WORKING paper

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Fiscal Requirements for Price Stability When Households are Not Ricardian[1]

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Abstract

Are restrictions on fiscal policy necessary for monetary policy to be able to deliver price stability? When households are Ricardian, the Fiscal Theory of the Price Level asserts that the government's intertemporal budget constraint must hold for on-target inflation. We show that when households are not Ricardian, fiscal requirements still exist but are very different. (1) Intertemporal budget constraints no longer impose any significant requirement on fiscal policy for price stability. A requirement derived from intertemporal budget constraints still exists, but it does not correspond to the one of the government and imposes much looser requirements on fiscal policy than in the Ricardian case. (2) Yet a second requirement exists, specific to non-Ricardian households: Public debt must remain below a threshold. Above it, its wealth effect on aggregate spending can no longer be counter-balanced by interest rate hikes, however large. This second requirement puts an upper bound on public debt regardless of future fiscal surpluses. Assessing risks of fiscal dominance is therefore very different when households are not Ricardian.

Keywords: De-Anchoring; Taylor Principle; Bounded Rationality.

JEL classification: E52, E31, E43

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Non-Technical Summary

Give the central bank a clear mandate of price stability. Grant it full independence from the government. And appoint at its head a steadfast governor who will not let anything but its mandate influence its policy. Does the central bank then have all it needs to deliver price stability? Or must requirements on the government's fiscal policy also be imposed? The question is a cornerstone of monetary-fiscal interactions, determining whether monetary policy has the power to insulate inflation from imprudent fiscal decisions, or is ultimately dependent on a well-behaved fiscal authority.

The existence of fiscal requirements for price stability is at the root of the convergence criteria of the Stability and Growth Pact in the euro area, but they have no consensual basis in economic theory. Today, the main rationale for fiscal requirements is the Fiscal Theory of the Price Level (FTPL) (Leeper 1991, Sims 1994, Woodford 2001, Cochrane 2001). Yet the FTPL remains controversial to this day. While early skepticism of the FTPL focused on its strong reliance on equilibrium selection, recent skepticism points to its strong reliance on the assumption of Ricardian households. As the Heterogeneous Agents New Keynesian (HANK) literature has become an appealing alternative to the Ricardian representative household, the robustness of the FTPL has been put into question.

We show that when households are not Ricardian, fiscal requirements for price stability still exist, but take a very different form. Under Ricardian households, the FTPL asserts that if the government does not satisfy its intertemporal budget constraint in the absence of above-target inflation, then the central bank cannot deliver price stability. Inflation must set in to dilute the real value of public debt until it matches the level of real future primary surpluses.

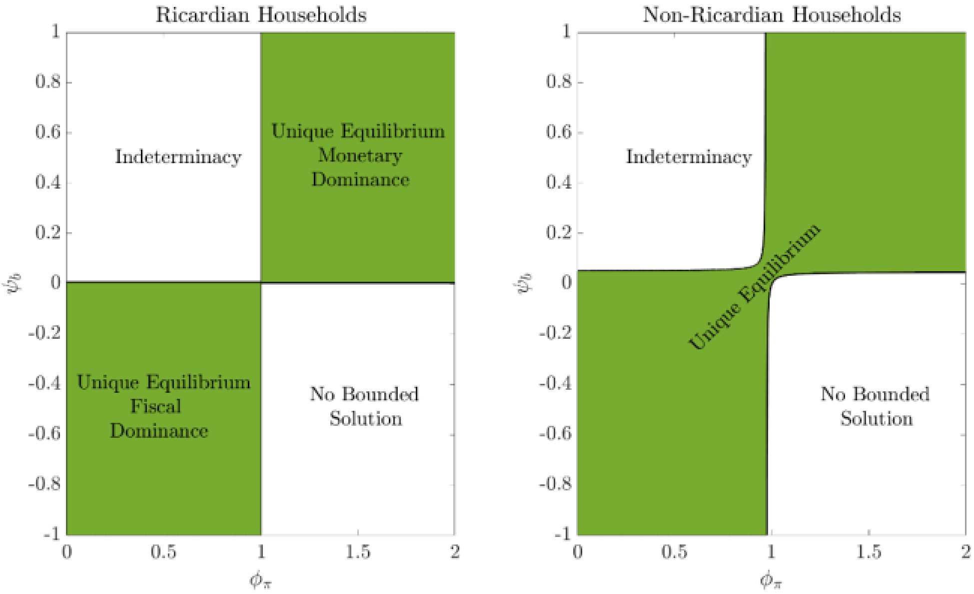
In the paper, we move from Ricardian to non-Ricardian households. Following the tradition of the FTPL, we first consider what requirements arise from intertemporal budget constraints. We show that when households are not Ricardian, households' intertemporal budget constraints impose only very weak requirement. In particular, if, from any current level of public debt, the government plans on never raising any tax to repay it, this violates no household's intertemporal budget constraint.

While intertemporal budget constraints no longer pose any significant constraint, we show that a new requirement on fiscal policy for price stability arises. When households are not Ricardian, higher public debt increases aggregate demand through a wealth effect, putting upward pressure on inflation. This in itself poses no constraint on the ability of the central bank to maintain price stability. It can counter the inflationary effect of higher debt with higher interest rates, just like it can counter any other demand shock with higher interest rates, retaining the ultimate control over inflation.

We show, however, that there exists a debt limit beyond which no interest rate, even very high, can counter-balance the effect of higher debt on aggregate demand and bring it back in line with aggregate supply. We therefore recover a fiscal requirement for price stability when households are not Ricardian. But it is very different from the requirement of the FTPL obtained when households are Ricardian.

We conclude by analyzing how the central bank can implement price stability once these fiscal requirements are satisfied. In doing so we reconsider Leeper (1991)'s local version of the FTPL in the case of non-Ricardian households. We show that when the central bank follows a standard Taylor rule that responds to inflation, fiscal shocks always affect inflation, however strong the response of monetary policy to inflation. It it no longer possible to distinguish between a monetary regime and a fiscal regime (see Figure below). Yet, we show that monetary policy can implement price stability if it responds to both inflation and the level of public debt. Inflation is then insulated from fiscal shocks.

Equilibrium Uniqueness



Note: The diagram represents when there exists a unique bounded equilibrium (in green), no bounded equilibrium, or a multiplicity of equilibria (indeterminacy) as a function of the degree of responsiveness of monetary policy to inflation Φπ and the degree of responsiveness of fiscal policy to public debt Ψb. The left panel represents the case of Ricardian households, with a strict distinction between a regime of fiscal dominance and a regime of monetary dominance. The right panel represents the case of non-Ricardian households, with no such strict distinction.

Prérequis fiscaux pour la stabilité des prix quand les ménages ne sont pas ricardiens

RÉSUMÉ

Des restrictions sur la politique budgétaire sont-elles nécessaires pour que la politique monétaire soit en mesure de garantir la stabilité des prix ? Lorsque les ménages sont ricardiens, la théorie budgétaire du niveau des prix énonce que la contrainte budgétaire intertemporelle du gouvernement doit être respectée pour une inflation à sa cible. Nous montrons que, lorsque les ménages ne sont pas « ricardiens », des prérequis budgétaires existent toujours, mais sont très différents. (1) Les contraintes budgétaires intertemporelles n'imposent plus de prérequis significatifs sur la politique budgétaire pour garantir la stabilité des prix. Un prérequis dérivé des contraintes budgétaires intertemporelles existe encore, mais il ne correspond pas à celle du gouvernement et impose des contraintes beaucoup moins fortes sur la politique budgétaire que dans le cas ricardien. (2) Il existe cependant un deuxième prérequis, spécifique aux ménages non ricardiens : la dette publique doit rester inférieure à une limite. Au-dessus, son effet de richesse sur la dépense agrégée ne peut plus être contrebalancé par des hausses des taux d'intérêt, aussi amples soient elles. Ce deuxième prérequis impose une limite sur la dette publique, indépendante des excédents budgétaires futurs. Évaluer les risques de dominance fiscale est donc très différent lorsque les ménages ne sont pas ricardiens.

Mots-clés : théorie budgétaire du niveau des prix, HANK, interactions fiscales/monétaires.

Introduction

Give the central bank a clear mandate of price stability. Grant it full independence from the government. And appoint at its head a steadfast governor who will not let anything but its mandate influence its policy. Does the central bank then have all it needs to deliver price stability? Or must requirements on the government's fiscal policy also be imposed? The question is a cornerstone of monetary-fiscal interactions, determining whether monetary policy has the power to insulate inflation from imprudent fiscal decisions, or is ultimately dependent on a well-behaved fiscal authority.

Policy-makers typically see fiscal requirements as necessary, yet fiscal requirements have no uncontroversial basis in economic theory. At the creation of the euro area, the convergence criteria of the Maastricht treaty and then the Stability and Growth Pact introduced fiscal rules on national governments under the assumption they were necessary to allow the ECB to deliver on its price stability mandate. Yet in the economic literature, the monetarist view long prevailed that the control of the price level is ultimately always in the realm of the central bank. Only with Sargent and Wallace (1981)'s unpleasant monetarist arithmetic did this view start to be questioned. Today, the main rationale for fiscal requirements is the Fiscal Theory of the Price Level (FTPL)—developed primarily *after* the introduction of debt breaks in the Maastricht treaty (Leeper, 1991; Sims, 1994; Woodford, 1995, 2001; Bassetto, 2002; Cochrane, 2001, 2005).

Yet the FTPL remains controversial to this day. While early skepticism of the FTPL focused on its strong reliance on equilibrium selection, recent skepticism points to its strong reliance on the assumption of Ricardian households, when finite lives, financial frictions, or limited foresight are enough to make households non Ricardian.[1] As the Heterogeneous Agents New Keynesian (HANK) literature has become an appealing alternative to the Ricardian representative household, the robustness of the FTPL has been put into question. For instance, Angeletos, Lian, and Wolf (2023, 2024) question how well the FTPL accounts for the inflationary effects of deficits compared to HANK models. It remains an open question whether the fiscal requirements for price stability asserted by the FTPL remain valid when households are not Ricardian, and whether there is any fiscal requirement at all in this case.

We show that when households are not Ricardian, fiscal requirements for price stability still exist, but take a very different form. Under Ricardian households, the FTPL asserts that for the central bank to be able to deliver on-target inflation, the government's intertemporal budget constraint

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{1}$$

must hold for on-target inflation $P_t = P^*$, where B_{t-1} is nominal public debt, T_{t+k} future real fiscal surpluses, and $\mathcal{R}_{t,t+k}$ the real interest rate from t to $t+k$. If equation (1) is satisfied for $P_t = P^*$, an equilibrium with on-target inflation exists, that the central bank is then free to implement by appropriately setting interest rates. But if the government does not plan on sufficient future fiscal surpluses to make equation (1) hold,

[1] In the FTPL, a key distinction is made between Ricardian and non-Ricardian *fiscal policies* (Woodford, 1995). A Ricardian fiscal policy is one that satisfies the intertemporal budget constraint of the government for any price level. Whether a fiscal policy is Ricardian or not is however different from whether households are Ricardian or not—they always are in the FTPL.

no equilibrium with on-target inflation exists. The central bank has therefore no chance of delivering it. To restore an equilibrium, inflation must set in to dilute the real value of public debt until it matches the level of real future surpluses.

We move from Ricardian households to non-Ricardian households and ask the same question: Are there constraints on fiscal policy—on the current level of public debt and on the future path of taxes—for a stable price equilibrium to exist? At this stage, we do not consider how the central bank may be able to implement the stable price equilibrium if it exists—we turn to this question in Section 5. We consider the case of non-Ricardian households through the tractable set-up of Blanchard (1985)'s perpetual youth model, which breaks the Ricardian equivalence by assuming that households face a mortality risk. The set-up has gained increasing appeal to study economies with non-Ricardian households, as households' mortality risk can be interpreted either literally as biological death—making it an overlapping generations model—or as the risk of hitting borrowing constraints—the financial frictions that are the focus of the HANK literature (e.g. Farhi and Werning, 2019; Wolf, 2021).[2]

Following the tradition of the FTPL, we first consider what requirements arise from intertemporal budget constraints. In an equilibrium, all households' intertemporal budget constraints must hold with equality. Otherwise, household would be leaving cash on the table. Under a representative Ricardian household, the unique intertemporal budget constraint is the one of the unique representative household, and it is the mirror image of the one of the government. It must therefore hold with equality, imposing equation (1) as a fiscal requirement for price stability.

We show that when households are not Ricardian, households' intertemporal budget constraints impose only very weak requirements for a stable price equilibrium to exist. In particular, if, from any current level of public debt, the government plans on never raising any tax to repay it, this violates no household's intertemporal budget constraint. For non-Ricardian households, holding a lot of public debt today does not mean they will end up with unspent wealth tomorrow, because they will be able to sell it to new generations tomorrow. As a result, there is no need for inflation to make intertemporal budget constraints hold. Intertemporal budget constraints do impose a restriction on the path of future taxes and transfers, but it is a very weak one—never raising taxes satisfies it. In addition, higher inflation does nothing to ease this constraint and to restore the possibility of an equilibrium.

Does this imply that there exists no fiscal requirement on price stability when households are not Ricardian? We show, second, that it does not. While intertemporal budget constraints no longer pose any significant constraint, a new requirement arises. When households are not Ricardian, higher public debt increases aggregate demand, and puts upward pressure on inflation. This in itself poses no constraint on the ability of the central bank to maintain price stability. The central bank can counter the inflationary effect of higher debt with higher interest rates, just like it can counter an inflationary oil-price shock with higher interest rates, retaining the ultimate control over inflation. At higher interest rates, households are willing to hold more public debt without spending their extra wealth, making aggregate demand in line with aggregate

[2] Our results do not depend on the existence of a mortality risk however. We generalize Blanchard (1985)'s set-up to allow for population growth, which breaks the Ricardian equivalence even when households face no mortality risk.

supply.

We show however that there exists a debt limit beyond which no interest rate, however high, can counterbalance the effect of higher debt on aggregate demand and bring it back in line with aggregate supply. If public debt is higher than this threshold, there exists no stable price equilibrium. Specifically, there exists a threshold d^* on the amount of debt $B_{t-1}/P_t - T_t$ the government must borrow relative to GDP Y_t,

$$\frac{B_{t-1}}{P_t Y_t} - \frac{T_t}{Y_t} \leq d^*. \tag{2}$$

When the debt-to-GDP ratio is above the threshold d^*, no stable price equilibrium exists. Inflation must set in to dilute the real value of public debt, lowering households' real wealth and therefore aggregate demand.

This limit on the debt-to-GDP ratio is typically very high. When calibrating the model according to its overlapping-generation interpretation, we find it to be 1600 times GDP. When calibrated the model according to its HANK interpretation—through an extension that allows to match intertemporal MPCs, as in Wolf (2021)—we find it to be lower, but still 10 times GDP. But because the debt limit must hold every period, it precludes the existence of a stable price equilibrium when fiscal policy is such that it lets public debt inexorably grow. For instance, when the initial level of public debt is high enough to have already brought the real interest rate r above the growth rate of the economy g, a government that never raises taxes necessarily puts public debt on a growing trajectory, and there exists no stable price equilibrium.

More generally, equation (2) puts a constraint on fiscal policy for a stable price equilibrium to exist. We therefore recover a fiscal requirement for price stability when households are not Ricardian. It is however very different from the requirement (1) of the FTPL obtained when households are Ricardian. In particular, in the FTPL high public debt does not in itself preclude price stability. Only public debt that is not backed by future surpluses does. When households are not Ricardian in contrast, a debt-to-GDP ratio above the d^* limit precludes price stability, irrespective of fiscal surpluses to come. The assessment of fiscal dominance is therefore very different when households are not Ricardian.

We derive the fiscal requirements for price stability abstracting from how the central bank can ensure price stability once they are satisfied. We conclude by addressing the question of implementation, assuming that the central bank sets nominal interest rates according to a standard Taylor rule, and the government sets taxes according to a similar feedback rule that responds to the level of public debt. In doing so we reconsider Leeper (1991)'s local version of the FTPL in the case of non-Ricardian households. We characterize analytically for which degree of responsiveness of fiscal and monetary policy there exists a unique bounded equilibrium.

We show that under a standard Taylor rule where monetary policy responds to inflation only, it it no longer possible to distinguish between a monetary regime and a fiscal regime. Within the region of parameters for which the equilibrium is unique, fiscal shocks always affect inflation, however strong the response of monetary policy to inflation. Yet, we show that monetary policy can insulate inflation from fiscal shocks and implement the stable price equilibrium if, on top of reacting to inflation, it directly responds to the level of public debt—not just to the higher inflation that higher debt generates. To insulate inflation from fiscal shocks, monetary policy must therefore monitor the level of public debt, in contrast to the idea that monetary dominance obtains when the central bank abstracts from fiscal developments.

By considering what the fiscal requirements for price stability are, this paper builds on the FTPL literature, developed by Leeper (1991), Sims (1994), Woodford (1995), Woodford (2001), Bassetto (2002), Cochrane (2001), and Cochrane (2005).[3] Among the founding papers of the FTPL, we connect in particular to Woodford (2001), to which the title of the present paper is a reference. We depart from the FTPL by considering non-Ricardian households.

An important stream of the recent FTPL literature considers whether the dynamics of US inflation can be accounted for by models that feature elements of both monetary and fiscal dominance, either because the economy oscillates between regimes of fiscal and monetary dominance (Davig and Leeper, 2007; Bianchi and Ilut, 2017; Bianchi and Melosi, 2017, 2019; Schmidt, 2024), or because some fiscal shocks are funded while others are not (Bianchi, Faccini, and Melosi, 2023; Smets and Wouters, 2024). We show that when households are not Ricardian the distinction between monetary and fiscal dominance is already blurred even absent regime switching and different reactions to different fiscal shocks. Elfsbacka-Schmoller and McClung (2024) consider the FTPL in a model with endogenous growth, a dimension from which we abstract. Maćkowiak and Schmidt (2024) extend the FTPL to a monetary union. Corsetti and Maćkowiak (2024) consider the FTPL when fiscal imbalances may be corrected with some probability in the future. Barro and Bianchi (2023) analyze the drivers of the recent inflation surge in OECD countries through the lens of the FTPL.

A fast growing literature analyses fiscal-monetary interactions in models with non-Ricardian households. Angeletos, Lian, and Wolf (2023) analyze how much deficits can finance themselves through the higher tax receipts that higher aggregate demand generates. Angeletos, Lian, and Wolf (2024) compare the inflationary effects of deficits in the FTPL and in a perpetual youth model similar to the one we study. Differently from them, we consider the fiscal requirements for price stability and how the central bank can insulate inflation from fiscal shocks.

Bassetto and Cui (2018) argue that the FTPL cannot determine the price level in a two-generation overlapping generation model. In contrast, we consider a perpetual youth set-up where intertemporal budget constraints put limits on the existence of equilibria, yet they do not correspond to the one of the FTPL, and we show that new fiscal requirements on price stability arise. We also point out in Section 4 that new requirements arise as well in a two-generation OLG model. Blanchard (2019) spurred the renewed interest in the possibility for the government to maintain deficits without ever raising taxes when the interest rate is less than the growth rate of the economy (r<g). Reis (2021) analyzes constraints on fiscal policy when $r < g$ arises from a safety premium of public debt over capital. Kaplan, Nikolakoudis, and Violante (2023) show that r can be less than g and the government can run constant primary deficit in a HANK model. Farmer and Zabczyk (2018, 2019) show that the same applies in a standard OLG model. Brunnermeier, Merkel, and Sannikov (2022) show the same applies in a perpetual youth model similar that the one we use. Hagedorn (2024) contends that the FTPL fails in models with non-Ricardian households because markets are then incomplete.

[3] For extensive reviews of the FTPL, see Leeper and Leith (2016) and Cochrane (2023), as well as Barthelemy, Mengus, and Plantin (2024). For a review of the FTPL with a special focus on empirical studies, see Bianchi, Melosi, and Rogantini Picco (2024).

Section 1 presents the model with non-Ricardian households. Section 2 derives the fiscal requirements arising from intertemporal budget constraints. Section 3 derives the new requirements specific to an economy with non-Ricardian households. Section 4 shows that the new requirement takes the similar form of a debt limit in a standard two-generation OLG model. Section 5 considers implementation issues following Leeper's local version of the FTPL.

1 An Economy with Non-Ricardian Households

In this section, we lay out the model of non-Ricardian households we rely on. It is a Blanchard-Yaari perpetual youth set-up (Blanchard, 1985) in discrete time, under perfect foresight. Relative to Blanchard (1985), we add two features. First, we allow for population growth at rate g, both to meaningfully talk about r and g and in order to stress that our results do not depend on the assumption that households die and buy life-insurance contracts—the Ricardian equivalence will break and all results hold even if households are infinitely lived, provided there is population growth. Second, we allow for individual incomes to shrink over time, both for realism to capture the need to save for retirement, and in order to allow for negative interest rates. We assume that the supply-side of the economy is given by an exogenous path for Y_t. Under flexible prices, this exogenous path can be interpreted as the supply-determined level of GDP. Under sticky prices, it can be interpreted as the exogenous path for natural output.

1.1 Of Life and Death

The economy is populated by an infinity of households of various ages. Each household faces a probability λ of dying each period, independent of how long it has been alive. As a consequence, each period a fraction λ of households dies. Households face the risk of dying with positive wealth. Insurance companies provide them with actuarially fair contracts to insure them against this risk. The wealth of households that die at t is redistributed to households still alive, in proportion to the financial wealth they had at the end of the previous period. The redistributed amount λB_{t-1} is therefore redistributed to the savings $(1-\lambda)B_{t-1}$ of the surviving households. Each dollar of saving therefore receives $\lambda/(1-\lambda)$ dollar of annuity.

New households are born every period, with no wealth. We allow for population growth. We assume that at t a number $(\lambda+g)N_{t-1}$ of households are born, so the population grows at rate g. Since λ of households die every period regardless of their ages, the number of households of age n at t is:

$$N_t(n) = (1-\lambda)^n(\lambda+g)N_{t-n-1} = \frac{\lambda+g}{1+g}\left(\frac{1-\lambda}{1+g}\right)^n N_t. \tag{3}$$

The particular case of the Ricardian representative household arises when there is neither new births nor death, $\lambda = 0$ and $g = 0$, in which cases all households are identical. Either $\lambda > 0$ or $g > 0$ is enough to break the Ricardian equivalence. From now on, we denote this non-Ricardian case as $\lambda + g > 0$.

1.2 Households

A household i has preferences over its consumption path

$$\sum_{k=0}^{\infty}(\beta(1-\lambda))^k \log(C^i_{t+k}), \tag{4}$$

where β is the preference discount factor. The household effectively discounts the future at the stronger rate $\beta(1-\lambda)$ because it factors in the chance that it won't be there to enjoy consumption tomorrow.

Household i maximizes its utility (4) subject its flow budget constraints and a No-Ponzi-scheme constraint. Having saved nominal wealth B^i_{t-1} from period $t-1$, it starts period t with wealth $1/(1-\lambda)B^i_{t-1}$. Its flow budget constraint is

$$C^i_t + \frac{1}{R_t}\frac{B^i_t}{P_{t+1}} = \frac{1}{1-\lambda}\frac{B^i_{t-1}}{P_t} + (Y^i_t - T^i_t), \tag{5}$$

where C^i_t is its real consumption, Y^i_t its real income, T^i_t the real taxes it has to pay, and where R_t is the real interest rate. Its No-Ponzi-scheme constraint is

$$\lim_{k\to\infty}\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{B^i_{t+k}}{P_{t+k+1}} \geq 0, \tag{6}$$

where $\mathcal{R}_{t,t+k}$ is the compounded real rate over k periods

$$\mathcal{R}_{t,t+k+1} = \prod_{j=0}^{k} R_{t+j}. \tag{7}$$

Like in the flow budget constraint (5) the factor $(1-\lambda)$ enters the No-Ponzi condition. It takes into account that the wealth the household holds grows at a rate $R/(1-\lambda)$ and not just R since the household also receives the annuity on its wealth.

1.3 Income Distribution

We let a household's income depend on its age. Specifically, we assume that a household of age n receives an income that decreases exponentially with its age, as the amount of labor it supplies decreases. This creates a desire to save for one's old age—present in the baseline two-period OLG model—that allows for the possibility of negative real interest rates. The household's income is

$$Y^i_t(n) = \kappa(1-\zeta)^n\left(\frac{Y_t}{N_t}\right), \tag{8}$$

where Y_t is aggregate income, $\zeta \in [0,1]$, and κ is a constant determined by the condition that individual incomes must sum to aggregate incomes Y_t. Given the age distribution in the population (3), the constant must be

$$\kappa = \frac{1+g-(1-\lambda)(1-\zeta)}{\lambda+g} \text{ if } \lambda+g>0. \tag{9}$$

The assumption that individual incomes decrease with age, $\zeta > 0$, is possible only in the non-Ricardian case $\lambda + g > 0$. In the Ricardian case of the representative agent $\lambda + g = 0$, all households necessarily get the same income, $Y_t^i = Y_t$. We can then only assume $\zeta = 0$ and $\kappa = 1$.

1.4 Fiscal Policy

We assume no government spending G. We do so to keep focus: we are interested in the effect of the financing of government expenditures through taxes for a fixed path for G which we set to zero to simplify. We therefore take fiscal policy to consist in a inherited debt level B_{-1} and a path for the level of aggregate taxes $(T_t)_{t\geq 0}$. The flow budget constraint of the government is

$$\frac{1}{R_t}\frac{B_t}{P_{t+1}} + T_t = \frac{B_{t-1}}{P_t}, \tag{10}$$

and determines the path for public debt B_t from the government's fiscal policy.

We assume that individual taxes are imposed proportionally on income, so that they follow the same age profile as income. A household of age n pays taxes

$$T_t^i(n) = \kappa(1-\zeta)^n \left(\frac{T_t}{N_t}\right). \tag{11}$$

1.5 Equilibrium and Stable Price Equilibrium

An equilibrium is defined in the standard way.

Definition 1. *For a given inherited level of public debt B_{-1}, distributed as an arbitrary $(B_{-1}^i)_i$ in the population, and a given fiscal policy $(T_t)_{t\geq 0}$, an equilibrium is an interest rate path $(R_t)_{t\geq 0}$ and an allocation $(C_t^i)_{t\geq 0,i}$ s.t.*

1. *All households behave optimally: Given interest rates $(R_t)_{t\geq 0}$, aggregate incomes $(Y_t)_{t\geq 0}$ and aggregate taxes $(T_t)_{t\geq 0}$—of which individual incomes and individual taxes depend according to (8) and (11)—each household maximizes its utility (4) subject to its flow budget constraints (5) and its No-Ponzi-scheme constraint (6).*

2. *The goods market clears:*

$$C_t = \int_i C_t^i di = Y_t. \tag{12}$$

Among all equilibria, we are interested in stable price equilibria. A stable price equilibrium is an equilibrium with on-target inflation. For convenience and without loss of generality, we assume that the inflation target is equal to 0, so that keeping inflation on target means keeping the price level constant, and we normalize the level of prices last period to one, $P_{-1} = 1$. A stable price equilibrium is then an equilibrium with $P_t = 1$ in all periods.

Definition 2. *A stable price equilibrium is an equilibrium where the price level is constant to $P_t = 1$ at all t.*

1.6 Characterization of Individual Optimality

The optimal behavior of a household can be characterized in the standard way through its consumption function and No-Ponzi-scheme constraint holding with equality.

Lemma 1. *A household i behaves optimally if and only if*

1. *Its consumption is given by the consumption function*

$$C_t^i = \mu \left(\frac{1}{1-\lambda} \frac{B_{t-1}^i}{P_t} + H_t^i \right), \tag{13}$$

where

$$\mu = 1 - \beta(1-\lambda) \tag{14}$$

is the household's marginal propensity to consume, and

$$H_t^i = \sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} (Y_{t+k}^i - T_{t+k}^i) \tag{15}$$

is its human capital.

2. *Its No-Ponzi constraint (6) holds with equality*

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{B_{t+k}^i}{P_{t+k+1}} = 0. \tag{16}$$

See Appendix A for a derivation. Since condition (16) states that the household leaves no cash on the table, we refer to it as the No-Cash-on-the-Table condition.

2 Fiscal Requirements from Intertemporal Budget Constraints

The rest of the paper derives conditions on B_{-1} and $(T_t)_{t\geq 0}$ for a stable price equilibrium to exist. In this section, we follow the tradition of the Fiscal Theory of the Price Level (FTPL) in the Ricardian case and consider what fiscal requirements arise from intertemporal budget constraints.

2.1 The Ricardian Case and the FTPL

In the Ricardian case $\lambda + g = 0$, the No-Cash-on-the-Table condition (16) is equivalent to that of a representative household. Combined with the flow budget constraints (5) of the household it imposes its intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = \frac{B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}). \tag{17}$$

Adding market-clearing (12) it imposes

$$\frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}. \tag{18}$$

Equation (18) is the cornerstone of the Fiscal Theory of the Price Level (FTPL). For a stable price equilibrium to exist, the intertemporal budget constraint (18) must hold for on-target inflation $P_t = 1$.

Because the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government coincide in the Ricardian case, the FTPL can be stated as a requirement on the intertemporal budget constraint of the government. The necessity of condition (18) as an equilibrium requirement comes from the optimality condition of the household however, not the government's. When the central bank stands ready to unconditionally buys government debt so that the government cannot default, nothing forces the government to satisfy any intertemporal budget constraint. But in an equilibrium, households cannot hold wealth that they plan never to spend. If the wealth they own in government bonds is not matched by future taxes to pay, households will spend it. Intuitively, the resulting inflation will dilute their real wealth on the left-hand-side of (18) until condition (18) holds.

2.2 Individual Debt Holdings of a Household

We now consider the non-Ricardian case $\lambda + g > 0$. To determine what constraints arise from the No-Cash-on-the-Table conditions (16), we first determine individual debt holdings in equilibrium. Appendix B shows the following lemma.

Lemma 2. *Consider the non-Ricardian case $\lambda + g > 0$.*

If all households are on their consumption function (13) and the goods market clears (12), then at time t a household of age $n \leq t$ has holdings of public debt

$$\frac{B_t^i(n)}{P_{t+1}} = \psi_B(n) \frac{B_t}{P_{t+1} N_t} - \sum_{k=0}^{n} \phi_T(n,k) \left(\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}} \right), \tag{19}$$

where

$$\psi_B(n) = \kappa(1-\zeta)^n \mu \frac{1-x^{n+1}}{1-x}, \tag{20}$$

$$\phi_T(n,k) = \kappa(1-\zeta)^n x^k \left(1 - \mu \frac{1-x^{n-k+1}}{1-x}\right), \tag{21}$$

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{22}$$

Lemma 2 gives the holding of one individual household of age n. Taken together, households of age n who are in number $N_t(n)$ given by (3) hold $N_t(n)B_t^i(n)$ of the public debt.[4] The expression for the coefficient $\psi_B(n)$ shows that abstracting from the effect of the past path of taxes, the amount of public debt held by a household either monotonically increases with age when $\zeta = 0$, or is a single-peaked function of age when $\zeta > 0$. When $\zeta = 0$, it increases monotically to converge to a constant fraction of public debt. When $\zeta > 0$, it first increases in the household's early years starting from zero, peaks in middle-age and then gradually shrinks to zero.

2.3 Characterization of the No-Cash-on-the-Table Condition

We now determine a necessary and sufficient condition for the No-Cash-on-the-Table conditions of all households to be satisfied. Appendix C shows the following lemma.

Lemma 3. *Consider the non-Ricardian case $\lambda + g > 0$.*

Assume all households consume according to the consumption function (13).

All the No-Cash-on-the-Table constraints (16) of all households are satisfied if and only if

$$\lim_{k\to\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{\bar{B}_{t+k}}{P_{t+k+1}} = 0, \tag{23}$$

where $\bar{B}_{t+k}$ is the average debt held at $t+k$ by households alive at t and still alive at $t+k$.

Intuitively, Lemma 3 states that the No-Cash-on-the-Table conditions of all households are satisfied if and only if the No-Cash-on-the-Table condition of a fictitious average household is satisfied, where the fictitious average household has debt holdings equal to the average debt holding among households that were already alive at t.

2.4 Requirement from Intertemporal Budget Constraints

We now use Lemma 2 and Lemma 3 to determine the requirements imposed by intertemporal budget constraints for an equilibrium to exist. Appendix D shows the following proposition.

Proposition 1. *Consider the non-Ricardian case $\lambda + g > 0$.*

Consider an inherited debt level B_{-1} and a future tax path $(T_t)_{t\geq 0}$. In an equilibrium, the following condition must hold

$$\lim_{k\to\infty} \sum_{j=0}^{k} \Omega(k,j) \frac{1}{\mathcal{R}_{t,t+j}} T_{t+j} = 0, \tag{24}$$

[4] Since we consider any arbitrary initial distribution of public debt (B_{-1}^i), expression (19) holds only for households born after period $t = 0$. For households born before $t = 0$, their debt holdings depend on their initial debt holdings B_{t-1}^i. If the initial distribution of public debt has been determined in the same way before $t = 0$ as after $t = 0$ however, expression (19) is valid for all households of any age.

where

$$\Omega(k,j) = (\beta(1-\lambda))^k \theta(j) - \xi(k), \tag{25}$$

$$\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{26}$$

$$\theta(0) = 0, \tag{27}$$

$$\forall j \geq 1, \theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\left(\beta(1-\lambda)\right)^j\right)\right]. \tag{28}$$

Proposition 1 puts only a very weak requirement on fiscal policy. In particular, the current debt level B_{t-1} does not appear in condition (24). As a consequence, for any debt level, the fiscal policy of never raising any taxes to repay public debt ($T_t = 0$ at all times) is consistent with all households' intertemporal budget constraints being satisfied. When households are not Ricardian, no level of public debt however high prevents any household's intertemporal budget constraint from being satisfied. In addition, condition (24) does not feature inflation. Inflation does nothing to ease this constraint and restore the possibility of an equilibrium.

3 Fiscal Requirements from the IS Curve

Away from the Ricardian case, intertemporal budget constraints do not impose any significant constraint on fiscal policy in order for monetary policy to be free to deliver price stability. In this section, we show however that there exists other fiscal requirements.

3.1 The IS Curve

To consider what additional restrictions are imposed on a stable price equilibrium, Appendix E derives the dynamic IS curve of the model.

Lemma 4. *In an equilibrium, the following dynamic IS curve holds*

$$Y_t = \frac{1-\zeta}{\beta(1+g)}\frac{1}{R_t}Y_{t+1} + \chi\left(\frac{B_{t-1}}{P_t} - T_t\right), \tag{29}$$

where

$$\chi = \frac{\mu}{1-\mu}\left(1-\frac{(1-\lambda)(1-\zeta)}{(1+g)}\right). \tag{30}$$

In the Ricardian case $\lambda + g = 0$ (and so $\zeta = 0$), the model reduces to the standard representative agent model and the dynamic IS curve reduces to the standard Euler equation where public debt does not appear as $\chi = 0$. Away from the Ricardian case, $\lambda + g > 0$, wealth matters for consumption, $\chi > 0$.

3.2 Countering an Expansive Fiscal Policy with a Tightening Monetary Policy

The IS curve (29) captures the fact that when households are born and/or die $\lambda + g > 0$, they are no longer Ricardian. Public debt is no longer irrelevant for aggregate demand and therefore for inflation. This in itself does not mean that the central bank loses control over inflation however. Many things can affect the level of aggregate demand, for instance exogenous shocks to households' preferences. The claim that monetary policy remains ultimately in control of the price level does not deny that other factors than monetary policy can affect inflation. It only argues that the central bank can use interest rates to counter-balance these shocks and deliver on-target inflation.

The same argument applies in principle to public debt: high public debt increases aggregate demand, but if it creates too much demand, the central bank can increase rates to bring it down. Specifically, if the central bank delivers the real interest rate

$$R_t = \frac{1-\zeta}{\beta(1+g)} \frac{Y_{t+1}}{Y_t} \left(1 - \chi \left(\frac{B_{t-1}}{Y_t} - \frac{T_t}{Y_t}\right)\right)^{-1}, \tag{31}$$

in all periods, a stable price equilibrium obtains.

But does there always exist an interest rate level that allows the central bank to bring down demand in line with supply? We now show that there does not when public debt is too high.

3.3 A Debt Limit

The following proposition states a limit on the level of real public debt for an equilibrium to exist when households are not Ricardian.

Proposition 2. *Consider the non-Ricardian case $\lambda + g > 0$.*

Consider an inherited debt level B_{-1} and a future tax path $(T_t)_{t\geq 0}$. In an equilibrium, the real-debt-to-GDP ratio must be below the threshold

$$d^* = \frac{1}{\chi} \tag{32}$$

at all periods,

$$\frac{B_{t-1}}{P_t Y_t} - \frac{T_t}{Y_t} < d^*. \tag{33}$$

The proposition follows from the fact that when the debt-to-GDP ratio is above the limit d^*, no interest rate, however large, can make the IS equation (29) hold. Even an infinitely large real interest rate $R_t = \infty$ is not enough to counter the wealth effect of public debt on aggregate consumption and bring aggregate demand down to supply.

It follows from Proposition 2 that a stable price equilibrium only exists if the debt-to-GDP ratio remains below the threshold

$$\frac{B_{t-1}}{Y_t} - \frac{T_t}{Y_t} < d^*. \tag{34}$$

Intuitively, if it does not, then demand is greater than supply and inflation ensues until it has diluted real public debt enough for there to exist a real interest rate that equates demand and supply.

Condition (33) shares some of the flavor of the FTPL condition (18). As in the FTPL, in equation (33) inflation still has the potential to restore an equilibrium when one does not exist, by decreasing the value of real debt. The idea of fiscal dominance as a situation in which the price level is what stabilizes real debt when fiscal policy does not is therefore still present. We will return to this point when studying implementation in Section 5. But the condition is thoroughly different from the FTPL condition (18). Contrary to the FTPL condition where only high public debt not backed by future surplus runs a risk of fiscal dominance, condition (33) bears on the level of public debt regardless of future surpluses.

Only in the Ricardian case $\lambda + g = 0$ (and so $\zeta = 0$) does the constraint (33) disappear. When households are Ricardian, public debt does not enter the IS equation and a higher level of public debt does not put upward pressure on equilibrium interest rates. The Ricardian case appears therefore as a very special one, where fiscal requirements take a thoroughly different form. This is summed up in the following main proposition of the paper, which also shows that the requirements we have derived as necessary conditions in Propositions 1 and 2 are also sufficient for a stable price equilibrium to exist (see Appendix F for a proof).

Proposition 3. *For a given initial level of debt B_{-1}, consider a future tax path $(T_t)_{t\geq 0}$.*

- *If households are Ricardian $\lambda + g = 0$, there exists a stable price equilibrium for this fiscal policy if and only if the following condition holds*

$$B_{t-1} = \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} T_{t+k}, \tag{35}$$

  for the interest rate path $R_t = Y_{t+1}/(\beta Y_t)$.

- *If households are not Ricardian $\lambda + g > 0$, there exists a stable price equilibrium for this fiscal policy if and only if the following two conditions hold*

  1. *Condition 1*

$$\lim_{k\to\infty} \sum_{j=1}^{k} \left((\beta(1-\lambda))^k \chi(j) - \psi(k) \right) \frac{1}{\mathcal{R}_{t,t+j}} T_{t+j} - \psi(k) T_t = 0, \tag{36}$$

  2. *Condition 2*

$$\frac{B_{t-1}}{Y_t} - \frac{T_t}{Y_t} < d^*, \tag{37}$$

  both of which for the interest rate path defined by the IS curve (29) together with the government's flow *budget constraint* (10).

The fiscal requirements in the Ricardian case are also discontinuous at the limit when the non-Ricardian model tends towards its Ricardian limit. For instance, setting $\zeta = 0$ (so that the limit to the Ricardian case can be taken) and setting $\lambda = 0$, the fiscal requirements in the non-Ricardian model for any $g > 0$ take the form of a debt limit that becomes larger and larger and tends to infinity as g tends towards $g = 0$. Only in the Ricardian case $g = 0$ does the new requirement of the intertemporal budget constraint of the government appear.

3.4 Allowing for Hand-to-Mouth Households

In the perpetual youth model, the MPC common to all households is $\mu = 1-\beta(1-\lambda)$. It can therefore be less than in the Ricardian case, but is tied to the factor $\beta(1-\lambda)$ that also determines the degree of discounting in households' consumption function. This limits the ability of the perpetual youth set-up to match the profile of intertemporal MPC (iMPC) in the data (Auclert, Rognlie, and Straub, 2018). As argued by Wolf (2021) however, the perpetual youth model augmented with a fraction of hand-to-mouth households can provide a very good approximation to a HANK model with explicit borrowing constraints and match the empirical iMPC. We show that the fiscal requirements of Proposition 3 can be easily generalized to this augmented version of the model.

Assume that a fraction α_Y of aggregate income Y_t goes to hand-to-mouth households that consume their income every period $C_t^i = Y_t^i$.[5] The remaining households behave as assumed so far in the paper. We assume that hand-to-mouth households collectively pay $\alpha_T T_t$ in taxes, with α_T possibly distinct from α_Y (when taxes are proportional to income, $\alpha_T = \alpha_Y$). We continue to assume among non hand-to-mouth households, taxes are imposed proportionally on income as per (11), where T_t is to be replaced by $(1-\alpha_T)T_t$.

Appendix G show the following extension of Proposition 3

Corollary 1. *Consider the perpetual youth model with a fraction α of hand-to-mouth households. For a given initial level of debt B_{-1}, consider a future tax path $(T_t)_{t\geq 0}$.*

- *If the non hand-to mouth households are Ricardian $\lambda + g = 0$, there exists a stable price equilibrium for this fiscal policy if and only if the intertemporal budget constraint of the government (35) holds for the interest rate path $R_t = (Y_{t+1} + \alpha_T/(1-\alpha_Y)T_{t+1})/(\beta(Y_t + \alpha_T/(1-\alpha_Y)T_t))$.*
- *If the non hand-to mouth households are not Ricardian $\lambda + g > 0$, there exists a stable price equilibrium for this fiscal policy if and only if are satisfied both condition (36) and the amended condition*

$$\frac{B_{t-1}}{Y_t} - \frac{\chi + \alpha_T}{\chi}\frac{T_t}{Y_t} < d^*, \tag{38}$$

[5] As far as the aggregate economy is concerned, how the incomes of hand-to-mouth households is distributed among themselves, e.g. according to age, does not matter.

where

$$d^* = \frac{1-\alpha_Y}{\chi}, \tag{39}$$

for the interest rate path defined by the IS curve (40) *together with the government's flow budget constraint* (10).

The first part of the corollary states that the FTPL condition continues to hold in a standard TANK model where non hand-to-mouth households are standard permanent-income households. Since such the Ricardian equivalence breaks in a standard TANK model, this result is not completely obvious. But Appendix G shows that the intertemporal budget constraint of non hand-to-mouth households still reduces to the intertemporal budget constraint once market-clearing is assumed. The FTPL can therefore still hold in some simple non-Ricardian models.

The second part of the corollary states that when the households that hold public debt are not Ricardian, the presence of hand-to-mouth households does affect fiscal requirements for price stability. Fiscal requirements still take the form of a debt limit however, but it is now lower by a factor $(1 - \alpha_Y)$. To understand why, note that the IS curve (29) is now

$$Y_t = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right)\left(Y_{t+1} + \frac{\alpha_T}{1-\alpha_Y}T_{t+1}\right) + \frac{\chi}{1-\alpha_Y}\frac{B_{t-1}}{P_t} - \frac{\alpha_T+\chi}{1-\alpha_Y}T_t. \tag{40}$$

To determine how much of public debt households want to spend, the MPC that matters is still the MPC μ of non hand-to-mouth households. Hand-to-mouth households have a higher MPC our of their income, but since they do not own wealth, their MPC does not directly matters. The presence of hand-to-mouth households is not irrelevant however. Because they consume a share α_Y of income, the consumption of the non hand-to-mouth households must now be lower. This puts a lower threshold on the maximum level of public debt they can hold in a stable price equilibrium.

3.5 Quantitative Assessment

How high is the debt limit d^*? We consider two calibrations of the model, corresponding to two interpretations of the model. In both quarterly calibrations, we set $\beta = 0.995$, $g = 0.005$ to correspond to a growth rate of 2%, and $\zeta = 0.005$, so that individual income shrinks by 2% per year.

The two calibrations differ as to the value of the death probability λ and α_Y. In the first calibration, we interpret the death probability λ literally as the probability of biological death, in line with the standard OLG interpretation of the model. We set it to $\lambda = 0.005$ or a 2% annual death probability. We abstract from hand-to-mouth households $\alpha_Y = \alpha_T = 0$. In the second calibration, we interpret λ as the probability of hitting one's borrowing constraints in line with the interpretation of the model as a HANK model as in Del Negro, Giannoni, and Patterson (2023), Farhi and Werning (2019), Angeletos, Lian, and Wolf (2023) and Wolf (2021). As in Angeletos, Lian, and Wolf (2023), we set $\lambda = 0.135$ and set α_Y to match an instantaneous

Table 1: Calibration and Debt Limits

β	0.995	
ζ	0.005	
g	0.005	
	OLG interpretation	HANK interpretation
λ	0.005	0.135
α_Y	0	0.07
Average MPC	0.01	0.20
d^*	1665.2	10.0
$\underline{d}$	8.3	0.05

Note: The table gives the two calibrations used in the paper, under two interpretations of the model. The OLG interpretation interprets the model literally as an OLG model, with the death probability the probability of actual biological death. The financial frictions interpretation interprets death as hitting one's financial constraint. The matching iMPC model is the version of the model with a share to hand-to-mouth households calibrated to match iMPC. The calibration is quarterly. d^* and $\underline{d}$ are expressed as debt to annual GDP.

quarterly MPC of 0.2 in order to match the empirical iMPC in Fagereng, Holm, and Natvik (2021). This gives $\alpha_Y = 0.07$. Note that the fraction of hand to mouth households is relatively small. Absent hand-to-mouth households, the model with $\lambda = 0.135$ already gives a substantial annual MPC of 0.44. The calibrations are summed up in Table 1.

Under the literal interpretation of the model, the debt limit d^* is extremely high, at about 1665 times annual GDP. In this case, the average MPC is very small (0.01 quarterly) so a very high level of public debt has only a limited effect on the level of aggregate demand. Under the HANK interpretation of the model, the debt limit d^* is much lower at 10.0. Since the non hand-to-mouth households who hold public debt have a much higher MPC (0.14 quarterly), they spend a much higher proportion of their wealth, and an equilibrium can only sustain a much lower level of public debt. The presence of hand-to-mouth households lowers the debt limit further still. As explained above however, the presence of hand-to-mouth households only has a limited impact, if only because there is only 7% of them. Absent hand-to-mouth households, the debt limit is only marginally higher, at 10.8.

3.6 Can the Government Never Repay its Debt and Prices Remain Stable?

With Proposition 3 in hands, we first turn to a particular but central question about the fiscal requirements for price stability: If the government intends to never repay its debt, does it threaten the ability of the central bank to deliver price stability?

A simplistic answer to this question is that the central bank's control over the price level is not threatened as long as the debt to GDP ratio is below the threshold d^*. The answer is incomplete however, since for a stable price equilibrium to exist the condition must hold at all future periods. Even if the current debt to GDP ratio is less than d^* at t, a tax path that lets the debt-to-GDP ratio inexorably increase will violate the debt limit at some point, make the fiscal policy inconsistent with a stable price equilibrium. To assess whether a stable price equilibrium exists, we therefore need to consider the future dynamic of public debt, and whether it remains below the threshold d^* at all times.

We consider the dynamics of public debt assuming the government never repays its debt and the central bank seeks to maintain price stability $P_t = 1$ in each period by delivering the real interest rate (31). Combining the interest rate (31) with the flow budget constraint of the government (10) with taxes set to zero gives the following dynamics of public debt

$$B_t = \frac{1-\zeta}{\beta}\left(\frac{1}{\frac{1}{B_{t-1}} - \frac{\chi}{Y_t}}\right). \tag{41}$$

Defining $y_t = Y_t/N_t$ the GDP per capita, and $b_{t-1} = B_{t-1}/Y_t$ the debt to GDP ratio, this rewrites as the dynamics of the debt-to-GDP ratio

$$b_t = \frac{1-\zeta}{\beta(1+g)}\left(\frac{1}{\frac{1}{b_{t-1}} - \chi}\right) \tag{42}$$

This dynamics is represented on Figure 1. On the figure, GDP per capita is assumed to be constant $y_t = y$ to focus on the debt dynamics. The shaded region corresponds to the one where the debt to GDP ratio is above the debt limit d^*. If the dynamics of the debt to GDP ration ends up in this region at any time in the future, there exists no stable price equilibrium.

This dynamics allows to determine under which conditions on the current debt to GDP ratio b_{-1} the debt to GDP ratio b_t ultimately reaches the debt limit precluding a stable price equilibrium, and under which conditions a stable price equilibrium does exist. From the government's flow budget constraint

$$b_t = \frac{R_t}{1+g} b_{t-1}, \tag{43}$$

the public debt grows and eventually reaches the debt limit d^* if the interest rate is greater than the growth rate of the economy. In turn, from the expression of the interest rate (31) in the tentative stable price equilibrium, the interest rate is increasing in the debt to GDP ratio. It follows that b_t increases and eventually reaches the debt limit if and only if b_{-1} is currently above the threshold on b that brings the interest rate (31) above the growth rate of the economy,

$$\underline{d} = \left(1 - \frac{1-\xi}{\beta(1+g)}\right) d^*. \tag{44}$$

As long as the deb to GDP ratio is initially below the threshold $\underline{d}$, the real interest rate (31) remains below the growth rate of the economy and the debt to GDP ratio shrinks back by itself even though the government never raises taxes. If at any time however, public spending or tax decreases bring the debt to GDP ratio above the threshold $\underline{d}$, the real interest rate (31) and the debt to GDP ratio then continually grow until public debt reaches the limit d^*. There is no stable price equilibrium.

This is illustrated on Figure 2, which plots the evolution over time of the debt to GDP ratio and of the real interest rate (31), depending on the initial level of b_{-1}. When the initial level of the debt to GDP ratio is

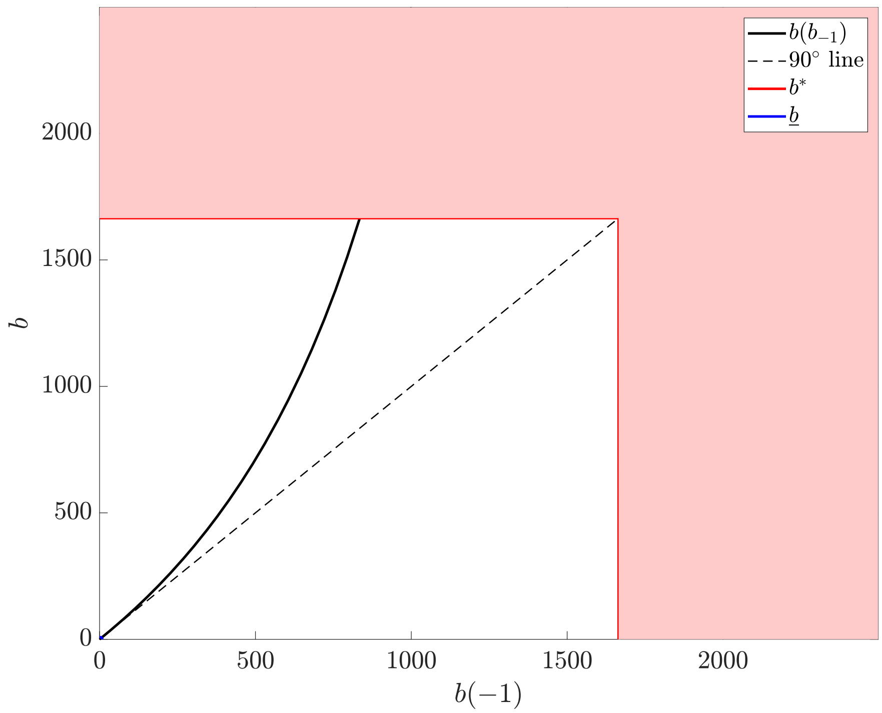


Figure 1: Diagram of Debt Dynamics

Note: The black thick line represents the mapping (42) giving the debt-to-annualized-GDP ratio in t as a function of the debt-to-GDP ratio at $t-1$, when there are no taxes. The shaded area in pink represents the area where the debt level if so large that no interest rate however large can reduce aggregate demand enough to equate demand to natural output. The calibration is given is Table 1, in the OLG interpretation of the model.

below $\underline{d}$, it then shrinks back to zero and a stable price equilibrium exists. When the initial level of the debt to GDP ratio is above $\underline{d}$, it then continually increases to go over d^* at some point. There exists no stable price equilibrium where the government never raises taxes.

The value of $\underline{d}$ is a fraction of the value of d^*. For the calibration corresponding to the OLG interpretation of the model, $\underline{d} = 8.3$, or 830% of GDP. For the calibration corresponding to the HANK interpretation of the model, $\underline{d} = 0.05$, or a debt-to-GDP ratio of just 5% of annual GDP.

Note that $\underline{d}$ is positive if and only if

$$1 - \xi < \beta(1 + g), \tag{45}$$

i.e. if and only the growth rate of the economy is large enough, or if households' income shrinks sufficiently fast with their age. In the latter case, the desire to save for the old age creates enough of a desire to save to bring the interest rate below one. When $1 - \xi > \beta(1 + g)$—in particular when $\xi = 0$ and $g = 0$—the interest rate always remains above 1 and there never exists a stable price equilibrium where the government never raises taxes.

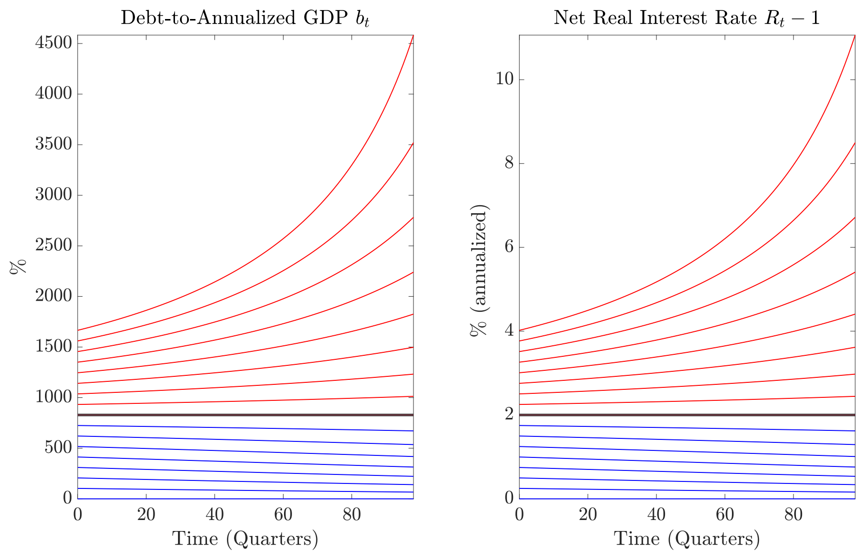


Figure 2: Debt Paths

Note: The right (left) panel plots the dynamics of the debt to annualized GDP ratio (the net real interest rate) over time when the governement never levies taxes, for different initial levels of the debt to annualized GDP ratio. For an initial debt-to-GDP ratio below $\underline{d}$, the debt-to-GDP ratio gradually shrinks back to zero (blue curves). For an initial debt-to-GDP ratio above $\underline{d}$, the debt-to-GDP ratio gradually increases and eventually diverges to infinity. The calibration is given is Table 1, in OLG interpretation of the model.

3.7 Comparison with the FTPL Condition

How does the condition for a stable price equilibrium from the debt limit compare to the one of the FTPL? First, neither is overall stricter than the other. The FTPL condition can fail while there exists a stable price equilibrium. The example of never repaying debt above is a primary example. Conversely, the FTPL condition can be satisfied yet no stable price equilibrium exists. This is the case if the current debt level is backed by future fiscal surpluses yet the debt to GDP ratio is above the limit d^*. In this case, the FTPL condition asserts there exists a stable price equilibrium, while there exists none.

Crucial to this different prediction is that in the FTPL high debt levels are not a problem in itself. Only high public debt unbacked by future fiscal surpluses is. Not with non-Ricardian households. The absolute level of the debt-to-GDP ratio is then a constraint on the existence of a stable price equilibrium. Future taxes still matter to assess the existence of a stable price equilibrium, since future taxes influence the future level of public debt. But they matter in a very different way than in the FTPL. A plan to raise taxes in the future is not enough to guarantee the existence of a stable price equilibrium if these tax increases are so far in the future that the debt to GDP ratio increases above the debt limit before these future taxes are collected.

4 Fiscal Requirements in a Standard Two-Generation OLG Model

In the section, we show that fiscal requirements for price stability take a similar form in a standard OLG model where agents live for two periods. Like the perpetual youth model, the standard OLG model can be interpreted either literally as overlapping generations, or the result of borrowing constraints and financial frictions (e.g. Woodford, 1990).

4.1 A Standard Samuelson OLG Model

We consider a standard overlapping-generation model (Samuelson, 1958). Households live for two periods. In period t, households are thus divided between young households born at t and old households born at $t-1$. The population N_t grows at rate g, and is therefore divided between $N_t^{old} = \frac{1}{2+g} N_t$ households born at $t-1$ and $N_t^{young} = \frac{1+g}{2+g} N_t$ households born at t.

A household born at t has preferences over its consumption in periods t and $t+1$

$$\log(C_t^y) + \beta \log(C_{t+1}^o). \tag{46}$$

The household is born with no wealth and maximizes its utility (46) subject to the flow budget constraints

$$C_t^y + \frac{1}{R_t} \frac{B_t^y}{P_{t+1}} = Y_t^y - T_t^y, \tag{47}$$

$$C_{t+1}^o = \frac{B_t^y}{P_t} + Y_{t+1}^o - T_{t+1}^o. \tag{48}$$

They can be combined into the intertemporal budget constraint

$$C_t^y + \frac{1}{R_t} C_{t+1}^o = Y_t^y - T_t^y + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o). \tag{49}$$

We assume that young households receive collectively a share γ of the economy's total income Y_t, while old households collectively receive the remaining share $1-\gamma$,

$$Y_t^y = \gamma \frac{Y_t}{N_t^y}, \tag{50}$$

$$Y_t^o = (1-\gamma) \frac{Y_t}{N_t^o}. \tag{51}$$

The government still sets aggregate taxes subject to the flow budget constraint (10) and taxes are still imposed proportionally to income

$$T_t^y = \gamma \frac{T_t}{N_t^y}, \tag{52}$$

$$T_t^o = (1-\gamma) \frac{T_t}{N_t^o}. \tag{53}$$

An equilibrium and a stable-price equilibrium are still defined as in Definition 1 and 2, where goods-market clearing now takes the form

$$N_t^y C_t^y + N_t^o C_t^o = Y_t. \tag{54}$$

4.2 Fiscal Requirements

When households live for two periods, their intertemporal budget constraints (49) necessarily hold once they behave optimally. As a consequence, no fiscal requirement arises from intertemporal budget constraints, as pointed out by Bassetto and Cui (2018). The following proposition shows however that a debt limit similar to (33) exists (Appendix H for a proof).

Proposition 4. *For a given initial level of debt B_{-1}, consider a future tax path $(T_t)_{t\geq 0}$.*

There exists a stable price equilibrium for this fiscal policy if and only if at all time t

$$\frac{B_{t-1}}{Y_t} - \left(1 - \frac{\gamma\beta}{1+\beta}\right)\frac{T_t}{Y_t} \leq \frac{\gamma\beta}{1+\beta}. \tag{55}$$

Taxes enter with a different coefficient than debt because young and old households have different MPC in the standard OLG model, but otherwise the fiscal requirement for price stability takes the overall form of a limit of the debt-to-GDP ratio.

5 Implementation and Leeper's Local FTPL

The previous sections have derived fiscal requirements for price stability abstracting from how the central bank can then implement price stability. In this section, we turn to the implementation question. In doing so, we reconsider Leeper (1991)'s analysis of fiscal and monetary dominance in the case of non Ricardian households.

5.1 Local Dynamics

We assume that prices are flexible, as in Leeper (1991)'s original analysis. The dynamics of the aggregate economy reduces to only two equations: the IS curve (29) and the flow budget constraint of the government (10), which can be written in per capital terms

$$y_t = \frac{1-\zeta}{\beta}\frac{\pi_{t+1}}{I_t}y_{t+1} + \chi\left(\frac{b_{t-1}}{\pi_t} - \tau_t\right), \tag{56}$$

$$b_t\frac{1+g}{I_t} + \tau_t = \frac{b_{t-1}}{\pi_t}. \tag{57}$$

where $b_{t-1} = B_{t-1}/(P_tN_t)$, $y_t = Y_t/N_t$ and $\tau_t = T_t/N_t$ are real public debt per capita, GDP per capita, and taxes per capita, π_t is the inflation rate and I_t is the nominal interest rates.

Following Leeper, we take the exogenous level of GDP under flexible prices to be constant, and consider the log-linearized version of these two equations around a steady state with real interest rate R and debt to

GDP ratio b/y,[6]

$$\hat{i}_t - \hat{\pi}_{t+1} = \eta \left(\hat{b}_{t-1} - \frac{b}{y} \hat{\pi}_t - \hat{\tau}_t \right), \tag{58}$$

$$\hat{b}_t = \frac{R}{1+g} \left(\hat{b}_{t-1} - \hat{\tau}_t \right) + \frac{b}{y} \left(\hat{i}_t - \frac{R}{1+g} \hat{\pi}_t \right), \tag{59}$$

where

$$\eta = \frac{\chi}{\left(\frac{1-\zeta}{\beta R} \right)}. \tag{60}$$

The analysis therefore reduces to the same two-equation system in b_t and π_t as in Leeper (1991). The only difference is that when households are not Ricardian $\eta > 0$, public debt now enters the IS curve (58), as higher public debt increases aggregate demand.

Following Leeper, we assume monetary and fiscal policies are conducted according to feedback rules. Taxes respond to the level of public debt through

$$\hat{\tau}_t = \psi_b \hat{b}_{t-1} - \nu_t^g, \tag{61}$$

and monetary policy follows a standard Taylor rule

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \nu_t^i. \tag{62}$$

Plugging in the policy equations (61) and (62) into the system (58)-(59) gives the system

$$\left(\phi_\pi + \eta \frac{b}{y} \right) \hat{\pi}_t + \nu_t^i = \hat{\pi}_{t+1} + \eta \left(\left(1 - \psi_b \right) \hat{b}_{t-1} + \nu_t^g \right), \tag{63}$$

$$\hat{b}_t = \frac{R}{1+g} \left(\left(1 - \psi_b \right) \hat{b}_{t-1} + \nu_t^g \right) + \frac{b}{y} \left(\left(\phi_\pi - \frac{R}{1+g} \right) \hat{\pi}_t + \nu_t^i \right). \tag{64}$$

5.2 Blurred Lines

Appendix I extends the result of Leeper (1991) on equilibrium determinacy to the case of non Ricardian households. It shows that when households are not Ricardian, the sharp distinction between a monetary and fiscal regime is no longer applicable.

Proposition 5. *Assume monetary and fiscal policy are given by the feedback rules (61) and (62).*

The economy has a unique bounded equilibrium if and only if

$$\left(1 - \phi_\pi \right) \left(1 - \frac{R}{1+g} (1 - \psi_b) \right) + \eta \frac{b}{y} \left((1 - \psi_b) \phi_\pi - 1 \right) < 0. \tag{65}$$

- *When households are Ricardian $\eta = 0$ or when steady-state public debt is zero $b/y = 0$, then when a*

[6] As is standard, when loglinearizing we define $\hat{b}_t = db_t / y^*$ to allow for the possibility of a zero level of public debt in steady-state.

unique bounded equilibrium exists the economy is either in a monetary regime $\phi_\pi > 1, \psi_b > 1 - 1/R$ where fiscal shocks ν_t^g have no effect on inflation, or in a fiscal regime $\phi_\pi < 1, \psi_b < 1 - 1/R$ where they do.

- *When households are not Ricardian $\eta > 0$ and when steady-state public debt is positive $b/y > 0$, then fiscal shocks ν_t^g always have an effect on inflation.*

The first item of Proposition 5 is Leeper's classical result. When households are Ricardian $\eta = 0$ (and so $g = 0$), condition (65) becomes

$$\Big(1 - \phi_\pi\Big)\Big(1 - R(1 - \psi_b)\Big) < 0. \tag{66}$$

It is satisfied either if $\phi_\pi > 1, \psi_b > 1 - (1+g)/R$ or if $\phi_\pi < 1, \psi_b < 1 - (1+g)/R$. The first case is what Leeper calls a regime of monetary dominance. In this case fiscal shocks ν_t^g have no effect on inflation. Indeed, the unique bounded solution can then be obtained by iterating forward the IS curve (58) to give equilibrium inflation as

$$\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{k+1} \nu_{t+k}^i, \tag{67}$$

which is independent of the fiscal shocks ν_t^g. The second case is what Leeper calls a regime of fiscal dominance. In this case fiscal shocks ν_t^g affect inflation. The unique bounded solution can be obtained by iterating forward the flow budget constraint of the government. For instance, when the interest rate is fully pegged $i_t = 0$, iterating the flow budget constraint of the government gives

$$\hat{b}_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1-\psi_b}{R}\right)^{k+1} R\nu_{t+k}^g, \tag{68}$$

which gives the level of inflation necessary to restore the intertemporal budget constraint of the government for the path of fiscal surpluses implied by the fiscal shocks ν_t^g and the (insufficient) strength of tax increases ψ_b.

The second item of Proposition 5 considers how the Leeper result changes once we move away from the case of Ricardian households. The sharp distinction between a regime of fiscal dominance and a regime of monetary dominance disappears. It first disappears in a most literal sense, represented in Figure 3. The figure plots the region of parameters (ϕ_π, ψ_b) for which there exists a unique bounded equilibrium. While in the case of Ricardian households, this consists of two distinct regions, when households are not Ricardian there is no straightforward way to distinguish between two regimes.

The distinction between fiscal and monetary regimes also disappears in terms of the inflationary effect of a fiscal shock. Appendix I shows that when households are not Ricardian, inflation is given by

$$\hat{\pi}_t = \mu\hat{b}_{t-1} + \sum_{k=0}^{\infty} \left(\frac{1}{\lambda}\right)^{k+1} \left(\left(\mu\frac{b}{y} - 1\right)\nu_{t+k}^i + \left(\mu\frac{R}{1+g} + \eta\right)\nu_{t+k}^g\right), \tag{69}$$

where λ is the root of the economy that is greater than 1, and μ is a positive constant given in Appendix I. This gives in particular the effect on inflation of a contemporaneous fiscal shock ν_t^g.

In the particular case $\psi_b = 0$, a contemporaneous fiscal shock has the same impact effect on inflation as in the case of Ricardian households. Whenever $\phi_\pi < 1$,

$$\frac{\partial \pi_t}{\partial \nu_t^g} = \frac{1}{\frac{b}{y}}. \tag{70}$$

But whenever $\psi_b \neq 0$, the impact effect of a fiscal shock on inflation is not the same as when households are Ricardian. In particular, it is non-zero in policy configurations that deliver monetary dominance and a full insulation of inflation from fiscal shocks in the Ricardian case.

Figure 4 gives the inflationary effect of a contemporaneous fiscal shock as a function of ϕ_π for fixed values of ψ_b. The upper panel gives the impact effect of the shock on inflation. As monetary policy becomes more reactive, it decreases. But it does so gradually, approaching zero at the asymptotic limit where ϕ_π tends toward infinity but never reaching it. This is in contrast to the Ricardian case, where an active monetary policy ϕ_π is enough to insulate inflation from fiscal shocks. In addition, the cumulative impact on inflation can be much larger, and even increasing in ϕ_π. This is because increasing ϕ_π increases the backward-looking root of the economy and therefore the persistence of inflation, as shown in the middle panel of Figure 4. As a result, the effect of the shock on cumulative inflation can even be increasing in ϕ_π.

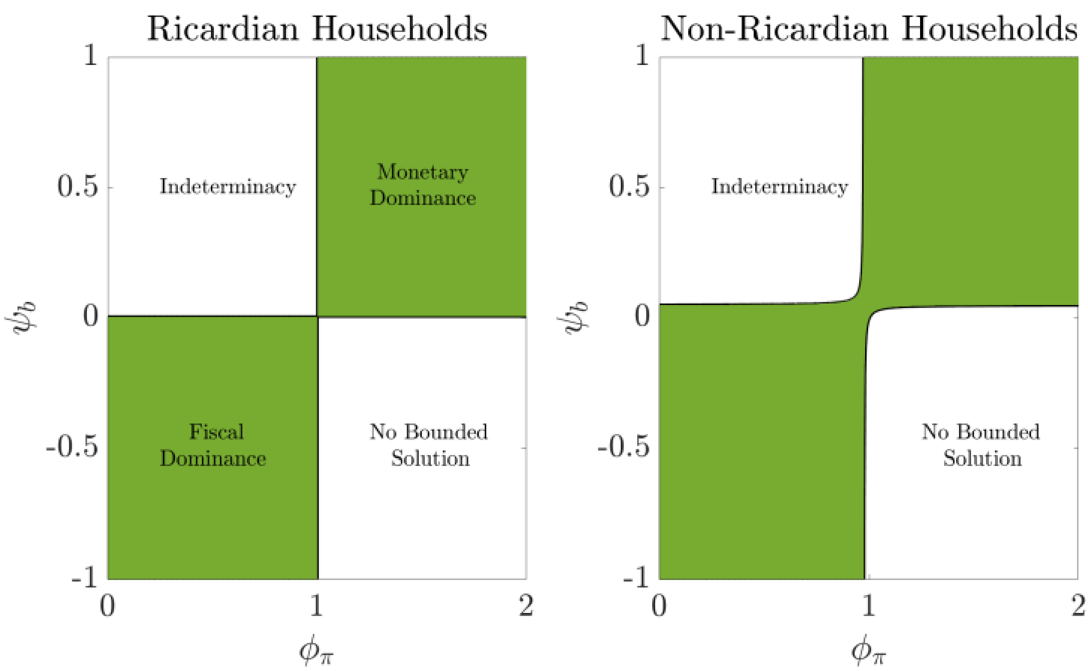


Figure 3: Condition for Equilibrium Uniqueness

Note: In green is the region of parameters (ϕ_π, ψ_b) *for which there exists a unique bounded equilibrium, in the economy with Ricardian households (left panel) and with non-Ricardian households. The calibration is* $\beta = 0.995$, $\zeta = 0.05$, $g = 0.05$, $b/y = 1.2$, $\lambda = 0.135$.

5.3 Implementing Price Stability by Letting Monetary Policy Respond to Debt

The analysis so far would conclude that it is impossible for monetary policy to insulate inflation from fiscal shocks. No degree of responsiveness to inflation ϕ_π in the Taylor rule (62) can fully prevent fiscal shocks from affecting inflation.

This is only if one restricts monetary policy to respond to inflation only however. If the central bank follows a policy rule that respond to public debt in addition to inflation, insulating inflation from fiscal shocks becomes possible again. If the reaction function of the central bank is

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \eta(\hat{b}_{t-1} - \hat{\tau}_t), \tag{71}$$

then a regime of monetary dominance—in the sense defined by the following proposition—-can implement the stable price equilibrium of the economy.

Proposition 6. *Assume monetary and fiscal policy are given by the feedback rules* (61) *and* (71)*.*

The economy has a unique bounded equilibrium if and only if it is in either of the two following regimes

- *Monetary regime* $\phi_\pi > 1 - \eta \frac{b}{y}$ *and* $\psi_b > 1 - \frac{1}{\frac{R}{1+g} + \frac{b}{y}\eta}$*. Fiscal shocks* ν_t^g *have then no effect on inflation.*
- *Fiscal regime* $\phi_\pi < 1 - \eta \frac{b}{y}$ *and* $\psi_b < 1 - \frac{1}{\frac{R}{1+g} + \frac{b}{y}\eta}$*. Fiscal shocks* ν_t^g *then affect inflation.*

See Appendix J for a proof. In the case of the monetary regime, the unique bounded solution can be obtained by solving forward the equation obtained by combining the IS curve (58) and monetary rule (71), to give

$$\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi + \eta \frac{b}{y}} \right)^{k+1} \nu_{t+k}^i \tag{72}$$

Under monetary dominance, monetary policy can be said to be active again, since it insulates inflation from fiscal shocks. If so however, this flips on its head the definition of an active monetary policy in Leeper (1991)

> *"I couch active and passive policy in terms of the constraints a policy authority faces. An active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks. Its behavior is constrained by private optimization and the active authority's actions."*

In contrast, the monetary policy (71) delivers monetary dominance *because* it reacts to public debt. While having monetary policy react to public debt is less standard, it follows naturally from the fact that the natural rate of interest—the real interest rate consistent with flexible prices—depends on the level of public debt once households are not Ricardian. The monetary rule (71) is of the form

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t, \tag{73}$$

where

$$\hat{r}_t^n = \eta(\hat{b}_{t-1} - \hat{\tau}_t). \tag{74}$$

Having the natural rate of interest as an intercept in the Taylor rule is a standard feature in specifications of monetary policy.

5.4 When $r < g$

A corollary of Proposition 6 is that, in contrast to the case of Ricardian households $\lambda + g = 0$, it is possible to be in a regime of monetary dominance even when the government never increases taxes in response to higher debt, $\psi_b = 0$. This is the case if and only if

$$\frac{R}{1+g} + \eta\frac{b}{y} < 1. \tag{75}$$

This is simply a consequence of the fact that in the stable price equilibrium of the economy with non Ricardian households, debt shrinks back by itself if the real interest rate r is below the growth rate of the economy g. The additional term $\eta\frac{b}{y}$ in equation (75) captures how the natural rate increases with the level of public debt at first order.

Condition (75) remains a local, first order, result however. In the full non-linear dynamics, the dependence of the natural rate on the level of public debt if convex. A large fiscal shock can increase the natural rate enough that it pushes R above $1 + g$. A large fiscal shock can then make debt explosive and prevent the existence of an equilibrium where the government never increases taxes. Deriving fiscal requirements for price stability require to go beyond the local dynamics to look at the non-linearized version of the model, as done in the previous sections.

6 Conclusion

Can the central bank deliver price stability whatever fiscal policy the government sets? The Fiscal Theory of the Price Level answers that it cannot, but the FTPL has remained controversial since its inception and relies on the strong assumption of Ricardian households. In this paper, we have argued that moving away from the particular case of Ricardian households provides a less controversial—and also more intuitive—answer to this question. With non Ricardian households, higher public debt makes households richer. As households spend their higher wealth, aggregate demand increases, putting inflationary pressure on the economy. While monetary policy can increase interest rates to counter the inflationary effects of public debt, we have shown that it can no longer do so when public debt is too high. When this level has been reached, monetary policy loses control over the price level. For the central bank to keep control over the price level, fiscal policy must be such that it never lets debt reach that level.

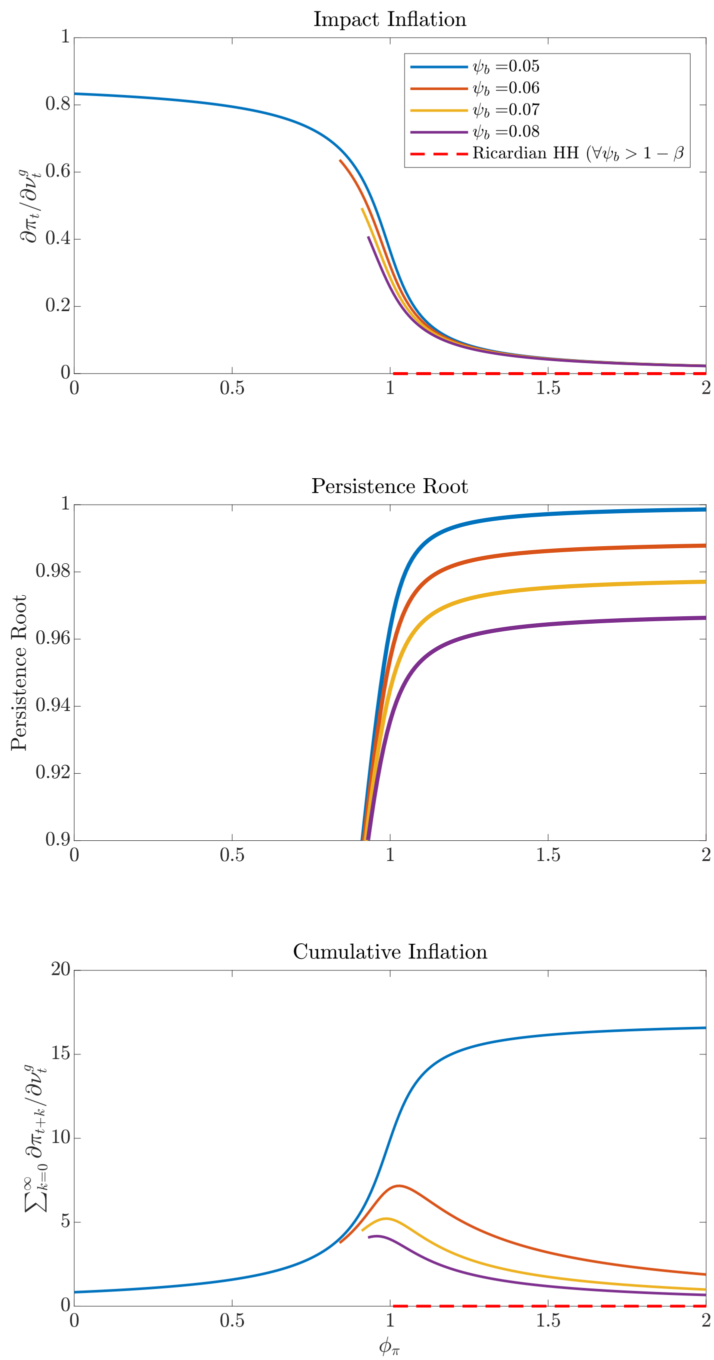


Figure 4: Inflationary Effect of Contemporaneous Fiscal Shock

Note: The figure plots the impact inflationary effect of a fiscal shock, the persistence root, and the cumulative effect of a fiscal shock, all three as a function of the reactivity of the Taylor rule ϕ_π*, and for a parameter in the fiscal rule* $\psi_b = 1 - \frac{1}{\frac{R}{1+g} + \eta \frac{b}{y}}$ *or higher. For this first value of* ψ_b *there exists an equilibrium under all values of* ϕ_π *in the case of Non Ricardian households. In the case of Ricardian households, there exists an equilibrium only for* $\phi_\pi > 0$*. The calibration is* $\beta = 0.995$, $\zeta = 0.05$, $g = 0.05$, $b/y = 1.2$, $\lambda = 0.135$.

References

Angeletos, G.-M., C. Lian, and C. K. Wolf (2023): "Can Deficits Finance Themselves?," NBER Working Papers 31185, National Bureau of Economic Research, Inc.

——— (2024): "Deficits and Inflation: HANK meets FTPL," Working paper.

Auclert, A., M. Rognlie, and L. Straub (2018): "The Intertemporal Keynesian Cross," NBER Working Papers 25020, National Bureau of Economic Research, Inc.

Barro, R., and F. Bianchi (2023): "Fiscal Influences on Inflation in OECD Countries, 2020-2022," *NBER Working Paper*.

Barthelemy, J., E. Mengus, and G. Plantin (2024): "Fiscal Dominance: Implications for Bond Markets and Central Banking," Discussion paper, CEPR Discussion Paper No. 19591.

Bassetto, M. (2002): "A Game-Theoretic View of the Fiscal Theory of the Price Level," *Econometrica*, 70(6), 2167–2195.

Bassetto, M., and W. Cui (2018): "The Fiscal Theory of the Price Level in a World of Low Interest Rates," *Journal of Economic Dynamics and Control*, 89(C), 5–22.

Bianchi, F., R. Faccini, and L. Melosi (2023): "A Fiscal Theory of Persistent Inflation," *Quarterly Journal of Economics*, 138(4), 21272179.

Bianchi, F., and C. Ilut (2017): "Monetary/Fiscal Policy Mix and Agents' Beliefs," *Review of Economic Dynamics*, 26, 113–139.

Bianchi, F., and L. Melosi (2017): "Escaping the Great Recession," *American Economic Review*, 107(4), 1030–58.

——— (2019): "The Dire Effects of the Lack of Monetary and Fiscal Coordination," *Journal of Monetary Economics*, 104(C), 1–22.

Bianchi, F., L. Melosi, and A. Rogantini Picco (2024): "Monetary/Fiscal Policy Mix and Inflation Dynamics," in *Handbook of Inflation*, ed. by G. Ascari, and R. Trezzi.

Blanchard, O. (2019): "Public Debt and Low Interest Rates," *American Economic Review*, 109(4), 1197–1229.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223–247.

Brunnermeier, M. K., S. Merkel, and Y. Sannikov (2022): "The Fiscal Theory of the Price Level with a Bubble," *NBER Working Paper*.

Cochrane, J. (2023): *The Fiscal Theory of the Price Level*. Princeton University Press.

Cochrane, J. H. (2001): "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69(1), 69–116.

Cochrane, J. H. (2005): "Money as stock," *Journal of Monetary Economics*, 52(3), 501–528.

Corsetti, G., and B. Maćkowiak (2024): " Gambling to Preserve Price (and Fiscal) Stability," *IMF Economic Review*, 72, 32–57.

Davig, T., and E. M. Leeper (2007): "Fluctuating Macro Policies and the Fiscal Theory," in *NBER Macroeconomics Annual 2006, Volume 21*, NBER Chapters, pp. 247–316. National Bureau of Economic Research, Inc.

Del Negro, M., M. P. Giannoni, and C. Patterson (2023): "The Forward Guidance Puzzle," *Journal of Political Economy Macroeconomics*, 1(1), 43–79.

Elfsbacka-Schmoller, M., and N. McClung (2024): "Can Growth Stabilize Debt? A Fiscal Theory Perspective," Discussion paper, Bank of Finland Research Discussion Paper No. 2/2024.

Fagereng, A., M. B. Holm, and G. J. Natvik (2021): "MPC Heterogeneity and Household Balance Sheets," *American Economic Journal: Macroeconomics*, 13(4), 1–54.

Farhi, E., and I. Werning (2019): "Monetary Policy, Bounded Rationality, and Incomplete Markets," *American Economic Review*, 109(11), 3887–3928.

Farmer, M. R., and P. Zabczyk (2019): "A Requiem for the Fiscal Theory of the Price Level," IMF Working Papers 2019/219, International Monetary Fund.

Farmer, R. E., and P. Zabczyk (2018): "The Household Fallacy," *Economics Letters*, 169, 83–86.

Hagedorn, M. (2024): "The Failed Theory of the Price Level," Working paper.

Kaplan, G., G. Nikolakoudis, and G. Violante (2023): "Price Level and Inflation Dynamics in Heterogeneous Agent Economies," *NBER Working Paper*.

Leeper, E. (1991): "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27(1), 129–147.

Leeper, E., and C. Leith (2016): "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2 of *Handbook of Macroeconomics*, chap. 0, pp. 2305–2415. Elsevier.

Maćkowiak, B., and S. Schmidt (2024): " Fiscal Backing for Price Stability in a Monetary Union," *American Economic Journal: Macroeconomics, Forthcoming.*

Reis, R. (2021): "The constraint on public debt when r," Discussion Papers 2111, Centre for Macroeconomics (CFM).

Samuelson, P. A. (1958): "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66(6), 467–467.

Sargent, T. J., and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*, 5(Fall).

Schmidt, S. (2024): "Monetary-Fiscal Policy Interactions when Price Stability Occasionally Takes a Back Seat," *CEPR Working Paper*.

Sims, C. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4(3), 381–99.

Smets, F., and R. Wouters (2024): "Fiscal Backing, Inflation and US Business Cycles," Discussion paper, Working Paper.

Wolf, C. K. (2021): "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result," NBER Working Papers 29193, National Bureau of Economic Research, Inc.

Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80(2), 382–88.

——— (1995): "Price-level Determinacy Without Control of a Monetary Aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43(1), 1–46.

——— (2001): "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 33(3), 669–728.

A Proof of Lemma 1

Maximizing (4) subject to the flow budget constraint (5) and the no Ponzi scheme constraint (6) gives the Euler equation

$$C_t^i = (\beta R_t)^{-1} C_{t+1}^i. \tag{A.1}$$

and the No-Ponzi-scheme condition holding with equality, i.e. the No-Cash-on-the-Table condition (16). Iterating the Euler equation (A.1) forward gives for all $k \geq 0$

$$C_{t+k}^i = (\beta^k \mathcal{R}_{t,t+k}) C_t^i. \tag{A.2}$$

Combining the FBC (5) and the No-Cash-on-the-Table condition (16), the intertemporal budget constraint is

$$\sum_{k=0}^{\infty} \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} C_{t+k}^i = \frac{1}{1-\lambda} \frac{B_{t-1}^i}{P_t} + H_t^i \tag{A.3}$$

where

$$H_t^i = Y_t^i - T_t^i + \frac{1-\lambda}{R_t} H_{t+1}^i \tag{A.4}$$

is the household's intertemporal wealth, or human capital. Iterated forward, it writes as equation (15).

Injecting (A.2) into the IBC (A.3) gives the consumption function (13). Conversely, equations (13) and (16) are sufficient for individual optimality.

B Proof of Lemma 2

Preliminary: Market-Clearing in the Debt Market

We first simply prove Walras's Law: the assumption of market-clearing in the goods market (12) implies market-clearing in the debt market when combined with flow budget constraints, i.e.

$$\int_i B_t^i di = B_t, \tag{B.1}$$

where the integral is over households alive at t.

By assumption, market-clearing held in period $t = -1$. We show that market-clearing holds in the debt market in all periods by showing that if it holds at $t-1$ then it holds at t. Summing the households' FBC (5) across households alive at t gives

$$C_t + \frac{1}{R_t} \frac{\int_i B_t^i di}{P_{t+1}} = \frac{B_{t-1}}{P_t} + Y_t - T_t, \tag{B.2}$$

where we used the fact that $\int_i B_{t-1}^i di = (1-\lambda) B_{t-1}$, since the remaining λB_{t-1} belonging to households that passed away has been taken by the insurance fund and redistributed to remaining households as annuities.

Combining (B.2) with goods market-clearing condition (12) gives

$$\frac{1}{R_t}\frac{\int_i B_t^i di}{P_{t+1}} = \frac{B_{t-1}}{P_t} - T_t, \tag{B.3}$$

which combined with the FBC of the government (10) implies

$$\int_i B_t^i di = B_t, \tag{B.4}$$

which ends the proof.

Preliminary: Defining Aggregate Human Capital

Define aggregate human capital as the sum of individual human capitals

$$H_t = \sum_{k=0}^{\infty} N_t(k) H_t^i(k). \tag{B.5}$$

The human capital (15) of a household of age n can then be written as a function of aggregate human capital per capita

$$H_t^i(n) = \kappa(1-\zeta)^n \left(\frac{H_t}{N_t}\right), \tag{B.6}$$

where aggregate human capital per capita can be written

$$\frac{H_t}{N_t} = \sum_{k=0}^{\infty} \frac{((1-\lambda)(1-\zeta))^k}{\mathcal{R}_{t,t+k}} \left(\frac{Y_{t+k} - T_{t+k}}{N_{t+k}}\right), \tag{B.7}$$

or recursively

$$\frac{H_t}{N_t} = \left(\frac{Y_t - T_t}{N_t}\right) + \frac{(1-\lambda)(1-\zeta)}{R_t}\left(\frac{H_{t+1}}{N_{t+1}}\right). \tag{B.8}$$

Aggregate human capital solves the recursion

$$H_t = (Y_t - T_t) + \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t} H_{t+1}. \tag{B.9}$$

An Intermediary Lemma

We now move to the core of the proof. To prove Lemma 2, we first prove the following lemma:

Lemma B.1. *If all households are on their consumption function (13) and the goods market clears (12), then at time t, a household of age $n \leq t$ has holdings of public debt*

$$\frac{B_t^i(n)}{P_{t+1}} = \kappa(1-\zeta)^n \left[\mu \frac{1-x^{n+1}}{1-x} \frac{\mathcal{R}_{t-n,t+1}}{(1+g)^{n+1}} \frac{B_{t-(n+1)}}{N_{t-(n+1)}P_{t-n}} - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right], \tag{B.10}$$

where

$$x = \frac{\beta(1+g)}{1-\zeta}. \tag{B.11}$$

Proof. The proof is by induction. For future abundant use, first notice that combining the FBC (5) and the consumption function (13) gives the debt-holding function of a household at t, i.e. how many bonds to hold as a function of inherited bonds and future and present incomes

$$\frac{B_t^i}{P_{t+1}} = R_t \left(\beta \frac{B_{t-1}^i}{P_t} + Y_t^i - T_t^i - \mu H_t^i \right). \tag{B.12}$$

Base Case(s)

To initialize the recursion, we determine the debt holdings of households of age 0 and of age 1. (The inductive step will show the property for $n+1$ assuming it holds for n and $n-1$, so it requires a double-initialization.) Applying (B.12) to households of age 0, who inherit no financial wealth from the previous period $B_{t-1}^i(0) = 0$, gives

$$\frac{B_t^i(0)}{P_{t+1}} = R_t \kappa \left(\frac{Y_t - T_t - \mu H_t}{N_t} \right). \tag{B.13}$$

Aggregating equation (B.12) across all households alive at t and using market-clearing in the debt market gives

$$\frac{B_t}{P_{t+1}} = R_t \left(\beta(1-\lambda) \frac{B_{t-1}}{P_t} + Y_t - T_t - \mu H_t \right). \tag{B.14}$$

Taking the difference between (B.14) and κ/N_t times (B.13) gives

$$\frac{B_t^i(0)}{P_{t+1}} - \kappa \frac{B_t}{N_t P_{t+1}} = -\kappa \frac{\beta(1-\lambda)}{(1+g)} R_t \frac{B_{t-1}}{P_t N_{t-1}}. \tag{B.15}$$

The government's flow budget constraint (10) can be rewritten in per-capita terms as

$$\frac{B_t}{N_t P_{t+1}} = R_t \left(\frac{1}{1+g} \frac{B_{t-1}}{N_{t-1} P_t} - \frac{T_t}{N_t} \right). \tag{B.16}$$

Using (B.16) to replace $B_t/(N_t P_{t+1})$, equation (B.15) can be written

$$\frac{B_t^i(0)}{P_{t+1}} = \kappa \left(\mu \frac{R_t}{(1+g)} \frac{B_{t-1}}{N_{t-1} P_t} - R_t \frac{T_t}{N_t} \right), \tag{B.17}$$

which proves the base case $n = 0$. The base case $n = 1$ is shown very similarly to the inductive step below, noting that a household of age 0 at t had no debt at $t-1$.

Inductive Step

Assume the property holds for households of age n and $n-1$. We show it then holds for households of age $n+1$. Taking the difference between equation (B.12) applied to a household of age $n+1$ and $(1-\zeta)$ times equation (B.12) applied to a household of age n gives

$$\frac{B_t^i(n+1)}{P_{t+1}} - (1-\zeta) \frac{B_t^i(n)}{P_{t+1}} = R_t \beta \left(\frac{B_{t-1}^i(n)}{P_t} - (1-\zeta) \frac{B_{t-1}^j(n-1)}{P_t} \right). \tag{B.18}$$

We now use the fact that the property holds for n and $n-1$ to rewrite the term on the right-hand side of (B.18) as

$$\frac{B^i_{t-1}(n)}{P_t} - (1-\zeta)\frac{B^i_{t-1}(n-1)}{P_t} = \kappa(1-\zeta)^n \left[\mu\left(\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-1-(n+1),t}}{(1+g)^{n+1}}\frac{B_{t-1-(n+1)}}{N_{t-1-(n+1)}P_{t-(n+1)}} - \frac{1-x^n}{1-x}\frac{\mathcal{R}_{t-1-n,t}}{(1+g)^n}\frac{B_{t-1-n}}{N_{t-1-n}P_{t-n}}\right) - \left(x^n + \mu\frac{1-x^n}{1-x}\right)\frac{\mathcal{R}_{t-1-n,t}}{(1+g)^n}\frac{T_{t-1-n}}{N_{t-1-n}}\right] \tag{B.19}$$

Using the government flow budget constraint (B.16) to replace $B_{t-1-n}/(N_{t-1-n}P_{t-n})$, this rewrites

$$\frac{B^i_{t-1}(n)}{P_t} - (1-\zeta)\frac{B^i_{t-1}(n-1)}{P_t} = \kappa(1-\zeta)^n \left[\mu x^n \frac{\mathcal{R}_{t-(n+2),t}}{(1+g)^{n+1}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - x^n\frac{\mathcal{R}_{t-(n+1),t}}{(1+g)^n}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right]. \tag{B.20}$$

Multiplying by βR_t,

$$\beta R_t\left(\frac{B^i_{t-1}(n)}{P_t} - (1-\zeta)\frac{B^i_{t-1}(n-1)}{P_t}\right) = \kappa(1-\zeta)^{n+1}\left[\mu x^{n+1}\frac{\mathcal{R}_{t-(n+2),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}} - x^{n+1}\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}}\right]. \tag{B.21}$$

Meanwhile, the term $(1-\zeta)B^i_t(n)/P_{t+1}$ on the left-hand side of equation (B.18) can be rewritten, using (B.10) and the government flow budget constraint (B.16) to eliminate $B_{t-(n+1)}/(N_{t-(n+1)}P_{t-n})$, as

$$(1-\zeta)\frac{B^i_t(n)}{P_{t+1}} = \kappa(1-\zeta)^{n+1}\left[\mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \mu\frac{1-x^{n+1}}{1-x}\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}} - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right]. \tag{B.22}$$

Injecting equations (B.21) and (B.22) into equation (B.18) gives

$$\frac{B^i_t(n+1)}{P_{t+1}} = \kappa(1-\zeta)^{n+1}\left[\mu\left(\frac{1-x^{n+1}}{1-x} + x^{n+1}\right)\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \left(x^{n+1} + \mu\frac{1-x^{n+1}}{1-x}\right)\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+1}}\frac{T_{t-(n+1)}}{N_{t-(n+1)}} - \sum_{k=0}^{n}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right]. \tag{B.23}$$

It rewrites

$$\frac{B^i_t(n+1)}{P_{t+1}} = \kappa(1-\zeta)^{n+1}\left[\mu\left(\frac{1-x^{n+2}}{1-x}\right)\frac{\mathcal{R}_{t-(n+1),t+1}}{(1+g)^{n+2}}\frac{B_{t-(n+2)}}{N_{t-(n+2)}P_{t-(n+1)}} - \sum_{k=0}^{n+1}\left(x^k + \mu\frac{1-x^k}{1-x}\right)\frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k}\frac{T_{t-k}}{N_{t-k}}\right], \tag{B.24}$$

which ends the proof. □

Finishing the Proof

To get from Lemma B.1 to Lemma 2, we just need to express $B_t^i(n)$ as a function of B_t instead of B_{t-n}. Iterate the government's flow budget constraint (B.16) from $t-(n+1)$ to t to obtain

$$\frac{B_t}{N_t P_{t+1}} = \frac{\mathcal{R}_{t-n,t+1}}{(1+g)^{n+1}} \frac{B_{t-(n+1)}}{N_{t-(n+1)} P_{t-n}} - \sum_{k=0}^{n} \frac{\mathcal{R}_{t-k,t+1}}{(1+g)^k} \frac{T_{t-k}}{N_{t-k}}. \tag{B.25}$$

Injecting it in equation (B.10) gives (19).

Sanity Check

Note as a sanity check that we have

$$\sum_{n=0}^{\infty} \psi_B(n) \frac{N_t(n)}{N_t} = 1, \tag{B.26}$$

$$\forall k \in [\![0, n]\!], \sum_{n=k}^{\infty} \phi_T(n,k) N_t(n) = 0, \tag{B.27}$$

so that when enough time has passed so that all the debt holdings of households of all ages is given by (19), the sum of debt holdings by all households sums to B_t.

C Proof of Lemma 3

If all households alive at t satisfy their No-Cash-on-the-Table condition (16), then (23) necessarily holds by taking the average over all households alive at t. The lengthier part is to show that (23) is also sufficient.

Let i be a household alive at t, and $n \geq 0$ its age at time t. It is therefore of age $n+k$ at $t+k$. We can decompose its debt holding at $t+k$ into the two following terms

$$\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} = \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} + \left(\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} \right). \tag{C.1}$$

It follows that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \frac{B_{t+k}^i(n+k)}{P_{t+k+1}} = \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} + \frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k}} \left(\frac{B_{t+k}^i(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} \right). \tag{C.2}$$

The proof amounts to showing that the second term in (C.2) necessarily tends to zero as $k \to \infty$. Condition (23) then implies that the first term tends to zero since $\kappa(1-\zeta)^n$ is a constant.

We denote with the superscript *old* the variables aggregated over all households already alive at t. Let $k \geq 0$. Aggregating (B.12) at period $t+k$ across households already alive at t gives

$$\frac{B_{t+k}^{old}}{P_{t+k+1}} = R_{t+k} \left(\beta(1-\lambda) \frac{B_{t+k-1}^{old}}{P_{t+k}} + Y_{t+k}^{old} - T_{t+k}^{old} - \mu H_{t+k}^{old} \right), \tag{C.3}$$

where

$$
\begin{aligned}
Y_{t+k}^{old} - T_{t+k}^{old} - \mu H_{t+k}^{old} &= \sum_{n=k}^{\infty} N_{t+k}(n)(Y_{t+k}^{i}(n) - T_{t+k}^{i}(n) - \mu H_{t+k}^{i}(n)) \\
&= \kappa \left(\frac{(1-\zeta)(1-\lambda)}{(1+g)} \right)^{k} (Y_{t+k} - T_{t+k} - \mu H_{t+k}) . \qquad \text{(C.4)}
\end{aligned}
$$

Injecting (C.4) into (C.3)

$$
\frac{B_{t+k}^{old}}{P_{t+k+1}} = R_{t+k} \left(\beta(1-\lambda) \frac{B_{t+k-1}^{old}}{P_{t+k}} + \kappa \left(\frac{(1-\zeta)(1-\lambda)}{(1+g)} \right)^{k} (Y_{t+k} - T_{t+k} - \mu H_{t+k}) \right) . \qquad \text{(C.5)}
$$

The number of households alive at t that are still alive at $t+k$ is $(1-\lambda)^k N_t$. Dividing (C.5) by this to get the average debt holding:

$$
\frac{\bar{B}_{t+k}}{P_{t+k+1}} = R_{t+k} \left(\beta \frac{\bar{B}_{t+k-1}}{P_{t+k}} + \kappa (1-\zeta)^{k} \left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu \frac{H_{t+k}}{N_{t+k}} \right) \right) . \qquad \text{(C.6)}
$$

Applying equation (B.12) at $t+k$ to household i gives

$$
\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} = R_{t+k} \left(\beta \frac{B_{t+k-1}^{i}(n+k-1)}{P_{t+k}} + \kappa (1-\zeta)^{n+k} \left(\frac{Y_{t+k}}{N_{t+k}} - \frac{T_{t+k}}{N_{t+k}} - \mu \frac{H_{t+k}}{N_{t+k}} \right) \right) . \qquad \text{(C.7)}
$$

Taking the difference between (C.7) and $\kappa(1-\zeta)^n$ times (C.6) gives

$$
\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} = \beta R_{t+k} \left(\frac{B_{t+k-1}^{i}(n+k-1)}{P_{t+k}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k-1}}{P_{t+k}} \right) \qquad \text{(C.8)}
$$

Iterating backward:

$$
\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} = \beta^k \mathcal{R}_{t,t+k+1} \left(\frac{B_{t-1}^{i}(n)}{P_{t}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t-1}}{P_{t}} \right) \qquad \text{(C.9)}
$$

This implies that the second term in (C.2) can be written as

$$
\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \left(\frac{B_{t+k}^{i}(n+k)}{P_{t+k+1}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t+k}}{P_{t+k+1}} \right) = (\beta(1-\lambda))^k \left(\frac{B_{t-1}^{i}(n)}{P_{t}} - \kappa(1-\zeta)^n \frac{\bar{B}_{t-1}}{P_{t}} \right), \qquad \text{(C.10)}
$$

which tends to zero as $k \to \infty$. This ends the proof.

D Proof of Proposition 1

In an equilibrium, all households must be on their consumption functions (13) so we can apply Lemma 3. In an equilibrium the No-Cash-on-the-Table conditions of all households must be satisfied, so from Lemma 3 condition (23) must hold. We show that it implies Proposition 1. To derive $\bar{B}_{t+k}$, we calculate B_{t+k}^{old} by calculating its complement: the quantity of public debt that is held at $t+k$ by households born at $t+1$ or later. We denote it B_{t+k}^{young}. It is equal to the sum of the debt held by households of age $0 \leq n \leq k-1$ at $t+k$. Since in an equilibrium all households are on their consumption function and the goods-market clears, we can use the expression for debt holdings in Lemma 2 to calculate it.

$$\begin{aligned}
\frac{B_{t+k}^{young}}{P_{t+k+1}} &= \sum_{n=0}^{k-1} \frac{B_{t+k}^{i}(n)}{P_{t+k+1}} N_{t+k}(n) \\
&= \left[1 - \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\right]\frac{B_{t+k}}{P_{t+k+1}} \\
&- \sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\Big(\beta(1-\lambda)\Big)^j\right)\right] \\
&\times \mathcal{R}_{t+j,t+k+1}T_{t+j}.
\end{aligned} \tag{D.1}$$

We can therefore express $\bar{B}_{t+k}$ as

$$\begin{aligned}
\frac{\bar{B}_{t+k}}{P_{t+k+1}} &= \frac{B_{t+k} - B_{t+k}^{young}}{P_{t+k+1}(1-\lambda)^k N_t} \\
&= \frac{1}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right)\frac{B_{t+k}}{N_t P_{t+k+1}} \\
&+ \frac{1}{(1-\lambda)^k}\sum_{j=1}^{k}(\beta(1-\lambda))^{k-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\Big(\beta(1-\lambda)\Big)^j\right)\right] \\
&\times \mathcal{R}_{t+j,t+k+1}\frac{T_{t+j}}{N_t}.
\end{aligned} \tag{D.2}$$

Condition (23) therefore implies that

$$\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}}\frac{\bar{B}_{t+k}}{P_{t+k+1}} = \xi(k)\frac{B_{t+k}}{N_t\mathcal{R}_{t,t+k+1}P_{t+k+1}} + (\beta(1-\lambda))^k\left(\sum_{j=1}^{k}\theta(j)\frac{1}{\mathcal{R}_{t,t+j}}\frac{T_{t+j}}{N_t}\right) \tag{D.3}$$

must tend to zero as $k \to \infty$, where

$$\xi(k) = \frac{(1-\lambda)^k}{1-x}\left(\mu\left(\frac{1-\zeta}{1+g}\right)^k - \left(\frac{(\lambda+g)\kappa\beta}{1-\zeta}\right)\beta^k\right), \tag{D.4}$$

$$\theta(j) = (\beta(1-\lambda))^{-j}\left[\left(1-\frac{\mu}{1-x}\right)\left(1-\left(\frac{(1-\zeta)(1-\lambda)}{1+g}\right)^j\right) + \kappa\frac{\lambda+g}{1+g}\frac{x}{1-x}\left(1-\Big(\beta(1-\lambda)\Big)^j\right)\right]. \tag{D.5}$$

Using the budget constraint of the government

$$\frac{1}{\mathcal{R}_{t,t+k+1}}\frac{B_{t+k}}{P_{t+k+1}} = \frac{B_{t-1}}{P_t} - \sum_{j=0}^{k}\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} \tag{D.6}$$

to replace B_{t+k}/P_{t+k+1}, and multiplying by N_t, this implies equivalently that

$$\xi(k)\frac{B_{t-1}}{P_t} + \left(\sum_{j=1}^{k}\left((\beta(1-\lambda))^k\theta(j) - \xi(k)\right)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j}\right) - \xi(k)T_t \tag{D.7}$$

must tend to zero as $k \to \infty$.

Since $\lambda > 0$ or $g > 0$ in the non-Ricardian case, $\xi(k)$ tends to zero as $k \to \infty$, and so the term in B_{t-1} tends to zero. The condition is therefore equivalent to

$$\lim_{k\to\infty}\sum_{j=1}^{k}\left((\beta(1-\lambda))^k\theta(j) - \xi(k)\right)\frac{1}{\mathcal{R}_{t,t+j}}T_{t+j} - \xi(k)T_t = 0. \tag{D.8}$$

Defining

$$\Omega(k,j) = (\beta(1-\lambda))^k\theta(j) - \xi(k) \text{ for } j \geq 1, \tag{D.9}$$

$$\Omega(k,0) = -\xi(k) \text{ for } j = 0, \tag{D.10}$$

this ends the proof.

E Proof of Lemma 4

In an equilibrium, all households are on their consumption function (13). Aggregating the consumption function (13) across households gives the aggregate consumption function

$$C_t = \mu\left(\frac{B^d_{t-1}}{P_t} + H_t\right). \tag{E.1}$$

where we denote by B^d_t the aggregate demand for public debt at t, $B^d_t = \int_i B^i_t di$ so that it does not assume market-clearing in the debt market, and where we used the fact that $\int_i B^i_{t-1}di = (1-\lambda)B^d_{t-1}$ since only $(1-\lambda)$ of household who demanded debt at $t-1$ are still alive at t.

Differentiate the aggregate consumption function (E.1), using $(1-\lambda)(1-\zeta)/((1+g)R_t)$ as the discount factor:

$$C_t - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}C_{t+1} = \mu\left[\left(\frac{B^d_{t-1}}{P_t} - \frac{(1-\lambda)(1-\zeta)}{(1+g)R_t}\frac{B^d_t}{P_{t+1}}\right) + Y_t - T_t\right]. \tag{E.2}$$

It simplifies using the aggregate flow budget constraint (B.2) into

$$C_t = \frac{1}{\beta R_t}\left(\frac{1-\zeta}{1+g}\right)C_{t+1} + \chi\frac{B^d_t}{R_t P_{t+1}}, \tag{E.3}$$

where χ is defined in equation (30). Note that equation (E.3) is a purely decision-theoretic object. It relies on no assumption of market clearing.

In equilibrium, market clearing holds in the debt market $B^d_t = B_t$. Replacing B_t by B_{t-1} using the flow budget of the government (10) gives equation (29).

F Proof of Proposition 3

Ricardian Case

The text already showed that condition (18) is necessary for an equilibrium, so (35) is necessary for a stable price equilibrium. We show that it is sufficient. The proof is constructive. Given an initial level of public debt B_{-1} distributed as $(B^i_{-1})_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define the following path for the interest rate

$$R_t = \frac{1}{\beta}\frac{Y_{t+1}}{Y_t}. \tag{F.1}$$

For this interest rate path, define individual consumption allocations through the consumption function (13), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-clearing

Start with market-clearing. Since by construction all households are on the consumption function (13), aggregate consumption is given by the aggregate consumption function (E.1), which writes since $\lambda = g = \zeta = 0$ in the Ricardian case

$$C_t = (1-\beta)\left(B^d_{t-1} + \sum_{k=0}^{\infty}\frac{1}{\mathcal{R}_{t,t+k}}(Y_{t+k} - T_{t+k})\right). \tag{F.2}$$

Given this interest rate path, we have that

$$\frac{Y_{t+k}}{\mathcal{R}_{t,t+k}} = \frac{Y_{t+k}}{R_{t+k}\mathcal{R}_{t,t+k-1}} = \beta\frac{Y_{t+k-1}}{\mathcal{R}_{t,t+k-1}}, \tag{F.3}$$

and by iteration

$$\frac{Y_{t+k}}{\mathcal{R}_{t,t+k}} = \beta^k Y_t. \tag{F.4}$$

The consumption function (E.1) therefore implies

$$C_t = Y_t + (1-\beta)\left(B^d_{t-1} - \sum_{k=0}^{\infty}\frac{T_{t+k}}{\mathcal{R}_{t,t+k}}\right). \tag{F.5}$$

Starting from $t=0$, given that $B^d_{-1} = B_{-1}$ initially, the FTPL condition (35) at $t=0$ guarantees market clearing in the goods market at $t=0$. It therefore implies market clearing in the debt market at $t=1$, $B^d_0 = B_0$. Continuing by induction, its proves market clearing in all periods.

Individual Optimality

We now check individual optimality, which is characterized in Lemma 1. Condition (13) is satisfied by construction. Combined with the flow budget constraint of the representative household (5), its No-Cash-on-the-Table constraint is equivalent to its intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k} = B_{t-1}^{d} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}). \tag{F.6}$$

Since the goods market clears in all periods, it is equivalent to condition (35), which is satisfied by assumption.

Non-Ricardian Case

Proposition 1 and 2 already showed that conditions (36) and (37) are necessary for a stable price equilibrium. We show that they are sufficient. The proof is constructive. Given an initial level of public debt B_{-1} distributed as $(B_{-1}^{i})_i$ in the population, consider a path for taxes $(T_t)_{t\geq 0}$. Define recursively the following path for the interest rate

$$R_t = \frac{\frac{1-\zeta}{\beta(1+g)} Y_{t+1}}{Y_t - \chi\left(B_{t-1} - T_t\right)}, \tag{F.7}$$

which in combination with the FBC (10) of the government defines an entire path for interest rates and public debt. Because condition 1 is satisfied, equation (F.7) defines a positive finite interest rate. For this interest rate path, define individual consumption allocations through the consumption function (13), which in combination with the FBC (5) of household i defines an entire path for its consumption and debt holdings.

We show that this allocation is a stable price equilibrium.

Market-Clearing

Start with market-clearing. Since by construction all households are on the consumption function (13), aggregate consumption is given by the aggregate consumption function (E.1), which writes injecting the definition of aggregate human capital (B.7)

$$C_t = \mu \left(\frac{B_{t-1}^{d}}{P_t} + \sum_{k=0}^{\infty} \frac{a^k}{\mathcal{R}_{t,t+k}} (Y_{t+k} - T_{t+k}) \right), \tag{F.8}$$

where

$$a = \frac{(1-\lambda)(1-\zeta)}{1+g}. \tag{F.9}$$

Define

$$Z_{t+k} = a^k \frac{Y_{t+k}}{\mathcal{R}_{t,t+k}}. \tag{F.10}$$

Using the definition of the interest rate path (F.7) which can be written

$$\frac{Y_{t+1}}{R_t} = \frac{\beta(1+g)}{1-\zeta}\left(Y_t - \chi\frac{B_t}{R_t P_{t+1}}\right), \tag{F.11}$$

Z_{t+k} satisfies the recursion

$$Z_{t+k} = bZ_{t+k-1} - \chi b\frac{a^{k-1}}{\mathcal{R}_{t,t+k}}B_{t+k-1}, \tag{F.12}$$

where

$$b = \beta(1-\lambda). \tag{F.13}$$

By iteration, it gives

$$Z_{t+k} = b^k Y_t - \chi\left(\sum_{j=0}^{k-1} b^{k-j}a^j\frac{B_{t+j}}{\mathcal{R}_{t,t+j+1}}\right). \tag{F.14}$$

It follows that (after permuting a double sum)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}\left(Y_t - \chi\beta(1-\lambda)\left(\sum_{j=0}^{k-1} a^j\frac{B_{t+j}}{\mathcal{R}_{t,t+j+1}}\right)\right). \tag{F.15}$$

Using the budget constraint of the government

$$\frac{B_{t+j}}{P_{t+j+1}\mathcal{R}_{t,t+j+1}} = \frac{B_{t-1}}{P_t} - \sum_{i=0}^{j}\frac{1}{\mathcal{R}_{t,t+i}}T_{t+i}, \tag{F.16}$$

it can be rewritten (after permuting a double sum again)

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}\left(Y_t - \frac{\chi\beta(1-\lambda)}{1-a}\left(\frac{B_{t-1}}{P_t} - \sum_{i=0}^{\infty} a^i\frac{T_{t+i}}{\mathcal{R}_{t,t+i}}\right)\right), \tag{F.17}$$

or noticing that $\chi\beta(1-\lambda)/(1-a) = \mu$,

$$\sum_{k=0}^{\infty} Z_{t+k} = \frac{1}{\mu}Y_t - \left(\frac{B_{t-1}}{P_t} - \sum_{i=0}^{\infty} a^i\frac{T_{t+i}}{\mathcal{R}_{t,t+i}}\right). \tag{F.18}$$

Injecting equation (F.18) into the aggregate consumption function (F.8) implies

$$C_t = Y_t + \mu\left(\frac{B^d_{t-1}}{P_t} - \frac{B_{t-1}}{P_t}\right). \tag{F.19}$$

Because $B^d_{-1} = B_{-1}$, this implies market clearing in the goods market at $t = 0$, and therefore in the debt market, and so on at all periods by induction.

Individual Optimality

We now check individual optimality, which is characterized in Lemma 1. Condition (13) is satisfied by construction. We need to show that the No-Cash-on-the-Table constraints of all households are satisfied. Since all households behave according to the consumption function (13), we can apply Lemma 3 and simply show that condition (23) is satisfied. Since all households are on their consumption function and the goods market clears, we can use Lemma 2. That $\frac{(1-\lambda)^k}{\mathcal{R}_{t,t+k+1}} \frac{\bar{B}_{t+k}}{P_{t+k+1}}$ tends to zero as $k \to \infty$ can therefore be rewritten as equation (D.8), which holds by assumption from condition (37).

G Proof of Corollary 1

Ricardian Case $\lambda + g = 0$: The intertemporal budget constraint of non hand-to-mouth households is

$$\sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} C_{t+k}^{PI} = \frac{B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \frac{1}{\mathcal{R}_{t,t+k}} (Y_{t+k}^{PI} - T_{t+k}^{PI}). \tag{G.1}$$

Injecting the expression for income $Y_t^{PI} = (1-\alpha_Y)Y_t$ and taxes $T_t^{PI} = (1-\alpha_T)T_t$, as well as the equilibrium requirement that $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$, gives the intertemporal budget constraint of the government (18).

The proof of sufficiency follows the proof of proposition 3.

Non-Ricardian Case $\lambda + g > 0$: Condition (36) can be derived following exactly the same steps are in the case without hand-to-mouth households. Indeed, in the proof, taxes only intervene through the flow budget constraint of the government.

We show that condition (38) is necessary. Following the same steps as without hand-to-mouth households, one can show that equation (E.3) holds for the aggregate consumption of non hand-to-mouth households,

$$C_t^{PI} = \frac{1}{\beta R_t} \left(\frac{1-\zeta}{1+g} \right) C_{t+1}^{PI} + \chi \frac{B_t^d}{R_t P_{t+1}}. \tag{G.2}$$

Since non hand-to-mouth households collectively hold all of public debt, assuming the debt market clears and using the flow budget constraint of the government (10) gives

$$C_t^{PI} = \frac{1}{\beta R_t} \left(\frac{1-\zeta}{1+g} \right) C_{t+1}^{PI} + \chi \left(\frac{B_{t-1}}{P_t} - T_t \right). \tag{G.3}$$

Using the fact that in equilibrium $C_t^{PI} = Y_t - C_t^{HTM} = (1-\alpha_Y)Y_t + \alpha_T T_t$ gives the IS curve (40). Condition (38) is obtained when R_t to infinity.

The proof of sufficiency follows the proof of proposition 3.

H Proof of Proposition 4

The optimal behavior of households is characterized by the following consumption function,

$$C_t^y = \frac{1}{1+\beta}\left((Y_t^y - T_t^y) + \frac{1}{R_t}(Y_{t+1}^o - T_{t+1}^o)\right), \tag{H.1}$$

$$C_t^o = \frac{B_{t-1}^y}{P_t} + (Y_t^o - T_t^o). \tag{H.2}$$

Given that households have finite lives and leave no wealth when they die, there is no extra No-Cash-on-the-Table constraint as part of individual optimality.

Necessary Condition: We first show that condition (55) is necessary in equilibrium. Summing up (H.1) and (H.2), and using the fact that the debt market cleared in period $t-1$, $B_{t-1} = N_{t-1}^{young} B_{t-1}^{young}$, total aggregate consumption is given by

$$C_t = \frac{1}{1+\beta}\left(\gamma(Y_t - T_t) + \frac{1}{R_t}(1-\gamma)(Y_{t+1} - T_{t+1})\right) + \frac{B_{t-1}}{P_t} + (1-\gamma)(Y_t - T_t). \tag{H.3}$$

Imposing market clearing on the goods market, we get the dynamic IS curve

$$Y_t = \frac{1-\gamma}{\gamma\beta}\frac{1}{R_t}(Y_{t+1} - T_{t+1}) + \frac{1+\beta}{\gamma\beta}B_{t-1} - \left(\frac{1+\beta}{\gamma\beta} - 1\right)T_t. \tag{H.4}$$

Setting the interest rate to infinity gives the condition (55).

Sufficient Condition: If condition (55) is satisfied in all periods, define a path for the real interest rate through

$$R_t = \frac{\frac{1-\gamma}{\gamma\beta}(Y_{t+1} - T_{t+1})}{Y_t - \frac{1+\beta}{\gamma\beta}B_{t-1} + \left(\frac{1+\beta}{\gamma\beta} - 1\right)T_t} \tag{H.5}$$

and the government flow budget constraint (10). For this interest rate path, define the consumption of young and old households through the consumption function (H.1) and (H.2). Individual optimality is satisfied by construction. Rewriting the expression of aggregate consumption (H.3) using the expression (H.5) for R_t shows market clearing $C_t = Y_t$. This ends the proof.

I Proof of Proposition 5

The system (63)-(64) can be written in matrix form

$$A\begin{bmatrix}\hat{\pi}_t \\ \hat{b}_{t-1}\end{bmatrix} = \begin{bmatrix}\hat{\pi}_{t+1} \\ \hat{b}_t\end{bmatrix} + B\begin{bmatrix}\nu_t^i \\ \nu_t^g\end{bmatrix}, \tag{I.1}$$

where

$$A = \begin{bmatrix} \phi_\pi + \eta \frac{b}{y} & -\eta(1-\psi_b) \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \frac{R}{1+g}(1-\psi_b) \end{bmatrix} \tag{I.2}$$

$$B = \begin{bmatrix} -1 & \eta \\ -\frac{b}{y} & -\frac{R}{1+g} \end{bmatrix} \tag{I.3}$$

The economy has a unique bounded solution if and only if the matrix A has one eigenvalue outside the unit circle and one within. Calculating its trace and determinant, its roots are the solution to the quadratic equation

$$P(\lambda) = \lambda^2 - \left(\frac{R}{1+g}(1-\psi_b) + \phi_\pi + \eta\frac{b}{y}\right)\lambda + (1-\psi_b)\phi_\pi\left(\frac{R}{1+g} + \eta\frac{b}{y}\right). \tag{I.4}$$

<u>Necessary Condition:</u> If the economy has a unique equilibrium, then one root is outside the unit circle and one is inside. If so, the roots are necessarily real (complex roots have the same modulus). If roots are real, they are both positive, so having one inside the unit circle and the other outside is equivalent to having one root greater than one and the other smaller than one. In turn, this is equivalent to $P(1) < 0$, which is condition (65).

<u>Sufficient Condition:</u> Conversely, if condition (65) is satisfied, then it means $1 - tr + det < 0$. In this case the determinant of the quadratic equation $\Delta = tr^2 - 4det > (det+1)^2 - 4det = (det-1)^2 > 0$, so both roots are positive. Given that condition (65) is satisfied, it means one root is greater than one and the other outside. Hence there is a unique equilibrium.

When there is a unique equilibrium, let λ be the root of A that is greater than one and let $(1, -\mu)$ be its left eigenvector. The constant μ can be solved to be

$$\mu = \frac{\eta(1-\psi_b)}{\lambda - \frac{R}{1+g}(1-\psi_b)}. \tag{I.5}$$

Since $\lambda > \frac{R}{1+g}(1-\psi_b)$, the constant μ is positive. Note that the expression for μ is ill-defined in the case of fiscal dominance of the Ricardian case, $\eta = 0$ and $\lambda = \frac{R}{1+g}(1-\psi_b)$. In this case, the constant μ can be expressed as

$$\mu = \frac{\phi_\pi - \frac{R}{1+g}(1-\psi_b)}{\frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right)}. \tag{I.6}$$

Equation (I) implies

$$\lambda(\hat{\pi}_t - \mu\hat{b}_{t-1}) = (\hat{\pi}_{t+1} - \mu\hat{b}_t) + \left(\left(\mu\frac{b}{y} - 1\right)\nu_t^i + \left(\eta + \mu\frac{R}{1+g}\right)\nu_t^g\right). \tag{I.7}$$

Iterating forward gives equations (69).

Finally, we show that in the particular case $\psi_b = 0$, the impact effect of fiscal shocks on inflation is the same as in the Ricardian case. When $\psi_b = 0$, we have that the two roots of the polynomial (I.4) are ϕ_π and $\lambda = \frac{R}{1+g} + \eta \frac{b}{y}$. It follows that $\mu = 1/(b/y)$. Replacing the expression for μ in equation (69) gives

$$b_{t-1} - \frac{b}{y}\hat{\pi}_t = -\sum_{k=0}^{\infty} \left(\frac{1}{\frac{R}{1+g} + \eta \frac{b}{y}} \right)^k \nu^g_{t+k}, \tag{I.8}$$

which implies that in particular $\frac{\partial \pi_t}{\partial \nu^g_t} = \frac{1}{\frac{b}{y}}$.

J Proof of Proposition 6

Equations (63)-(64) are now

$$\left(\phi_\pi + \eta \frac{b}{y}\right) \hat{\pi}_t + \nu^i_t = \hat{\pi}_{t+1} + (\eta - \phi_b)\left(\left(1 - \psi_b\right)\hat{b}_{t-1} + \nu^g_t\right), \tag{J.1}$$

$$\hat{b}_t = \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)\left(\left(1 - \psi_b\right)\hat{b}_{t-1} + \nu^g_t\right) + \frac{b}{y}\left(\left(\phi_\pi - \frac{R}{1+g}\right)\hat{\pi}_t + \nu^i_t\right). \tag{J.2}$$

The system (J.1)-(J.2) can be written as in equation (I) but replacing matrix A with the matrix

$$A' = \begin{bmatrix} \phi_\pi + \eta \frac{b}{y} & 0 \\ \frac{b}{y}\left(\phi_\pi - \frac{R}{1+g}\right) & \left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1 - \psi_b) \end{bmatrix} \tag{J.3}$$

The two roots of A' are $\phi_\pi + \eta \frac{b}{y}$ and $\left(\frac{R}{1+g} + \frac{b}{y}\eta\right)(1 - \psi_b)$, from which the result follows.