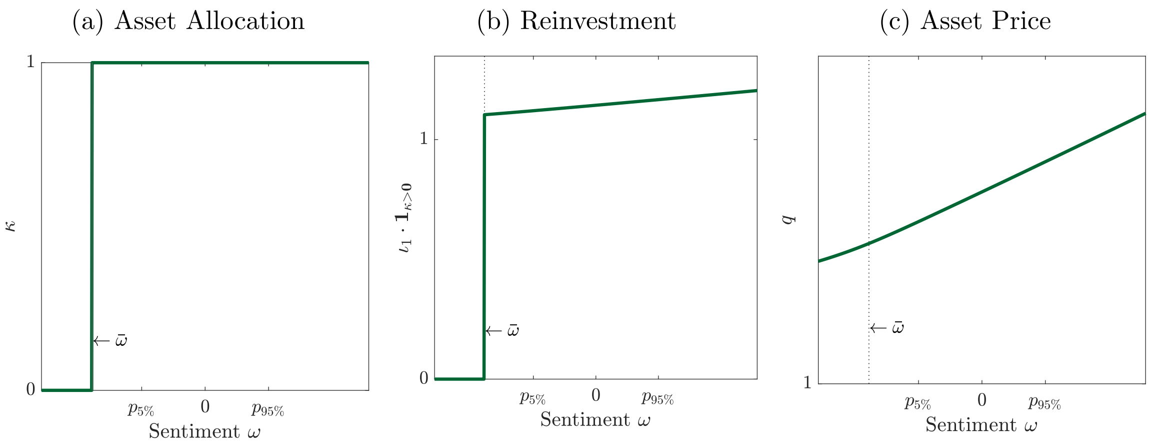
$$\begin{array}{ll} \textit{if } \omega < \bar{\omega} \Rightarrow & \kappa = 0 \; , \; \alpha_1 < 0 \; , \; \alpha_2 = 0 \; ; \\ \textit{if } \omega > \bar{\omega} \Rightarrow & \kappa = 1 \; , \; \alpha_1 = 0 \; , \; \alpha_2 < 0 \; ; \end{array} \; . \tag{31}$$

The threshold state $\bar{\omega} < 0$ is the solution to

$$\alpha_1 = \alpha_2 = 0 \Rightarrow \frac{A_1 - \iota_1 - A_2}{q} + \mathcal{I}_1(\iota_1) + (\sigma_q + \hat{\mu}\omega)\,\sigma_1 = 0 \; . \tag{32}$$

The equilibrium outcome features two distinct regimes. When $\omega < \bar{\omega}$, the economy operates in a *precautionary* regime, in which the aggregate quantity of the asset is allocated to the unproductive technology and the asset is priced according to $\alpha_2 = 0$. In this regime, households are the marginal buyers of the asset, whereas financiers strictly prefer to acquire debt rather than to operate the productive technology. Output flows $y/k = A_2 < A_1$ are low, as are asset price q, aggregate growth rate $\mu_k = 0$, and aggregate risk $\sigma_k = 0$. By contrast, when $\omega > \bar{\omega}$, the economy operates in a *non-precautionary* and *financially unconstrained* regime, in which the aggregate quantity of the asset is allocated to the productive technology, financiers are the marginal buyers of the asset, and households strictly prefer to hold debt rather than to operate the unproductive technology. Formally, $\kappa = 1$ and $\alpha_1 = 0$. In this case, aggregate output flows $y/k = A_1$, asset price q, aggregate growth rate $\mu_k = \mathcal{I}_1(\iota_1)$, and aggregate risk $\sigma_k = \sigma_1$ are high.[22] The outcome repeatedly alternates between these two regimes according to the law of motion (29). This law of motion generates a stationary distribution of sentiment of $\omega \sim \mathcal{N}\left[0, 1/\left(2\delta\right)\right]$.[23]

Figure 1: Diagnostic Expectations and No Financial Frictions



Notes. The figure plots the allocation of the asset between the technologies (panel a), the reinvestment rate conditional on a positive allocation of the asset to the productive technology (panel b), and the asset price (panel c) as a function of the relevant state of the economy, i.e., sentiment ω. Variables are normalized by their respective value in the economy in Section 3.1. Threshold $\bar{\omega} < 0$ separates the precautionary and the non-precautionary regimes. Point $p_{x\%}$ in the x-axis, with $x \in \{5; 95\}$, indicates the $x\%$-percentile of the limiting distribution of sentiment.

[22] The threshold state that separates the two regimes is negative because (i) the asset price is positively related to sentiment—that is, $\sigma_q > 0$—and (ii) the productive technology is riskier but yields higher dividend returns than the unproductive technology.

[23] Note that the law of motion of sentiment is *perceived* to fluctuate according to $d\omega = -\left(\delta - \hat{\mu}\right)\omega dt + dZ$. This implies that, for any given level of sentiment, agents expect sentiment to revert less strongly to its unconditional mean $E(\omega) = 0$ than what it actually does under the true data generating process. The law of motion also implies a perceived stationary distribution of sentiment of $\omega \sim \mathcal{N}\left[0, 1/\left(2\left(\delta - \hat{\mu}\right)\right)\right]$. As a consequence, relative to the rational expectations benchmark, agents assign less likelihood to moderate values of sentiment (i.e., values around $E(\omega) = 0$) in the long run, but more likelihood to extreme values. These properties naturally follow from the non-rational form of extrapolation assumed in expectation formation.

This economy exhibits recurrent boom-bust cycles in aggregate output, asset prices, and economic growth rates. The driver of the cycles is sentiment. Risk-taking in real investments κ, price q, growth rate $\mu_k = \kappa \mathcal{I}_1(\iota_1)$, and aggregate risk $\sigma_k = \kappa\sigma_1$ are pro-cyclical. Forecast errors $-\hat{E}[dZ] = -\hat{\mu}\omega \neq 0$ are instead counter-cyclical. Finally, procyclical asset prices amplify the positive feedback between prices and reinvestment when sentiment is sufficiently high. This effect contributes to generate higher asset prices and reinvestment rates relative to their corresponding levels in the first economy (Figure 1, panel c).[24]

3.3 Financial Frictions but Rational Expectations

Now consider an economy with rational expectations and financial frictions. That is, expectation weight $\hat{\mu} = 0$ is null, net worth $n \geq 0$ cannot be negative, and leverage limit $\lambda < +\infty$ is bounded.

Corollary 3. *In the economy with rational expectations and financial frictions, wealth share η is the only relevant state that influences the equilibrium outcome. A threshold state $\bar{\eta} \in (0,1)$ exists such that*

$$\begin{array}{lll} \text{if } \eta < \bar{\eta} \Rightarrow & \kappa = \lambda\eta < 1 \ , & \alpha_1 > 0 \ , \ \alpha_2 = 0 \ ; \\ \text{if } \eta > \bar{\eta} \Rightarrow & \kappa = 1 \ , & \alpha_1 = 0 \ , \ \alpha_2 < 0 \ ; \end{array} . \tag{33}$$

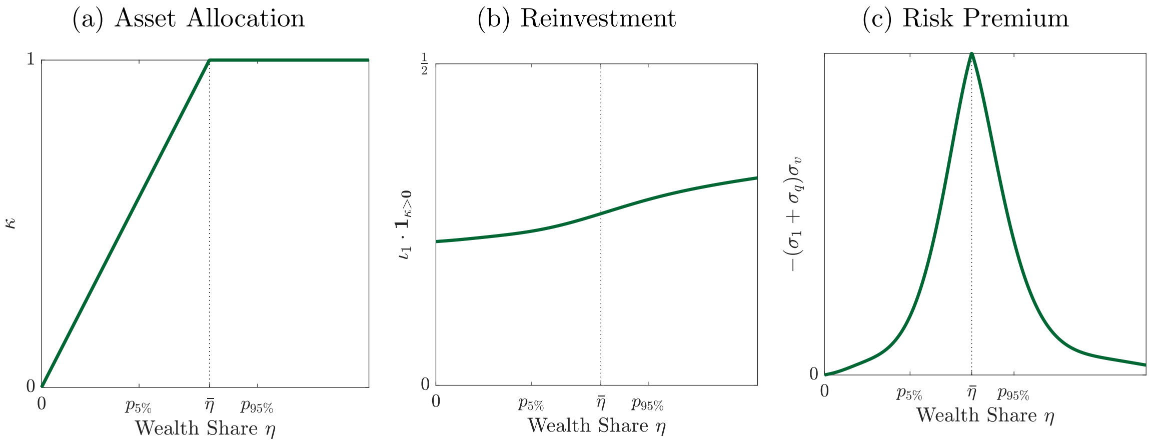
The threshold state $\bar{\eta} \in (0,1)$ is $\bar{\eta} = \frac{1}{\lambda}$.

As in the economy presented in section 3.2, the equilibrium outcome features two well-delimited regimes. In contrast to that economy, however, the regimes are only determined by the financial capacity of financiers to acquire assets and operate the productive technology. Specifically, when $\lambda\eta < 1$, the economy operates in a *financially constrained* regime in which financiers are constrained by their leverage limit to acquire assets. Accordingly, households hold the remnant share of the asset and are the marginal buyers. That is, $\kappa = \lambda\eta < 1$ and $\alpha_2 = 0 < \alpha_1$ (Figure 2, panel (a)). By contrast, when $\lambda\eta \geq 1$, the economy operates in a *financially unconstrained* regime in which financiers are marginal

[24] In Figure 1, the price and reinvestment exceed their levels in the frictionless economy when sentiment is low. This effect is a consequence of an asymmetric effect of sentiment on the price. High sentiment exerts upward pressure on the price, whereas low sentiment exerts downward pressure. These pressures are not only exerted on impact, but also effective throughout the state space, because the price is forward-looking. The upward pressure is relatively stronger, because when sentiment is low, the aggregate quantity of the asset is allocated to the unproductive technology, which eliminates the exposure of the asset to quality risk as well as the direct negative effect of low sentiment on the price.

buyers, hold the aggregate quantity of the asset, and households only hold debt issued by financiers.

Figure 2: Rational Expectations and Financial Frictions

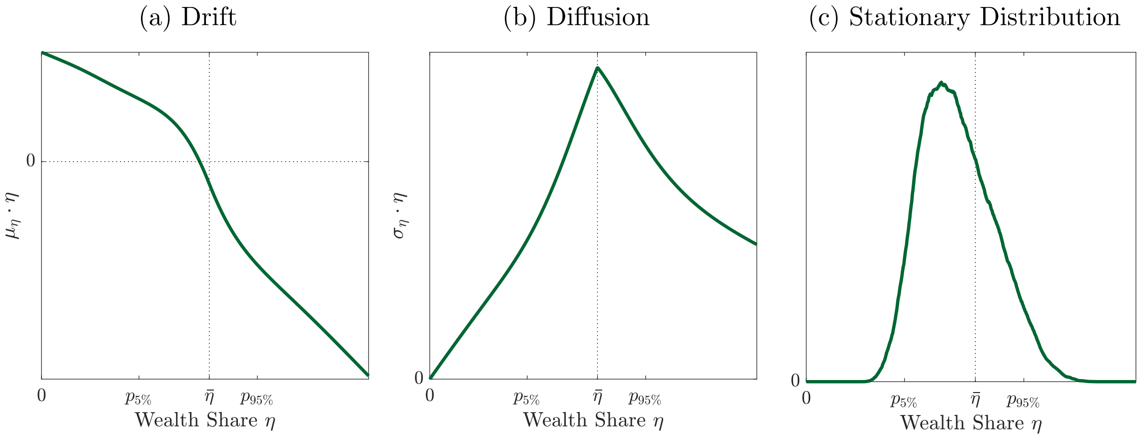


Notes. The figure plots the allocation of the asset between the technologies (panel a), the reinvestment rate conditional on a positive allocation of the asset to the productive technology (panel b), and the price of risk (panel c) as a function of the relevant state of the economy, i.e., financiers' wealth share η. The variables in the figure (except the risk premium) are normalized by their values in the economy in section (3.1). Threshold $\bar{\eta}$ separates the financially constrained and unconstrained regimes. Point $p_{x\%}$ indicates the x%-percentile of the limiting distribution of the wealth share of financiers.

Aggregate output and the asset price are increasing in the wealth share—as is investment rate ι_1—because financiers operate the productive technology. By contrast, value v is decreasing in the wealth share, because the rents from operating the productive technology are positive $\alpha_1\lambda > 0$ when the wealth share is low $\eta < 1/\lambda$, and null otherwise. A counter-cyclical marginal net worth v creates a negative risk-premium term in (19), $(\sigma_q + \sigma_1)\,\sigma_v \leq 0$, which reflects financiers' effective risk aversion in the presence of financial constraints (Figure 2, panel (c)).

The equilibrium outcome repeatedly alternates between the two financial regimes (Figure 3) according to the law of motion (26). Fluctuations display two properties. First, fluctuations are mean reverting around a stochastic steady state (i.e., η such that $\mu_\eta\eta = 0$—panel a). This is a consequence of the counter-cyclicality of rents $\alpha_1\lambda$ and the a-cyclicality of dividend payouts. Second, fluctuations are stochastic (panel b), which is a consequence of a positive interaction between net-worth risk $\sigma_\eta = (\phi - 1)\,\sigma_q + (1 - \eta)\,\phi\sigma_1$ and price risk $\sigma_q = \varepsilon_q\sigma_\eta$, where $\varepsilon_q \equiv (\partial q/\partial\eta)\,(\eta/q) \geq 0$ is the elasticity of the asset price with respect

Figure 3: Rational Expectations and Financial Frictions
Dynamics of Wealth Share



Notes. The figure plots the drift (panel a), the diffusion (panel b), and the limiting distribution (panel c) of the wealth share that satisfies (26) $d\eta = \mu_\eta \eta dt + \sigma_\eta \eta dZ$. Threshold $\bar{\eta}$ separates the financially constrained and the financially unconstrained regimes. Point $p_{x\%}$ indicates the x%-percentile of the limiting distribution of the wealth share of financiers.

to the wealth share.[25] This interaction generates endogenous financial amplification of disturbances to the wealth share and the asset price according to

$$\frac{\sigma_\eta}{\sigma_1} = \frac{\phi - \phi\eta}{1 - (\phi - 1)\,\varepsilon_q} \geq 0 \quad \text{and} \quad \frac{\sigma_q}{\sigma_1} = \frac{(\phi - \phi\eta)\,\varepsilon_q}{1 - (\phi - 1)\,\varepsilon_q} \geq 0, \text{ with } \phi = \min\left\{\frac{1}{\eta}, \lambda\right\}. \quad (34)$$

In the financially constrained regime, notably, this amplification is characterized by fire sales and a reallocation of the asset from the productive to the unproductive technology. Indeed, when negative disturbances erode the wealth share, i.e., $d\eta < 0$, financiers are forced to sell the asset to households at discount prices to meet a tighter collateral constraint.

Overall, like its counterpart in subsection 3.2, this economy exhibits recurrent boom-bust cycles in aggregate output, asset prices, and economic growth rates. Risk-taking in real investments κ, price q, growth rate $\mu_k = \kappa \mathcal{I}_1(\iota_1)$, and aggregate risk $\sigma_k = \kappa\sigma_1$ are also pro-cyclical. By contrast, sentiment does not influence economic cycles, and forecasts are not subject to systematic errors. Rather, the wealth share of financiers is the driver of the cycles, and conditional forecast errors on average are null. Finally, these cycles feature fire

[25] Formula $\sigma_q = \varepsilon_q \sigma_\eta$ follows from Ito's Lemma.

sales and asset reallocation with recessionary implications when the collateral constraint is binding and negative disturbances hit the technologies. Endogenous risk is time-varying and peaks when the collateral constraint is locally occasionally binding.

3.4 Diagnostic Expectations and Financial Frictions

Finally, consider the whole economy presented in section 2, with diagnostic expectations and financial frictions. The following corollary from proposition 1 describes the equilibrium outcome.

Corollary 4. *In the economy with diagnostic expectations and financial frictions, both sentiment* ω *and wealth share* η *influence the equilibrium outcome. Thresholds* $\bar{\omega} < 0$ *and* $\bar{\eta} \in (0, 1)$ *partition the state space as follows:*

$$\begin{aligned} &\textit{if } \omega < \bar{\omega} \Rightarrow && \kappa = 0\ , && \alpha_1 < 0\ ,\ \alpha_2 = 0\ ; \\ &\textit{if } \omega > \bar{\omega} \textit{ and } \eta < \bar{\eta} \Rightarrow && \kappa = \lambda\eta\ , && \alpha_1 > 0\ ,\ \alpha_2 = 0\ ; \\ &\textit{if } \omega > \bar{\omega} \textit{ and } \eta > \bar{\eta} \Rightarrow && \kappa = 1\ , && \alpha_1 = 0\ ,\ \alpha_2 < 0\ ; \end{aligned} \tag{35}$$

Threshold process $\bar{\omega}$ *is the solution to*

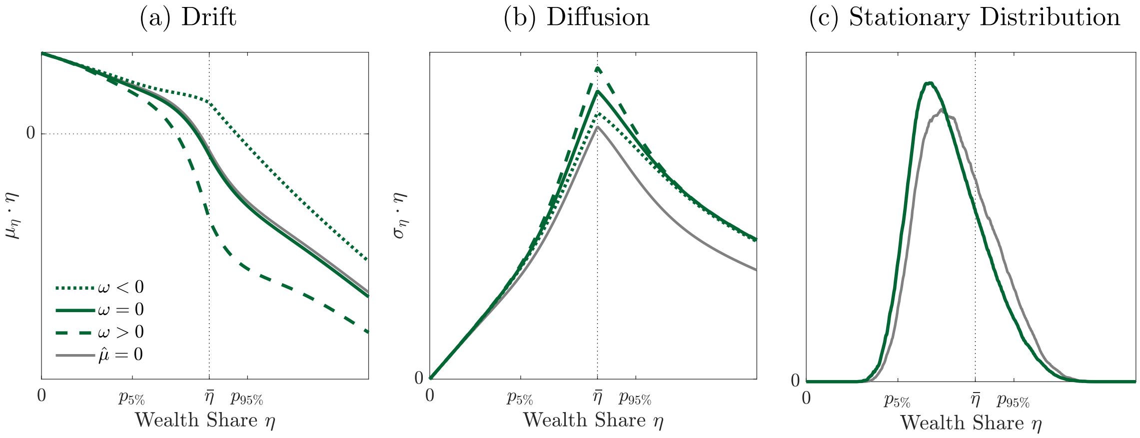
$$\alpha_1 = \alpha_2 = 0 \Rightarrow \frac{A_1 - \iota_1 - A_2}{q} + \mathcal{I}_1(\iota_1) + (\sigma_q + \hat{\mu}\omega)\,\sigma_1 + (\sigma_q + \sigma_1)\,\sigma_v = 0\ , \tag{36}$$

and the threshold state $\bar{\eta} \in (0, 1)$ *is* $\bar{\eta} = \frac{1}{\lambda}$*.*

With financial frictions, the characterization of sentiment threshold $\bar{\omega}$ includes a risk-premium term $(\sigma_q + \sigma_1)\,\sigma_v \leq 0$. Everything else being the same, this term reduces the perceived relative value of the productive technology, because operating that technology exposes de facto risk-averse financiers to aggregate risk. The equilibrium outcome repeatedly alternates among the aforementioned three regimes—*precautionary*, *financially constrained*, and *financially unconstrained*—according to the laws of motion (26) and (29). In these cycles, sentiment and the wealth share co-move positively, because current disturbances positively affect both the perceived likelihood of future disturbances and the excess returns earned by financiers—as respectively shown by equations (29) and (34).

Figure (4) highlights the key takeaways from this economy and our main positive result: diagnostic expectations intensify financial instability relative to the rational expectation benchmark—as measured by a leftward shift in the stationary marginal distribution of the

Figure 4: Diagnostic Expectations and Financial Frictions
Dynamics of Wealth Share



Notes. The figure plots the drift (panel a), the diffusion (panel b), and the limiting distribution (panel c) of the wealth share that satisfies (26) $d\eta = \mu_\eta \eta dt + \sigma_\eta \eta dZ$. Grey lines refer to the economy with rational expectations (i.e., section 3.3), whereas blue lines refer to the economy with diagnostic expectations (i.e., section 3.4). For the latter economy, variables are plotted for three different values of sentiment.

wealth share of financiers (Figure 4, panel c and Table 2).[26] This additional instability results from the following two interactions between diagnostic expectations and financial frictions.

First, compared with rational expectations, the positive co-movement of sentiment and the wealth share amplifies the interaction between risks σ_η and σ_q. For example, after an adverse shock, falling sentiment drives down asset prices and erodes the wealth share, further strengthening this feedback (Figure 4, panel b).[27] In the financially constrained regime, sufficiently large shocks thus trigger deeper fire sales and more pronounced recessionary asset reallocation.

Second, everything else being the same, when both sentiment and financiers' wealth share are high, inflated forecasts $\hat{E}[dZ|\cdot] = \hat{\mu}\omega > 0$ exert upward pressure on the asset price. The higher price then deteriorates excess return $E[dR_1 - r|\cdot]$, which eventually hurts the profitability of financiers—as measured by conditional average growth rate $\mu_\eta \eta =$

[26]We also verify that the stationary distribution under rational expectations first-order stochastic dominates the stationary distribution under diagnostic expectations, namely, that the probability that financial conditions are lower than any $\eta \in (0, 1)$ is higher under the latter expectations.

[27]Put more formally, $dZ < 0$ exerts downward pressure on ω on impact, which reduces the first term in $\sigma_q = \varepsilon_{q,\omega}/\omega + \varepsilon_{q,\eta}\sigma_\eta$, with $\varepsilon_{q,\omega} \equiv (\partial q/\partial \omega)(\omega/q) \geq 0$ and $\varepsilon_{q,\eta} \equiv (\partial q/\partial \eta)(\eta/q) \geq 0$. This reduction, in turn, intensifies the interaction between the first term in $\sigma_\eta = (\phi - 1)\sigma_q + \phi\sigma_1 - \sigma_k$ and σ_q.

$E[d\eta|\cdot]$. The opposite naturally happens when sentiment and the wealth share are instead low. However, because forecast errors and aggregate risk $\sigma_k = \kappa\sigma_1$ are negatively related, the effects of forecast errors on the wealth share are asymmetric, with the former effects dominating (panel a).[28] This asymmetry then exerts leftward pressure on the stationary marginal distribution of the wealth share relative to the rational expectation benchmark.

Table 2, panel (a) reports key statistics to gauge the quantitative importance of the increased financial instability and the deterioration of economic activity resulting from diagnostic expectations. Relative to the benchmark economy with rational expectations, the average level of financial conditions (as measured by average wealth share $E[\eta]$) falls by around 7%, while the average level of detrended output and economic growth rate decrease by around 1.6% and 1 percentage point (pp) respectively. Both financial and economic volatility increase, as reflected in the increase of the standard deviations of asset price, output and growth. Downside risks in output and economic growth (as measured by a fall in skewness) increase by approximately 0.12 and 0.17 points. In a related vein, tail risks in financial conditions, output and growth (as measured by an increase in kurtosis) increase by approximately 0.27, 0.83 and 0.42 points respectively. The frequencies of the financially constrained and precautionary regimes increase by around 10.94 pp and 0.23 pp, respectively. Diagnostic expectations imply an annualized loss in lifetime discounted consumption of 2.66%, indicating a quantitatively large macroeconomic impact.

To evaluate the contribution of the behavioral and financial frictions to the amplification of financial instability, Table 2, panels b and c report sensitivities of economic outcomes to the diagnostic weight $\hat{\mu}$ and the leverage constraint λ, respectively. In panel b, a 10 % increase in $\hat{\mu}$ further degrades financial conditions and economic performance—output and growth decline, and asset-price and real-variable volatility rise—resulting in more frequent precautionary and constrained regimes (panel d). By contrast, panel c shows that an increase in the the leverage constraint λ has little impact on diagnostic amplification

[28]The asymmetry is also reflected in the high variation in $E[d\eta|\eta,\omega]$ across sentiment ω when wealth share $\eta > \bar{\eta}$ is high and in the reduction of the variation as the wealth share shrinks. In our simulations, when the wealth share is high and sentiment is non-negative, the aggregate quantity of the asset is allocated to the productive technology. Because in that region the exposure of the asset to aggregate risk is then high, variations in sentiment have a large effect on the asset price and consequently also on the conditional average growth rate of the wealth share, as suggested by the large difference between the solid and the dashed green lines in panel (a) when $\eta > \bar{\eta}$. By contrast, when wealth share $\eta < p_{5\%}$ is extremely low, almost all of the asset is allocated to the unproductive technology, even when sentiment is high. Because the exposure of the asset to aggregate risk is then low, variations in sentiment have only a marginal effect on the asset price and the conditional average growth rate, as indicated by the negligible difference between the green lines in that region.

Table 2: Economic Implications of Intensified Financial Instability under Diagnostic Beliefs

Panel (a) Baseline Economy

	Av.	Median.	Std. Dev.	Skew.	Kurt.
Asset price q	+4.45%	+4.34%	+8.81%	+0.16	+0.40
Financial condition η	-6.18%	-7.14%	-2.79%	+0.13	+0.22
Leverage ϕ	+1.16%	+0.00%	+1.13%	-3.45	+48.88
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-1.45%	-2.22%	+8.93%	+0.08	+0.82
Growth $I \cdot \kappa$	-0.003 p.p.	-0.02 p.p.	+0.02 p.p.	+0.14	+0.49

Panel (b) Higher Diagnostic Weight $\hat{\mu}$ (+10%)

	Av.	Median.	Std. Dev.	Skew.	Kurt.
Asset price q	+5.00%	+4.88%	+8.91%	+0.20	+0.52
Financial condition η	-7.70%	-8.48%	-4.47%	+0.15	+0.29
Leverage ϕ	+1.18%	+0.00%	+15.88%	-4.76	+61.53
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-1.84%	-2.63%	+12.17%	-0.05	+1.70
Growth $I \cdot \kappa$	-0.01 p.p.	-0.03 p.p.	+0.02 p.p.	+0.05	+1.06

Panel (c) Higher Leverage Constraint Parameter λ (+10%)

	Av.	Median.	Std. Dev.	Skew.	Kurt.
Asset price q	+4.58%	+4.39%	+10.76%	+0.19	+0.26
Financial condition η	-7.24%	-8.30%	-1.15%	+0.15	+0.03
Leverage ϕ	+2.23%	+0.00%	-3.81%	-1.64	+14.93
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-1.50%	-2.79%	+15.88%	+0.21	+0.41
Growth $I \cdot \kappa$	-0.002 p.p.	-0.03 p.p.	+0.03 p.p.	+0.24	+0.11

Panel (d) Equilibrium regimes

	Baseline	Higher $\hat{\mu}$	Higher λ
Precautionary regime	+0.19 p.p.	+0.44 p.p.	+0.21 p.p.
Constrained regime	+8.21 p.p.	+10.27 p.p.	+11.04 p.p.
Unconstrained regime	-8.41 p.p.	-10.71 p.p.	-11.25 p.p.

Notes. The table reports in panels (a) to (c) the percentage change of economic outcomes in the economy with diagnostic beliefs relative to the economy with rational expectations. The bottom panel reports the change in the frequency of equilibrium regimes.

compared with the rational-expectations benchmark.

Overall, the economy with financial frictions and diagnostic beliefs features both systematic forecast errors and quantitatively large amplification of financial frictions on economic activity. Relative to the economy with rational expectations, fire sales are stronger and reallocation of assets across technologies is swifter. Moreover, because of countercyclical forecast errors and pro-cyclical risk-taking, financial markets are more unstable, economic cycles are more volatile, and economic performances are degraded. These results are closely in line with the views espoused by the FIH, and in particular highlight the substantial contribution of diagnostic expectations to financial instability, relative to the rational expectation counterpart economy.

4 Robustness

The key takeaway from the positive analysis is that diagnostic expectations intensify financial instability relative to the rational expectations benchmark. In this section, we demonstrate that this finding is robust to alternative modeling specifications.

4.1 Alternative Production Technologies

The baseline model proposes three differences between production technologies: productivity A_j, reinvestment opportunity $\mathcal{I}_j(\iota_j)$, and exposure to capital-quality risk σ_j. In this subsection, we investigate the sensitivity of the amplification result to each of these. To do so, we derive the equilibrium for three alternative specifications, each of which eliminates one of the differences. Table 3 presents the percentage change in financial variables under diagnostic beliefs relative to an economy with rational expectations. The table reports also Anderson-Darling (AD) statistics to compare related cumulative distribution functions.[29]

The primary factor driving the amplification of financial instability under diagnostic beliefs is the difference in risk across technologies, i.e., the risk-taking channel. Indeed, eliminating the gap in risk exposure ($\sigma_2 = \sigma_1$) results in a modest decrease in asset prices, the smallest contraction in average financial conditions, the lowest increase in the volatility of these variables, and nearly identical stationary distributions for financial variables under diagnostic and rational beliefs.

[29] Formally, the AD statistics computes the sum of squared differences between two cumulative distribution functions. Smaller values indicate more similar distributions.

Table 3: Sensitivity of Diagnostic Amplification to Production Technologies

	$\%\Delta E(q)$	$\%\Delta\sigma(q)$	AD q	$\%\Delta E(\eta)$	$\%\Delta\sigma(\eta)$	AD η
Baseline	+4.45%	+8.81%	5.56	-6.188%	-2.79%	1.10
No Productivity Gap	+4.66%	+64.82%	14.53	-5.49%	-4.99%	0.82
No Reinvestment Gap	+6.91%	+18.28%	17.96	-8.80%	-5.58%	2.01
No Risk Gap	+0.24%	+6.49%	0.02	-2.73%	-0.37%	0.24

Notes. The table presents statistics comparing the moments and distributions of economic outcomes under diagnostic expectations versus rational expectations for both the baseline economy and alternative specifications that eliminate differences across technologies. The Anderson-Darling (AD) statistics report the sum of squared differences between cumulative distribution functions (CDFs).

In contrast, closing either the productivity or reinvestment gap does lead to substantial diagnostic amplification of financial instability. Equalizing productivity across technologies ($A_2 = A_1$) results in increased asset price and volatility, while equalizing reinvestment opportunities ($\chi_2 = \chi_1$) primarily leads to a deterioration of financial conditions.

These results emphasize that the risk-taking channel – which operates through differences in the exposure of production technologies to aggregate risk – is the central mechanism explaining how diagnostic expectations contribute to increased financial instability. This underscores the importance of the interplay between counter-cyclical forecast errors and endogenous pro-cyclical aggregate risk in accounting for heightened financial instability.

4.2 Generic Process in Diagnostic Expectation Formation

In the baseline model, agents use a weighted average of exogenous past disturbances to form expectations about future disturbances. We now characterize equilibrium with a general process for diagnostic expectation formation, where agents rely on a generic Ito path $\{dX_s\}_{s<t}$ to form expectations about a generic Ito variable dY_t. The proposition shows how a generic expectation process translates into a forecast operator for disturbances and an Ito process for sentiment.

Proposition 3. *If agents rely on Ito path $\{dX_s\}_{s<t}$ to form diagnostic expectations about Ito variable dY_t, the implied diagnostic expectation operator over disturbance dZ_t is*

$$\hat{E}_t \left[dZ_t\right] = \hat{\mu} \frac{\omega_t}{\sigma_{Y,t} Y_t} dt \; , \tag{37}$$

where $\sigma_{Y,t} \in \mathbb{R}$ is the diffusion of the variable and where sentiment $\omega_t \in \mathbb{R}$ is given by

$$\omega_t = \int_0^t e^{-\delta(t-s)} dX_s \ . \tag{38}$$

This result allows us to characterize equilibrium for any alternative specification of diagnostic expectations. To illustrate the proposition, we follow the literature (e.g., Maxted (2023)) and present in Table 4, panel (a), economic statistics for when agents form diagnostic beliefs based on the endogenous evolution of the capital stock, rather than on the exogenous capital quality shock as in the baseline (Table 2, panel a).[30] The main findings reported in Section 3 are maintained: diagnostic expectations intensify financial instability relative to the rational expectations benchmark, with adverse implications for output and growth.

4.3 Endogenous Collateral Constraint

In the baseline model, the collateral constraint imposes an exogenous limit on leverage, $\phi_t \leq \lambda$. In this subsection, we consider an alternative constraint that generates an endogenous leverage limit $\phi_t \leq \nu v_t$, where $\nu \geq 1$ is a parameter and $v_t \geq 1$ represents the marginal value of net worth.[31]

An endogenous leverage limit νv_t introduces additional interactions between diagnostic expectations and financial frictions. Specifically, higher sentiment ω_t enhances perceived rents $\alpha_{1,t}\phi_t > 0$, increases the marginal value v_t, and thereby relaxes the leverage constraint $\phi_t \leq \nu v_t$. Conversely, lower sentiment has the opposite effect. This interaction has two notable consequences for the allocation of the asset. In the financially constrained regime, higher sentiment increases the share of the asset allocated to the productive technology. Second, outside the precautionary regime, higher sentiment lowers the threshold $\bar{\eta}$ that distinguishes the two financial regimes.

To study how these effects shape the equilibrium outcome and influence financial stability, we set $\nu = 2.7$, to achieve comparability with the same average leverage as in the baseline specification. Table 4, panel (b) presents financial and real equilibrium moments

[30] For comparability with the baseline economy, we set $\mu_h = 0.006$ and $\delta = 0.8$ in the economy with diagnostic beliefs on capital. These parameters yield a correlation between wealth share and sentiment of 0.71 and an output bias of 0.78%, as in the baseline economy

[31] This constraint is common in the literature, e.g., Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). It is derived from an agency problem, where, upon default, financiers lose access to their company, whose value is V_t.

Table 4: Alternative Specifications

Panel (a) Diagnostic Beliefs on Capital

	Av.	Median	Std. Dev.	Skew.	Kurt.
Asset price q	-0.64%	-0.82%	+0.66%	+0.32	+0.71
Financial condition η	-3.84%	-4.91%	-2.69%	+0.18	+0.36
Leverage ϕ	+0.44%	+0.00%	+25.06%	-3.66	+44.69
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-0.94%	-1.52%	+7.61%	-0.29	+2.58
Growth $I \cdot \kappa$	-0.02 p.p.	-0.04 p.p.	+0.01 p.p.	-0.13	+1.57

Panel (b) Endogenous Collateral Constraint

	Av.	Median	Std. Dev.	Skew.	Kurt.
Asset price q	+2.38%	+2.291%	-4.05%	+0.27	+0.69
Financial condition η	-5.93%	-6.55%	-5.31%	+0.19	+0.36
Leverage ϕ	+0.02%	+0.32%	-11.90%	-1.78	+17.16
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-1.65%	-1.98%	+12.11%	-0.02	+1.86
Growth $I \cdot \kappa$	-0.02 p.p.	-0.03 p.p.	+0.01 p.p.	-0.06	+1.11

Panel (c) Equilibrium regimes

	DB on dk	Endo CC
Precautionary regime	+0.46 p.p.	+0.16 p.p.
Constrained regime	+5.90 p.p.	+8.03 p.p.
Unconstrained regime	-6.35 p.p.	-8.19 p.p.

Notes. The table reports in panels (a) and (b) statistics related to changes of economic outcomes in the economy with diagnostic beliefs relative to the economy with rational expectations. The bottom panel reports the change in the frequency of equilibrium regimes.

for the economy with endogenous collateral constraints. The main takeaway is that the endogenous leverage limit νv_t does not significantly change the equilibrium outcome or financial instability. Indeed, rents $\alpha_{1,t}\phi_t$ are weakly correlated with sentiment ω_t, and their average value is low. These features combined make the value v_t insensitive to sentiment, as reflected by an average elasticity $E[\varepsilon_{v,\omega}] = -0.4\%$, where $\varepsilon_{v,\omega} \equiv \frac{\partial v}{\partial \omega}\frac{\omega}{v}$. These results emphasize that the amplifying effects of diagnostic expectations on financial instability primarily operate through sentiment-based fluctuations in asset prices, rather than through sentiment-driven variations in leverage limits.

5 Normative Analysis

The positive analysis has shown how diagnostic expectations intensify financial instability compared to the rational expectations benchmark. From a normative perspective, this increased instability may exacerbate allocative inefficiencies, as the economy may experience pecuniary externalities arising from financial frictions (Dávila and Korinek 2017). To explore these considerations, we characterize in this section a constrained-efficient, socially optimal allocation and compare its allocative properties with those of the competitive equilibrium presented in section 3.

5.1 Socially Optimal Allocation

To characterize a constrained-efficient allocation, we consider a social welfare problem that aligns with the incentive constraints of private agents and satisfies the resource constraints of the competitive equilibrium. The social planner evaluates welfare using an expectation weight $\tilde{\mu}$ within the interval $[0, \hat{\mu}]$. This assumption ensures that the planner's expectations can be more rational but not more diagnostic than those of private agents. The socially optimal allocation is defined as follows.

Definition 2. *The socially optimal allocation is the solution to the optimization problem in the following dynamic program:*

$$
\begin{aligned}
r\tilde{u} \;=\; & \max_{\{\iota_1,\kappa\}} \Bigg\{ \kappa \left\{ A_1 - \iota_1 + \left[\mathcal{I}_1(\iota_1) + \sigma_1\tilde{\mu}\omega\right]\tilde{u} \right\} + (1-\kappa)\, A_2 + \frac{\partial \tilde{u}}{\partial \omega}\left(-\delta\omega + \tilde{\mu}\omega + \kappa\sigma_1\right) \\
& + \frac{\partial \tilde{u}}{\partial \eta}\left(\mu_\eta \eta + \sigma_\eta \eta \tilde{\mu}\omega + \sigma_\eta \eta \kappa \sigma_1\right) + \frac{1}{2}\frac{\partial^2 \tilde{u}}{(\partial \omega)^2} + \frac{\partial^2 \tilde{u}}{\partial \omega \partial \eta}\sigma_\eta \eta + \frac{1}{2}\frac{\partial^2 \tilde{u}}{(\partial \eta)^2}\left(\sigma_\eta \eta\right)^2 \Bigg\}, \qquad (39)
\end{aligned}
$$

with

$$\iota_1 \in [0, A_1] \ \ \textit{and}\ \kappa \in [0, \min\{\lambda\eta, 1\}] \ , \tag{40}$$

where expressions for drift μ_η *and diffusion* σ_η *satisfy* (27) *and* (28)*, with* μ_q *and* σ_q *being characterized by Ito's Lemma. Mapping* v *satisfies* (2) *and mapping* q *is consistent with the following three mutually exclusive relationships:*

$$\begin{array}{lllll} \textit{Relationship \#1:} & \lambda\eta \geq 1 \ , & \kappa = 1 \ , & \alpha_1 = 0 \ , & \alpha_2 < 0 \ ; \\ \textit{Relationship \#2:} & \lambda\eta \geq 1 \ , & \kappa \in [0, 1) \ , & & \alpha_2 = 0 \ ; \\ \textit{Relationship \#3:} & \lambda\eta < 1 \ , & \kappa \in [0, \lambda\eta] \ , & & \alpha_2 = 0 \ ; \end{array} \ , \tag{41}$$

where α_1 *and* α_2 *satisfy* (19) *and* (16)*.*

The mapping $\tilde{u} \geq 0$ is the present discounted value of consumption per unit of the asset under the expectation weight $\tilde{\mu} \in [0, \hat{\mu}]$. This definition assumes the planner has no commitment, in the sense that controls $\{\iota_1, \kappa\}$ in program (39) are set state by state, treating mappings $\{\tilde{u}, q, v\}$ and their partial derivatives with respect to the states as given. Expression (41) specifies three mutually exclusive relationships between asset allocation κ and private valuations of technologies α_1 and α_2. These relationships differ from those in the competitive equilibrium (21) in two ways. First, the planner can allocate the asset to the productive technology (i.e., $\kappa > 0$) even when the expected excess return of that technology is negative according to private valuations ($\alpha_1 < 0$). Second, the planner can restrict the allocation of the asset to the productive technology ($\kappa < \lambda\eta$), even when the expected excess return of the productive technology is positive ($\alpha_1 > 0$). This flexibility allows the planner to address externalities, as reflected by differences between social and private valuations of the asset and its allocation between the production technologies.

Proposition 4. *The socially optimal reinvestment rate solves*

$$\mathcal{I}_1'(\iota_1) = \frac{1 + \frac{1}{1-(\phi-1)\varepsilon_{q,\eta}} \frac{1}{q} \frac{\partial \tilde{u}}{\partial \eta}}{\tilde{u} + \frac{1-\eta}{1-(\phi-1)\varepsilon_{q,\eta}} \frac{\partial \tilde{u}}{\partial \eta}} \ . \tag{42}$$

The socially optimal share κ *maximizes the RHS in (39). The candidate solutions are*

$\kappa = 0$, $\kappa = \min\{\lambda\eta, 1\}$ *and any interior* $\kappa \in (0, \min\{\lambda\eta, 1\})$ *that solves*

$$0 = \left[\frac{A_1 - \iota_1 - A_2}{\tilde{u}} + \mathcal{I}_1(\iota_1) + (\sigma_{\tilde{u}} + \tilde{\mu}\omega)\,\sigma_1\right] + \varepsilon_{\tilde{u},\eta}\left[\frac{\partial \mu_\eta}{\partial \kappa} + (\tilde{\mu}\omega + \kappa\sigma_1)\frac{\partial \sigma_\eta}{\partial \kappa}\right] + \frac{1}{\tilde{u}}\left(\frac{\partial^2 \tilde{u}}{(\partial \eta)^2}\sigma_\eta \eta + \frac{\partial^2 \tilde{u}}{\partial \eta \partial \omega}\right)\frac{\partial \sigma_\eta}{\partial \kappa}\eta\,, \quad (43)$$

where $\frac{\partial \mu_\eta}{\partial \kappa}$ *and* $\frac{\partial \sigma_\eta}{\partial \kappa}$ *are the partial derivatives of* μ_η *and* σ_η *with respect to* κ*, respectively.*

Comparing these expressions with their counterparts of the competitive equilibrium—that is, (17) and (21)— reveals two key differences between the socially optimal and the equilibrium allocations. First, the planner internalizes the collective impact of individual decisions on aggregate variables and dynamics, whereas individual agents in the competitive equilibrium do not. This difference is evident in the expression for the socially optimal rate of reinvestment (42), where the second terms in the numerator and denominator, absent in (17), reflect this consideration. Second, the planner bases her decisions on the social value of the asset as perceived under her own expectations, $\tilde{u}$, whereas private agents in the competitive equilibrium make decisions based on the asset price, q. Consequently, any wedge between q and $\tilde{u}$ indicates inefficiencies in the competitive equilibrium due to pecuniary externalities.

Together, the optimality conditions in Proposition 4 and Definition 2 analytically characterize a system of second-order PDEs for the socially optimal allocation.

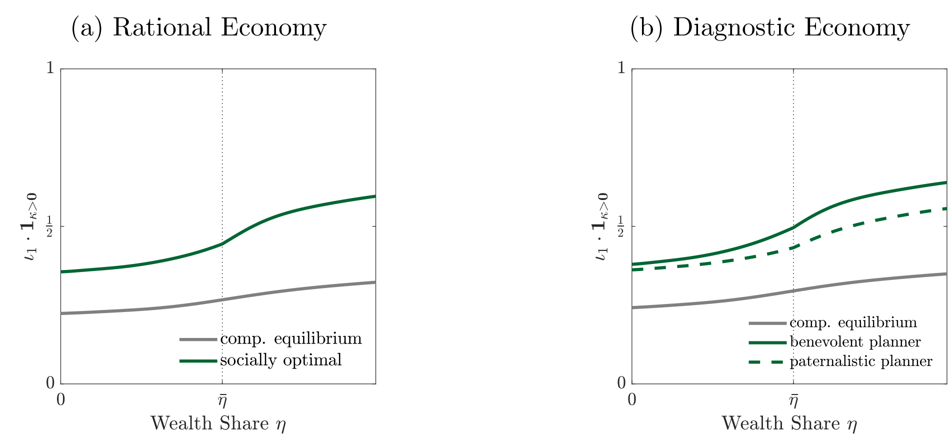
Proposition 5. *The socially optimal allocation and its associated mappings* $\{\tilde{u}, v, q\}$ *are analytically characterized by a system of second-order PDEs for these mappings in terms of the state variables* $\{\omega, \eta\}$*.*

5.2 Properties of The Economy under Socially Optimal Allocation

We now describe the behavior of the economy under the socially optimal allocation. For clarity, we focus on planners with expectation weights $\tilde{\mu} = 0$ and $\tilde{\mu} = \hat{\mu}$, whom we define as *paternalistic* and *benevolent*, respectively. Planners with intermediate degrees of diagnostic expectations favor allocations that lie between those implemented by these two types of planners.

Figure 5 illustrates competitive equilibrium and socially optimal reinvestment rates as a function of the wealth share of the financiers, for economies with rational (panel a) and diagnostic expectations (panel b). Even under rational expectations, the socially optimal reinvestment rate exceeds that of the competitive equilibrium. This discrepancy arises from a pecuniary externality affecting the asset price. Specifically, in the competitive equilibrium, the collateral constraint combined with non-contingent debt results in an excessively depressed asset price relative to what is socially desirable, reflected by $q < u$. This depressed asset price leads to excessively low reinvestment rates through the optimal rule $\mathcal{I}_j'(\iota_j) = 1/q$. In the economy with diagnostic expectations (panel b), a similar pecuniary externality explains why competitive reinvestment rates are lower than those chosen by a social planner. A benevolent planner selects higher reinvestment rates than a paternalistic planner because the benevolent planner perceives sentiment as fundamental information for setting the allocation. In particular, given the asymmetric effects of sentiment on perceived risk and return (see Figure 1), a benevolent planner systematically stimulates reinvestment more than what a paternalistic planner would.

Figure 5: Socially Optimal Reinvestment Rates



Notes. The figure plots the reinvestment rates of the competitive equilibrium allocation (grey lines) and the socially optimal allocation for the economy with rational expectations (panel a) and the economy with diagnostic expectations (panel b). In the latter case, it contrasts the optimal reinvestment rates under a benevolent planner (blue line), and under a paternalistic planner (red line). All of the reinvestment rates are deflated by the first-best value in the corresponding economy.

Consider next the allocation of the asset between the technologies. When agents form rational expectations, the competitive equilibrium allocation is socially optimal under our calibration. In other words, the planner has no incentive to distort the competitive equilibrium rule $\kappa = \min\{\lambda\eta, 1\}$. This finding is consistent with in the literature (e.g., Brunnermeier and Sannikov 2014), as linear preferences over consumption imply that usually leverage restrictions do not yield welfare gains from enhanced consumption smoothing or reduced consumption volatility.[32]

In the economy with diagnostic expectations and amplified financial instability, by contrast, the asset allocation is not socially optimal, for either the benevolent or the paternalistic planner, but the desired adjustments differ across them.[33] For the benevolent planner, the socially optimal allocation restricts the share of the asset allocated to the productive technology when sentiment is moderately low, as shown in Figure 6, panel (a). In other words, the precautionary regime expands. In this regime, a diagnostic benevolent planner perceives that allocating the asset to the productive technology deteriorates the expected recovery rate of the wealth share, thereby affecting future financial conditions. In contrast, a paternalistic planner reduces the share of the asset allocated to the productive technology precisely when financial amplification peaks (i.e., around threshold state $\bar{\eta}$ and when sentiment is moderately high), as shown in Figure 6, panel b. Indeed, the paternalistic planner is cautious about the financial amplification effects that arise from the interaction between diagnostic expectations and financial frictions, and therefore seeks to moderate the asset-price boom that precedes a fire-sale episode. Importantly, the benefits of these restrictions are not limited to the states where they are binding, but extend throughout the economic cycle via their general equilibrium effect on the dynamics of the asset price.

Finally, to assess the equilibrium allocative and welfare properties of a planner's intervention, Table 5 presents key variable moments in the socially optimal economy relative to the competitive equilibrium. In a rational economy, a rational planner's higher reinvestment rates slightly slow financiers' net worth accumulation, leading to a marginal decrease in average output but a significant increase in growth. Additionally, the reduction in financial volatility resulting from these interventions is beneficial. In a diagnostic economy, both benevolent and paternalistic interventions have similar effects: a marginal reduction in output and financial conditions, alongside increased growth and decreased financial volatility. However, a paternalistic planner, mindful of diagnostic amplifications

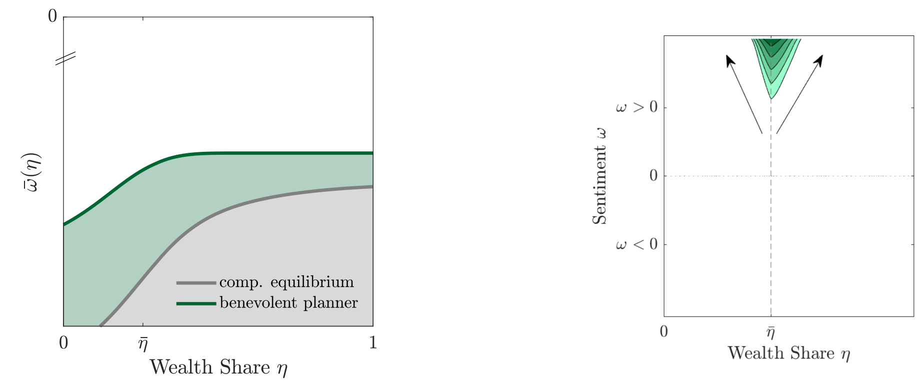
[32] Specifically, in an environment similar to ours, Brunnermeier and Sannikov (2014) also finds negligible welfare gains from altering the asset allocation relative to the competitive equilibrium. In contrast, Van der Ghote (2021) finds large welfare gains, but in his economy preferences over consumption are logarithmic.

[33] Note that one cannot infer from the asset allocative efficiency in the rational economy that a benevolent planner in a diagnostic economy would not distort the allocation of the asset. This is because diagnostic expectations generate additional financial amplification relative to the rational expectations benchmark, thereby strengthening pecuniary externalities and exacerbating allocative inefficiencies.

Figure 6: Socially Optimal Asset Allocation

(a) Benevolent Planner
Frontier Precautionary Regime

(b) Paternalistic Planner
Leverage Restrictions



Notes. This figure illustrates socially optimal restrictions on the allocation of the asset to the productive technology relative to the competitive equilibrium presented in section 3.4. Panel (a) reports the occurrence of the precautionary regime for the competitive equilibrium (grey area) and the socially optimal allocation when the planner is benevolent (blue area). Panel (b) reports restrictions implemented by a paternalistic planner in the non-precautionary regime. A darker shade means the planner imposes stronger restrictions on the share κ relative to the upper bound $\min\{\lambda\eta, 1\}$ that applies in a competitive equilibrium. The white color means no reduction in the share below $\kappa = \min\{\lambda\eta, 1\}$.

on economic outcomes, implements more prudent reinvestment as well as pro-cyclical asset restrictions. This leads to a smaller decline in average output and financial conditions, and a larger reduction in financial volatility. Importantly, compared with a benevolent planner, the paternalistic planner supports a consumption process with higher unconditional mean and substantially lower volatility, thereby enhancing overall welfare.

Table 5: Socially Optimal Allocations

Panel (a) Rational Economy

	Av.	Median	Std. Dev.
Financial condition η	-1.34%	-1.33%	-3.33%
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-0.21%	-0.42%	-0.25%
Growth $I \cdot \kappa$	+28.25 p.p.	+26.15 p.p.	+37.61 p.p.
Consumption c	-10.74%	-6.65%	-34.47%

Panel (b) Diagnostic Economy — Benevolent Planner

	Av.	Median	Std. Dev.
Financial condition η	-1.96%	-1.90%	-5.14%
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-0.40%	-0.57%	+1.46%
Growth $I \cdot \kappa$	+26.76 p.p.	+25.13 p.p.	+36.02 p.p.
Consumption c	-11.21%	-9.41%	-26.77%

Panel (c) Diagnostic Economy — Paternalistic Planner

	Av.	Median	Std. Dev.
Financial condition η	-1.59%	-1.43%	-2.18%
Output $\kappa \cdot y_1 + (1-\kappa) \cdot y_2$	-0.29%	-0.42%	-1.28%
Growth $I \cdot \kappa$	+19.81 p.p.	+18.89 p.p.	+20.15 p.p.
Consumption c	-8.01%	-35.78%	-54.19%

Notes. The table reports, in panels (a) to (c), the percentage change of economic outcomes in the socially optimal allocation relative to the competitive equilibrium. In the economies with diagnostic beliefs (panels b and c), outcomes are evaluated under the actual (physical) distribution.

Altogether, compared to an economy with rational expectations, the interplay between financial frictions and diagnostic beliefs requires additional restrictions on financial risk-taking. The nature of these restrictions depends on the planner's degree of diagnosticity: a paternalistic planner imposes leverage restrictions during economic expansions, while a benevolent planner does so during economic downturns.

6 Conclusion

This paper explores the combined implications of diagnostic expectations and external financing frictions for financial stability and regulation in an environment with a real risk-taking channel. We find that interactions between these elements exacerbate financial market instability compared to the rational expectations benchmark, aligning closely with classical writings on the Financial Instability Hypothesis. Consequently, the socially optimal regulation imposes additional restrictions on leverage and risk-taking, regardless of the planner's degree of diagnosticity, compared to an economy under rational expectations. This analysis has focused on expectations deviating from the full information rational expectations (FIRE) benchmark. Future research could investigate the effects of imperfect information on financial stability and regulation.

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