WORKING paper

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Fiscal Requirements for Price Stability When Households are Not Ricardian[1]

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Abstract

Are restrictions on fiscal policy necessary for monetary policy to be able to deliver price stability? When households are Ricardian, the net present value of future fiscal surpluses needs to equate the real value of government debt absent inflation. We show that when households are not Ricardian, fiscal requirements still exist but take the very different form of a limit on the debt-to-GDP ratio. The debt-to-GDP limit captures the idea that public debt cannot be so large that the wealth effect of public debt on aggregate spending can no longer be counter-balanced by interest rate hikes, however large. To implement price stability when the debt-to-GDP requirement is satisfied, monetary policy must respond to the level of public debt, not just to the inflation it creates.

Keywords: Monetary-Fiscal Interactions, Non-Ricardian Households, Price Stability.

JEL classification: E52, E62

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NON-TECHNICAL SUMMARY

Give the central bank a clear mandate of price stability. Grant it full independence from the government. And appoint at its head a steadfast governor who will not let anything but its mandate influence its policy. Does the central bank then have all it needs to deliver price stability? Or must requirements on the government's fiscal policy also be imposed? The question is a cornerstone of monetary-fiscal interactions, determining whether monetary policy has the power to insulate inflation from imprudent fiscal decisions, or is ultimately dependent on a well-behaved fiscal authority.

The existence of fiscal requirements for price stability is at the root of the convergence criteria of the Stability and Growth Pact in the euro area, but they have no consensual basis in economic theory. Today, the main rationale for fiscal requirements is stipulated by Leeper (1991), Sims (1994), Woodford (2001), and the subsequent Fiscal Theory of the Price Level (FTPL): for the central bank to be able to deliver price stability, the real value of public debt at stable prices must be equal to the net present value (NPV) of future real fiscal surpluses.

Yet the NPV requirement has remained controversial to this day. In particular, recent skepticism points out that it is derived under the strong assumption of Ricardian households, when finite lives, financial frictions, or limited foresight are enough to make households non Ricardian. Whether fiscal policy can make monetary policy lose control over inflation when households are not Ricardian is heavily debated.

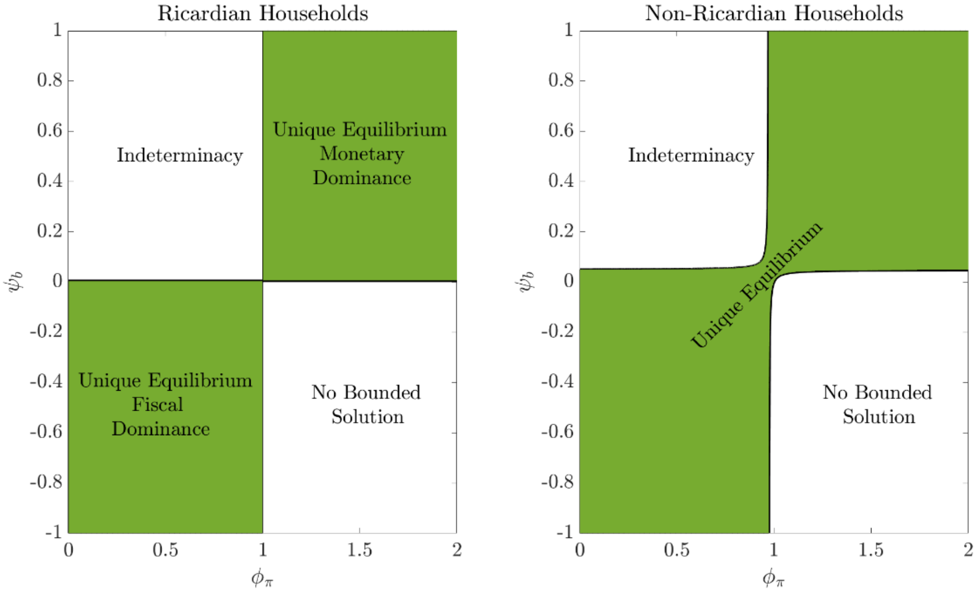
In this paper, we show that when households are not Ricardian, fiscal requirements for price stability do exist, but that they reduce to the very different form of a limit on the real-debt-to-GDP ratio. When the debt-to-GDP ratio is above the threshold or projected to grow above this threshold in the future, no stable price equilibrium exists. The debt-to-GDP limit arises because above it, no interest rate however high can counter-balance the effect of higher debt on aggregate demand and bring it back in line with aggregate supply. To restore an equilibrium, inflation must necessarily set in to erode the real value of public debt, lowering households' real wealth and therefore aggregate demand.

We derive the debt-to-GDP requirement in two steps. First, following the derivation of the NPV requirement when households are Ricardian, we consider what requirements arise from households' intertemporal budget constraints. We show that when households are not Ricardian, households' intertemporal budget constraints impose only very weak requirements. In particular, if, from any current level of public debt, the government plans on never raising any tax to repay it, this violates no household's intertemporal budget constraint.

Second, we show that a new requirement arises when households are not Ricardian. Higher public debt increases aggregate demand through a wealth effect, and puts upward pressure on inflation. This in itself poses no constraint on the ability of the central bank to maintain price stability. The central bank can counter the inflationary effect of higher debt with higher interest rates, just like it can counter any other inflationary shock with higher interest rates, retaining the ultimate control over inflation. Yet we show that there exists a threshold on the debt-to-GDP ratio above which even infinitely high interest rates are not enough to counter the wealth effect of public debt, resulting in the limit on the debt-to-GDP ratio.

We conclude by analyzing how the central bank can implement price stability once these fiscal requirements are satisfied. In doing so we reconsider Leeper (1991)'s local version of the FTPL in the case of non-Ricardian households. We show that when the central bank follows a standard Taylor rule that responds to inflation, fiscal shocks always affect inflation, however strong the response of monetary policy to inflation. It it no longer possible to distinguish between a monetary regime and a fiscal regime (see Figure below). Yet, we show that monetary policy can implement price stability if, on top of reacting to inflation, it directly responds to the level of public debt—not just to the higher inflation that higher debt generates.

Figure:



Note: The diagram represents when there exists a unique bounded equilibrium (in green), no bounded equilibrium, or a multiplicity of equilibria (indeterminacy) as a function of the degree of responsiveness of monetary policy to inflation Φπ and the degree of responsiveness of fiscal policy to public debt Ψb. The left panel represents the case of Ricardian households, with a strict distinction between a regime of fiscal dominance and a regime of monetary dominance. The right panel represents the case of non-Ricardian households, with no such strict distinction.

Prérequis Fiscaux pour la Stabilité des Prix Quand les Ménages ne sont pas Ricardiens

RÉSUMÉ

Des restrictions sur la politique budgétaire sont-elles nécessaires pour que la politique monétaire soit en mesure de garantir la stabilité des prix ? Lorsque les ménages sont ricardiens, la valeur actuelle nette des excédents budgétaires futurs doit être égale à la valeur réelle de la dette publique en l'absence d'inflation. Nous montrons que lorsque les ménages ne sont pas ricardiens, des prérequis budgétaires existent toujours, mais qu'ils prennent la forme très différente d'une limite sur le ratio dette/PIB. Cette limite capture l'idée que la dette publique ne peut être si élevée que son effet de richesse sur la dépense agrégée ne peut plus être contrebalancé par des hausses de taux d'intérêt, aussi amples soient elles. Pour assurer la stabilité des prix lorsque la contrainte sur le ratio dette/PIB est respectée, la politique monétaire doit réagir au niveau de la dette publique, et pas seulement à l'inflation qu'elle engendre.

Mots-clés : interactions fiscales/monétaires, ménages non-ricardiens, stabilité des prix.

Introduction

Give the central bank a clear mandate of price stability. Grant it full independence from the government. And appoint at its head a steadfast governor who will not let anything but its mandate influence its policy. Does the central bank then have all it needs to deliver price stability? Or must requirements on the government's fiscal policy also be imposed? The question is a cornerstone of monetary-fiscal interactions, determining whether monetary policy has the power to insulate inflation from imprudent fiscal decisions, or is ultimately dependent on a well-behaved fiscal authority.

In policy circles fiscal requirements are typically seen as necessary, yet economic theory provides no uncontroversial basis for them. At the creation of the euro area, the convergence criteria of the Maastricht treaty and then the Stability and Growth Pact introduced fiscal rules on national governments under the assumption they were necessary to allow the ECB to deliver on its price stability mandate. But the economic literature provides no consensus on whether such requirements are necessary. On the one hand, the monetarist view long prevailed that the control of the price level is ultimately always in the realm of the central bank. On the other hand, Leeper (1991), Sims (1994), Woodford (1995, 2001) and the subsequent Fiscal Theory of the Price Level (FTPL) literature argue that for the central bank to be able to deliver price stability, current public debt must be backed by future fiscal surpluses.[1] Namely, the real value of public debt B_{t-1} at stable prices $P_t = P^*$ must be equal to the net present value (NPV) of future real surpluses T_{t+k},

$$\frac{B_{t-1}}{P^*} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{1}$$

where $\mathcal{R}_{t,t+k}$ is the real interest rate from t to $t+k$. If the government does not plan on sufficient future fiscal surpluses to make equation (1) hold, no equilibrium with stable prices exists. The central bank has therefore no chance of delivering it. To restore an equilibrium inflation must set in to erode the real value of public debt until it matches the level of real future surpluses.

Yet the NPV requirement (1) has remained controversial to this day. Early on, the controversies centered on the ability of the NPV equation (1) to determine the price level, a central tenet of the FTPL (e.g. Kocherlakota and Phelan, 1999; Buiter, 2002). Regardless of its ability to determine the price level, recent skepticism points out that the NPV requirement relies on the assumption of Ricardian households, when finite lives, financial frictions, or limited foresight are enough to make households non Ricardian.[2] Whether fiscal policy can make monetary policy lose control over inflation when households are not Ricardian is heavily debated. Bassetto and Cui (2018) show that the NPV requirement can no longer determine inflation when households are not Ricardian and the real interest rate is below the growth rate of the economy. Blanchard (2019) argues that when the real interest rate is below the growth rate of the economy the NPV equation no

[1] Distinct from the Fiscal Theory of the Price Level, the monetarist view was first challenged by Sargent and Wallace (1981)'s unpleasant monetarist arithmetic, which provides a distinct argument based on seigniorage revenues. In the present paper we stick to a cashless economy in which there is no seignoriage revenues from money holdings.

[2] A note on terminology: In the FTPL, a key distinction is made between Ricardian and non-Ricardian *fiscal policies* (Woodford, 1995). A Ricardian fiscal policy is one that satisfies the intertemporal budget constraint of the government for any price level. Whether a fiscal policy is Ricardian or not is however different from whether households are Ricardian or not—they always are in the FTPL. To avoid confusion, in the paper we use the term *Ricardian* to refer to households only, and avoid using the term to refer to policies.

longer poses a constraint on fiscal policy. Reis (2021, 2022) and Brunnermeier, Merkel, and Sannikov (2020) argue that this does not imply there is no constraint on fiscal policy, but instead that the NPV requirement (1) needs to be augmented with a "bubble", or convenience yield, term. As this extra term is endogenous to monetary policy however, adding it to the NPV equation (1) leaves open the question of what fiscal policies are consistent with price stability and which ones are not.

In this paper, we show that when households are not Ricardian, fiscal requirements for price stability do exist, but that they reduce to the very different form of a limit on the real-debt-to-GDP ratio

$$\frac{B_{t-1}}{P^* Y_t} - \frac{T_t}{Y_t} \leq d^*. \tag{2}$$

When the debt-to-GDP ratio is above the threshold d^* or projected to grow above this threshold in the future, no stable price equilibrium exists. The debt-to-GDP limit arises because above it, no interest rate however high can counter-balance the effect of higher debt on aggregate demand and bring it back in line with aggregate supply. To restore an equilibrium, inflation must necessarily set in to erode the real value of public debt, lowering households' real wealth and therefore aggregate demand.

In our main result, we derive the fiscal requirement (2) in Blanchard (1985)'s model of perpetual youth, which breaks the Ricardian equivalence by assuming that households face a mortality risk.[3] We also show that fiscal requirements take the similar form of a limit on debt-to-GDP in a standard two-generation overlapping-generation model. We focus on the perpetual-youth set-up because it has gained increasing appeal to study economies with non-Ricardian households, as households' mortality risk can be interpreted either literally as biological death—making it an overlapping generations model—or as the risk of hitting borrowing constraints—the financial frictions that are the focus of the HANK literature (e.g. Farhi and Werning, 2019; Wolf, 2021).

We derive condition (2) in two steps. First, following the derivation of the NPV requirement (1) when households are Ricardian, we consider what requirements arise from households' intertemporal budget constraints. In an equilibrium, all households' intertemporal budget constraints must hold with equality. Otherwise, household would be leaving cash on the table. Under a representative Ricardian household, the unique intertemporal budget constraint is the one of the unique representative household, and it is the mirror image of the one of the government. It must therefore hold with equality, imposing equation (1) as a fiscal requirement for price stability.

We show that when households are not Ricardian, households' intertemporal budget constraints impose only very weak requirements for a stable price equilibrium to exist. In particular, if, from any current level of public debt, the government plans on never raising any tax to repay it, this violates no household's intertemporal budget constraint. This does not mean individual households accumulate explosive amounts of debt that they intend never to spend. Instead, they sell it to new generations. As a result, there is no need for inflation to make households' intertemporal budget constraints hold. Intertemporal budget constraints

[3] Our results do not depend on the existence of a mortality risk however. We generalize Blanchard (1985)'s set-up to allow for population growth, which breaks the Ricardian equivalence even when households face no mortality risk.

do impose a restriction on the path of future taxes and transfers, but it is a very weak one—never raising taxes satisfies it.

Second, we show this does not imply there exists no fiscal requirement for price stability. While intertemporal budget constraints no longer pose any significant constraint, a new requirement arises when households are not Ricardian. Higher public debt increases aggregate demand, and puts upward pressure on inflation. This in itself poses no constraint on the ability of the central bank to maintain price stability. The central bank can counter the inflationary effect of higher debt with higher interest rates, just like it can counter any other inflationary shock with higher interest rates, retaining the ultimate control over inflation. At higher interest rates, households are willing to hold more public debt without spending their extra wealth, making aggregate demand in line with aggregate supply. Yet we show that there exists a threshold on the debt-to-GDP ratio above which even infinitely high interest rates are not enough to counter the wealth effect of public debt, resulting in the limit (2).

We show that the debt-to-GDP limit (2) can be both more and less stringent than the NPV requirement (1). On the one hand, the debt-to-GDP limit makes high levels of public debt inconsistent with price stability even when public debt is backed by future surpluses. This is in contrast to the NPV requirement, according to which only public debt that is not backed by future surpluses threatens price stability. On the other hand, when the interest rate is less than the growth rate of the economy ($r < g$), it is possible for public debt not to be backed by future surpluses without threatening price stability. However, we show that even in this case there is a limit on how much debt the government can issue without planning on future fiscal surpluses. Because the real interest rate increases with the level of public debt, there exists a level of public debt above which r is necessarily greater than g, so that in the absence of fiscal surpluses debt-to-GDP necessarily ends up crossing the limit d^*.

Quantitatively, the limit d^* on the debt-to-GDP ratio is typically very high. When calibrating the model according to its overlapping-generation interpretation, we find it to be 1600 times GDP. When calibrated the model according to its HANK interpretation—allowing to match intertemporal MPCs, as in Wolf (2021)—we find it to be lower, but still 10 times GDP.

We derive the fiscal requirement for price stability (2) abstracting from how the central bank can ensure price stability once it is satisfied. We turn to this question of implementation in Section 5, assuming that the central bank sets nominal interest rates according to a standard Taylor rule, and the government sets taxes according to a similar feedback rule that responds to the level of public debt. In doing so we reconsider Leeper (1991)'s local version of the FTPL in the case of non-Ricardian households. We characterize analytically for which degree of responsiveness of fiscal and monetary policy there exists a unique bounded equilibrium, and when so whether inflation is insulated from fiscal shocks.

We show that under a standard Taylor rule where monetary policy responds to inflation only, it it no longer possible to distinguish between a monetary regime that insulates inflation from fiscal shocks and a fiscal regime that does not—the central result of Leeper (1991). For all pairs of monetary/fiscal rules that deliver a unique equilibrium, fiscal shocks always affect inflation, however strong the response of monetary policy to inflation. Yet, we show that monetary policy can insulate inflation from fiscal shocks and implement the stable price equilibrium if, on top of reacting to inflation, it directly responds to the level of public debt—not just to the higher inflation that higher debt generates. To insulate inflation from fiscal shocks, monetary policy must therefore monitor the level of public debt, in contrast to the idea that monetary dominance obtains when the central bank abstracts from fiscal developments.

By considering what the fiscal requirements for price stability are, this paper connects to the papers that have derived them in the case of Ricardian households, many of them associated to the FTPL literature developed by Leeper (1991), Sims (1994), Woodford (1995, 2001), Bassetto (2002), and Cochrane (2001, 2005).[4] Among these, we connect in particular to Woodford (2001), to which the title of the present paper is a reference. We depart from these by considering non-Ricardian households.

An important stream of the recent FTPL literature considers whether the dynamics of US inflation can be accounted for by models that feature elements of both monetary and fiscal dominance, either because the economy oscillates between regimes of fiscal and monetary dominance (Davig and Leeper, 2007; Bianchi and Ilut, 2017; Bianchi and Melosi, 2017, 2019; Schmidt, 2024), or because some fiscal shocks are funded while others are not (Cochrane, 2022; Bianchi, Faccini, and Melosi, 2023; Smets and Wouters, 2024). We show that when households are not Ricardian the distinction between monetary and fiscal dominance is already blurred even absent regime switching and different reactions to different fiscal shocks. Elfsbacka-Schmoller and McClung (2024) consider the FTPL in a model with endogenous growth, a dimension from which we abstract. Maćkowiak and Schmidt (2024) extend the FTPL to a monetary union. Corsetti and Maćkowiak (2024) consider the FTPL when fiscal imbalances may be corrected with some probability in the future. Barro and Bianchi (2023) analyze the drivers of the recent inflation surge in OECD countries through the lens of the FTPL.

A fast-growing literature analyses fiscal-monetary interactions in models with non-Ricardian households. Kaplan, Nikolakoudis, and Violante (2023) show that r can be less than g and the government can run permanent primary deficits in a HANK model. Farmer and Zabczyk (2018, 2019) show that the same applies in a standard OLG model. Brunnermeier, Merkel, and Sannikov (2022) show the same applies in a perpetual youth model similar that the one we use. Hagedorn (2024) contends that the FTPL fails in models with non-Ricardian households because markets are then incomplete. In a perpetual-youth set-up similar to the one we consider, Angeletos, Lian, and Wolf (2023) analyze how much deficits can finance themselves through both the higher tax receipts and the inflation that higher aggregate demand generates, and Angeletos, Lian, and Wolf (2024) compare the inflationary effects of deficits in HANK models and in the FTPL. Instead, we use the set-up to derive when and how the central bank can guarantee that deficits do *not* have an effect on inflation. We do so allowing for the possibility that r be less than g, the possibility emphasized by the papers cited above, which Angeletos and coauthors rule out by assumption.

Section 1 presents the main model of perpetual-youth with non-Ricardian households. Section 2 derives the fiscal requirements for price stability in this model, as well as in a standard two-generation OLG model.

[4]For extensive reviews of the FTPL, see Leeper and Leith (2016) and Cochrane (2023), as well as Barthelemy, Mengus, and Plantin (2024). For a review of the FTPL with a special focus on empirical studies, see Bianchi, Melosi, and Rogantini Picco (2024).

Section 3 discusses the new debt-to-GDP limit and compares it to the NPV requirement. Section 4 considers extensions of the model that generate higher marginal propensities to consume than the baseline model. Section 5 considers how the central bank can implement the stable price equilibrium once it exists.

1 An Economy with Non-Ricardian Households

In this section, we lay out the model of non-Ricardian households we rely on. It is a Blanchard-Yaari perpetual youth set-up (Blanchard, 1985) in discrete time, under perfect foresight. Relative to Blanchard (1985), we add two features. First, we allow for population growth at rate g, both to meaningfully talk about r and g and in order to stress that our results do not depend on the assumption that households die and buy life-insurance contracts—the Ricardian equivalence will break and all results hold even if households are infinitely lived, provided there is population growth. Second, we allow for individual incomes to shrink over time, both for realism to capture the need to save for retirement, and in order to allow for negative interest rates. We assume that the supply-side of the economy is given by an exogenous path for Y_t. Under flexible prices, this exogenous path can be interpreted as the supply-determined level of GDP. Under sticky prices, it can be interpreted as the exogenous path for natural output.

1.1 Of Life and Death

The economy is populated by an infinity of households of various ages. Each household faces a probability λ of dying each period, independent of how long it has been alive. As a consequence, each period a fraction λ of households dies. Households face the risk of dying with positive wealth. Insurance companies provide them with actuarially fair contracts to insure them against this risk. The wealth of households that die at t is redistributed to households still alive, in proportion to the financial wealth they had at the end of the previous period. The redistributed amount λB_{t-1} is therefore redistributed to the savings $(1-\lambda)B_{t-1}$ of the surviving households. Each dollar of saving therefore receives $\lambda/(1-\lambda)$ dollar of annuity.

New households are born every period, with no wealth. We allow for population growth. We assume that at t a number $(\lambda+g)N_{t-1}$ of households are born, so the population grows at rate g. Since λ of households die every period regardless of their ages, the number of households of age n at t is:

$$N_t(n) = (1-\lambda)^n(\lambda+g)N_{t-n-1} = \frac{\lambda+g}{1+g}\left(\frac{1-\lambda}{1+g}\right)^n N_t. \tag{3}$$

The particular case of the Ricardian representative household arises when there is neither new births nor death, $\lambda = 0$ and $g = 0$, in which cases all households are identical. Either $\lambda > 0$ or $g > 0$ is enough to break the Ricardian equivalence. From now on, we denote this non-Ricardian case as $\lambda + g > 0$.

1.2 Households

A household i has preferences over its consumption path

$$\sum_{k=0}^{\infty}(\beta(1-\lambda))^k \log(C_{t+k}^i), \tag{4}$$

where β is the preference discount factor. The household effectively discounts the future at the stronger rate $\beta(1-\lambda)$ because it factors in the chance that it won't be there to enjoy consumption tomorrow.

Household i maximizes its utility (4) subject its flow budget constraints and a No-Ponzi-scheme constraint. Having saved nominal wealth B_{t-1}^i from period $t-1$, it starts period t with wealth $1/(1-\lambda)B_{t-1}^i$. Its flow budget constraint is

$$C_t^i + \frac{1}{R_t}\frac{B_t^i}{P_{t+1}} = \frac{1}{1-\lambda}\frac{B_{t-1}^i}{P_t} + (Y_t^i - T_t^i), \tag{5}$$

where C_t^i is its real consumption, Y_t^i its real income, T_t^i the real taxes it has to pay, and where R_t is the real interest rate. Its No-Ponzi-scheme constraint is

$$\lim_{k\to\infty}\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{B_{t+k}^i}{P_{t+k+1}} \geq 0, \tag{6}$$

where $\mathcal{R}_{t,t+k}$ is the compounded real rate over k periods

$$\mathcal{R}_{t,t+k+1} = \prod_{j=0}^{k} R_{t+j}. \tag{7}$$

Like in the flow budget constraint (5) the factor $(1-\lambda)$ enters the No-Ponzi condition. It takes into account that the wealth the household holds grows at a rate $R/(1-\lambda)$ and not just R since the household also receives the annuity on its wealth.

1.3 Income Distribution

We let a household's income depend on its age. Specifically, we assume that a household of age n receives an income that decreases exponentially with its age, as the amount of labor it supplies decreases. This creates a desire to save for one's old age—present in the baseline two-period OLG model—that allows for the possibility of negative real interest rates. The household's income is

$$Y_t^i(n) = \kappa(1-\zeta)^n\left(\frac{Y_t}{N_t}\right), \tag{8}$$

where Y_t is aggregate income, $\zeta \in [0,1]$, and κ is a constant determined by the condition that individual incomes must sum to aggregate incomes Y_t. Given the age distribution in the population (3), the constant must be

$$\kappa = \frac{1+g-(1-\lambda)(1-\zeta)}{\lambda+g} \text{ if } \lambda+g>0. \tag{9}$$

The assumption that individual incomes decrease with age, $\zeta > 0$, is possible only in the non-Ricardian case $\lambda + g > 0$. In the Ricardian case of the representative agent $\lambda + g = 0$, all households necessarily get the same income, $Y_t^i = Y_t$. We can then only assume $\zeta = 0$ and $\kappa = 1$.

1.4 Fiscal Policy

We assume no government spending G. We do so to keep focus: we are interested in the effect of the financing of government expenditures through taxes for a fixed path for G which we set to zero to simplify. We therefore take fiscal policy to consist in a inherited debt level B_{-1} and a path for the level of aggregate taxes $(T_t)_{t\geq 0}$. The flow budget constraint of the government is

$$\frac{1}{R_t}\frac{B_t}{P_{t+1}} + T_t = \frac{B_{t-1}}{P_t}, \tag{10}$$

and determines the path for public debt B_t from the government's fiscal policy.

We assume that individual taxes are imposed proportionally on income, so that they follow the same age profile as income. A household of age n pays taxes

$$T_t^i(n) = \kappa(1-\zeta)^n \left(\frac{T_t}{N_t}\right). \tag{11}$$

1.5 Equilibrium and Stable Price Equilibrium

An equilibrium is defined in the standard way.

Definition 1. *For a given inherited level of public debt B_{-1}, distributed as an arbitrary $(B_{-1}^i)_i$ in the population, and a given fiscal policy $(T_t)_{t\geq 0}$, an equilibrium is an interest rate path $(R_t)_{t\geq 0}$ and an allocation $(C_t^i)_{t\geq 0,i}$ s.t.*

1. *All households behave optimally: Given interest rates $(R_t)_{t\geq 0}$, aggregate incomes $(Y_t)_{t\geq 0}$ and aggregate taxes $(T_t)_{t\geq 0}$—of which individual incomes and individual taxes depend according to (8) and (11)—each household maximizes its utility (4) subject to its flow budget constraints (5) and its No-Ponzi-scheme constraint (6).*

2. *The goods market clears:*

$$C_t = \int_i C_t^i di = Y_t. \tag{12}$$

Among all equilibria, we are interested in stable price equilibria. A stable price equilibrium is an equilibrium with on-target inflation. For convenience and without loss of generality, we assume that the inflation target is equal to 0, so that keeping inflation on target means keeping the price level constant, which we denote P^*. A stable price equilibrium is then an equilibrium with $P_t = P^*$ in all periods.

Definition 2. *A stable price equilibrium is an equilibrium where the price level is constant to $P_t = P^*$ at all t.*

1.6 Characterization of Individual Optimality

The optimal behavior of a household can be characterized in the standard way through its consumption function and No-Ponzi-scheme constraint holding with equality.

Lemma 1. *A household i behaves optimally if and only if*

1. *Its consumption is given by the consumption function*

$$C_t^i = \mu \left(\frac{1}{1-\lambda} \frac{B_{t-1}^i}{P_t} + H_t^i \right), \tag{13}$$

where $\mu = 1 - \beta(1-\lambda)$ is the household's marginal propensity to consume, and H_t^i is its human capital,

$$H_t^i = \sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} (Y_{t+k}^i - T_{t+k}^i). \tag{14}$$

2. *Its No-Ponzi constraint* (6) *holds with equality*

$$\lim_{k \to \infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{B_{t+k}^i}{P_{t+k+1}} = 0. \tag{15}$$

See Appendix A for a derivation. Since condition (15) states that the household leaves no cash on the table, we refer to it as the No-Cash-on-the-Table condition.

2 Fiscal Requirements for Price Stability

This section derives the conditions on B_{-1} and $(T_t)_{t \geq 0}$ for a stable price equilibrium to exist. We first rederive the fiscal requirements in the case of Ricardian households—the NPV requirement—then move to the main result of the paper: the debt-limit requirement in the case of non-Ricardian households. We show that fiscal requirements take the similar form of a debt limit in a simple two-generation OLG model.

2.1 The Case of Ricardian Households

For further reference, we first rederive fiscal requirements for price stability in the traditional case where households are Ricardian (Leeper, 1991; Sims, 1994; Woodford, 1995, 2001).

Proposition 1. *Assume households are Ricardian $\lambda + g = 0$. Consider an initial level of debt B_{-1} and a* *future tax path $(T_t)_{t\geq 0}$. There exists a stable price equilibrium for this fiscal policy if and only if*

$$\frac{B_{t-1}}{P^*} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{16}$$

where the real interest rate is given by $R_t = \frac{Y_{t+1}}{\beta Y_t}$.

The necessity of condition (16) follows from the intertemporal bubget constraint of the representative household

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = \frac{B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}), \tag{17}$$

which is obtained by combining its flow budget constraint (5) and No-Cash-on-the-Table constraint (15). Adding market-clearing (12) it implies (16) when prices are stable $P_t = P^*$. The value of the real interest rate follows from the Euler equation of the representative household. Appendix B proves that condition (16) is also sufficient for a stable price equilibrium to exist.

Proposition 1 states that in an equilibrium, the real value of public debt must be equal to the net present value of future fiscal surpluses. Because the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government coincide in the Ricardian case, the requirement (16) can be stated as a requirement on the intertemporal budget constraint of the government. The necessity of condition (16) as an equilibrium requirement comes from the optimality condition of the household however, not the government's. When the central bank stands ready to unconditionally buys government debt so that the government cannot default, nothing forces the government to satisfy any intertemporal budget constraint. But in an equilibrium, households cannot hold wealth that they plan never to spend. If the wealth they own in government bonds is not matched by future taxes to pay, households will spend it. Intuitively, the resulting inflation will dilute their real wealth on the left-hand-side of (16) until it equates the NPV of future surpluses.

2.2 The Case of Non-Ricardian Households

The following proposition states the main result of the paper.

Proposition 2. *Assume households are non-Ricardian $\lambda + g > 0$. Consider an initial level of debt B_{-1} and a future tax path $(T_t)_{t\geq 0}$. There exists a stable price equilibrium for this fiscal policy if and only at all time t the following two conditions hold:*

$$\frac{B_{t-1}}{P^* Y_t} - \frac{T_t}{Y_t} < d^*, \tag{18}$$

where

$$d^* = \left(\frac{\mu}{1-\mu} \left(1 - \frac{(1-\lambda)(1-\zeta)}{(1+g)} \right) \right)^{-1}, \tag{19}$$

and

$$\lim_{k\to\infty} \sum_{j=1}^{k} \Omega(k,j) \frac{1}{\mathcal{R}_{t,t+j}} T_{t+j} = 0, \tag{20}$$

for coefficient $\Omega(k,j)$ *given further in the text.*

Proposition 2 states that when households are not Ricardian, fiscal requirements for price stability take the form of a limit (18) of the debt-to-GDP ratio. Technically there exists a second requirement in the form of condition (20), but it can be disregarded in practice. It only poses extremely weak constraints on fiscal policy—never raising taxes satisfies it. The rest of this section derives the fiscal requirement of Proposition 2, explaining where they arise from. Section 3 discusses this new requirement and how it compares to the NPV requirement (16) that prevails when households are Ricardian.

2.2.1 Fiscal Requirement from Households' Intertemporal Budget Constraints

We first follow the logic of the case of Ricardian households and consider what fiscal requirements arise from households' intertemporal budget constraints, or equivalently their No-Cash-on-the-Table conditions (15). Because there is now an infinity of such constraints, we first need to determine the debt holdings of an individual household in equilibrium. Appendix C shows the following lemma.[5]

Lemma 2. *Assume households are not Ricardian* $\lambda + g > 0$.

If all households are on their consumption function (13) *and the goods market clears* (12)*, then at time t a household of age* $n \leq t$ *has holdings of public debt*

$$\frac{B_t^i(n)}{P_{t+1}} = \psi_B(n) \frac{B_t}{P_{t+1} N_t} - \sum_{k=0}^{n} \phi_T(n,k) \left(\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}} \right), \tag{21}$$

where

$$\psi_B(n) = \kappa(1-\zeta)^n \mu \frac{1-x^{n+1}}{1-x}, \tag{22}$$

$$\phi_T(n,k) = \kappa(1-\zeta)^n x^k \left(1 - \mu \frac{1-x^{n-k+1}}{1-x}\right), \tag{23}$$

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{24}$$

The expression for the coefficient $\psi_B(n)$ on aggregate public debt shows that abstracting from the effect of the past path of taxes, the amount of public debt held by a household either monotonically increases with age when $\zeta = 0$, or is a single-peaked function of age when $\zeta > 0$. It is plotted on Figure 1 for different values of ζ. When $\zeta = 0$, it increases monotonically to converge to a constant fraction of public debt. When $\zeta > 0$,

[5] Since we consider any arbitrary initial distribution of public debt (B_{-1}^i), expression (21) holds only for households born after period $t = 0$. For households born before $t = 0$, their debt holdings depend on their initial debt holdings B_{t-1}^i. If the initial distribution of public debt has been determined in the same way before $t = 0$ as after $t = 0$ however, expression (21) is valid for all households of any age.

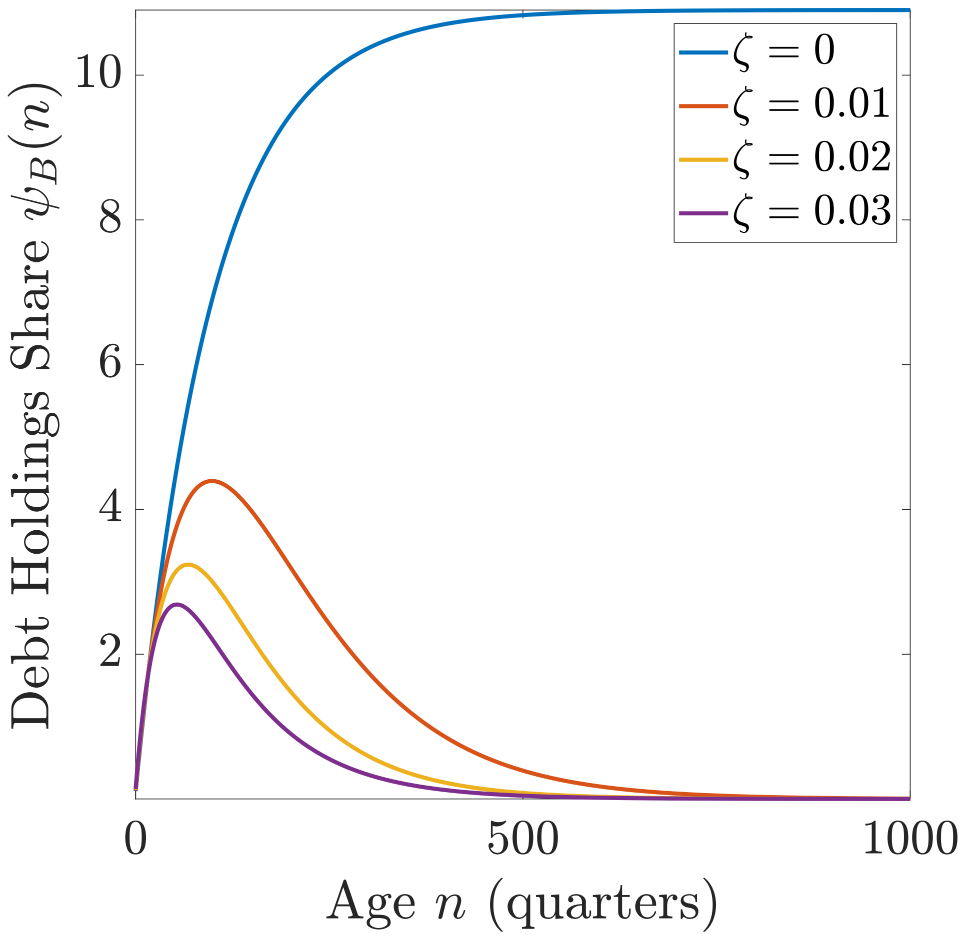


Figure 1: Diagram of Debt Dynamics

Note: The figure represents the coefficient $\psi_B(n)$ in the expression of individual debt holdings (21) as a function of the household's age, for different values of the parameter ζ defined in equation (8). The three other parameters are set to $\beta = 0.995$, $\lambda = 0.005$ and $g = 0.005$.

it first increases in the household's early years starting from zero, peaks in middle-age and then gradually shrinks to zero.

We now determine a necessary and sufficient condition for the No-Cash-on-the-Table conditions of all households to be satisfied. Appendix D shows the following lemma.

Lemma 3. *Assume households are not Ricardian $\lambda + g > 0$.*

Assume all households consume according to the consumption function (13).

All the No-Cash-on-the-Table constraints (15) of all households are satisfied if and only if

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{\bar{B}_{t+k}}{P_{t+k+1}} = 0, \tag{25}$$

where $\bar{B}_{t+k}$ is the average debt held at $t+k$ by households alive at t and still alive at $t+k$.

Intuitively, Lemma 3 states that the No-Cash-on-the-Table conditions of all households are satisfied if and only if the No-Cash-on-the-Table condition of a fictitious average household is satisfied, where the fictitious average household has debt holdings equal to the average debt holding among households that were already alive at t.

Lemmas 2 and 3 can be combined to determine what requirements households' intertemporal budget constraints impose on fiscal policy. Appendix E shows that they impose condition (20) in Proposition (2).

Lemma 4. *Assume households are not Ricardian $\lambda + g > 0$.*

Consider an inherited debt level B_{-1} and a future tax path $(T_t)_{t\geq 0}$. In an equilibrium, condition (20) must hold, where

$$\Omega(k,j) = (\beta(1-\lambda))^k \theta(j) - \xi(k), \tag{26}$$

$$\xi(k) = \frac{(1-\lambda)^k}{1-x} \left(\mu \left(\frac{1-\zeta}{1+g} \right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta} \right) \beta^k \right), \tag{27}$$

$$\theta(0) = 0, \tag{28}$$

$$\forall j \geq 1, \theta(j) = (\beta(1-\lambda))^{-j} \left[\left(1 - \frac{\mu}{1-x}\right) \left(1 - \left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa \frac{\lambda+g}{1+g} \frac{x}{1-x} \left(1 - \left(\beta(1-\lambda)\right)^j\right)\right]. \tag{29}$$

As mentioned above, condition (20) only puts a very weak requirement on fiscal policy, contrary to the NPV equation (16) in the case of Ricardian households. In particular, the current debt level B_{t-1} does not appear in condition (20). As a consequence, for any current level of public debt, the fiscal policy of never raising any tax to repay public debt ($T_t = 0$ at all times) is consistent with all households' intertemporal budget constraints being satisfied. When households are not Ricardian, this does not mean individual households accumulate explosive amounts of debt they intend never to spend. Instead, they sell it to new generations. In addition, condition (20) does not feature inflation. Inflation does nothing to ease this constraint and restore the possibility of an equilibrium.

2.2.2 Fiscal Requirements from the IS Curve

When households are not Ricardian, households' intertemporal budget constraints no longer impose any significant constraint on fiscal policy for price stability. But other fiscal requirements arise, specific to the case of non-Ricardian households. They arise from the IS curve of the model, which Appendix F derives.

Lemma 5. *In an equilibrium, the following dynamic IS curve holds*

$$Y_t = \frac{1-\zeta}{\beta(1+g)} \frac{1}{R_t} Y_{t+1} + \chi \left(\frac{B_{t-1}}{P_t} - T_t \right), \tag{30}$$

where

$$\chi = \frac{\mu}{1-\mu} \left(1 - \frac{(1-\lambda)(1-\zeta)}{(1+g)} \right). \tag{31}$$

In the case of Ricardian households $\lambda + g = 0$ (and so $\zeta = 0$), the model reduces to the standard representative agent model and the dynamic IS curve reduces to the standard Euler equation where public debt does not appear as $\chi = 0$. When households are not Ricardian $\lambda + g > 0$, net aggregate wealth matters for aggregate consumption $\chi > 0$, and therefore for inflation.

The presence of aggregate net wealth in the IS curve (30) does not in itself mean that the central bank loses control over inflation however. Many things can affect the level of aggregate demand, for instance exogenous shocks to households' preferences. The claim that monetary policy remains ultimately in control of the price level does not deny that other factors than monetary policy can affect inflation. It only argues that the central bank can use interest rates to counter-balance these shocks and deliver on-target inflation.

The same argument applies in principle to public debt: high public debt increases aggregate demand, but if it creates too much demand, the central bank can increase rates to bring it down. Specifically, if the central bank delivers the real interest rate

$$R_t = \frac{1-\zeta}{\beta(1+g)} \frac{Y_{t+1}}{Y_t} \left(1 - \chi \left(\frac{B_{t-1}}{P^* Y_t} - \frac{T_t}{Y_t}\right)\right)^{-1}, \tag{32}$$

in all periods, a stable price equilibrium obtains.

But does there always exist an interest rate level that allows the central bank to bring down demand in line with supply? The following lemma shows there does not when the debt-to-GDP ratio is too high, delivering the fiscal requirement (18).

Lemma 6. *Assume households are not Ricardian $\lambda + g > 0$.*

Consider an inherited debt level B_{-1} and a future tax path $(T_t)_{t \geq 0}$. In a stable price equilibrium, the real-debt-to-GDP ratio for on target inflation $P_t = P^*$ must be below the threshold

$$d^* = \frac{1}{\chi} \tag{33}$$

at all periods, resulting in equation (18).

The debt-to-GDP limit follows from the fact that when the debt-to-GDP ratio is above the limit d^*, no interest rate, however large, can make the IS equation (30) hold. Even an infinitely large real interest rate $R_t = \infty$ is not enough to counter the wealth effect of public debt on aggregate consumption and bring aggregate demand down to supply.

Lemmas 4 and 6 together show that conditions (18) and (20) are necessary in a stable price equilibrium. Appendix G shows that they are also sufficient, proving the characterization of the stable price equilibrium in Proposition 2.

2.3 Similar Characterization in a Standard Two-Generation OLG Model

Before discussing the new fiscal requirement of the debt-to-GDP ratio, we show that a similar characterization applies in a standard two-generation OLG model. We consider a standard overlapping-generation model à la

Samuelson (1958). Households live for two periods. In period t, households are thus divided between young households born at t and old households born at $t-1$. The population N_t grows at rate g, and is therefore divided between $N_t^{old} = \frac{1}{2+g} N_t$ households born at $t-1$ and $N_t^{young} = \frac{1+g}{2+g} N_t$ households born at t.

A household born at t has preferences over its consumption in periods t and $t+1$

$$\log(C_t^y) + \beta \log(C_{t+1}^o). \tag{34}$$

The household is born with no wealth and maximizes its utility (34) subject to the flow budget constraints

$$C_t^y + \frac{1}{R_t} \frac{B_t^y}{P_{t+1}} = Y_t^y - T_t^y, \tag{35}$$

$$C_{t+1}^o = \frac{B_t^y}{P_{t+1}} + Y_{t+1}^o - T_{t+1}^o. \tag{36}$$

We assume that young households receive collectively a share γ of the economy's total income Y_t, while old households collectively receive the remaining share $1-\gamma$,

$$Y_t^y = \gamma \frac{Y_t}{N_t^y}, \tag{37}$$

$$Y_t^o = (1-\gamma) \frac{Y_t}{N_t^o}. \tag{38}$$

The government still sets aggregate taxes subject to the flow budget constraint (10) and taxes are still imposed proportionally to income

$$T_t^y = \gamma \frac{T_t}{N_t^y}, \tag{39}$$

$$T_t^o = (1-\gamma) \frac{T_t}{N_t^o}. \tag{40}$$

An equilibrium and a stable-price equilibrium are still defined as in Definitions 1 and 2, where goods-market clearing now takes the form

$$N_t^y C_t^y + N_t^o C_t^o = Y_t. \tag{41}$$

When households live for two periods, their intertemporal budget constraints necessarily hold once they behave optimally. As a consequence, no fiscal requirement arises from intertemporal budget constraints, as pointed out by Bassetto and Cui (2018). The following proposition shows however that a real-debt-to-GDP limit similar to (18) characterizes the fiscal policies consistent with price stability (see Appendix K for a proof).

Proposition 3. *For a given initial level of debt B_{-1}, consider a future tax path $(T_t)_{t \geq 0}$.*

There exists a stable price equilibrium for this fiscal policy if and only if at all time t

$$\frac{B_{t-1}}{P^*Y_t} - \left(1 - \frac{\gamma\beta}{1+\beta}\right)\frac{T_t}{Y_t} \leq \frac{\gamma\beta}{1+\beta}. \tag{42}$$

Taxes now enter with a different coefficient than debt because young and old households have different MPC in the standard OLG model. But otherwise the fiscal requirement for price stability takes the similar form of a limit of the debt-to-GDP ratio.

3 Discussion and Comparison to the NPV Requirement

In this section, we discuss the new debt-to-GDP limit (18) and compare it to the NPV equation (16), and answer common questions on the threat fiscal policy can pose to price stability through the lens of the model. In particular, we discuss whether the new fiscal requirement of the debt-to-GDP limit is more or less stringent than the NPV requirement. We show that overall it is neither more or less stringent. The NPV equation can fail while there exists a stable price equilibrium, and conversely the NPV equation can be satisfied yet no stable price equilibrium exist.

Note first that the debt-to-GDP limit (18) shares some of the flavor of the NPV requirement (16). Like in the NPV equation, in equation (18) inflation still has the potential to restore an equilibrium when one does not exist, by decreasing the value of real debt. The way this is brought to happen is also similar. If condition (18) fails to be satisfied for stable prices $P_t = P^*$, then demand is greater than supply and inflation must arise to erode real public debt enough to equate demand and supply. If anything, this mechanism is more explicit in the case of Ricardian households, since public debt explicitly increases aggregate demand through the IS equation (30). The idea of fiscal dominance as a situation in which the price level adjusts to stabilizes the level of real public debt is therefore still present. We will return to this point in Section 5 when studying implementation. For the moment, we compare both conditions as what they are in Propositions 1 and 2: characterizations of which fiscal policies are consistent with price stability.

3.1 Does it Mean the NPV Equation Never Needs to Hold?

No. In some cases the debt-to-GDP limit implies the NPV equation, so that any fiscal policy that delivers a stable price equilibrium must be such that the NPV equation holds. This is the case if the model parameters are such that, for any level of public debt, the real interest rate is necessarily above the growth rate of the economy

$$R_t \geq \frac{Y_{t+1}}{Y_t}. \tag{43}$$

Appendix H provides a proof. In turn, the real interest rate is above the growth rate of the economy for any level of public debt when the following conditions on parameters holds

$$\frac{1-\xi}{\beta} > 1 + g. \tag{44}$$

This follows from the dynamic IS curve (30). Condition (44) states that the population growth rate is not too large, and/or that households' income does not shrink too much with their age.

When condition (44) is satisfied, the NPV equation (16) necessarily holds in a stable price equilibrium.[6] Yet even in this case, the NPV equation is implied by the debt-to-GDP limit but is not equivalent to it. The debt-to-GDP limit is actually more stringent, as we explain in the next subsection.

3.2 Can Real-Debt-to-GDP Grow Unbounded yet Prices Remain Stable?

No, since there is a limit on it! But this is a big difference with the NPV requirement that applies when households are Ricardian. Indeed, the NPV requirement does not preclude a real-debt-to-GDP ratio that grows unbounded over time. Consider the case where households are Ricardian and assume for instance that from any positive debt position B_{-1} the government repays a share ψ_b of its public debt every period, with $0 < \psi_b < 1 - \beta$. It implies that taxes at $t + k$ are

$$T_{t+k} = \psi_b (1 - \psi_b)^k \mathcal{R}_{t,t+k} \frac{B_{t-1}}{P_t}, \tag{45}$$

so that the NPV equation (16) is satisfied and there exists a stable price equilibrium. Yet the real-debt-to-GDP ratio is given by

$$\frac{B_{t+k}}{Y_{t+k} P_{t+k+1}} = \left(\frac{1}{\beta} (1 - \psi_b) \right)^k \frac{B_{t-1}}{P_t Y_t}, \tag{46}$$

which diverges to infinity over time.

The debt-to-GDP limit that applies when households are not Ricardian precludes this. The NPV equation is not enough to guarantee price stability. This has implications for the interpretation of fiscal requirements for price stability as corresponding to fiscal backing of the currency, as we discuss in the next subsection.

3.3 Is Fiscal Backing of the Public Debt enough to Guarantee Price Stability?

Not if fiscal backing is understood as the NPV equation (16). When households are Ricardian, the NPV requirement suffices to guarantee the existence of a stable price equilibrium. High public debt in itself does not threaten price stability, only high public debt unbacked by future fiscal surpluses does.

This is no longer the case when households are not Ricardian, as the case of an exploding real-debt-to-GDP ratio showed. The absolute level of the debt-to-GDP ratio is then a constraint on the existence

[6] Angeletos, Lian, and Wolf (2023, 2024) restrict to such cases.

of a stable price equilibrium. Obviously, future taxes still matter to assess the existence of a stable price equilibrium, since future taxes influence the future level of the debt-to-GDP ratio. But a plan to raise taxes in the future is not enough to guarantee the existence of a stable price equilibrium if future taxes are planned to be collected so far into the future that the debt-to-GDP ratio increases above the debt limit before taxes are collected.

The debt-to-GDP limit can therefore be more stringent than the NPV equation. But it can also be less stringent, as we discuss in the next subsection.

3.4 When Can the NPV Equation Not Hold?

When r can be less than g, for at least some levels of public debt. This is the case that has attracted much attention recently (e.g. Blanchard, 2019; Reis, 2021; Brunnermeier, Merkel, and Sannikov, 2020). As these and other papers have pointed out, it is then sometimes possible for the government to run positive public debt that it never intends to repay, without threatening price stability. In this case, there is a stable price equilibrium yet the NPV equation (16) is not satisfied. However this does not mean that never repaying debt is never a threat to price stability, as we explain in the next subsection.

3.5 Can the Government Never Repay its Debt and Prices Remain Stable?

Only when the level of public debt is low enough. But when public debt is above a threshold, it no longer is. There is therefore a limit on the amount of spending the government can make without raising taxes nor threatening price stability.

It is *not* the case that the government does not need to raise taxes as long as public debt is below the threshold d^*. For a stable price equilibrium to exist, the debt-to-GDP limit (18) must hold in all future periods, not just today. Even if the current debt-to-GDP ratio is less than d^* today, a tax path that lets the debt-to-GDP ratio inexorably increase will violate the debt limit at some point, making the fiscal policy inconsistent with a stable price equilibrium. To assess whether a stable price equilibrium exists, we therefore need to consider the future dynamic of public debt, and whether it remains below the threshold d^* at all times.

When the government never repays its debt, combining the interest rate (32) that must prevail under price stability with the flow budget constraint of the government (10) with taxes set to zero gives the following dynamics of the real-debt-to-GDP ratio $b_{t-1} = B_{t-1}/(P^*Y_t)$,

$$b_t = \frac{1-\zeta}{\beta\frac{Y_{t+1}}{Y_t}}\left(\frac{1}{\frac{1}{b_{t-1}} - \chi}\right) \tag{47}$$

This dynamics is represented on Figure 2. On the figure, GDP-per-capita is assumed to be constant so that Y_{t+1}/Y_t is constant to $1+g$, to focus on the debt dynamics. The shaded region corresponds to the one where the debt to GDP ratio is above the debt limit d^*. If the dynamics of the debt-to-GDP ratio ends up in this region at any time in the future, there exists no stable price equilibrium. Whether this happens depends on the value of the initial debt-to-GDP ratio b_{-1}. This is illustrated on the left panel of Figure 3, which plots the evolution over time of the debt-to-GDP ratio depending on the initial level of b_{-1}. The exists a threshold on b_{-1} above which never raising taxes is inconsistent with price stability.

The threshold has a simple interpretation and can be easily obtained analytically. From the government's flow budget constraint

$$b_t = \frac{R_t}{1+g} b_{t-1}, \tag{48}$$

the public debt grows and eventually reaches the debt limit d^* if the interest rate is greater than the growth rate of the economy. In turn, from the expression of the interest rate (32), the interest rate is increasing in the debt-to-GDP ratio. It follows that b_t increases and eventually reaches the debt limit if and only if b_{-1} is currently above the threshold on b that brings the interest rate (32) above the growth rate of the economy,

$$\underline{d} = \left(1 - \frac{1-\xi}{\beta(1+g)}\right) d^*. \tag{49}$$

As long as the debt-to-GDP ratio is initially below the threshold $\underline{d}$, the real interest rate (32) remains below the growth rate of the economy and the debt-to-GDP ratio shrinks back by itself even when the government never raises taxes. However, if government spending or tax cuts bring the debt-to-GDP ratio above the threshold $\underline{d}$, then the debt-to-GDP ratio increases and continues to do so since a higher debt-to-GDP ratio only pushes the real interest rate further above the growth rate of the economy, as plotted on the right panel of Figure 3.

Note that $\underline{d}$ is positive if and only if we are not in the case defined by equation (44). Otherwise the real interest rate is above the growth rate of the economy for all levels of public debt, so that any level of public debt must be repaid with future surpluses, as discusses in subsection 3.1.

3.6 How High is the Debt-to-GDP Limit d^*? How High is the Threshold $\underline{d}$?

Under a literal OLG interpretation of the model, very high. We calibrate the model in the following way (on a quarterly frequency). We set $\beta = 0.995$, $g = 0.005$ to correspond to a growth rate of 2%, and $\zeta = 0.005$, so that individual income shrinks by 2% per year. The key parameter is the death probability λ, which in the literal OLG interpretation of the model we interpret as the probability of biological death. We set it to $\lambda = 0.005$ or a 2% annual death probability. The calibration is summed up in the leftmost column of Table 1.

Under the calibration, the debt limit d^* is extremely high, at about 1665 times annual GDP. This is because, while the MPC $\mu = \beta(1-\lambda)$ is larger than in the case of Ricardian households where it is just β, it is still very small (0.01 quarterly). As a consequence even very high levels of public debt have only a limited effect on the level of aggregate demand, and can still be counterbalanced by sufficiently high interest rates. The value of the threshold $\underline{d}$ is a fraction of the value of d^*. But it is still high, at $\underline{d} = 8.3$, or 830% of GDP.

Table 1: Calibration and Debt Limits

	OLG	OLG-HTM	HANK	HANK-HTM
β	0.995			
ζ	0.005			
g	0.005			
λ	0.005	0.005	0.135	0.135
α_Y	0	0.19	0	0.07
Average MPC	0.01	0.20	0.14	0.20
d^*	1665.2	1345.6	10.8	10.0
$\underline{d}$	8.3	6.7	0.05	0.05

Note: The table gives the calibrations of the four variants of the model used in the paper. The calibration is quarterly. The average MPC is the quarterly average MPC. d^* and $\underline{d}$ are expressed as debt to annual GDP. The OLG interpretation interprets the model literally as an OLG model, with the death probability the probability of actual biological death. The HANK interpretation interprets death as hitting one's financial constraint. The OLG-HTM and HANK-HTM add a fraction of hand-to-mouth households that receive a share α_Y of income.

These high numbers are informative but not the last word, since under its OLG interpretation the model generates an average MPC that is much lower than in the data. Arguably the fact that very high levels of public debt can be sustained without threatening price stability when households only spend a small fraction of their wealth is to be expected. The next section considers extensions of the model that can match the average MPC in the data.

4 Extensions with Higher MPC

In this section, we consider extensions and alternative interpretations of the model. In particular we add hand-to-mouth households and we consider the alternative interpretation and calibration of the model as a proxy HANK model, capturing households' liquidity constraints.

4.1 Standard TANK

Before extending our main Proposition 2 to the existence of hand-to-mouth households, we first extend Proposition 1. We add hand-to-mouth households to the standard model where non hand-to-mouth households are Ricardian—i.e. a standard TANK model—and show the NPV requirement continues to hold in this case.

Assume that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$. Assume that the remaining households are Ricardian permanent-income households ($\lambda + g = 0$). We assume that hand-to-mouth households collectively pay $\alpha_T T_t$ in taxes, with α_T possibly distinct from α_Y (when taxes are proportional to income, $\alpha_T = \alpha_Y$). Appendix I shows the following Proposition.

Proposition 4. *Consider a standard TANK model where non hand-to mouth households are Ricardian* $\lambda + g = 0$*. Consider an initial level of debt* B_{-1} *and a future tax path* $(T_t)_{t\geq 0}$*.*

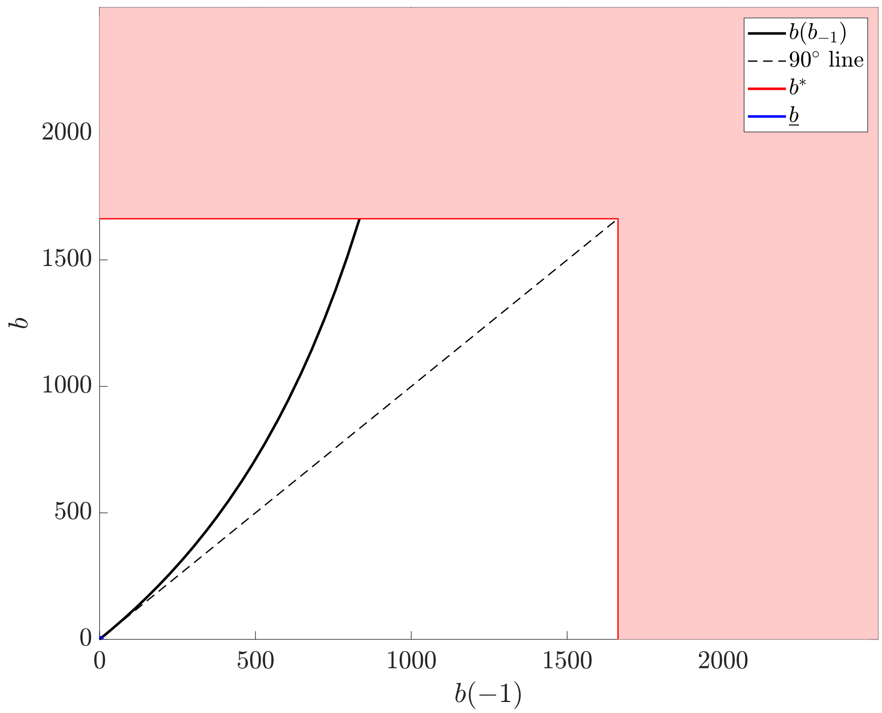


Figure 2: Diagram of Debt Dynamics

Note: The black thick line represents the mapping (47) giving the debt-to-annualized-GDP ratio in t as a function of the debt-to-GDP ratio at $t-1$, when there are no taxes. The shaded area in pink represents the area where the debt level if so large that no interest rate however large can reduce aggregate demand enough to equate demand to natural output. The calibration is given in Table 1, in the OLG interpretation of the model.

There exists a stable price equilibrium for this fiscal policy if and only if the NPV equation (16) holds for the interest rate path $R_t = (Y_{t+1} + \alpha_T/(1-\alpha_Y)T_{t+1})/(\beta(Y_t + \alpha_T/(1-\alpha_Y)T_t))$.

Proposition 4 states that in a standard TANK model the NPV requirement still characterizes the fiscal policies that are consistent with price stability. The only difference is that the equilibrium real interest rate takes a different value than in the absence of hand-to-mouth households.

A TANK model breaks the Ricardian equivalence yet still results in the NPV requirement (16). The result that fiscal requirements for price stability takes the form of a limit on the debt-to-GDP when households are not Ricardian therefore needs a more precise statement. What is required for fiscal requirements to take the form of the debt-to-GDP limit is that the households *that hold public debt* are non Ricardian. In a TANK model, the hand-to-mouth households are not Ricardian but they do not hold public debt, so their presence does not change the result that fiscal requirements take the form of the NPV equation. When households that hold public debt are not Ricardian, the presence of hand-to-mouth households makes little change to the model, as the next subsection shows.

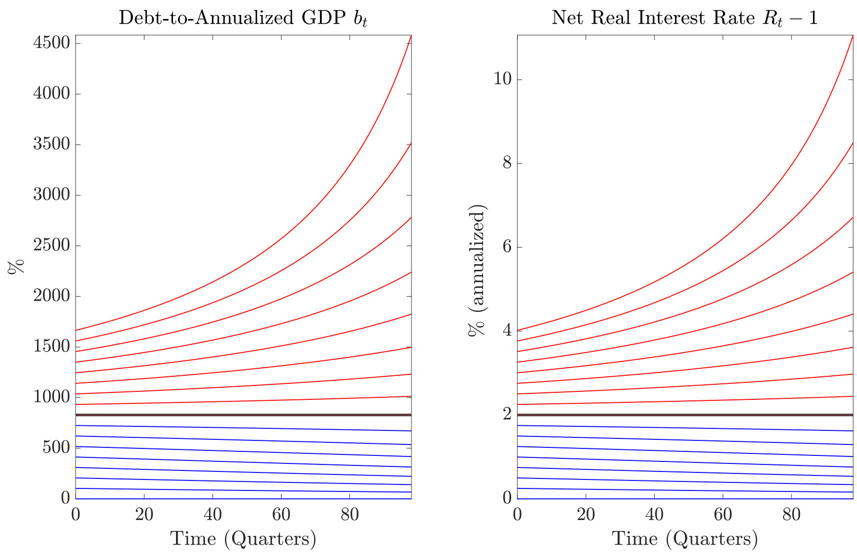


Figure 3: Debt Paths

Note: The left (right) panel plots the dynamics of the debt to annualized GDP ratio (the net real interest rate) over time when the governement never levies taxes, for different initial levels of the debt to annualized GDP ratio. For an initial debt-to-GDP ratio below $\underline{d}$, the debt-to-GDP ratio gradually shrinks back to zero (blue curves). For an initial debt-to-GDP ratio above $\underline{d}$, the debt-to-GDP ratio gradually increases and eventually diverges to infinity. The calibration is given in the left panel of Table 1.

4.2 OLG-HTM

In the perpetual-youth model of Section 1, the MPC of all households is $\mu = 1 - \beta(1 - \lambda)$. It can therefore be more than in the Ricardian case where it is $\mu = 1 - \beta$, but only in proportion to the death probability λ. To match a quarterly MPC of 0.20 as typically found in the data however (e.g. Fagereng, Holm, and Natvik, 2021), the quarterly death probability would need to be around 20%. If the death probability λ is taken to be the literal probability of dying, empirically plausible values of λ yield MPCs that are only marginally higher than in the Ricardian case—it is 0.01 in the calibration we considered in Section 3.6.

A simple way to match any average MPC is by adding hand-to-mouth households, like in a standard TANK model. Assume once again that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$, and that hand-to-mouth households collectively pay $\alpha_T T_t$ in taxes.[7] We continue to assume that among non hand-to-mouth households, taxes are imposed proportionally on income as per (11), where T_t is to be replaced by $(1 - \alpha_T)T_t$. Appendix J show the following extension of Proposition 2.

Corollary 1. *Consider the model with a share of hand-to-mouth households. Assume the non hand-to mouth households are not Ricardian $\lambda + g > 0$.*

Consider an initial level of debt B_{-1} and a future tax path $(T_t)_{t\geq 0}$. There exists a stable price equilibrium

[7] As far as the aggregate economy is concerned, how the incomes of hand-to-mouth households is distributed among themselves, e.g. according to age, does not matter.

for this fiscal policy if and only if are satisfied both condition (20) *and the amended condition*

$$\frac{B_{t-1}}{P^*Y_t} - \frac{\chi + \alpha_T}{\chi}\frac{T_t}{Y_t} < d^*, \tag{50}$$

where

$$d^* = \frac{1 - \alpha_Y}{\chi}. \tag{51}$$

Corollary 1 states that the requirements of Proposition 1 are robust to the addition of hand-to-mouth households. The debt-to-GDP limit now takes a different value, but fiscal requirements still take the form of a limit on the debt-to-GDP ratio.

Quantitatively as well, the addition of hand-to-mouth households makes little change to the value of the debt-to-GDP limit. They decrease it, but not by much. As reported in the second column of Table 1, when the model is calibrated as in Section 3.6 but with enough hand-to-mouth households to match an average MPC of 0.20 as in Fagereng, Holm, and Natvik (2021), the debt-to-GDP limit is still very high at 1347 times annual GDP, again 1665 without hand-to-mouth households. The value of $\underline{d}$ is similarly still very high at 6.7 times annual GDP, against 8.3.

At bottom, this is because fiscal requirements for price stability primarily depend on the behavior of the households that hold public debt. Since hand-to-mouth households do not, they only marginally affect fiscal requirements. The presence of hand-to-mouth households does lower the value of the threshold (51) on public debt, by a factor $(1 - \alpha_Y)$. But this is *not* because hand-to-mouth households increase the average MPC. Hand-to-mouth households have a higher MPC out of their income, but since they do not own wealth, their MPC does not directly matters. To determine how much of public debt households spend, the MPC that matters is still the MPC μ of non hand-to-mouth households. Hand-to-mouth households affect the debt-to-GDP limit only indirectly. Because they consume a share α_Y of income, the consumption of the non hand-to-mouth households must now be lower. This puts a lower threshold on the maximum level of public debt they can hold in a stable price equilibrium.

4.3 HANK Interpretation

When the perpetual-youth model is interpreted literally as an overlapping-generation model, the death probability λ cannot be set much higher than a quarterly 1%. But the model can alternatively be interpreted as a proxy for a HANK model (e.g. Del Negro, Giannoni, and Patterson, 2023; Farhi and Werning, 2019; Angeletos, Lian, and Wolf, 2023; Wolf, 2021). Under this alternative interpretation, the death probability is to be interpreted as the probability for households to hit their borrowing constraints. This rationalizes a much higher calibration of λ. We consider the model under this interpretation, calibrating $\lambda = 0.135$ and keeping all other parameters at the same values as in Section 3.6—the third column of table 1, abstracting for the moment from hand-to-mouth households. The MPC is then much higher at 0.14, an empirically plausible value.

The higher calibration of λ considerably reduces the debt limit d^*, dividing it by more than 100. It is now only 10.8 times annual GDP. Similarly, $\underline{d}$ is more than 100 smaller, at just 5% of annual GDP. Since households—which all hold public debt here—now have a much higher MPC, they spend a much higher proportion of their wealth, and an equilibrium can only sustain a much lower level of public debt.

4.4 HANK-HTM

The HANK interpretation of the model generates an empirically plausible MPC, but still ties it to the death probability λ. To exactly match any value for the average MPC it is still possible to consider the extension of the model to a share of hand-to-mouth households. As argued by Wolf (2021), in this case the model can actually provide a very good match not only to the instantaneous MPC, but also to the whole profile of intertemporal MPC (iMPC) (Auclert, Rognlie, and Straub, 2018). We consider this case by calibrating $\lambda = 0.135$ and setting α_Y to match an instantaneous quarterly MPC of 0.2—the last column of Table 1.

Once again, hand-to-mouth households somewhat lower d^*, from 10.8 to 10 times GDP, but once again not by much. This is in part because when $\lambda = 0.135$ the model only needs the addition of a small fraction (7%) of hand-to-mouth households to match an average MPC of 0.20. But primarily this is again because hand-to-mouth households have only a small effect on the debt-to-GDP limit, since they do not hold public debt.

4.5 HANK Interpretation: A Caveat

The HANK interpretation of the model has a considerable effect on the debt-to-GDP limit. As far as the debt limit is concerned however, there is a caveat to using the perpetual-youth model as a proxy for a HANK model. The HANK interpretation takes the probability of hitting one's borrowing constraint as exogenous, when in a full-fledged HANK model it is endogenous and depends on the level of public debt. Abstracting from this endogeneity is a reasonable assumption when considering the model locally around a given level of public debt (e.g. Angeletos, Lian, and Wolf, 2023, 2024). But in no longer is when the level of public debt can vary greatly, including when considering fiscal requirements for price stability.

We show the importance of the feedback of the level of public debt on the probability of hitting borrowing constraints through Woodford (1990)'s alternating-endowment model, a particular case of a HANK model which is tractable enough to be solved analytically. The economy is populated by two types of infinitely-lived households which cannot borrow. They face idiosyncratic risk because their individual incomes alternate between a low and a high endowment. In odd periods, type-A households receive the high endowment Y_t^h and pay taxes T_t^h while type-B households receive the low endowment Y_t^l and pay taxes T_t^l, and conversely in even periods. Both types of households have the same preferences

$$\sum_{k=0}^{\infty} \beta^k \log(C_{t+k}^i), \tag{52}$$

which they maximize subject to their flow budget constraint

$$\frac{1}{R_t}\frac{B_t^i}{P_t+1} + C_t^i = \frac{B_{t-1}^i}{P_t} + Y_t^i + T_t^i, \tag{53}$$

and the constraint that they cannot borrow

$$B_t^i \geq 0. \tag{54}$$

As Woodford (1990) shows, the model is exactly equivalent to the two-generation OLG model of Section 2.3 when the borrowing constraint of the low-endowment household is always binding. Indeed in this case, the low-endowment household holds no debt so the high-endowment household holds all of public debt. Since the high-endowment household holds positive public debt its borrowing constraint is not binding. At t, it decides how much to save facing the t and $t+1$ budget constraints

$$C_t^h + \frac{1}{R_t}\frac{B_t}{P_{t+1}} = Y_t^h - T_t^h, \tag{55}$$

$$C_{t+1}^l = \frac{B_t}{P_{t+1}} + Y_t^l - T_t^l, \tag{56}$$

where C_t^h and C_t^l are the consumption of high and low-endowment households and B_t is aggregate public debt. This is exactly the same program as in the OLG model of Section 2.3, with high-income households in the stead of young households and low-income households in the stead of old households (with no population growth, $g = 0$).

The exact equivalence between the alternating-endowment model and the two-generation OLG model buttresses the case for using the perpetual-youth model in its HANK interpretation. However, the equivalence only holds when the borrowing constraint of the low-endowment households is binding. Yet whether it is binding depends on the level of public debt. If public debt is high enough there is enough liquidity in the economy for households to self-insure and avoid ever being up against their borrowing constraint in the low-endowment state. The model is then no longer equivalent to the OLG model.

Figure 4 illustrates this by plotting the steady-state relationship between the real interest rate and the real-debt-to-GDP ratio in both the alternating-endowment model and the OLG model. Up to a threshold on the debt-to-GDP ratio, the borrowing constraint of the low-endowment households is binding and the two models are exactly equivalent. But once public debt is above this threshold, borrowing constraints are no longer binding in the alternating-endowment model. The model is then equivalent to one with a representative Ricardian household. As a result, the real rate is constant to $1/\beta$ and no longer increases with public debt. In the OLG model in contrast, the real interest rate keeps increasing with public debt. Appendix L provides further details.

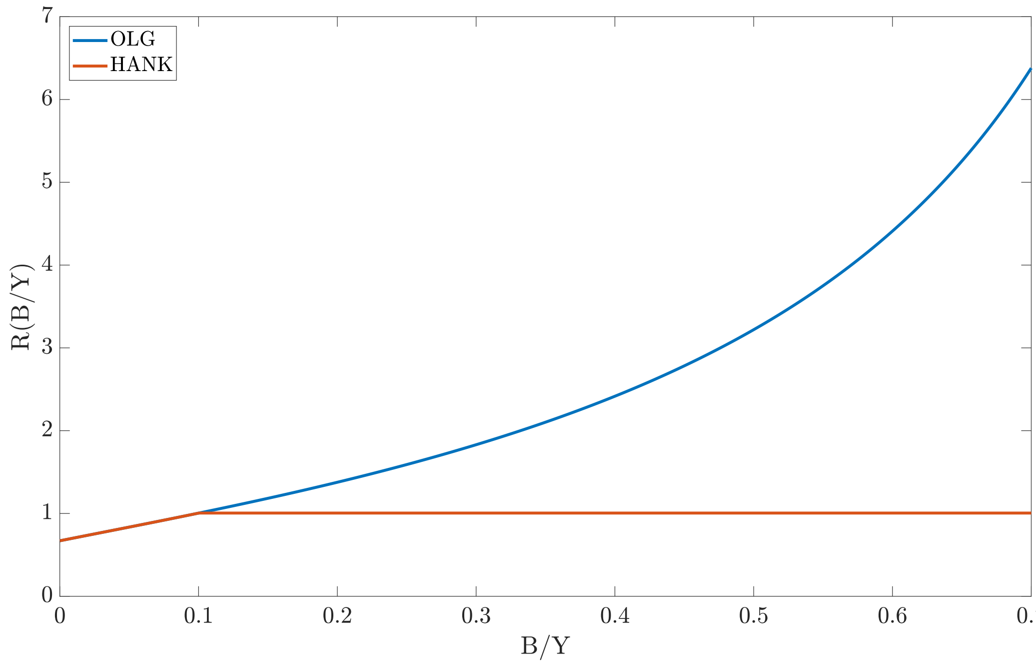


Figure 4: Steady-State $R(B/Y)$ Relationship, HANK vs. OLG

Note:The figure plots the steady-state relationship between the real interest rate and the real debt-to-GDP ratio in the OLG model of Section 2.3 and the alternating-endowment HANK model of Section 4.5. The models are calibrated with $\beta = 0.995$ and a difference between the high and low endowment equal to 20% of the aggregate endowment.

5 Implementation

Proposition 2 derived fiscal requirements for price stability abstracting from how the central bank can then implement price stability. In this section, we turn to the implementation question. In doing so, we reconsider Leeper (1991)'s analysis of fiscal and monetary dominance in the case of non Ricardian households.

5.1 Local Dynamics

We assume that prices are flexible, as in Leeper (1991)'s original analysis. The dynamics of the aggregate economy reduces to only two equations: the IS curve (30) and the flow budget constraint of the government (10), which can be written in per capital terms

$$y_t = \frac{1-\zeta}{\beta}\frac{\pi_{t+1}}{I_t}y_{t+1} + \chi\left(\frac{b_{t-1}}{\pi_t} - \tau_t\right), \tag{57}$$

$$b_t\frac{1+g}{I_t} + \tau_t = \frac{b_{t-1}}{\pi_t}. \tag{58}$$

where $b_{t-1} = B_{t-1}/(P_t N_t)$, $y_t = Y_t/N_t$ and $\tau_t = T_t/N_t$ are real public debt per capita, GDP per capita, and taxes per capita, π_t is the inflation rate and I_t is the nominal interest rates.

Following Leeper, we take the exogenous level of GDP under flexible prices to be constant, and consider the log-linearized version of these two equations around a steady state with real interest rate R and debt to

GDP ratio b/y,[8]

$$\hat{i}_t - \hat{\pi}_{t+1} = \eta\left(\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t - \hat{\tau}_t\right), \tag{59}$$

$$\hat{b}_t = \frac{R}{1+g}\left(\hat{b}_{t-1} - \hat{\tau}_t\right) + \frac{b}{y}\left(\hat{i}_t - \frac{R}{1+g}\hat{\pi}_t\right), \tag{60}$$

where

$$\eta = \frac{\chi}{\left(\frac{1-\zeta}{\beta R}\right)}. \tag{61}$$

The analysis therefore reduces to the same two-equation system in b_t and π_t as in Leeper (1991). The only difference is that when households are not Ricardian $\eta > 0$, public debt now enters the IS curve (59), as higher public debt increases aggregate demand.

Following Leeper, we assume monetary and fiscal policies are conducted according to feedback rules. Taxes respond to the level of public debt through

$$\hat{\tau}_t = \psi_b \hat{b}_{t-1} - \nu_t^g, \tag{62}$$

and monetary policy follows a standard Taylor rule

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \nu_t^i. \tag{63}$$

Plugging in the policy equations (62) and (63) into the system (59)-(60) gives the 2-by-2 system in $\hat{\pi}_t$ and $\hat{b}_t$

$$\left(\phi_\pi + \eta\frac{b}{y}\right)\hat{\pi}_t + \nu_t^i = \hat{\pi}_{t+1} + \eta\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right), \tag{64}$$

$$\hat{b}_t = \frac{R}{1+g}\left(\left(1-\psi_b\right)\hat{b}_{t-1} + \nu_t^g\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu_t^i\right). \tag{65}$$

5.2 Blurred Lines

Appendix M extends the result of Leeper (1991) on equilibrium determinacy to the case of non Ricardian households. It shows that when households are not Ricardian, the sharp distinction between a monetary and fiscal regime is no longer applicable.

Proposition 5. *Assume monetary and fiscal policy are given by the feedback rules* (62) *and* (63)*.*

The economy has a unique bounded equilibrium if and only if

$$\left(1-\phi_\pi\right)\left(1 - \frac{R}{1+g}(1-\psi_b)\right) + \eta\frac{b}{y}\left((1-\psi_b)\phi_\pi - 1\right) < 0. \tag{66}$$

[8] As is standard, when loglinearizing we define $\hat{b}_t = db_t/y^*$ to allow for the possibility of a zero level of public debt in steady-state.

- *When households are Ricardian $\eta = 0$ or when steady-state public debt is zero $b/y = 0$, then when a unique bounded equilibrium exists the economy is either in a monetary regime $\phi_\pi > 1, \psi_b > 1 - 1/R$ where fiscal shocks ν_t^g have no effect on inflation, or in a fiscal regime $\phi_\pi < 1, \psi_b < 1 - 1/R$ where they do.*

- *When households are not Ricardian $\eta > 0$ and when steady-state public debt is positive $b/y > 0$, then when a unique bounded equilibrium exists fiscal shocks ν_t^g always have an effect on inflation.*

The first item of Proposition 5 is Leeper's classical result. When households are Ricardian $\eta = 0$ (and so $g = 0$), condition (66) becomes

$$\Big(1 - \phi_\pi\Big)\Big(1 - R(1 - \psi_b)\Big) < 0. \tag{67}$$

It is satisfied either if $\phi_\pi > 1, \psi_b > 1 - (1+g)/R$ or if $\phi_\pi < 1, \psi_b < 1 - (1+g)/R$. The first case is what Leeper calls a regime of monetary dominance. In this case fiscal shocks ν_t^g have no effect on inflation. Indeed, the unique bounded solution can then be obtained by iterating forward equation (64) to give equilibrium inflation as

$$\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{k+1} \nu_{t+k}^i, \tag{68}$$

which is independent of the fiscal shocks ν_t^g. The second case is what Leeper calls a regime of fiscal dominance. In this case fiscal shocks ν_t^g affect inflation. For instance, when the interest rate is fully pegged $i_t = 0$, iterating equation (65) forward gives

$$(1 - \psi_b)\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{R(1-\psi_b)}\right)^k \nu_{t+k}^g, \tag{69}$$

which gives the level of inflation necessary to restore the intertemporal budget constraint of the government for the path of fiscal surpluses implied by the fiscal shocks ν_t^g and the (insufficient) strength of tax increases ψ_b.

The second item of Proposition 5 considers how the Leeper result changes once we move away from the case of Ricardian households. The sharp distinction between a regime of fiscal dominance and a regime of monetary dominance disappears. It first disappears in a most literal sense, represented in Figure 5. The figure plots the region of parameters (ϕ_π, ψ_b) for which there exists a unique bounded equilibrium. While in the case of Ricardian households, this consists of two distinct regions, when households are not Ricardian there is no straightforward way to distinguish between two regimes.

The distinction between fiscal and monetary regimes also disappears in terms of the inflationary effect of a fiscal shock. Appendix M shows that when households are not Ricardian, inflation is given by

$$\hat{\pi}_t = \mu\hat{b}_{t-1} + \sum_{k=0}^{\infty} \left(\frac{1}{\lambda}\right)^{k+1} \left(\left(\mu\frac{b}{y} - 1\right)\nu_{t+k}^i + \left(\mu\frac{R}{1+g} + \eta\right)\nu_{t+k}^g\right), \tag{70}$$

where λ is the root of the economy that is greater than 1, and μ is a positive constant given in Appendix M. This gives in particular the effect on inflation of a contemporaneous fiscal shock ν_t^g.

In the particular case where the government never raises taxes $\psi_b = 0$, a contemporaneous fiscal shock has the same impact effect on inflation as in the case of Ricardian households. Whenever $\phi_\pi < 1$,

$$\frac{\partial \pi_t}{\partial \nu_t^g} = \frac{1}{\frac{b}{y}}. \tag{71}$$

But whenever $\psi_b \neq 0$, the impact effect of a fiscal shock on inflation is not the same as when households are Ricardian. In particular, it is non-zero in policy configurations that deliver monetary dominance and a full insulation of inflation from fiscal shocks in the Ricardian case.

Figure 6 gives the inflationary effect of a contemporaneous fiscal shock as a function of ϕ_π for fixed values of ψ_b. The upper panel gives the impact effect of the shock on inflation. As monetary policy becomes more reactive, it decreases. But it does so gradually and without ever reaching zero, approaching it only as its asymptotic limit where ϕ_π tends toward infinity. This is in contrast to the Ricardian case, where an active monetary policy $\phi_\pi > 1$ is enough to perfectly insulate inflation from fiscal shocks. In addition, the *cumulative* impact on inflation can be much larger, and even increasing in ϕ_π, as shows on the bottom panel of Figure 6. This is because increasing ϕ_π increases the backward-looking root of the economy and therefore the persistence of inflation, as shown in the middle panel of Figure 6. As a result, the effect of the shock on cumulative inflation can even be increasing in ϕ_π.

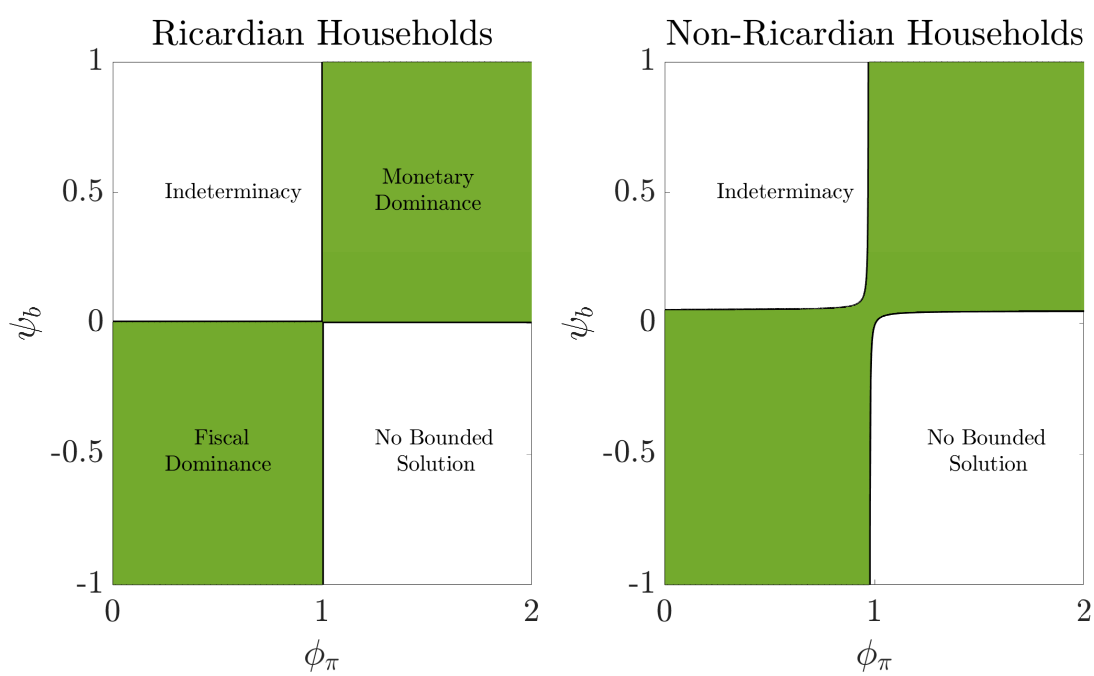


Figure 5: Condition for Equilibrium Uniqueness

Note: In green is the region of parameters (ϕ_π, ψ_b) for which there exists a unique bounded equilibrium, in the economy with Ricardian households (left panel) and with non-Ricardian households. The calibration is $\beta = 0.995$, $\zeta = 0.05$, $g = 0.05$, $b/y = 1.2$, $\lambda = 0.135$.

5.3 Implementing Price Stability by Letting Monetary Policy Respond to Debt

The analysis so far would conclude that it is impossible for monetary policy to insulate inflation from fiscal shocks. No degree of responsiveness to inflation ϕ_π in the Taylor rule (63) can fully prevent fiscal shocks from affecting inflation.

This is only if one restricts monetary policy to respond to inflation only however. If the central bank follows a policy rule that respond to public debt in addition to inflation, insulating inflation from fiscal shocks becomes possible again. If the reaction function of the central bank is

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \eta(\hat{b}_{t-1} - \hat{\tau}_t), \tag{72}$$

then a regime of monetary dominance—in the sense defined by the following proposition—-can implement the stable price equilibrium of the economy.

Proposition 6. *Assume monetary and fiscal policy are given by the feedback rules (62) and (72).*

The economy has a unique bounded equilibrium if and only if it is in either of the two following regimes

- *Monetary regime* $\phi_\pi > 1 - \eta\frac{b}{y}$ *and* $\psi_b > 1 - \frac{1}{\frac{R}{1+g} + \frac{b}{y}\eta}$. *Fiscal shocks* ν_t^g *have then no effect on inflation.*
- *Fiscal regime* $\phi_\pi < 1 - \eta\frac{b}{y}$ *and* $\psi_b < 1 - \frac{1}{\frac{R}{1+g} + \frac{b}{y}\eta}$. *Fiscal shocks* ν_t^g *then affect inflation.*

See Appendix N for a proof. In the case of the monetary regime, the unique bounded solution can be obtained by solving forward the equation obtained by combining the IS curve (59) and monetary rule (72), to give

$$\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi + \eta\frac{b}{y}} \right)^{k+1} \nu_{t+k}^i \tag{73}$$

Under monetary dominance, monetary policy can be said to be active again, since it insulates inflation from fiscal shocks. However, this flips on its head the definition of an active monetary policy in Leeper (1991)

> *"I couch active and passive policy in terms of the constraints a policy authority faces. An active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks. Its behavior is constrained by private optimization and the active authority's actions."*

In contrast, the monetary policy (72) delivers monetary dominance *because* it reacts to public debt. While having monetary policy react to public debt is less standard, it follows naturally from the fact that the natural rate of interest—the real interest rate consistent with flexible prices—depends on the level of public debt once households are not Ricardian. The monetary rule (72) is of the form

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t, \tag{74}$$

where

$$\hat{r}_t^n = \eta(\hat{b}_{t-1} - \hat{\tau}_t) \tag{75}$$

is the natural rate. Having the natural rate of interest as an intercept in the Taylor rule is a standard feature in specifications of monetary policy.

5.4 When $r < g$

A corollary of Proposition 6 is that, in contrast to the case of Ricardian households $\lambda + g = 0$, it is possible to be in a regime of monetary dominance even when the government never increases taxes in response to higher debt, $\psi_b = 0$. This is the case if and only if

$$\frac{R}{1+g} + \eta \frac{b}{y} < 1. \tag{76}$$

This is simply a consequence of the fact that in the stable price equilibrium of the economy with non Ricardian households, debt shrinks back by itself if the real interest rate r is below the growth rate of the economy g. The additional term $\eta \frac{b}{y}$ in equation (76) captures how the natural rate increases with the level of public debt at first order.

Condition (76) remains a local first-order result however. In the full non-linear dynamics, the dependence of the natural rate on the level of public debt is convex. A large fiscal shock can increase the natural rate enough that it pushes R above $1 + g$. A large fiscal shock can then make debt explosive and prevent the existence of an equilibrium where the government never increases taxes. Deriving fiscal requirements for price stability require to go beyond local dynamics to look at the non-linearized version of the model, as done in Section 2.

6 Conclusion

Can the central bank deliver price stability whatever fiscal policy the government sets? The NPV requirement that current government debt needs to be backed by future fiscal surpluses has remained controversial since its inception and relies on the strong assumption of Ricardian households. In this paper, we have argued that moving away from the particular case of Ricardian households provides a less controversial—and also more intuitive—answer to this question. With non Ricardian households, higher public debt makes households richer. As households spend their higher wealth, aggregate demand increases, putting inflationary pressure on the economy. While monetary policy can increase interest rates to counter the inflationary effects of public debt, it can no longer do so when public debt is too high. For the central bank to keep control over the price level, fiscal policy must be such that it never lets debt reach that level. Provided this fiscal requirement is satisfied, monetary policy can deliver price stability. But to do so, it needs to increase its policy rate with the level of public debt—not just increase it in response to the inflation that higher public debt generates.

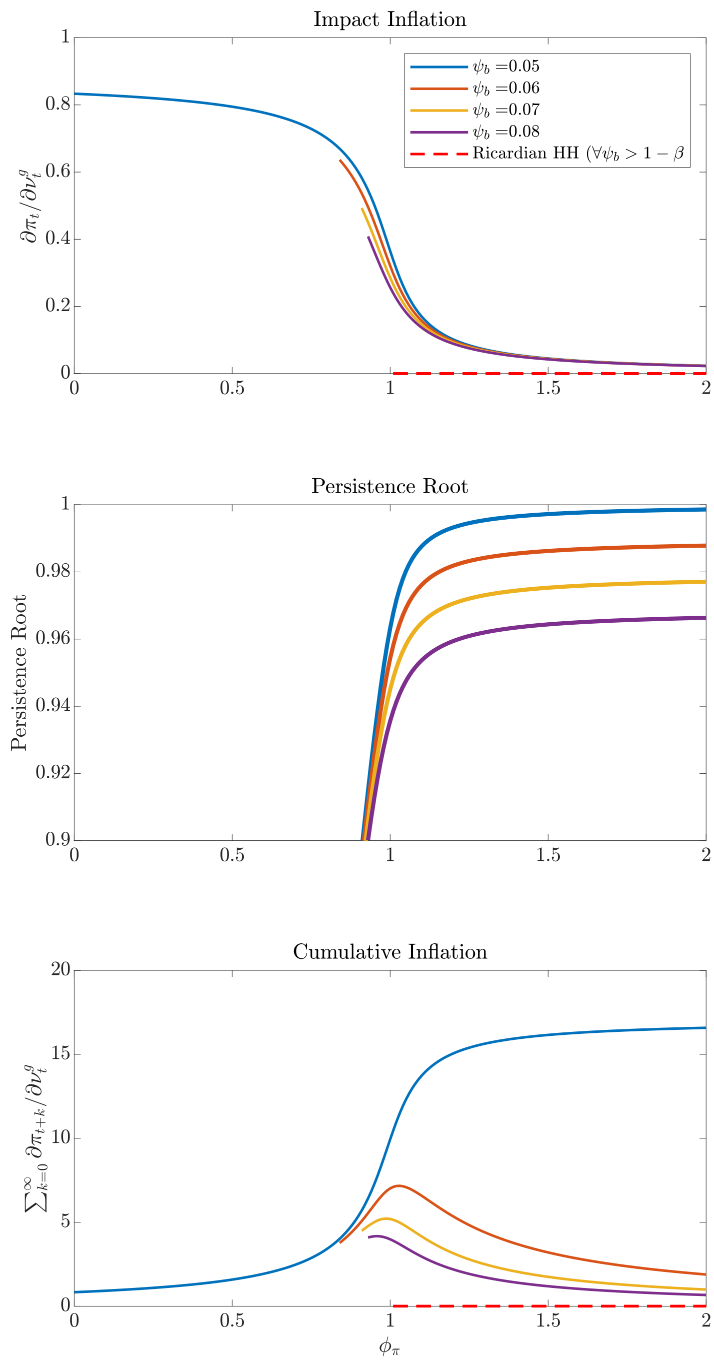


Figure 6: Inflationary Effect of Contemporaneous Fiscal Shock

Note: The figure plots the impact inflationary effect of a fiscal shock, the persistence root, and the cumulative effect of a fiscal shock, all three as a function of the reactivity of the Taylor rule ϕ_π, and for a parameter in the fiscal rule $\psi_b = 1 - \frac{1}{\frac{R}{1+g}+\eta\frac{b}{y}}$ or higher. For this first value of ψ_b there exists an equilibrium under all values of ϕ_π in the case of Non Ricardian households. In the case of Ricardian households, there exists an equilibrium only for $\phi_\pi > 0$. The calibration is $\beta = 0.995$, $\zeta = 0.05$, $g = 0.05$, $b/y = 1.2$, $\lambda = 0.135$.

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A Proof of Lemma 1

Maximizing (4) subject to the flow budget constraint (5) and the no Ponzi scheme constraint (6) gives the Euler equation

$$C_t^i = (\beta R_t)^{-1} C_{t+1}^i. \tag{A.1}$$

and the No-Ponzi-scheme condition holding with equality, i.e. the No-Cash-on-the-Table condition (15). Iterating the Euler equation (A.1) forward gives for all $k \geq 0$

$$C_{t+k}^i = (\beta^k \mathcal{R}_{t,t+k}) C_t^i. \tag{A.2}$$

Combining the FBC (5) and the No-Cash-on-the-Table condition (15), the intertemporal budget constraint is

$$\sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} C_{t+k}^i = \frac{1}{1-\lambda} \frac{B_{t-1}^i}{P_t} + H_t^i \tag{A.3}$$

where

$$H_t^i = Y_t^i - T_t^i + \frac{1-\lambda}{R_t} H_{t+1}^i \tag{A.4}$$

is the household's intertemporal wealth, or human capital. Iterated forward, it writes as equation (14).

Injecting (A.2) into the IBC (A.3) gives the consumption function (13). Conversely, equations (13) and (15) are sufficient for individual optimality.

B Proof of Proposition 1

The text already showed that condition (16) is necessary for a stable price equilibrium. We show that it is sufficient. The proof is constructive. Given an initial level of public debt B_{-1} distributed as $(B_{-1}^i)_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define the following path for the interest rate

$$R_t = \frac{1}{\beta} \frac{Y_{t+1}}{Y_t}. \tag{B.1}$$

For this interest rate path, define individual consumption allocations through the consumption function (13), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-clearing

Start with market-clearing. Since by construction all households are on the consumption function (13), aggregate consumption is given by the aggregate consumption function (F.1), which writes since $\lambda = g =$

$\zeta = 0$ in the Ricardian case

$$C_t = (1-\beta)\left(B^d_{t-1} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}}(Y_{t+k} - T_{t+k})\right). \tag{B.2}$$

Given this interest rate path, we have that

$$\frac{Y_{t+k}}{\mathcal{R}_{t,t+k}} = \frac{Y_{t+k}}{R_{t+k}\mathcal{R}_{t,t+k-1}} = \beta \frac{Y_{t+k-1}}{\mathcal{R}_{t,t+k-1}}, \tag{B.3}$$

and by iteration

$$\frac{Y_{t+k}}{\mathcal{R}_{t,t+k}} = \beta^k Y_t. \tag{B.4}$$

The consumption function (F.1) therefore implies

$$C_t = Y_t + (1-\beta)\left(B^d_{t-1} - \sum_{k=0}^{\infty} \frac{T_{t+k}}{\mathcal{R}_{t,t+k}}\right). \tag{B.5}$$

Starting from $t = 0$, given that $B^d_{-1} = B_{-1}$ initially, the NPV equation (16) at $t = 0$ guarantees market clearing in the goods market at $t = 0$. It therefore implies market clearing in the debt market at $t = 1$, $B^d_0 = B_0$. Continuing by induction, its proves market clearing in all periods.

Individual Optimality

We now check individual optimality, which is characterized in Lemma 1. Condition (13) is satisfied by construction. Combined with the flow budget constraint of the representative household (5), its No-Cash-on-the-Table constraint is equivalent to its intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = B^d_{t-1} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}}(Y_{t+k} - T_{t+k}). \tag{B.6}$$

Since the goods market clears in all periods, it is equivalent to condition (16), which is satisfied by assumption.

C Proof of Lemma 2

Preliminary: Market-Clearing in the Debt Market

We first simply prove Walras's Law: the assumption of market-clearing in the goods market (12) implies market-clearing in the debt market when combined with flow budget constraints, i.e.

$$\int_i B^i_t di = B_t, \tag{C.1}$$

where the integral is over households alive at t.

By assumption, market-clearing held in period $t = -1$. We show that market-clearing holds in the debt market in all periods by showing that if it holds at $t-1$ then it holds at t. Summing the households' FBC (5) across households alive at t gives

$$C_t + \frac{1}{R_t}\frac{\int_i B_t^i di}{P_{t+1}} = \frac{B_{t-1}}{P_t} + Y_t - T_t, \tag{C.2}$$

where we used the fact that $\int_i B_{t-1}^i di = (1-\lambda)B_{t-1}$, since the remaining λB_{t-1} belonging to households that passed away has been taken by the insurance fund and redistributed to remaining households as annuities.

Combining (C.2) with goods market-clearing condition (12) gives

$$\frac{1}{R_t}\frac{\int_i B_t^i di}{P_{t+1}} = \frac{B_{t-1}}{P_t} - T_t, \tag{C.3}$$

which combined with the FBC of the government (10) implies

$$\int_i B_t^i di = B_t, \tag{C.4}$$

which ends the proof.

Preliminary: Defining Aggregate Human Capital

Define aggregate human capital as the sum of individual human capitals

$$H_t = \sum_{k=0}^{\infty} N_t(k) H_t^i(k). \tag{C.5}$$

The human capital (14) of a household of age n can then be written as a function of aggregate human capital per capita

$$H_t^i(n) = \kappa(1-\zeta)^n \left(\frac{H_t}{N_t}\right), \tag{C.6}$$

where aggregate human capital per capita can be written

$$\frac{H_t}{N_t} = \sum_{k=0}^{\infty} \frac{((1-\lambda)(1-\zeta))^k}{\mathcal{R}_{t,t+k}} \left(\frac{Y_{t+k} - T_{t+k}}{N_{t+k}}\right), \tag{C.7}$$

or recursively

$$\frac{H_t}{N_t} = \left(\frac{Y_t - T_t}{N_t}\right) + \frac{(1-\lambda)(1-\zeta)}{R_t} \left(\frac{H_{t+1}}{N_{t+1}}\right). \tag{C.8}$$

Aggregate human capital solves the recursion

$$H_t = (Y_t - T_t) + \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t} H_{t+1}. \tag{C.9}$$

An Intermediary Lemma

We now move to the core of the proof. To prove Lemma 2, we first prove the following lemma:

Lemma C.1. *If all households are on their consumption function (13) and the goods market clears (12),* *then at time t, a household of age $n \leq t$ has holdings of public debt*

$$\frac{B_t^i(n)}{P_{t+1}} = \kappa(1-\zeta)^n \left[\mu \frac{1-x^{n+1}}{1-x} \frac{\mathcal{R}_{t-n,t+1}}{(1+g)^{n+1}} \frac{B_{t-(n+1)}}{N_{t-(n+1)}P_{t-n}} - \sum_{k=0}^{n} \left(x^k + \mu \frac{1-x^k}{1-x}\right) \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}}\right], \quad \text{(C.10)}$$

where

$$x = \frac{\beta(1+g)}{1-\zeta}. \quad \text{(C.11)}$$

Proof. The proof is by induction. For future abundant use, first notice that combining the FBC (5) and the consumption function (13) gives the debt-holding function of a household at t, i.e. how many bonds to hold as a function of inherited bonds and future and present incomes

$$\frac{B_t^i}{P_{t+1}} = R_t \left(\beta \frac{B_{t-1}^i}{P_t} + Y_t^i - T_t^i - \mu H_t^i\right). \quad \text{(C.12)}$$

Base Case(s)

To initialize the recursion, we determine the debt holdings of households of age 0 and of age 1. (The inductive step will show the property for $n+1$ assuming it holds for n and $n-1$, so it requires a double-initialization.) Applying (C.12) to households of age 0, who inherit no financial wealth from the previous period $B_{t-1}^i(0) = 0$, gives

$$\frac{B_t^i(0)}{P_{t+1}} = R_t \kappa \left(\frac{Y_t - T_t - \mu H_t}{N_t}\right). \quad \text{(C.13)}$$

Aggregating equation (C.12) across all households alive at t and using market-clearing in the debt market gives

$$\frac{B_t}{P_{t+1}} = R_t \left(\beta(1-\lambda) \frac{B_{t-1}}{P_t} + Y_t - T_t - \mu H_t\right). \quad \text{(C.14)}$$

Taking the difference between (C.14) and κ/N_t times (C.13) gives

$$\frac{B_t^i(0)}{P_{t+1}} - \kappa \frac{B_t}{N_t P_{t+1}} = -\kappa \frac{\beta(1-\lambda)}{(1+g)} R_t \frac{B_{t-1}}{P_t N_{t-1}}. \quad \text{(C.15)}$$

The government's flow budget constraint (10) can be rewritten in per-capita terms as

$$\frac{B_t}{N_t P_{t+1}} = R_t \left(\frac{1}{1+g} \frac{B_{t-1}}{N_{t-1}P_t} - \frac{T_t}{N_t}\right). \quad \text{(C.16)}$$

Using (C.16) to replace $B_t/(N_t P_{t+1})$, equation (C.15) can be written

$$\frac{B_t^i(0)}{P_{t+1}} = \kappa \left(\mu \frac{R_t}{(1+g)} \frac{B_{t-1}}{N_{t-1}P_t} - R_t \frac{T_t}{N_t}\right), \quad \text{(C.17)}$$

which proves the base case $n = 0$. The base case $n = 1$ is shown very similarly to the inductive step below, noting that a household of age 0 at t had no debt at $t-1$.

Inductive Step

Assume the property holds for households of age n and $n-1$. We show it then holds for households of age $n+1$. Taking the difference between equation (C.12) applied to a household of age $n+1$ and $(1-\zeta)$ times equation (C.12) applied to a household of age n gives

$$\frac{B_t^i(n+1)}{P_{t+1}} - (1-\zeta)\frac{B_t^i(n)}{P_{t+1}} = R_t\beta\left(\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^j(n-1)}{P_t}\right). \tag{C.18}$$

We now use the fact that the property holds for n and $n-1$ to rewrite the term on the right-hand side of (C.18) as

$$\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^i(n-1)}{P_t} = \kappa(1-\zeta)^n\left[\mu\left(\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-1-(n+1),t}}{(1+g)^{n+1}}\frac{B_{t-1-(n+1)}}{N_{t-1-(n+1)}P_{t-(n+1)}} - \frac{1-x^n}{1-x}\frac{\mathcal{R}_{t-1-n,t}}{(1+g)^n}\frac{B_{t-1-n}}{N_{t-1-n}P_{t-n}}\right)\right.$$
$$\left. - \left(x^n + \mu\frac{1-x^n}{1-x}\right)\frac{\mathcal{R}_{t-1-n,t}}{(1+g)^n}\frac{T_{t-1-n}}{N_{t-1-n}}\right] \tag{C.19}$$

Using the government flow budget constraint (C.16) to replace $B_{t-1-n}/(N_{t-1-n}P_{t-n})$, this rewrites

$$\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^i(n-1)}{P_t} = \kappa(1-\zeta)^n\left[\mu x^n\frac{\mathcal{R}_{t-(n+2),t}}{(1+g)^{n+1}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - x^n\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^n}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right]. \tag{C.20}$$

Multiplying by βR_t,

$$\beta R_t\left(\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta)\frac{B_{t-1}^i(n-1)}{P_t}\right) = \kappa(1-\zeta)^{n+1}\left[\mu x^{n+1}\frac{\mathcal{R}_{t-(n+2),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}} - x^{n+1}\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right]. \tag{C.21}$$

Meanwhile, the term $(1-\zeta)B_t^i(n)/P_{t+1}$ on the left-hand side of equation (C.18) can be rewritten, using (C.10) and the government flow budget constraint (C.16) to eliminate $B_{t-(n+1)}/(N_{t-(n+1)}P_{t-n})$, as

$$(1-\zeta)\frac{B_t^i(n)}{P_{t+1}} = \kappa(1-\zeta)^{n+1}\left[\mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right.$$
$$\left. - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right]. \tag{C.22}$$

Injecting equations (C.21) and (C.22) into equation (C.18) gives

$$\frac{B_t^i(n+1)}{P_{t+1}} = \kappa(1-\zeta)^{n+1}\left[\mu\left(\frac{1-x^{n+1}}{1-x} + x^{n+1}\right)\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}}\right.$$
$$\left. - \left(x^{n+1} + \mu\frac{1-x^{n+1}}{1-x}\right)\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}} - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right]. \tag{C.23}$$

It rewrites

$$\frac{B_t^i(n+1)}{P_{t+1}} = \kappa(1-\zeta)^{n+1}\left[\mu\left(\frac{1-x^{n+2}}{1-x}\right)\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \sum_{k=0}^{n+1}\left(x^k+\mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right], \tag{C.24}$$

which ends the proof. □

Finishing the Proof

To get from Lemma C.1 to Lemma 2, we just need to express $B_t^i(n)$ as a function of B_t instead of B_{t-n}. Iterate the government's flow budget constraint (C.16) from $t-(n+1)$ to t to obtain

$$\frac{B_t}{N_tP_{t+1}} = \frac{\mathcal{R}_{t-n,t+1}}{(1+g)^{n+1}}\frac{B_{t-(n+1)}}{N_{t-(n+1)}P_{t-n}} - \sum_{k=0}^{n}\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}. \tag{C.25}$$

Injecting it in equation (C.10) gives (21).

Sanity Check

Note as a sanity check that we have

$$\sum_{n=0}^{\infty}\psi_B(n)\frac{N_t(n)}{N_t} = 1, \tag{C.26}$$

$$\forall k\in[\![0,n]\!], \sum_{n=k}^{\infty}\phi_T(n,k)N_t(n) = 0, \tag{C.27}$$

so that when enough time has passed so that all the debt holdings of households of all ages is given by (21), the sum of debt holdings by all households sums to B_t.

D Proof of Lemma 3

If all households alive at t satisfy their No-Cash-on-the-Table condition (15), then (25) necessarily holds by taking the average over all households alive at t. The lengthier part is to show that (25) is also sufficient.

Let i be a household alive at t, and $n\geq 0$ its age at time t. It is therefore of age $n+k$ at $t+k$. We can decompose its debt holding at $t+k$ into the two following terms

$$\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} = \kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}} + \left(\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}}\right). \tag{D.1}$$

It follows that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} = \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}} + \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}}\left(\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n\frac{\bar{B}_{t+k}}{P_{t+k+1}}\right). \tag{D.2}$$

The proof amounts to showing that the second term in (D.2) necessarily tends to zero as $k\to\infty$. Condition (25) then implies that the first term tends to zero since $\kappa(1-\zeta)^n$ is a constant.

We denote with the superscript *old* the variables aggregated over all households already alive at t. Let $k \geq 0$. Aggregating (C.12) at period $t+k$ across households already alive at t gives

$$\frac{B_{t+k}^{old}}{P_{t+k+1}} = R_{t+k}\left(\beta(1-\lambda)\frac{B_{t+k-1}^{old}}{P_{t+k}} + Y_{t+k}^{old} - T_{t+k}^{old} - \mu H_{t+k}^{old}\right), \tag{D.3}$$

where

$$\begin{aligned} Y_{t+k}^{old} - T_{t+k}^{old} - \mu H_{t+k}^{old} &= \sum_{n=k}^{\infty} N_{t+k}(n)(Y_{t+k}^{i}(n) - T_{t+k}^{i}(n) - \mu H_{t+k}^{i}(n)) \\ &= \kappa\left(\frac{(1-\zeta)(1-\lambda)}{(1+g)}\right)^{k}(Y_{t+k} - T_{t+k} - \mu H_{t+k}). \end{aligned} \tag{D.4}$$

Injecting (D.4) into (D.3)

$$\frac{B_{t+k}^{old}}{P_{t+k+1}} = R_{t+k}\left(\beta(1-\lambda)\frac{B_{t+k-1}^{old}}{P_{t+k}} + \kappa\left(\frac{(1-\zeta)(1-\lambda)}{(1+g)}\right)^{k}(Y_{t+k} - T_{t+k} - \mu H_{t+k})\right). \tag{D.5}$$

The number of households alive at t that are still alive at $t+k$ is $(1-\lambda)^k N_t$. Dividing (D.5) by this to get the average debt holding:

$$\frac{\bar{B}_{t+k}}{P_{t+k+1}} = R_{t+k}\left(\beta\frac{\bar{B}_{t+k-1}}{P_{t+k}} + \kappa(1-\zeta)^{k}\left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu\frac{H_{t+k}}{N_{t+k}}\right)\right). \tag{D.6}$$

Applying equation (C.12) at $t+k$ to household i gives

$$\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} = R_{t+k}\left(\beta\frac{B_{t+k-1}^{i}(n+k-1)}{P_{t+k}} + \kappa(1-\zeta)^{n+k}\left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu\frac{H_{t+k}}{N_{t+k}}\right)\right). \tag{D.7}$$

Taking the difference between (D.7) and $\kappa(1-\zeta)^n$ times (D.6) gives

$$\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^{n}\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \beta R_{t+k}\left(\frac{B_{t+k-1}^{i}(n+k-1)}{P_{t+k}} - \kappa(1-\zeta)^{n}\frac{\bar{B}_{t+k-1}}{P_{t+k}}\right) \tag{D.8}$$

Iterating backward:

$$\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^{n}\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \beta^{k}\mathcal{R}_{t,t+k+1}\left(\frac{B_{t-1}^{i}(n)}{P_{t}} - \kappa(1-\zeta)^{n}\frac{\bar{B}_{t-1}}{P_{t}}\right) \tag{D.9}$$

This implies that the second term in (D.2) can be written as

$$\frac{(1-\lambda)^{k}}{\mathcal{R}_{t,t+k+1}}\left(\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^{n}\frac{\bar{B}_{t+k}}{P_{t+k+1}}\right) = (\beta(1-\lambda))^{k}\left(\frac{B_{t-1}^{i}(n)}{P_{t}} - \kappa(1-\zeta)^{n}\frac{\bar{B}_{t-1}}{P_{t}}\right), \tag{D.10}$$

which tends to zero as $k \to \infty$. This ends the proof.

E Proof of Proposition 4

In an equilibrium, all households must be on their consumption functions (13) so we can apply Lemma 3. In an equilibrium the No-Cash-on-the-Table conditions of all households must be satisfied, so from Lemma 3 condition (25) must hold. We show that it implies Proposition 4. To derive $\bar{B}_{t+k}$, we calculate B^{old}_{t+k} by calculating its complement: the quantity of public debt that is held at $t+k$ by households born at $t+1$ or later. We denote it B^{young}_{t+k}. It is equal to the sum of the debt held by households of age $0 \leq n \leq k-1$ at $t+k$. Since in an equilibrium all households are on their consumption function and the goods-market clears, we can use the expression for debt holdings in Lemma 2 to calculate it.

$$
\begin{aligned}
\frac{B^{young}_{t+k}}{P_{t+k+1}} &= \sum_{n=0}^{k-1} \frac{B^i_{t+k}(n)}{P_{t+k+1}} N_{t+k}(n) \\
&= \left[1 - \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\right]\frac{B_{t+k}}{P_{t+k+1}} \\
&- \sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right] \\
&\times \mathcal{R}_{t+j,t+k+1} T_{t+j}.
\end{aligned} \tag{E.1}
$$

We can therefore express $\bar{B}_{t+k}$ as

$$
\begin{aligned}
\frac{\bar{B}_{t+k}}{P_{t+k+1}} &= \frac{B_{t+k} - B^{young}_{t+k}}{P_{t+k+1}(1-\lambda)^k N_t} \\
&= \frac{1}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\frac{B_{t+k}}{N_t P_{t+k+1}} \\
&+ \frac{1}{(1-\lambda)^k}\sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right] \\
&\times \mathcal{R}_{t+j,t+k+1}\frac{T_{t+j}}{N_t}.
\end{aligned} \tag{E.2}
$$

Condition (25) therefore implies that

$$
\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \xi(k)\frac{B_{t+k}}{N_t \mathcal{R}_{t,t+k+1} P_{t+k+1}} + (\beta(1-\lambda))^k\left(\sum_{j=1}^{k}\theta(j)\frac{1}{\mathcal{R}_{t,t+j}}\frac{T_{t+j}}{N_t}\right) \tag{E.3}
$$

must tend to zero as $k \to \infty$, where

$$
\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{E.4}
$$

$$
\theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]. \tag{E.5}
$$

Using the budget constraint of the government

$$\frac{1}{\mathcal{R}_{t,t+k+1}}\frac{B_{t+k}}{P_{t+k+1}} = \frac{B_{t-1}}{P_t} - \sum_{j=0}^{k}\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} \tag{E.6}$$

to replace B_{t+k}/P_{t+k+1}, and multiplying by N_t, this implies equivalently that

$$\xi(k)\frac{B_{t-1}}{P_t} + \left(\sum_{j=1}^{k}\Big((\beta(1-\lambda))^k\theta(j) - \xi(k)\Big)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j}\right) - \xi(k)T_t \tag{E.7}$$

must tend to zero as $k \to \infty$.

Since $\lambda > 0$ or $g > 0$ in the non-Ricardian case, $\xi(k)$ tends to zero as $k \to \infty$, and so the term in B_{t-1} tends to zero. The condition is therefore equivalent to

$$\lim_{k\to\infty}\sum_{j=1}^{k}\Big((\beta(1-\lambda))^k\theta(j) - \xi(k)\Big)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} - \xi(k)T_t = 0. \tag{E.8}$$

Defining

$$\Omega(k,j) = (\beta(1-\lambda))^k\theta(j) - \xi(k) \text{ for } j \geq 1, \tag{E.9}$$

$$\Omega(k,0) = -\xi(k) \text{ for } j = 0, \tag{E.10}$$

this ends the proof.

F Proof of Lemma 5

In an equilibrium, all households are on their consumption function (13). Aggregating the consumption function (13) across households gives the aggregate consumption function

$$C_t = \mu\left(\frac{B^d_{t-1}}{P_t} + H_t\right). \tag{F.1}$$

where we denote by B^d_t the aggregate demand for public debt at t, $B^d_t = \int_i B^i_t di$ so that it does not assume market-clearing in the debt market, and where we used the fact that $\int_i B^i_{t-1}di = (1-\lambda)B^d_{t-1}$ since only $(1-\lambda)$ of household who demanded debt at $t-1$ are still alive at t.

Differentiate the aggregate consumption function (F.1), using $(1-\lambda)(1-\zeta)/((1+g)R_t)$ as the discount factor:

$$C_t - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}C_{t+1} = \mu\left[\left(\frac{B^d_{t-1}}{P_t} - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}\frac{B^d_t}{P_{t+1}}\right) + Y_t - T_t\right]. \tag{F.2}$$

It simplifies using the aggregate flow budget constraint (C.2) into

$$C_t = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right)C_{t+1} + \chi\frac{B^d_t}{R_t P_{t+1}}, \tag{F.3}$$

where χ is defined in equation (31). Note that equation (F.3) is a purely decision-theoretic object. It relies on no assumption of market clearing.

In equilibrium, market clearing holds in the debt market $B_t^d = B_t$. Replacing B_t by B_{t-1} using the flow budget of the government (10) gives equation (30).

G Proof of Proposition 2

Proposition 4 and 6 already showed that conditions (20) and (18) are necessary for a stable price equilibrium. We show that they are sufficient. The proof is constructive. Given an initial level of public debt B_{-1} distributed as $(B_{-1}^i)_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define recursively the following path for the interest rate

$$R_t = \frac{\frac{1-\zeta}{\beta(1+g)} Y_{t+1}}{Y_t - \chi\left(B_{t-1} - T_t\right)}, \tag{G.1}$$

which in combination with the FBC (10) of the government defines an entire path for interest rates and public debt. Because condition 1 is satisfied, equation (G.1) defines a positive finite interest rate. For this interest rate path, define individual consumption allocations through the consumption function (13), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-Clearing

Start with market-clearing. Since by construction all households are on the consumption function (13), aggregate consumption is given by the aggregate consumption function (F.1), which writes injecting the definition of aggregate human capital (C.7)

$$C_t = \mu\left(\frac{B_{t-1}^d}{P_t} + \sum_{k=0}^{\infty} \frac{a^k}{\mathcal{R}_{t,t+k}}(Y_{t+k} - T_{t+k})\right), \tag{G.2}$$

where

$$a = \frac{(1-\lambda)(1-\zeta)}{1+g}. \tag{G.3}$$

Define

$$Z_{t+k} = a^k \frac{Y_{t+k}}{\mathcal{R}_{t,t+k}}. \tag{G.4}$$

Using the definition of the interest rate path (G.1) which can be written

$$\frac{Y_{t+1}}{R_t} = \frac{\beta(1+g)}{1-\zeta}\left(Y_t - \chi\frac{B_t}{R_t P_{t+1}}\right), \tag{G.5}$$

Z_{t+k} satisfies the recursion

$$Z_{t+k} = bZ_{t+k-1} - \chi b \frac{a^{k-1}}{\mathcal{R}_{t,t+k}} B_{t+k-1}, \tag{G.6}$$

where

$$b = \beta(1-\lambda). \tag{G.7}$$

By iteration, it gives

$$Z_{t+k} = b^k Y_t - \chi \left(\sum_{j=0}^{k-1} b^{k-j} a^j \frac{B_{t+j}}{\mathcal{R}_{t,t+j+1}} \right). \tag{G.8}$$

It follows that (after permuting a double sum)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu} \left(Y_t - \chi\beta(1-\lambda) \left(\sum_{j=0}^{k-1} a^j \frac{B_{t+j}}{\mathcal{R}_{t,t+j+1}} \right) \right). \tag{G.9}$$

Using the budget constraint of the government

$$\frac{B_{t+j}}{P_{t+j+1}\mathcal{R}_{t,t+j+1}} = \frac{B_{t-1}}{P_t} - \sum_{i=0}^{j} \frac{1}{\mathcal{R}_{t,t+i}} T_{t+i}, \tag{G.10}$$

it can be rewritten (after permuting a double sum again)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu} \left(Y_t - \frac{\chi\beta(1-\lambda)}{1-a} \left(\frac{B_{t-1}}{P_t} - \sum_{i=0}^{\infty} a^i \frac{T_{t+i}}{\mathcal{R}_{t,t+i}} \right) \right), \tag{G.11}$$

or noticing that $\chi\beta(1-\lambda)/(1-a) = \mu$,

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu} Y_t - \left(\frac{B_{t-1}}{P_t} - \sum_{i=0}^{\infty} a^i \frac{T_{t+i}}{\mathcal{R}_{t,t+i}} \right). \tag{G.12}$$

Injecting equation (G.12) into the aggregate consumption function (G.2) implies

$$C_t = Y_t + \mu \left(\frac{B^d_{t-1}}{P_t} - \frac{B_{t-1}}{P_t} \right). \tag{G.13}$$

Because $B^d_{-1} = B_{-1}$, this implies market clearing in the goods market at $t = 0$, and therefore in the debt market, and so on at all periods by induction.

Individual Optimality

We now check individual optimality, which is characterized in Lemma 1. Condition (13) is satisfied by construction. We need to show that the No-Cash-on-the-Table constraints of all households are satisfied. Since all households behave according to the consumption function (13), we can apply Lemma 3 and simply show that condition (25) is satisfied. Since all households are on their consumption function and the goods market clears, we can use Lemma 2. That $\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{\bar{B}_{t+k}}{P_{t+k+1}}$ tends to zero as $k \to \infty$ can therefore be rewritten as equation (E.8), which holds by assumption from condition (18).

H Proof of the Implication in Section 3.1

Write the budget constraint of the government (10) in per-GDP term, and iterate it forward to give

$$\frac{B_{t-1}}{P_t Y_t} = \sum_{k=0}^{\infty} \left(\Pi_{i=0}^{k} \frac{\frac{Y_{t+i+1}}{Y_{t+i}}}{R_{t+i}} \right) \frac{T_{t+k}}{Y_{t+k}} + \lim_{k\to\infty} \left(\Pi_{i=0}^{k} \frac{\frac{Y_{t+i+1}}{Y_{t+i}}}{R_{t+i}} \right) \frac{B_{t+k}}{P_{t+k+1} Y_{t+k+1}}. \tag{H.1}$$

Assume that the debt-to-GDP limit (18) is satisfied and that the real interest rate is necessarily above the growth rate of the economy (43). The last bubble term is then the product of a term that tends to zero and of a bounded term, so it tends to zero. Multiplying what is left of equation (H.1) by Y_t gives the NPV equation (16).

I Proof of Proposition 4

The intertemporal budget constraint of non hand-to-mouth households, denoted with the superscript PI for permanent-incomers, is

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k}^{PI} = \frac{B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k}^{PI} - T_{t+k}^{PI}). \tag{I.1}$$

Injecting the expression for income $Y_t^{PI} = (1-\alpha_Y)Y_t$ and taxes $T_t^{PI} = (1-\alpha_T)T_t$, as well as the equilibrium requirement that $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$, gives the intertemporal budget constraint of the government (16).

The proof of sufficiency follows the proof of Proposition 1.

J Proof of Corollary 1

Condition (20) can be derived following exactly the same steps are in the case without hand-to-mouth households. Indeed, in the proof, taxes only intervene through the flow budget constraint of the government.

We show that condition (50) is necessary. Following the same steps as without hand-to-mouth households, one can show that equation (F.3) holds for the aggregate consumption of non hand-to-mouth households,

$$C_t^{PI} = \frac{1}{\beta R_t} \left(\frac{1-\zeta}{1+g} \right) C_{t+1}^{PI} + \chi \frac{B_t^d}{R_t P_{t+1}}. \tag{J.1}$$

Since non hand-to-mouth households collectively hold all of public debt, assuming the debt market clears and using the flow budget constraint of the government (10) gives

$$C_t^{PI} = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right) C_{t+1}^{PI} + \chi\left(\frac{B_{t-1}}{P_t} - T_t\right). \tag{J.2}$$

Using the fact that in equilibrium $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$ gives the IS curve

$$Y_t = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right)\left(Y_{t+1} + \frac{\alpha_T}{1-\alpha_Y}T_{t+1}\right) + \frac{\chi}{1-\alpha_Y}\frac{B_{t-1}}{P_t} - \frac{\alpha_T+\chi}{1-\alpha_Y}T_t. \tag{J.3}$$

Condition (50) is obtained when R_t to infinity.

The proof of sufficiency follows the proof of proposition 2.

K Proof of Proposition 3

The two flow budget constraints (35) and (36) can be combined into the intertemporal budget constraint

$$C_t^y + \frac{1}{R_t}C_{t+1}^o = Y_t^y - T_t^y + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o). \tag{K.1}$$

Combining it with the household's first-order condition, the optimal behavior of households is characterized by the following consumption function,

$$C_t^y = \frac{1}{1+\beta}\left((Y_t^y - T_t^y) + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o)\right), \tag{K.2}$$

$$C_t^o = \frac{B_{t-1}^y}{P_t} + (Y_t^o - T_t^o). \tag{K.3}$$

Given that households have finite lives and leave no wealth when they die, there is no extra No-Cash-on-the-Table constraint as part of individual optimality.

Necessary Condition: We first show that condition (42) is necessary in equilibrium. Summing up (K.2) and (K.3), and using the fact that the debt market cleared in period $t-1$, $B_{t-1} = N_{t-1}^{young}B_{t-1}^{young}$, total aggregate consumption is given by

$$C_t = \frac{1}{1+\beta}\left(\gamma(Y_t - T_t) + \frac{1}{R_t}(1-\gamma)(Y_{t+1} - T_{t+1})\right) + \frac{B_{t-1}}{P_t} + (1-\gamma)(Y_t - T_t). \tag{K.4}$$

Imposing market clearing on the goods market, we get the dynamic IS curve

$$Y_t = \frac{1-\gamma}{\gamma\beta}\frac{1}{R_t}(Y_{t+1} - T_{t+1}) + \frac{1+\beta}{\gamma\beta}B_{t-1} - \left(\frac{1+\beta}{\gamma\beta} - 1\right)T_t. \tag{K.5}$$

Setting the interest rate to infinity gives the condition (42).

Sufficient Condition: If condition (42) is satisfied in all periods, define a path for the real interest rate through

$$R_t = \frac{\frac{1-\gamma}{\gamma\beta}(Y_{t+1} - T_{t+1})}{Y_t - \frac{1+\beta}{\gamma\beta}B_{t-1} + \left(\frac{1+\beta}{\gamma\beta} - 1\right)T_t} \tag{K.6}$$

and the government flow budget constraint (10). For this interest rate path, define the consumption of young and old households through the consumption function (K.2) and (K.3). Individual optimality is satisfied by construction. Rewriting the expression of aggregate consumption (K.4) using the expression (K.6) for R_t shows market clearing $C_t = Y_t$. This ends the proof.

L Details on Woodford 1990 Model

Consider a steady-state where the high and low endowments Y^h and Y^l are constant, and where the government maintains a constant level of public debt B through constant taxes $T = (1 - 1/R)B$, which it splits between $T^h = Y^h/Y \times T$ and $T^l = Y^l/Y \times T$. We look for a steady-state equilibrium, i.e. values for R, C^h and C^l.

In both the OLG and the HANK models, we always have

$$R = \frac{1}{\beta}\frac{C^l}{C^h}. \tag{L.1}$$

In the OLG model, we always have in addition that

$$C^h = Y^h - T^h - \frac{1}{R}B, \tag{L.2}$$

$$C^l = B + Y^l - T^l. \tag{L.3}$$

Together, the three equations (L.1)-(L.2)-(L.3) determine an equation in R that gives the equilibrium real rate.

In the HANK model, the equilibrium is characterized by equations (L.1)-(L.2)-(L.3) only up to the level of public debt such that $C^h = C^h$ and $R = 1/\beta$. Beyond this level, the low-endowment household is no longer up against its borrowing constraint, and is instead on its Euler equation

$$R = \frac{1}{\beta}\frac{C^h}{C^l}, \tag{L.4}$$

so that $C^l = C^h$ and $R = 1\beta$. There is perfect self-insurance.

M Proof of Proposition 5

The system (64)-(65) can be written in matrix form

$$A \begin{bmatrix} \hat{\pi}_t \\ \hat{b}_{t-1} \end{bmatrix} = \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{b}_t \end{bmatrix} + B \begin{bmatrix} \nu_t^i \\ \nu_t^g \end{bmatrix}, \tag{M.1}$$

where

$$A = \begin{bmatrix} \phi_\pi + \eta \frac{b}{y} & -\eta(1-\psi_b) \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \frac{R}{1+g}(1-\psi_b) \end{bmatrix} \tag{M.2}$$

$$B = \begin{bmatrix} -1 & \eta \\ -\frac{b}{y} & -\frac{R}{1+g} \end{bmatrix} \tag{M.3}$$

The economy has a unique bounded solution if and only if the matrix A has one eigenvalue outside the unit circle and one within. Calculating its trace and determinant, its roots are the solution to the quadratic equation

$$P(\lambda) = \lambda^2 - \left(\frac{R}{1+g}(1-\psi_b) + \phi_\pi + \eta\frac{b}{y}\right)\lambda + (1-\psi_b)\phi_\pi\left(\frac{R}{1+g} + \eta\frac{b}{y}\right). \tag{M.4}$$

Necessary Condition: If the economy has a unique equilibrium, then one root is outside the unit circle and one is inside. If so, the roots are necessarily real (complex roots have the same modulus). If roots are real, they are both positive, so having one inside the unit circle and the other outside is equivalent to having one root greater than one and the other smaller than one. In turn, this is equivalent to $P(1) < 0$, which is condition (66).

Sufficient Condition: Conversely, if condition (66) is satisfied, then it means $1 - tr + det < 0$. In this case the determinant of the quadratic equation $\Delta = tr^2 - 4det > (det+1)^2 - 4det = (det-1)^2 > 0$, so both roots are positive. Given that condition (66) is satisfied, it means one root is greater than one and the other outside. Hence there is a unique equilibrium.

When there is a unique equilibrium, let λ be the root of A that is greater than one and let $(1, -\mu)$ be its left eigenvector. The constant μ can be solved to be

$$\mu = \frac{\eta(1-\psi_b)}{\lambda - \frac{R}{1+g}(1-\psi_b)}. \tag{M.5}$$

Since $\lambda > \frac{R}{1+g}(1-\psi_b)$, the constant μ is positive. Note that the expression for μ is ill-defined in the case of fiscal dominance of the Ricardian case, $\eta = 0$ and $\lambda = \frac{R}{1+g}(1-\psi_b)$. In this case, the constant μ can be expressed as

$$\mu = \frac{\phi_\pi - \frac{R}{1+g}(1-\psi_b)}{\frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right)}. \tag{M.6}$$

Equation (M) implies

$$\lambda(\hat{\pi}_t - \mu\hat{b}_{t-1}) = (\hat{\pi}_{t+1} - \mu\hat{b}_t) + \left(\left(\mu\frac{b}{y} - 1\right)\nu_t^i + \left(\eta + \mu\frac{R}{1+g}\right)\nu_t^g\right). \tag{M.7}$$

Iterating forward gives equations (70).

Finally, we show that in the particular case $\psi_b = 0$, the impact effect of fiscal shocks on inflation is the same as in the Ricardian case. When $\psi_b = 0$, we have that the two roots of the polynomial (M.4) are ϕ_π and $\lambda = \frac{R}{1+g} + \eta\frac{b}{y}$. It follows that $\mu = 1/(b/y)$. Replacing the expression for μ in equation (70) gives

$$b_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty}\left(\frac{1}{\frac{R}{1+g} + \eta\frac{b}{y}}\right)^k \nu_{t+k}^g, \tag{M.8}$$

which implies that in particular $\frac{\partial \pi_t}{\partial \nu_t^g} = \frac{1}{\frac{b}{y}}$.

N Proof of Proposition 6

Equations (64)-(65) are now

$$\left(\phi_\pi + \eta\frac{b}{y}\right)\hat{\pi}_t + \nu_t^i = \hat{\pi}_{t+1} + (\eta - \phi_b)\left(\left(1 - \psi_b\right)\hat{b}_{t-1} + \nu_t^g\right), \tag{N.1}$$

$$\hat{b}_t = \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)\left(\left(1 - \psi_b\right)\hat{b}_{t-1} + \nu_t^g\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu_t^i\right). \tag{N.2}$$

The system (N.1)-(N.2) can be written as in equation (M) but replacing matrix A with the matrix

$$A' = \begin{bmatrix} \phi_\pi + \eta\frac{b}{y} & 0 \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1 - \psi_b) \end{bmatrix} \tag{N.3}$$

The two roots of A' are $\phi_\pi + \eta\frac{b}{y}$ and $\left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1 - \psi_b)$, from which the result follows.