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A General Equilibrium Approach to Carbon Permit Banking

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Abstract

We study the general equilibrium effects of carbon permit banking during the transition to a climate-neutral economy by 2050. To this end, we develop an environmental dynamic stochastic general equilibrium model, in which the business sector is regulated by a generic emission trading system (ETS). Firms are authorized to transfer unused permits from one period to the next (banking), but the reverse direction (borrowing) is prohibited. Allowing for positive banking gives firms the opportunity to smooth their permit demand along the business cycle. Applications inspired by recent European Union-ETS regulations underscore the critical role of permit banking in shaping policy outcomes. For example, the 2023 cap reform would result in a more significant reduction in both permit banking and carbon emissions, as well as a 40% to 50% increase in the carbon price compared to pre-reform projections, without substantial additional GDP loss by 2060. Importantly, forgetting about permit banking when assessing cap policies would lead to both a significant underestimation of the total macroeconomic effects and an inaccurate representation of the carbon emission trajectory.

Keywords: Emission Trading Systems, Cap Policies, Carbon Permit Banking, Environmental DSGE Model, Occasionally-Binding Constraints, Nonlinear Estimation

JEL classification: C32, E32, Q50, Q52, Q58.

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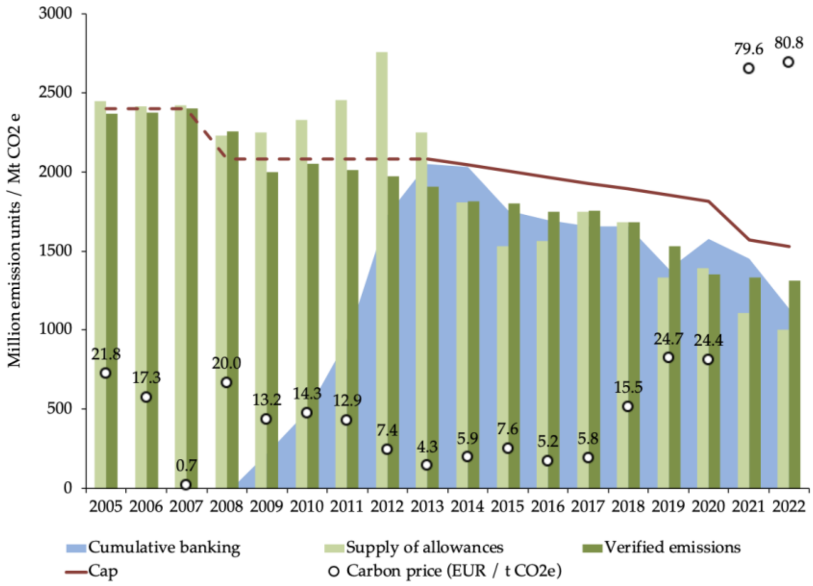
NON-TECHNICAL SUMMARY

A carbon cap policy, also known as an emission trading system (ETS), is a market-based approach implemented by regulators to adjust and reduce carbon emissions. It was designed to address the negative externalities associated with greenhouse gas emissions, particularly carbon dioxide emissions that contribute to climate change. Under a cap policy, the regulator sets a limit, or "cap", on the total amount of emissions allowed within a specific jurisdiction or industry. This emission cap is usually expressed in terms of a specific number of permits, with each permit representing the right to emit a certain amount of carbon dioxide or other greenhouse gases. In such a system, companies can strategically manage their emissions over time using *permit banking*. Firms that can reduce their emissions below the level implied by their allocated permits can bank surplus permits for future use or trade them to other entities. Cumulative banking -- that is, the total number of permits in circulation-- usually represents a considerable amount. For instance, in the European Union (EU)-ETS, it reached almost 2.1 billion at its peak in 2013 before decreasing to 1.1 billion in 2022 (blue area), which is higher than the same years' worth of market supply.

Carbon emissions, allowances, banking and prices in the EU-ETS



Sources: European Environment Agency and European Commission. Cumulative banking is defined as the difference between the allowances allocated for free, auctioned or sold plus international credits surrendered or exchanged from 2008 to 2022 minus the cumulative emissions. Between 2005 and 2012, the emissions cap was established in a decentralized manner and mainly relied on free allocations based on past emissions. Each member state established a national allocation plan to distribute allowances among covered installations, and the sum of these plans formed the overall cap (dashed line). Since 2013, the cap has been determined at the EU level and allocation takes place according to harmonized rules in which auctioning is the dominant allocation mechanism (plain line). MtCO2e: million tons of CO2 equivalent.

In the coming years, emission trading systems worldwide are expected to play a critical role in countries' efforts to achieve the goals of the Paris Agreement. Nevertheless, considerable uncertainty remains concerning future cap trajectories and the way companies adjust their permit banking strategies and production processes, ultimately affecting overall economic costs. Thus, a comprehensive economic assessment is crucial for designing and implementing effective environmental policies, as it enables policymakers to strike a balance between environmental targets and economic growth.

We propose a general equilibrium approach to carbon permit banking and assess the macroeconomic effects of various cap policies within this context. To this end, we first develop an *environmental dynamic stochastic general equilibrium* (E-DSGE) model that embeds a generic emission trading system with permit banking. The model includes (i) households that maximize intertemporal utility by choosing consumption, hours worked and capital accumulation, and (ii) firms that produce a homogeneous final good, which could in turn be used for consumption and investment. As firms' activities generate CO2 emissions, regulatory authorities implement an emission cap policy that grants them legal authorization to release a specified quantity. This allocated amount is contingent on the number of pollution permits issued by the regulators. We allow firms that purchase carbon emission permits to either use them directly or bank them for later use, without the possibility of borrowing between allowance periods. Finally, we assume that firms can reduce their carbon emissions by conducting expensive abatement activities. This *nonlinear model is estimated* by applying full-information methods to monthly EU data. Bringing the model to the data underscores the critical role of accounting for permit banking in understanding business cycle fluctuations. Notably, plugging the estimated shocks into an alternative version without permit banking results in excessive volatility for most of the variables. This empirical confrontation reveals that overlooking the intertemporal banking of permits would steer a policymaker toward an inappropriate representation of the economy.

Practical applications derived from recent decisions or regulations concerning the EU-ETS lead to the following results. Implementing the EU-ETS 2023 cap reform in the model, defined as a new sequence of linear reduction factors of the cap for stationary installations (4.3% from 2024 to 2027 and 4.4% from 2028 onward), leads to a more significant reduction in both permit banking and carbon emissions, as well as a 40% to 50% increase in the carbon price (depending on whether the market stability reserve is accounted for) compared to pre-reform projections, without substantial additional GDP loss by 2060. The market stability reserve, which triggers adjustments to annual auction volumes if the requirements based on the level of the aggregate bank of allowances are met, is a powerful tool that slows down firms' permit banking and reduces emissions more quickly. This approach facilitates the achievement of the objective of net zero emissions prior to 2050. Announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not at the time of its implementation. Importantly, forgetting permit banking when assessing cap policies would lead to a significant underestimation of total macroeconomic effects and an incorrect carbon emission path.

Une approche d'équilibre général de la mise en réserve des permis carbone

RÉSUMÉ

Nous étudions les effets d'équilibre général de la mise en réserve de permis carbone pendant la transition vers une économie climatiquement neutre d'ici 2050. À cette fin, nous développons un modèle d'équilibre général stochastique dynamique environnemental, dans lequel les entreprises sont régulées par un système générique d'échange de quotas d'émission (ETS). Les entreprises sont autorisées à transférer les permis non utilisés d'une période à l'autre (mise en réserve), mais la direction inverse (emprunt) est interdite. Autoriser une mise en réserve donne aux entreprises la possibilité de lisser leur demande de permis tout au long du cycle économique. Des applications inspirées des récentes réglementations de l'Union Européenne sur l'ETS soulignent le rôle essentiel de la mise en réserve de permis. Par exemple, la réforme du plafonnement des émissions de 2023 entraînerait une réduction plus importante de la mise en réserve des permis et des émissions de carbone, ainsi qu'une augmentation de 40 à 50 % du prix du carbone par rapport aux projections obtenues avant la réforme, sans perte supplémentaire de PIB d'ici 2060. Il est important de noter que l'oubli de la mise en réserve des permis carbone lors de l'évaluation des politiques de plafonnement conduirait à la fois à une sous-estimation significative des effets macroéconomiques totaux et à une représentation inexacte de la trajectoire des émissions de carbone.

Mots-clés : Systèmes d'échange de quotas d'émissions, politiques de plafonnement, mise en réserve de permis carbone, modèle DSGE environnemental, contraintes parfois mordantes, estimation non linéaire.

Codes JEL : E32, H23, Q50, Q55, Q58.

1. INTRODUCTION

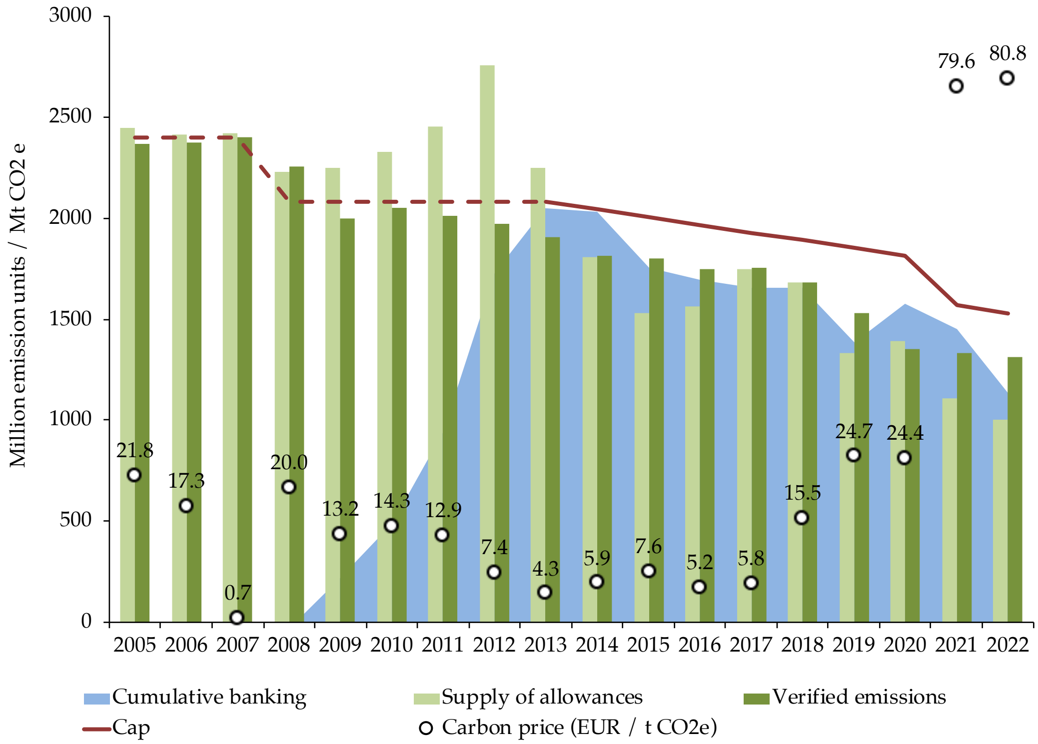
A carbon cap policy, also known as an emission trading system (ETS), is a market-based approach implemented by regulators to adjust and reduce carbon emissions. It was designed to address the negative externalities associated with greenhouse gas emissions, particularly carbon dioxide emissions that contribute to climate change. Under a cap policy, the regulator sets a limit, or "cap", on the total amount of emissions allowed within a specific jurisdiction or industry. This emission cap is usually expressed in terms of a specific number of permits, with each permit representing the right to emit a certain amount of carbon dioxide or other greenhouse gases. In such a system, companies can strategically manage their emissions over time using *permit banking*. Firms that can reduce their emissions below the level implied by their allocated permits can bank surplus permits for future use or trade them to other entities. Cumulative banking – that is, the total number of permits in circulation– usually represents a considerable amount. For instance, Figure 1 shows that in the European Union (EU)-ETS, it reached almost 2.1 billion at its peak in 2013 before decreasing to 1.1 billion in 2022 (blue area), which is higher than the same year's worth of market supply.[1]

In the coming years, emission trading systems worldwide are expected to play a critical role in countries' efforts to achieve the goals of the Paris Agreement. A significant portion of the reduction in carbon emissions to zero by 2050 must be driven by a decrease in permit supply. Nevertheless, considerable uncertainty remains concerning future cap trajectories and the way companies adjust their permit banking strategies and production processes, ultimately affecting overall economic costs. Thus, a comprehensive economic assessment is crucial for designing and implementing effective environmental policies, as it enables policymakers to strike a balance between environmental targets and economic growth.

In this paper, we propose a general equilibrium approach to carbon permit banking and assess the macroeconomic effects of various cap policies within this context. Permit banking provides firms with the opportunity to smooth their demand for permits along the business cycle. By spreading out emissions reduction and compliance costs, firms can minimize

[1]This substantial surplus of allowances originates from two primary sources. Firstly, due to the absence of reliable data on industry-wide and company-specific emissions of installations under the EU-ETS prior to 2005, the cap was initially established based on conservative emissions estimates. Consequently, the majority of member states implemented too generous caps and allocated an excessive number of free allowances during the initial years. Secondly, the 2008 financial crisis resulted in a significant decrease in demand for allowances, which was not correspondingly reflected in supply due to the inflexibility of auctioning and the grandfathering of free allowances. The total supply even exceeded the cap due to the influx of international credits between 2008 and 2013.

FIGURE 1. Carbon emissions, allowances, banking and prices in the EU-ETS



Sources: European Environment Agency and European Commission. Cumulative banking is defined as the difference between the allowances allocated for free, auctioned or sold plus international credits surrendered or exchanged from 2008 to 2022 minus the cumulative emissions. Between 2005 and 2012, the emissions cap was established in a decentralized manner and mainly relied on free allocations based on past emissions. Each member state established a national allocation plan to distribute allowances among covered installations, and the sum of these plans formed the overall cap (dashed line). Since 2013, the cap has been determined at the EU level and allocation takes place according to harmonized rules in which auctioning is the dominant allocation mechanism (plain line). MtCO2e: million tons of CO2 equivalent.

fluctuations in their marginal costs and maintain their production levels. Thus, effective permit banking can contribute to market stability, reduce uncertainty and encourage consumer spending and business investments. Nevertheless, the general equilibrium effects of permit banking are highly dependent on both its aggregate level (high in the EU, as Figure 1 shows) and the stringency of underlying cap policies. This suggests that to provide relevant recommendations to policymakers, it is valuable to confront the model with official regulatory decisions.

This study makes three major contributions to the literature. First, we develop an *environmental dynamic stochastic general equilibrium* (E–DSGE) model that embeds a generic emission trading system with permit banking. The model includes *(i)* households that maximize intertemporal utility by choosing consumption, hours worked and capital accumulation, and *(ii)* firms that produce a homogeneous final good, which could in turn be used for consumption and investment. As firms' activities generate CO_2 emissions, regulatory authorities implement an emission cap policy that grants them legal authorization to release a specified quantity. This allocated amount is contingent on the number of pollution permits issued by the regulators. We allow firms that purchase carbon emission permits to either use them directly or bank them for later use, without the possibility of borrowing between allowance periods. This non-borrowing constraint captures the realistic dynamics of intertemporal permit trading and generates additional forward-looking dynamics that impact firm behavior. Finally, we assume that firms can reduce their carbon emissions by conducting expensive abatement activities.[2] The resulting model has appealing properties that make it amenable to the analysis of alternative economic policies as well as to empirical testing or validation. Specifically, it *(i)* formalizes the behavior of economic agents based on explicit microfoundations, *(ii)* manages all interactions between them within general equilibrium, *(iii)* emulates how forward-looking agents form expectations about a future characterized by stochastic events or outcomes, and *(iv)* incorporates uncertainty into agents' decision-making processes, as suggested by Pindyck (2013). Crucially, this framework effectively addresses the impacts of regulatory measures when firms rely on expectations to determine the optimal use of allowances in the future, akin to the principles derived from the Hotelling (1931)'s rule.

Our second contribution is the *estimation of this nonlinear model* by applying full-information methods to monthly EU data. The presence of an occasionally-binding constraint arising from the non-negativity of permit banking breaks the linear assumption commonly used in the literature on estimated structural models (e.g., Smets and Wouters, 2007). Our chosen piecewise-linear solution method handles the constraint as two different regimes in which it is either slack or binding (Guerrieri and Iacoviello, 2015; Aruoba et al., 2021). A recursive representation of the solution is obtained, conditional on how long the constraint is expected to bind in the future. Once a set of measurement equations is specified to link the state variables to the observables, an inversion filter can be used to compute the likelihood function analytically by inverting the observation equations to compute structural shocks (Guerrieri and Iacoviello, 2017; Cuba-Borda et al., 2019). Bringing the model to the data underscores the critical role of accounting for permit banking in understanding business cycle fluctuations. Notably, plugging the estimated shocks into an alternative version without permit banking

[2]This study examines the cost-effectiveness of smoothing emission reductions over time through the utilization of permits without addressing the potential environmental benefits of avoiding climate damage. As the objective is not to determine the optimal path for carbon reduction, the framework does not incorporate the accumulation of atmospheric carbon or the associated economic damage to productivity.

results in excessive volatility for most of the variables. This occurs because, without the ability to store permits, firms struggle to insure themselves against future economic disruptions and smoothly adapt their production processes. This empirical confrontation reveals that overlooking the intertemporal banking of permits would steer a policymaker toward an inappropriate representation of the economy.

Our third contribution is to propose *practical applications* derived from recent decisions or regulations concerning the EU-ETS, which have been implemented or announced by the European Parliament. These projection exercises evaluate the impact of permit banking on the macroeconomic outcomes of cap policies. Six major findings emerge from these exercises. First, implementing the EU-ETS 2023 cap reform in the model, defined as a new sequence of linear reduction factors of the cap for stationary installations (4.3% from 2024 to 2027 and 4.4% from 2028 onward), leads to *(i)* a more pronounced decline in permit banking subsequent to 2033, *(ii)* a 40% increase in the carbon price, and *(iii)* an additional average GDP loss of approximately 0.4% by 2060, relative to the projections prior to the 2023 cap reform. Second, announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not only by the time of implementation. Third, a frontloading of permits (as announced in February 2023) results in a net drop in emissions during the implementation period but after an increase in the stock of pollution in the atmosphere, there is a net negative effect on the carbon price and a reduction in GDP over both periods of frontloading and withdrawal. Fourth, the market stability reserve, which triggers adjustments to annual auction volumes if requirements based on the level of the aggregate bank of allowances are met, is a powerful tool that slows down firms' banking of permits and thus reduces emissions more quickly. It achieves the objective of net-zero emissions before 2050 without incurring any additional costs in terms of GDP. Fifth, a policymaker can achieve the same emission reduction path as under cap policy by setting a carbon price that accounts for firms' forward-looking behavior implied by the ETS. This choice allows her to reduce GDP losses during the transition period. Finally, forgetting about permit banking leads to a significant underestimation of the macroeconomic effects of policy tightening, and an incorrect emission path. The latter misleadingly suggests that achieving net-zero emissions would occur by 2040.

Our paper is related to the literature on permit banking, which generally recognizes its importance in achieving cost-effective emission reductions and providing flexibility to regulated entities. In particular, Cronshaw and Kruse (1996), Rubin (1996), and Schennach (2000) show that the price of allowances should rise at the same rate as the real interest rate, consistent with Hotelling (1931). As the cost of emissions may increase over time owing to more stringent environmental regulations, firms can strategically use their banked permits to offset higher future costs. Recent contributions have extended these early studies in several directions, such as *(i)* the role of firms' market power (Liski and Montero, 2005), *(ii)* policy choices under uncertainty (Fell and Morgenstern, 2010; Fell et al., 2012), *(iii)* the role of delayed compliance (Holland and Moore, 2013), *(iv)* the interaction of the ETS with the electricity market (Pommeret and Schubert, 2018), *(v)* the proposal of various ETS stabilization mechanisms (Kollenberg and Taschini, 2016, 2019; Lintunen and Kuusela, 2018), *(vi)* putting into perspective the common features between pricing of allowances and financial claims (Hitzemann and Uhrig-Homburg, 2018; Jaccard et al., 2023), and *(vii)* the introduction of the market stability reserve (Perino and Willner, 2016; Quemin and Trotignon, 2021). However, these studies are interested in the role of banking in the functioning of the emissions market itself without paying significant attention to its broader effects on the economy. By contrast, we propose a general equilibrium approach to permit banking with a non-borrowing constraint that allows us to quantify the effects of cap policies on the real side of the economy. This nuanced view contributes to a deeper understanding of the interactions between permit banking and fundamental aspects of economic activity.

Our work also complements the burgeoning literature that focuses on climate issues using microfounded structural models. Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013) are among the first to introduce CO_2 emissions into real business cycle models. They assume that emissions stem from production and adversely impact utility, productivity, and output. Recent contributions have extended these models in several directions, including *(i)* multisector aspects (Golosov et al., 2014; Dissou and Karnizova, 2016), *(ii)* labor market frictions (Finkelstein Shapiro and Metcalf, 2023; Gibson and Heutel, 2023), *(iii)* distortionary fiscal policy (Barrage, 2020), *(iv)* endogenous entry (Annicchiarico et al., 2018; Finkelstein Shapiro and Metcalf, 2023), *(v)* public subsidies (Jondeau et al., 2023), and *(vi)* nominal rigidities and monetary policy (Annicchiarico and Di Dio, 2015; Annicchiarico and Di Dio, 2017; Diluiso et al., 2021; Carattini et al., 2023; Ferrari and Nispi Landi, 2024). These models have gained prominence in policy circles as they can be used to investigate the effects of environmental policies on aggregate variables in both short and medium terms. We contribute to this literature by offering a tractable framework that embeds an ETS with permit banking and estimate it using European data. By accounting for an additional regime in which firms are allowed to store permits, we generalize previous frameworks that represent a cap policy as a unique regime (e.g., Fischer and Springborn, 2011 and Heutel, 2012).

The remainder of this paper is organized as follows. Section 2 describes the E-DSGE model with carbon permit banking. Section 3 reports the estimation methodology and discusses the model's dynamic properties. Section 4 illustrates the general equilibrium effects of permit banking through a series of policy exercises inspired by the EU-ETS regulations. Section 5 compares the cap policy to different tax policies. Finally, Section 6 concludes the paper.

2. An environmental DSGE model with pollution permit banking

The economy is described by an environmental DSGE model.[3] There is a unit mass of atomistic, identical, and infinitely lived households that maximize intertemporal utility by choosing consumption, hours worked, and capital accumulation. On the production side, there is a unit mass of atomistic firms that hire labor services and physical capital to produce a homogeneous final good, which could in turn be used for consumption and investment. Firms' activities generate CO_2 emissions, and do not consider their effects on pollution or environmental damage. To force them to internalize this externality, a regulatory authority implements a cap policy. Specifically, this environmental policy gives firms the legal right to pollute a certain amount, which depends on the number of pollution permits issued by the regulator. These permits are bankable, that is, they can be stored for future use. Finally, firms can reduce their carbon emissions by conducting costly abatement activities.

2.1. **Household sector.** Each household indexed by $i \in [0,1]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on hours worked $n_{i,t}$:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(c_{i,t+s} - \varphi c_{t+s-1})^{1-\sigma} - 1}{1-\sigma} - \chi_t \frac{n_{i,t+s}^{1+\nu}}{1+\nu} \right\}, \tag{1}$$

subject to the sequence of real budget constraints

$$c_{i,t} + x_{i,t} + \mathcal{A}^x_{i,t} \leq w_t n_{i,t} + d_{i,t} + r_{k,t} k_{i,t-1}, \tag{2}$$

[3] The economy is assumed to grow at a constant gross rate γ_z. To maintain consistency with a balanced growth path, the utility function (Equation (1)) and the production function (Equation (8)) are modified to align with the economy's growth rate. The technology efficiency in abatement costs (Equation (12)) does not exhibit this attribute. Furthermore, the carbon emission process (Equation (9)) is adjusted to allow the emission trend to be decoupled from that of the output (in accordance with empirical data).

where E_t denotes the mathematical expectation operator conditional on the information available at t, $\beta \in (0,1)$ is the subjective discount factor, φ captures external habit formation ("catching up with the Joneses"), σ is the inverse of the elasticity of substitution in consumption, $\nu > 0$ is the inverse of the Frisch labor supply elasticity, and $\chi_t = \chi\gamma_z^{t(1-\sigma)}$ is a time-varying parameter that cancels out the effects of the labor-augmenting deterministic growth rate in the economy γ_z^t on labor supply, where χ is a scaling parameter.[4] Such a feature is necessary to obtain a balanced growth path for the hours worked. The variable $x_{i,t}$ is investment, $d_{i,t}$ is the equity payout received from the ownership of firms, and w_t is the real wage. Physical capital $k_{i,t}$ is rented to the firm at the rental rate $r_{k,t}$. $\mathcal{A}_{i,t}^x = \frac{\psi}{2}\left(\frac{x_{i,t}}{x_{i,t-1}} - \gamma_z\right)^2 x_{i,t-1}$ represents adjustment costs for investment, with $\psi > 0$. Physical capital accumulates according to

$$k_{i,t} = (1-\delta)\,k_{i,t-1} + x_{i,t}, \tag{3}$$

where $\delta \in [0,1]$ is the depreciation rate of capital.

The first-order conditions with respect to $c_{i,t}$, $n_{i,t}$, $x_{i,t}$, and $k_{i,t}$ are as follows:

$$\lambda_{h,i,t} = (c_{i,t} - \varphi c_{t-1})^{-\sigma}, \tag{4}$$

$$w_t = \frac{\chi_t n_{i,t}^{\nu}}{\lambda_{h,i,t}}, \tag{5}$$

$$q_{i,t} = 1 + \psi\left(\frac{x_{i,t}}{x_{i,t-1}} - \gamma_z\right) - \beta\mathrm{E}_t\left\{\frac{\lambda_{h,i,t+1}}{\lambda_{h,i,t}}\frac{\psi}{2}\left(\left(\frac{x_{i,t+1}}{x_{i,t}}\right)^2 - \gamma_z^2\right)\right\}, \tag{6}$$

$$q_{i,t} = \beta\mathrm{E}_t\left\{\frac{\lambda_{h,i,t+1}}{\lambda_{h,i,t}}\left((1-\delta)\,q_{i,t+1} + r_{k,t+1}\right)\right\}. \tag{7}$$

where $\lambda_{h,i,t}$ is the Lagrangian multiplier associated with household i's budget constraint and $q_{i,t}$ is the relative price of capital $k_{i,t}$ (i.e., the marginal Tobin's Q).

2.2. **Business sector.**

2.2.1. *Technology.* Each firm indexed by $j \in [0,1]$ produces a homogenous good using the following production function:

$$y_{j,t} = \varepsilon_{a,t} A k_{j,t-1}^{\alpha}(\gamma_z^t n_{j,t})^{1-\alpha}, \tag{8}$$

[4]Consumption habit is a key component in modern new Keynesian business cycle models (Del Negro et al., 2007; Smets and Wouters, 2007; Dennis, 2009). It enhances models' ability to *(i)* realistically capture consumer behavior and economic dynamics, *(ii)* explain and predict the co-movement between consumption, investment, and labor supply, *(iii)* increase the persistence of economic shock effects, and *(iv)* influence the intertemporal elasticity of substitution, which helps match the observed low elasticity found in empirical data.

where $\alpha \in (0,1)$ denotes the capital share, A is a scale factor, $k_{j,t}$ and $n_{j,t}$ denote the amounts of physical capital and labor services used by the firm respectively, and $\varepsilon_{a,t}$ is the total factor productivity shock common to all firms.

During its production process, a firm generates CO_2 emissions, denoted by $e_{j,t}$, which accumulate to increase the stock of pollutants in the air (Heutel, 2012):

$$e_{j,t} = \eta_t \left(1 - \mu_{j,t}\right) y_{j,t}^{1-\gamma_\mu}, \tag{9}$$

where $\mu_{j,t}$ represents the effort to abate emissions, $1 - \gamma_\mu$ is the elasticity of emissions with respect to output, and $\eta_t = \eta \gamma_z^{t(\gamma_\mu - 1)}$ is a time-varying parameter that allows the emission trend to be decoupled from that of the output, with η as a scaling parameter.

However, firms do not consider the effects of their activities on pollution or environmental damages. Because firms are atomistic, their marginal impact on total CO_2 emissions is zero. Therefore, the regulator implements a cap policy.[5] Specifically, the regulator sets an emission cap and issues a quantity of emissions permits ϑ_t consistent with that cap. Firms must hold permits for every ton of CO_2 they emit. To this end, a firm may buy permits on a specific market, thus establishing the permit (or equivalently carbon) price $p_{e,t}$. Firms that can reduce their current emissions at a lower cost may bank any excess permits for latter use. Two important properties relative to the dynamics of permit banking are as follows:

Assumption 1. *The law of motion of firm j's bank of permits $b_{j,t}$ is given by:*

$$b_{j,t} = b_{j,t-1} + \vartheta_{j,t} - e_{j,t}. \tag{10}$$

This equation states that the current stock of permits is the sum of the previous period bank $b_{j,t-1}$ and the newly bought permits $\vartheta_{j,t}$ minus the number of surrendered permits, measured in terms of emissions unit $e_{j,t}$. The possibility of trading permits between firms does not appear in this equation. Indeed, in an economy with homogeneous firms, the absence of idiosyncratic characteristics (e.g., differences in productivity, carbon intensity, or abatement costs) precludes them from exploiting the asymmetry in abatement to attain additional gains. As a result, in equilibrium, the amount traded is zero, justifying the exclusion of trade from the optimization problem.

[5] Our objectif is to focus on a cost-effectiveness analysis. Therefore, it is not necessary to explicitly represent the economic damage caused by the accumulation of pollutants in the atmosphere. Introducing an endogenous determination of the cap would be feasible, and this policy would be significantly different from the cap policy currently implemented in Europe. Because our approach is mainly positive, we leave normative evaluation for future research.

Assumption 2. *Firms are not allowed to borrow permits from the future, such that:*

$$b_{j,t} \geq 0. \tag{11}$$

This constraint effectively captures the operational characteristics of existing ETSs. First, while borrowing is expected to provide entities with flexibility in formulating their compliance strategies, it postpones the emissions reductions needed to achieve the ETS's environmental objectives by reducing mitigation action in the short term (Appendix A provides an illustration of this outcome in the context of our policy exercises). This is the reason why most ETSs have prohibited borrowing. Second, it generalizes the way firms are required to comply with environmental policy by introducing nonlinear effects into their profit optimization problems. When $b_{j,t} = 0$, the model is isomorphic to the standard linear versions used in the literature, e.g., Fischer and Springborn (2011) or Heutel (2012): each period, firms would buy $\vartheta_{j,t} = e_{j,t}$ permits to make up for their contemporaneous emissions. However, allowing for positive banking gives firms the opportunity to smooth their permit demand along the business cycle.

Finally, firms may substitute carbon-intensive technologies with low-carbon technologies; however this change in the existing lines of production is costly. We assume that the cost of abatement technology (in proportion to output) is given by:

$$\mathcal{A}^{\mu}_{j,t} = \theta_{1,t}\mu^{\theta_2}_{j,t} y_{j,t}\varepsilon_{\mu,t}. \tag{12}$$

where $\theta_{1,t}$ denotes the technology efficiency that reduces abatement costs. The decline rate of abatement cost is assumed to be 1% per year, as in Barrage and Nordhaus (2024). The term $\mu^{\theta_2}_{j,t}$ captures the long-term cost of reducing carbon emissions, as in DICE, where θ_2 is the abatement cost function curvature. Finally, $\varepsilon_{\mu,t}$ is an abatement shock.

2.2.2. *Profits maximization.* The objective of a firm is to maximize its intertemporal profits:

$$\mathrm{E}_t \sum_{t=s}^{\infty} \Omega_{t,t+s}\{y_{j,t+s} - w_{t+s}n_{j,t+s} - r_{k,t+s}k_{j,t+s-1} - p_{e,t+s}\vartheta_{j,t+s} - \mathcal{A}^{\mu}_{j,t+s}\}, \tag{13}$$

subject to constraints (8)–(12). In this expression, $\Omega_{t,t+s} = \beta^s \frac{\lambda_{h,t+s}}{\lambda_{h,t}}$ is the stochastic discount factor that converts future payoffs into current values, and $\lambda_{h,t}$ is the Lagrange multiplier associated with the budget constraint of the representative household.[6]

[6]Our approach aligns with the Modigliani-Miller theorem, which asserts that under specific conditions (perfect capital markets), a firm's capital structure does not influence its market value. In our model, we postulate the absence of explicit financial frictions on the firm side, implying that the return on capital assets perfectly co-varies with the risk-free rate. Furthermore, fluctuations in permit prices do not directly result in changes in

This problem yields the following first-order conditions for an optimal solution:

$$w_t = (1-\alpha)\frac{y_{j,t}}{n_{j,t}} mc_{j,t}, \tag{14}$$

$$\mathrm{E}_t\left\{r^k_{t+1}\right\} = \mathrm{E}_t\left\{\alpha\frac{y_{j,t+1}}{k_{j,t}} mc_{j,t+1}\right\}, \tag{15}$$

$$mc_{j,t} = 1 - \varepsilon_{\mu,t}\theta_{1,t}\mu_{j,t}^{\theta_2} - \lambda_{f_1,j,t}(1-\gamma_\mu)\frac{e_{j,t}}{y_{j,t}}, \tag{16}$$

$$\varepsilon_{\mu,t}\theta_{1,t}\theta_2\mu_{j,t}^{\theta_2-1} y_{j,t} = \lambda_{f_1,j,t}\eta_t y_{j,t}^{1-\gamma_\mu}, \tag{17}$$

$$\lambda_{f_1,j,t} = p_{e,t}, \tag{18}$$

$$\lambda_{f_1,j,t} = \mathrm{E}_t\left\{\Omega_{t,t+1}\lambda_{f_1,j,t+1}\right\} + \lambda_{f_2,j,t}, \tag{19}$$

$$(\lambda_{f_2,j,t} = 0 \quad \text{and} \quad b_t \geq 0) \quad \text{or} \quad (\lambda_{f_2,j,t} > 0 \quad \text{and} \quad b_t = 0), \tag{20}$$

where $\lambda_{f_1,j,t}$ is the Lagrange multiplier associated with constraints (9) and (10), which have been combined into one, and $\lambda_{f_2,j,t}$ is the Lagrange multiplier associated with the non-borrowing constraint (11), which is the shadow value of the carbon emission permits.

Equation (14) is the first-order condition with respect to labor. It states that real wage is equal to the marginal product of labor net of the marginal resources that must be spent on abatement and pollution permits. Indeed, as emissions are a by-product of output, any additional output creates the need for extra abatement and permits to comply with the cap policy.[7] Equation (15) represents the first-order condition with respect to capital, which indicates that the rental rate of capital is equal to its net marginal productivity. Equation (16) is the first-order condition with respect to output, which defines the marginal cost. Equation (17) is the first-order condition with respect to abatement, which equalizes the marginal benefits and costs of an additional abatement unit. This indicates the amount of resources a firm no longer needs to spend on purchasing permits. Equation (18) is the first-order condition with

firm profit. Given the assumption of efficient markets, it is feasible to introduce an insurance mechanism that aggregates the risk associated with unused permits, thereby providing firms with competitive and complete insurance against unanticipated price variations. This assumption enables us to simplify the model by treating changes in permit prices as fully insured and consequently neutral with respect to firm profits.

[7] If pollution was an input in the production function, this marginal cost component would disappear and an extra first-order condition relative to the optimal use of that input would appear.

respect to the demand for new permits, which simply states that the Lagrangian multiplier $\lambda_{f_1,j,t}$ is equal to the price of bankable carbon emissions. Equation (19) is the first-order condition with respect to the bank of permits. It is a forward-looking equation that relates the contemporaneous carbon price to the discounted carbon price expectation and Lagrangian multiplier $\lambda_{f_2,j,t}$. Finally, Equation (20) is the Karush-Kuhn-Tucker complementary slackness condition associated with the non-borrowing constraint.

2.2.3. *Implications of the non-borrowing constraint on permits.* Let us first look at the case $\lambda_{f_2,j,t} > 0$ and $b_t = 0$. Equation (19) indicates that the current permit price is above expectations for the next period. Therefore, firms have no incentive to bank permits because they expect to obtain them later at a cheaper price. Hence, banking does not occur: $b_t = 0$. When $\lambda_{f_2,j,t}$ tends toward zero, the current and expected prices become closer. Once $\lambda_{f_2,j,t}$ reaches zero, firms are indifferent to (i) buying a permit today for later use and (ii) buying it later. Banking can occur and b_t is allowed to be positive. Note that, because $\lambda_{f_2,j,t}$ is not negative, the current price is never below the expected price. If this were the case, an arbitrage opportunity would lead firms to buy an infinite number of permits and bank them accordingly. Instead, the banking opportunity creates an additional demand for permits at time t and contributes to increasing the current price to at least the value of $\mathrm{E}_t\{\Omega_{t,t+1}p_{e,t+1}\}$, consistently with the Hotelling principle. Thus, the economy can be in one of the following two regimes: (i) a regime without banking where the current price is higher than the future price of a permit, or (ii) a regime with banking where the two prices are equal. This characteristic introduces nonlinearity, which translates to the occasionally-binding constraint $\lambda_{f_2,j,t} \geq 0$.

2.3. **Regulatory authority.** To incentivize firms to reduce their emissions, a regulatory authority sets a cap $\bar{\vartheta}$ on the maximum level of emissions and creates permits for each unit of emissions allowed under the cap:

$$\int_{j=0}^{1} \vartheta_{j,t}\mathrm{d}j = \bar{\vartheta}\varepsilon_{\vartheta,t}, \tag{21}$$

where $\varepsilon_{\vartheta,t}$ is a shock that captures the possible discrepancy between the official cap and the actual supply of allowances.[8]

[8]Figure 1 illustrates this phenomenon. Indeed, the supply of allowances exceeded the cap between 2008 and 2013 due to the extensive utilization of carbon credits from the Clean Development Mechanism and Joint Implementation (two UN-organized crediting programs established under the Kyoto Protocol) towards fulfilling a portion of operators' EU-ETS obligations. Subsequently, the supply of allowances fell below the cap from 2014 onward as a measure to rebalance supply and demand and reduce the substantial surplus. Specifically, it was

2.4. **The government.** Revenues from cap policies are collected by governments and used in several ways (ICAP, 2024). Some governments channel collected revenues towards their general budget (including debt reduction), while others prefer to earmark revenues for specific uses (e.g., funding climate mitigation and low-carbon innovation, pursuing developmental objectives, such as education and health). In our model, the government does not impose taxes on households or firms, engage in investment activities, or issue debt. Consequently, for the purpose of simplification, we assume that all collected revenues $p_{e,t}\vartheta_t$ are used in the form of generic public expenditure.[9]

2.5. **Market clearing and equilibrium conditions.** The aggregate resource constraint of the economy is obtained by integrating across households and firms:

$$\int_{j=0}^{1} y_{j,t}\mathrm{d}j = \int_{i=0}^{1}\int_{j=0}^{1} \left(c_{i,t} + x_{i,t} + p_{e,t}\vartheta_{j,t} + \mathcal{A}^{x}_{i,t} + \mathcal{A}^{\mu}_{j,t}\right) \mathrm{d}i\mathrm{d}j \tag{22}$$

Regarding the properties of the stochastic variables, all shocks follow an AR(1) process $\varepsilon_{x,t} = 1 - \rho_x + \rho_x \varepsilon_{x,t-1} + \zeta_{x,t}$, with $x \in \{a, \mu, \vartheta\}$. In all cases, $\zeta_{x,t} \sim i.i.d.\mathcal{N}\left(0, \sigma_x^2\right)$.

3. Dynamic properties of the model

This section discusses the dynamic properties of the general equilibrium model with permit banking. To this end, we first estimate the model parameters using the maximum-likelihood methodology and then provide an analysis through *(i)* the impulse response functions of key variables to the three underlying shocks (total factor productivity, abatement costs, and permit supply) and *(ii)* a counterfactual exercise that provides insights into the effects of not accounting for permit banking. Due to the fluctuation of numerous variables around a deterministic balanced growth path, the model is initially reformulated in terms of detrended variables. Subsequently, the stationary nonlinear model is solved using the piecewise linear perturbation approach proposed by Guerrieri and Iacoviello (2015), which is a variant of the extended perfect-foresight path method introduced by Fair and Taylor (1983). A detailed description of the model resolution method is provided in Appendix B.

determined that the auctioning quantities would be reduced by 400 million in 2014, 300 million in 2015, and 200 million in 2016.

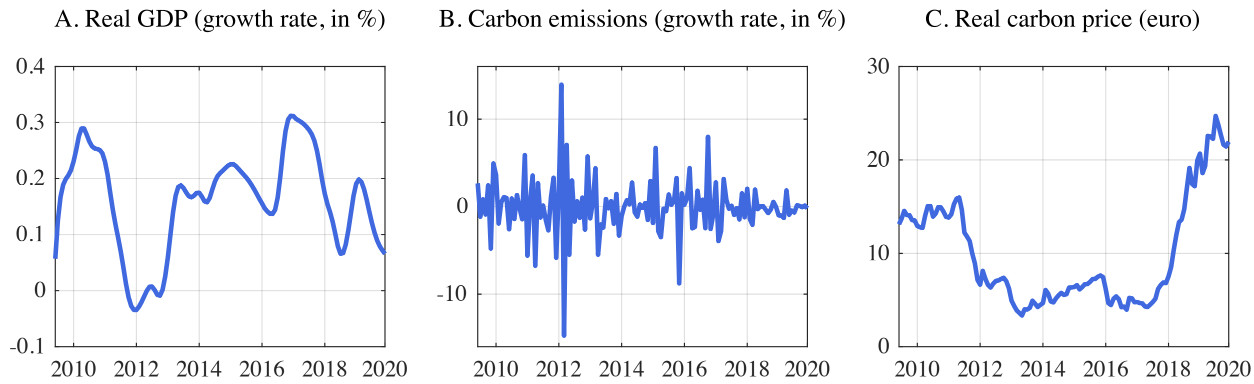
[9] While less realistic, these funds could potentially be distributed as lump-sum transfers to households. However, this approach would not significantly alter our primary conclusions.

3.1. **Data description.** The model is estimated using monthly data for the European Union from June 2009 to December 2019. Carbon emission data is taken from the Emissions Database for Global Atmospheric Research (Crippa et al., 2020), which provides estimates of global anthropogenic emissions and emissions trends, based on publicly available statistics (https://data.jrc.ec.europa.eu/collection/edgar). We build an aggregate time series of fossil CO2 emissions by summing the emissions of the 27 member countries of the European Union plus Iceland, Norway and the United Kingdom, the latter being part of the EU-ETS in our sample. The resulting series exhibits seasonal patterns. Thus, the data are seasonally adjusted using the X-13 ARIMA-SEATS filter from the Census Bureau (Lengwiller, 2022). The carbon price is obtained from the International Carbon Action Partnership (https://icapcarbonaction.com/fr/node/839), which offers a historical daily series, updated quarterly from the European Energy Exchange (the common auction platform of the EU-ETS designated by the European Commission). It is a spot price stemming from primary market auctions, i.e., the price at which permits are supplied directly from the government to firms to be either used directly, or banked for later use. The monthly time series is obtained by taking the average price for each month. Real GDP is taken from the OECD Main Economic Indicators, which offer a monthly proxy for OECD-Europe.[10] It is retrieved from the Federal Reserve of Saint Louis website (https://fred.stlouisfed.org/series/OECDELORSGPORIXOBSAM). Finally, we use the GDP deflator to construct a real carbon price, i.e., adjusted for the effects of price inflation. We extract the quarterly series of the GDP deflator for the European Union from the OECD Main Economic Indicators, retrieved from the Federal Reserve of Saint Louis website (https://fred.stlouisfed.org/series/NAGIGP01EUQ661S) and convert it into monthly data using the Chow and Lin (1971) approach and the monthly consumer price index for OECD-Europe (https://fred.stlouisfed.org/series/OECDECPALTT01IXOBM). Figure 2 shows the retrieved variables used for the estimation.

3.2. **Parameter values.** A first set of parameters is calibrated and is listed in Table 1. To be consistent with the monthly frequency, the discount rate β is set to 0.997, and the capital depreciation rate is set to 0.005 (i.e., an annual rate of 6%). The capital share in the production

[10]There is no monthly GDP series for the European Union. However, we found that the year-on-year GDP growth obtained from the monthly series (OECD-Europe) was very close to that of the official quarterly series for the European Union. Therefore, they have the same business cycle characteristics.

FIGURE 2. Observable variables



Sources: Organization for Economic Cooperation and Development (GDP and deflator), Emissions Database for Global Atmospheric Research (carbon emissions), and International Carbon Action Partnership (carbon price).

function is set to $1/3$ and the parameters $(\theta_1; \theta_2)$ associated with the abatement costs are $(0.1; 2.6)$, in line with Barrage and Nordhaus (2024).[11]

TABLE 1. Calibrated parameters

	PARAMETER	VALUE
Discount factor	β	0.997
Capital depreciation rate	δ	0.005
Capital share of output	α	0.333
Abatement cost parameter (scale)	θ_1	0.100
Abatement cost parameter (elasticity)	θ_2	2.600

A second set of parameters is estimated using the full information maximum likelihood methodology. Specifically, we use an inversion filter to recursively extract shock innovations by inverting the observation equations conditional on the initial state. This approach allows for easy computation of the likelihood function in the context of a model with an occasionally-binding constraint (Guerrieri and Iacoviello, 2017; Kollmann, 2017). The last column of Table 2 reports the parameter estimates and their associated P-values.

All values are significant and consistent with those reported in the existing literature. In particular, usual parameters such as habit formation, elasticities in the utility function and adjustment costs on investment are close to those found in Smets and Wouters (2007). In addition, the elasticity of emissions with respect to output is estimated to be 0.79. This value lies in the interval (0.69–0.86) obtained by Heutel (2012) from regressions of the log of emissions

[11]Note that $\theta_{1,t}$ is kept fix at the estimation stage.

on the log of GDP with three different data treatments (ARIMA, seasonally adjusted, and HP filters).

TABLE 2. Estimated parameters

	PARAMETER	ESTIMATES
Panel A: Structural parameters		
Inv. of elasticity of substitution in consumption	σ	1.9881 *[0.00]*
Inv. of Frisch labor supply elasticity	ν	1.7199 *[0.00]*
Habit formation	φ	0.7083 *[0.00]*
Elasticity of emissions with respect to output	$1-\gamma_\mu$	0.7976 *[0.00]*
Steady-state abatement effort	$\bar{\mu}$	0.3031 *[0.00]*
Adjustement cost on investment	ψ	6.4157 *[0.00]*
Growth rate of the economy	γ_z	1.0013 *[0.00]*
Panel B: Shock processes		
AR(1) productivity	ρ_a	0.9898 *[0.00]*
AR(1) abatement cost	ρ_μ	0.9238 *[0.00]*
AR(1) permit supply	ρ_ϑ	0.9398 *[0.00]*
Std dev. productivity	σ_a	0.0013 *[0.00]*
Std dev. abatement	σ_μ	0.1286 *[0.00]*
Std dev. permit supply	σ_ϑ	0.0243 *[0.00]*
Log likelihood		-491.6508

Note: P-values are in brackets (null hypothesis of being equal to zero).

The abatement effort is estimated to be 0.30. Combined with the values of θ_1 and θ_2, this estimates leads to steady-state abatement costs that amount to 0.44% of GDP. Based on the values obtained for the other parameters, the scaling parameters χ and A are close to 0 and the carbon intensity η is 0.5149. Finally, we estimate the parameters pertaining to the dynamics of the three shocks introduced in the model (ϵ_a, ϵ_μ, ϵ_ϑ). As is usually found in estimated dynamic stochastic general equilibrium models, shocks are highly autocorrelated (above 0.9).

3.3. **Impulse response functions.** Figure 3 displays the responses of the main macroeconomic and environmental variables to the three shocks embedded in the model for *(i)* the baseline model with permit banking (plain blue line) and *(ii)* an alternative version without banking (dotted green line). The latter is a linear version of the baseline model without permit banking. It is obtained by eliminating the lower bound on $\lambda_{f_2,t}$ and setting $b_t = 0$, which leads to $e_t = \vartheta_t$ at all times (see Equation (10)). Hence, impulse responses are expected to differ when the baseline model enters its second regime in which permit banking arises.

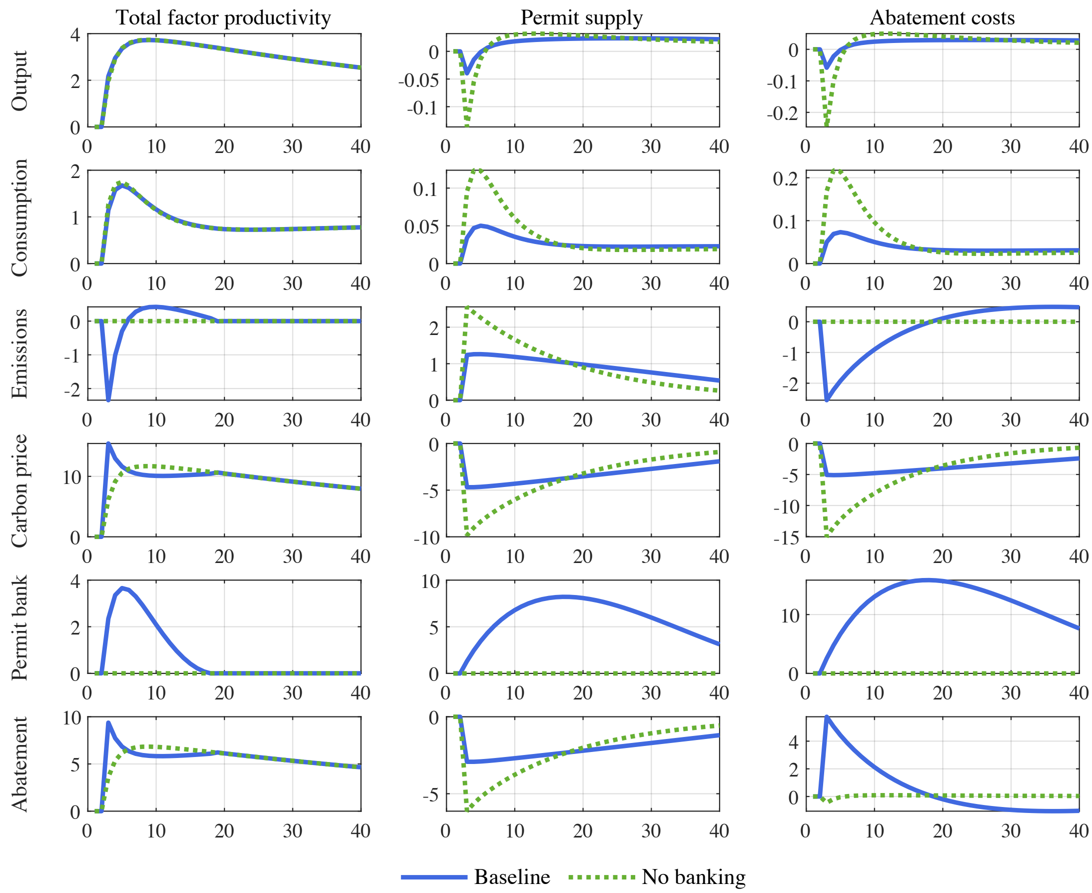
The first shock is a positive disturbance to total factor productivity (first column).[12] As emissions are a by-product of output, the shock automatically increases firms' pre-abatement pollution. This translates into increased demand for permits, a higher carbon (or equivalently permit) price, and a higher required abatement effort. Banking opportunities create an additional dynamics in the baseline model. The realization of the shock and gradual capital accumulation cause peak productivity to materialize only after a few periods. Meanwhile, forward-looking firms begin to build a bank of permits to be used during the most profitable times. This additional demand for permits in the short run raises the carbon price relatively more than in the model without banking. Abatement also increases relatively more during the early stages. Once peak productivity is reached, firms start depleting the bank which reduces their demand for newly issued permits. Thus, both the carbon price and abatement effort are lower than in the no-banking case until full depletion. Recall also that emissions are allowed to differ from the constant permit supply in our framework. They decrease early during the build-up of the bank and increase when the peak productivity is reached.

The second shock is a positive permit supply shock (second column). In both models, the shock increases carbon emissions and reduces carbon price and abatement. However, in the baseline model, the temporarily reduced carbon price creates an incentive for firms to store permits. For a few periods after the impact, the reduction in the permit price in the baseline model is less pronounced than that in the model without banking. Banking opportunities create an additional demand for permits in the short run. Abatement is also reduced less because of both the lower drop in carbon price and the incentive for firms to fill the bank. In the medium run, when firms start to use banked permits, the demand for newly issued permits declines and the price stays below the path obtained in the model without banking. After firms have finished filling their reserves and start depleting them, carbon emissions remain at a higher level than in the no banking case for many periods. At this point, firms need less abatement to comply with the policy.

The third shock is a negative shock to abatement costs (third column). The reduction in abatement costs implies that fewer resources must be devoted to abatement goods in the economy, leading to a decrease in output (cf. the equilibrium resource constraint given by Equation (22)). Indeed, at general equilibrium, the resources used to abate emissions are accounted for in production. Without banking, lower output for the same level of emissions implies less abatement. Hence, instead of allowing firms to produce more while conducting

[12]This shock is explicitly calibrated to have the constraint binding, to differentiate the dynamics of the two models.

FIGURE 3. Impulse response functions



Note: The figure displays the impulse response functions (IRFs) of several variables to three shocks: total factor productivity (Column 1), permit supply (Column 2), and abatement costs (Column 3). Each IRF is expressed in percentage deviations from the steady-state, except for the bank of permits and carbon price.

more abatement, reduced abatement costs allow them to produce less but more efficiently. This means that, while total production decreases, the production net of abatement costs increases, as does consumption. Following this shock, the carbon price decreases, driven by *(i)* the reduced cost of the substitute of permits for compliance with the policy, and *(ii)* the lower demand for permits due to reduced production.

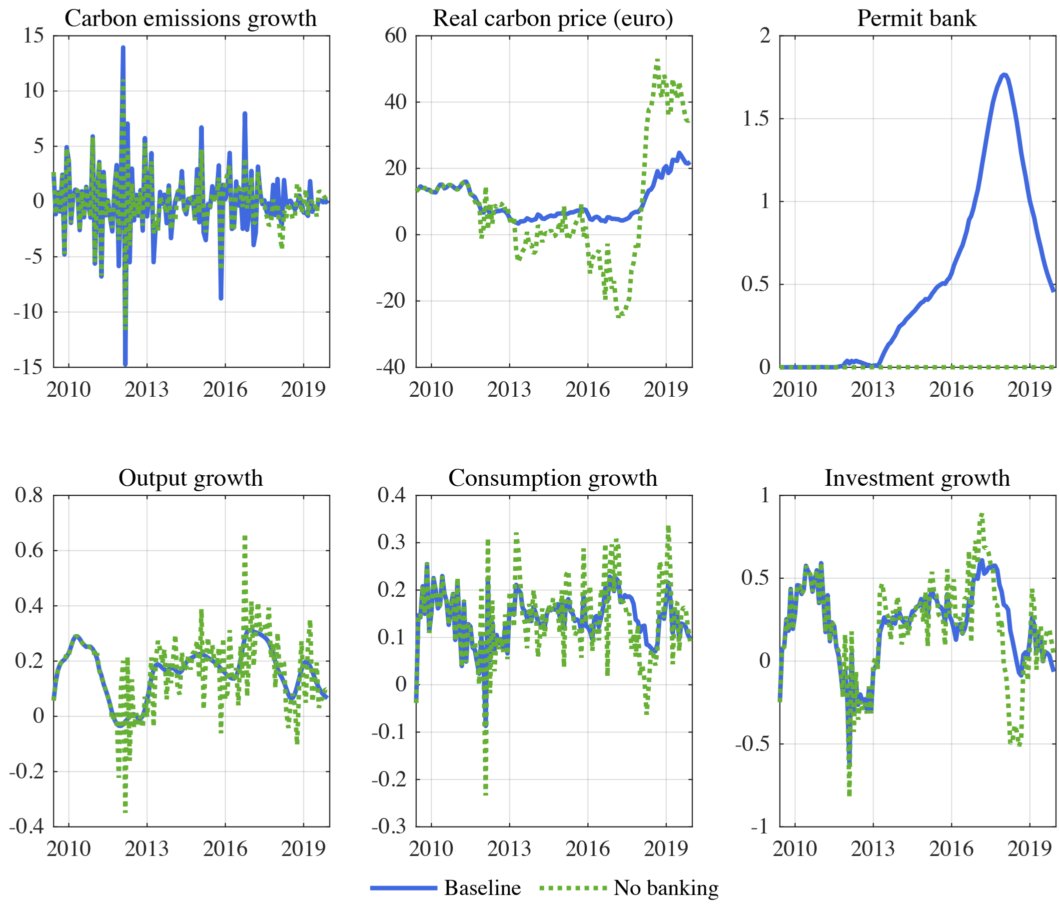
This shock is interesting because it puts forward a puzzle (we refer to it as the *abatement puzzle*) usually found in the literature, which can be solved by introducing a permit banking system. The standard environmental general equilibrium model unexpectedly predicts that a reduction in abatement costs yields a reduction (of low amplitude in our example) in abatement efforts. This counterintuitive outcome originates from the restriction that emissions are always equal to contemporaneous permit supply. Interestingly, the *abatement puzzle* is solved under permit banking because our model offers more intuitive abatement dynamics. While the general equilibrium effect described above is still at play, the banking channel modifies firms' behavior. Lower abatement costs incentivize companies to immediately increase their abatement efforts, leaving them with the option of doing less later when costs rise. The carbon price is driven down but to a lesser extent than in the no-banking case, due to the additional demand for banking. To make the best use of the shock, firms increase abatement and fill their banks with the saved permits. Later, when firms use banked permits, abatement decreases more than it does in the no-banking model. The path of output is, therefore, modified with a lower loss at the beginning and then higher afterwards, when fewer resources are needed to conduct abatement and buy permits. The emission dynamics is no longer constrained to be the same as permit supply dynamics. Following the shock, increased abatement leads to lower emissions. Later, both the decrease in abatement and use of stored permits induce higher emissions. Thus, our model can reproduce the capacity of firms to smooth emissions along the business cycle.

Overall, these impulse response functions illustrate the ability of the banking model to properly replicate the idea of Cronshaw and Kruse (1996): firms are willing to bank carbon permits when they expect that either the price will be higher later or that abatement will be costlier later compared to the current situation. The nonlinearity embedded in the model eliminates the restrictive assumption that emissions are equal to the contemporaneous permit supply at all times and adds more realism to the behavior of firms and the whole economy. Finally, the banking model can solve the *abatement puzzle* that typically appears in standard environmental models.

3.4. **The pitfall of assuming no permit banking.** In this section, we perform a counterfactual exercise to understand the importance of the nonlinearities generated by the intertemporal banking of permits. It consists of plugging the smoothed shocks obtained from the estimated baseline model into an alternative version without permit banking. In this alternative scenario, the parameter values are set to those estimated by the baseline model, which is assumed to be the true representation of the economy. Figure 4 displays the results of this exercise (conditional on the estimated sequence of shocks from the baseline model), and Figure 5 reports the simulated second-order moments for each model version.

First, a model without banking leads to higher volatility for all variables. This results in standard deviations of two of the three observable variables (output growth and real carbon

FIGURE 4. The counterfactual exercise

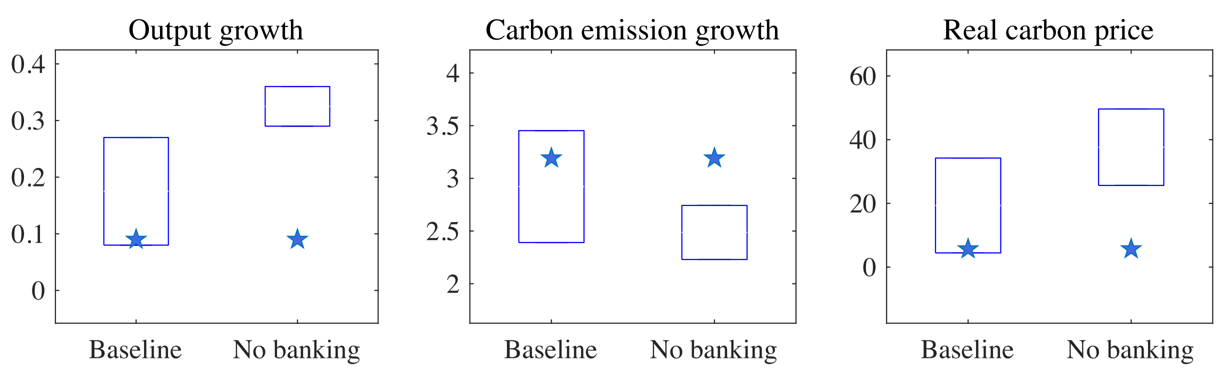


Note: The figure presents the trajectories of the main environmental and macroeconomic variables conditional on the estimated shocks, with and without (counterfactual) permit banking.

price) that are far beyond those of their empirical counterparts, while the third (carbon emission growth) is far below, as shown in Figure 5. Firms that are not allowed to store permits are unable to insure themselves against fluctuations in permit supply, abatement costs, and productivity. Emissions are always equal to the contemporaneous cap level, and the price of carbon is determined solely by the interaction between current permit supply and demand. Thus, any change in the cap level has an immediate and strong effect on the carbon price. This effect lasts only as long as the change in level does.

Similarly, any change in permit demand (e.g., through shocks to productivity or abatement costs) directly affects the carbon price. In a version of the model without banking, these effects cannot be mitigated neither by additional demand for permits that could be used in the future nor by the firm already having a reserve of paid-for permits. This leads to more volatile carbon-price dynamics. When confronted with smoothed shocks, the alternative model predicts that this price would be negative at several points. This would mean that the ETS subsidizes the pollution. Indeed, in these instances the demand for permits is so much lower than the supply that the regulator pays for firms to maintain emissions at the cap level. In our baseline model, firms can reduce their emissions below this level and bank a surplus of permits for later use. This generates additional demand and maintains a positive carbon price. When shocks increase the carbon price, the latter also reaches greater heights if firms are not allowed to use some banked permits. Increased carbon price volatility translates to increased macroeconomic volatility. Firms with no forward-looking abilities are highly dependent on the price they must pay to maintain their emissions at the level implied by the stringency of the current policy. Consequently, the volatility of output growth is predicted to be approximately four times higher than that found in the data (Figure 5). Likewise, consumption and investment are more volatile when permit banking is not accounted for.

FIGURE 5. Empirical and model-implied standard errors



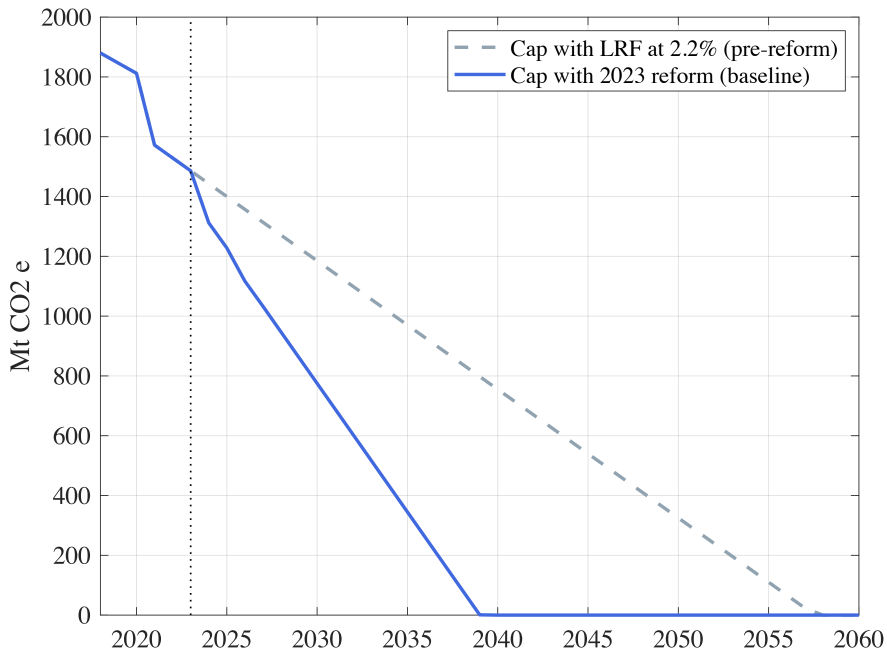
Note: The two models were simulated 300 times for 127 periods (same size as the data sample). The stars represent the values obtained from the data. The rectangles represent the range of values simulated from the baseline model.

This counterfactual exercise highlights that considering permit banking is essential for accurately examining the interaction between a cap policy and economic fluctuations at the business cycle frequency.

4. POLICY APPLICATIONS INSPIRED BY EU-ETS REGULATIONS

In this section, we use the estimated model to conduct policy exercises inspired by environmental regulations in the European Union's economy. Despite the apparent simplicity of the model, these practical policy exercises provide valuable insights into the interplay between carbon cap policies and permit banking choices, while simultaneously offering a comprehensive understanding of their economic impacts. Specifically, we simulate the recent resolutions of the European Parliament associated with the emission trading system, which aim to change the overall cap on emissions and to propose new rules for auctioning and distributing emission allowances.[13]

FIGURE 6. European-Union emission trading system cap



Note: The figure starts in 2018, when the cap was at 1,892 millions tonnes of CO2 equivalent (MtCO2e). A linear reduction factor of 1.74%, which translates into a year-on-year reduction of the cap by approximately 38 million allowances, is applied from 2018 to 2021. The cap is adjusted in 2021 to reflect the exit of the UK from the EU ETS. For the baseline scenario, we annually deduct 43 MtCO2e between 2021 and 2023 (LRF of 2.2%) and adjust this value to an LRF of 4.3% for 2024-2027 and of 4.4% from 2028 onward. In addition, a one-off reduction of 90 MtCO2e is applied in 2024, followed by a one-off reduction of 27 MtCO2e in 2026.

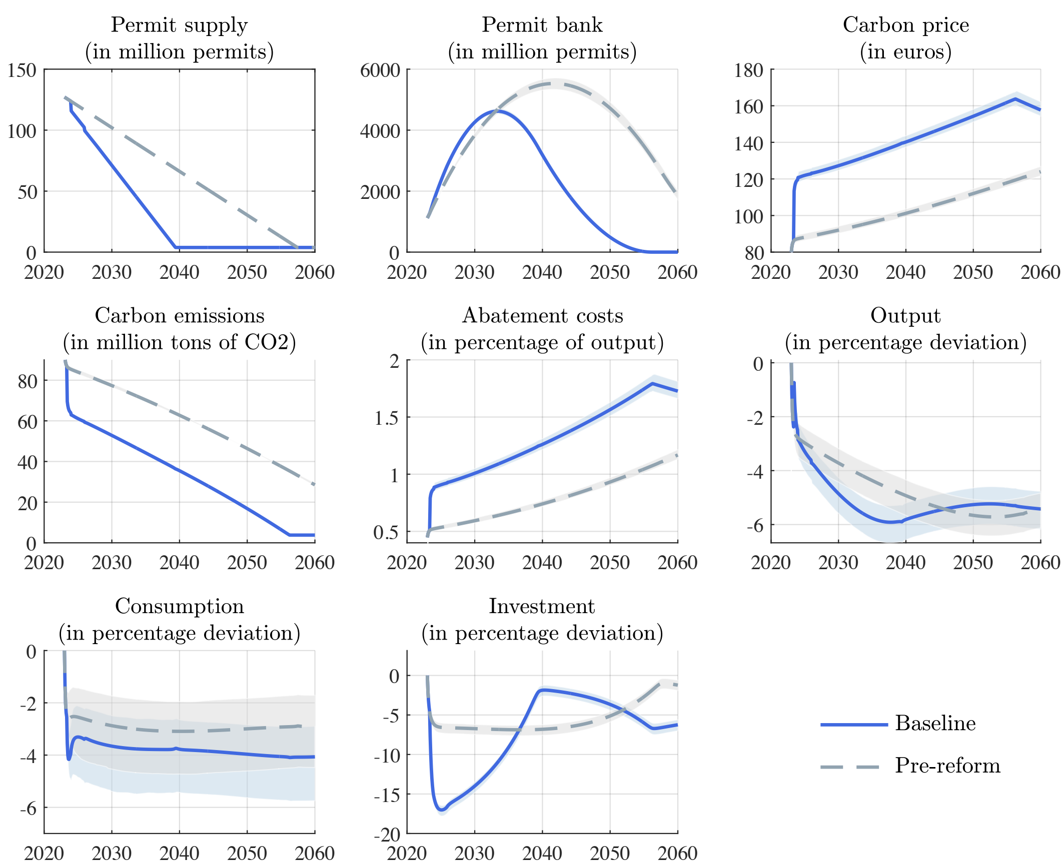
4.1. **Baseline scenario.** An emission trading system requires that the cap set on carbon emissions would diminish over time to respect pre-determined climate goals. After setting a linear reduction factor (LRF) on the cap for stationary installations at 1.74% during ETS Phase 3 (2013-2020), the European Parliament announced in 2018 that from Phase 4 (2021-2030) onwards, the LRF would increase to 2.2% (Directive EU 2018/410). This *pre-reform* path is represented by the dashed gray line in Figure 6. It was then realized that this LRF would not make it possible to reduce emissions by 55% by 2030 from 1990 levels (62% from 2005 levels), and thus would not be in line with the European Green Deal's emissions reduction targets.

[13]The European Parliament, along with the Council of the European Union, shares the responsibility for adopting EU legislation, including policies related to climate change mitigation. It reviews, amends, and votes on proposals put forth by the European Commission, which form the basis of EU climate policy.

Consequently, the regulators decided in May 2023, as part of the 'Fit for 55' package, to increase the LRF to 4.3% from 2024 to 2027 and to 4.4% from 2028 on (Directive (EU) 2023/959). It was further decided to apply two one-off cap reductions of 90 and 27 millions tons of CO2 equivalent (MtCO2e) in 2024 and 2026, respectively. These successive changes (present and future) constitute our *baseline scenario* for the 2023-2060 period (blue plain line in Figure 6).

To analyze the general equilibrium effects of such a decrease, we run perfect foresight simulations, starting from the union-wide cap for stationary installations fixed at 1,529 million allowances in 2022. This type of simulations captures the fact that regulatory changes are typically announced years in advance, leaving time for firms to adjust their expectations and behavior.

FIGURE 7. Projections under pre and post EU-ETS 2023 cap reform



Note: This figure displays the trajectories of the main environmental and macroeconomic variables under (i) the post EU-ETS 2023 cap reform (baseline scenario, plain blue line) and (ii) the pre-reform scenario (dashed gray line). Variables are expressed in monthly terms. Permit supply is assumed to be equal to the cap policy. The blue and gray areas represent the 90% confidence intervals of each scenario based on 300 draws in a Normal distribution of the parameter estimates.

The results are shown in Figure 7. Following the announcement of a tightening of emission targets, the carbon price rapidly increases from 80 to 120 euros, ultimately reaching 160 euros in 2060. This results in a price 38 euros higher than that obtained in the pre-reform scenario. In anticipation of this gradual increase, firms bank a portion of the permits they acquire until they are compelled to use them to maintain their production level, which occurs approximately in 2033 (10 years earlier than before the reform). Without the reform, firms would have also banked significantly more permits for an extended period. The revised targets necessitate increased abatement efforts from firms, resulting in a substantial increase in the total abatement costs. This phenomenon is reinforced early by firms' incentives to store permits. Following the announcement, there is an abatement cost differential of 0.4% of output, and nearly 1.2% in 2060 (relative to 2022). The latter figure is 0.53 percentage points higher than that which would be obtained under the pre-reform scenario. This results in an immediate 16% decrease in investment, whereas the reduction would be 6% without the reform. The effect takes approximately 15 years to diminish. After the reform, consumption also declines immediately and subsequently remains persistently 4% lower than its 2022 level. Consequently, total output (i.e., the sum of the demand component and all costs) experiences an average loss of approximately 5% by 2060.[14] This indicates that the reform incurs a mean relative cost that represents 0.4% of output, 0.9% of consumption, and 10% of investment. Furthermore, we observe that this scenario enables the European Union to achieve the desired reduction of more than 62% of the emissions from their 2005 level (2,369 MtCO2e) by 2030, without reaching net zero by 2050. The situation would have been clearly less favorable without the reform, as net zero would have been attained around 2070.

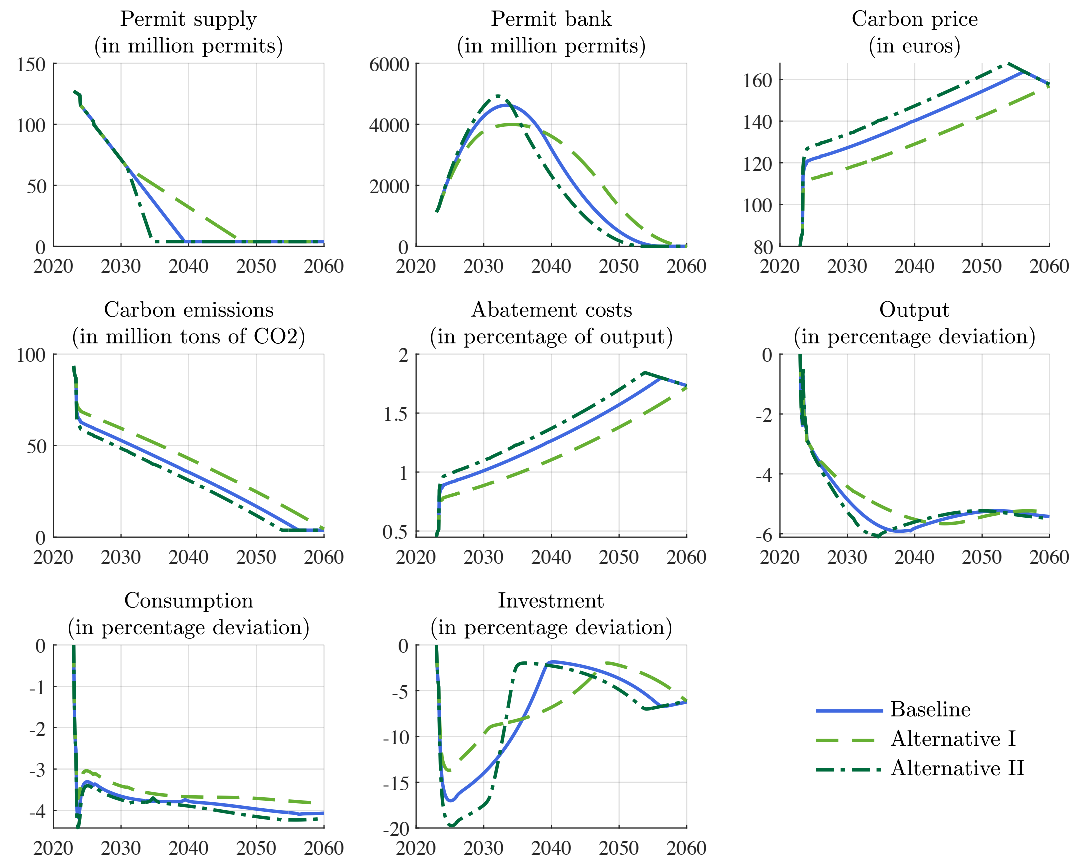
Result 1. *The baseline cap scenario results in (i) a more pronounced decline in permit banking subsequent to 2033, (ii) a 40% increase in the carbon price, and (iii) an additional average output loss of approximately 0.4% by 2060, relative to the projections prior to the 2023 cap reform.*

4.2. **What about after 2030?** As indicated above, the European Parliament has announced trajectories for what is called Phase 4 of the EU-ETS (2021-2030). However, at this stage, there is no indication of the characteristics of Phase 5 which will begin in 2031. Therefore, there is considerable uncertainty regarding the future trajectories of permit supply. With respect to the baseline characterized by a 4.4% LRF after 2031, we propose two credible alternative scenarios that differ after this date. In a first scenario called "Alternative I", we decrease the

[14] Note that the reported effects on GDP do not account for any productivity effects resulting from differences in pollution levels or co-pollutants.

LRF to 2.2% to return to a situation similar to that which prevailed in 2022. In the second scenario called "Alternative II", we increase the LRF to 10% to reach a cap of virtually 0 in 2035, to evaluate an early phase-out from carbon emissions.

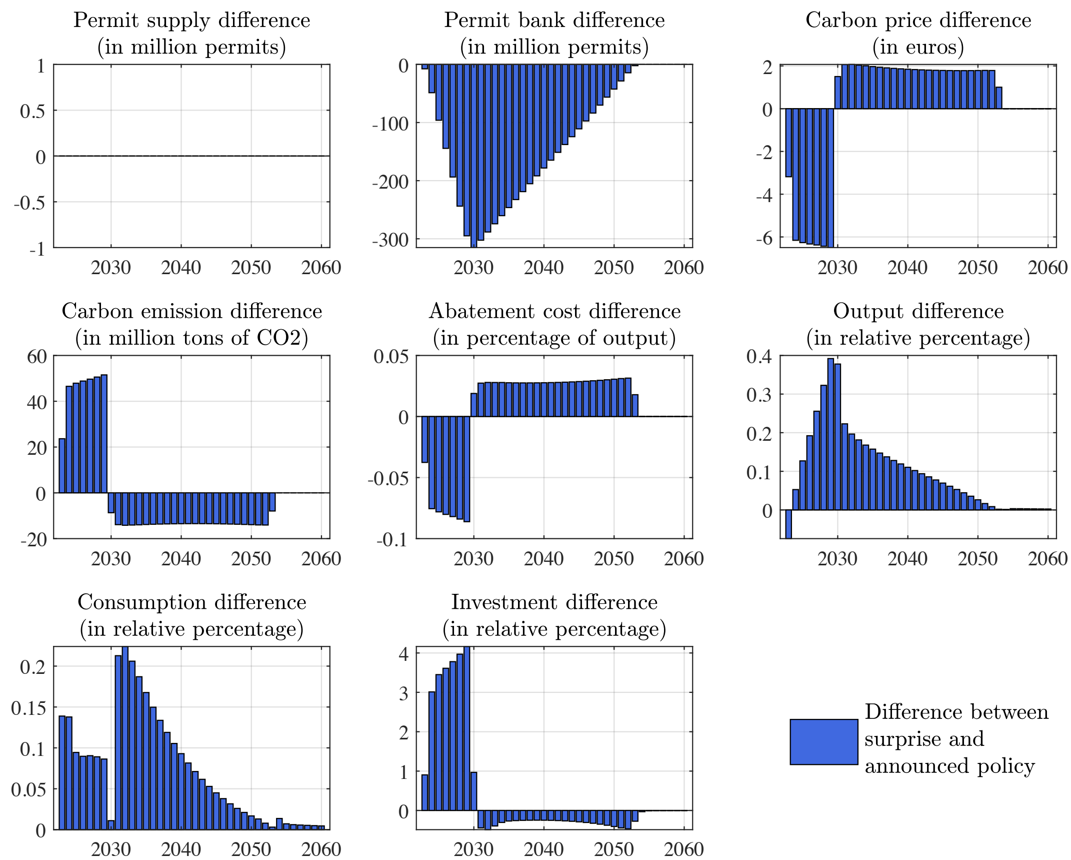
FIGURE 8. Alternative cap scenarios from 2031



Note: This figure displays the trajectories of the main environmental and macroeconomic variables by applying alternative cap scenarios (Alternative I sets a LFR at 2.2% after 2031 and Alternative II sets a LFR at 10% after 2031). Variables are expressed in monthly terms. Permit supply is assumed to be equal to the cap policy.

Figure 8 shows the results for the three credible scenarios. As expected, decreasing the LRF is less restrictive for firms that reduce their emissions less and store their permits longer (green dashed line). This results in a smaller drop in GDP, at the cost of greater accumulation of CO2 in the atmosphere. Conversely, increasing the LRF from 4.4% to 10% from 2031 allows the world to suffer fewer emissions, although not by a large amount compared with the baseline scenario, at the cost of a slightly greater GDP loss in the short term. Indeed, both consumption and investment would reach lower levels than those in the baseline scenario. In this context, firms can reduce their bank of permits more quickly and the carbon price is above that of the baseline case. Adapting their permit banking strategies allow firms to spread the greater (smaller) cost of the transition implied by the increased (decreased) LRF over time and not suffer (benefit) from it sharply only from 2031 onward. Note also that due to the presence of the permit bank, it is not because the permit supply is zero that the emissions are. In fact, stored permits lead to pollution in the future. It would take a much greater drop in supply to be close to net zero by 2050.

FIGURE 9. The effects of the timing of a policy announcement



Note: This figure displays the differences (expressed in level or relative percentage) between the trajectories of the main variables resulting from a surprise cap reduction policy and those resulting from an announced policy. Each blue bar represents the difference over the year. Permit supply is assumed to be equal to the cap policy.

Thus, we understand from these simulations that not only does the amount of an announced regulatory change matter when firms can bank carbon emission allowances, but also the timing of this announcement. Although changes are typically announced years in advance, they can also be rather short notice, as was the case with the 2024 cap decrease of 90 million allowances. To assess the role of expectations in shaping policy outcomes, we now compare the pre-announced "Alternative II" scenario with a surprise "Alternative II" scenario.[15] In the latter case, agents believe that they are on the track of the baseline scenario until 2030. At that point, they are surprised with an announcement informing them of the faster cap decrease to come from 2031 on. Figure 9 shows the differences between the two paths, taking the surprise scenario minus the announced scenario.

We see that the effects at the time of policy implementation are similar, regardless of how the agents learn about it (announced or surprised). However, significant differences are observed during the pre-implementation period. This means that surprising agents would entail gains in aggregate demand, mainly through investment, until 2029. These economic gains stem from inaction until the policy is implemented, allowing firms to act less against climate change. The accumulation of extra capital allowed by a lower carbon price, lower efforts to conduct abatement, and bank permits before the policy announcement translates into long-lasting, small gains in GDP and consumption. However, these gains must also be rebalanced against the possible climate-related costs from an increased stock of carbon as emissions continue to grow.

Thus, this exercise underlines the importance of an environmental policy announcement to act on the behavior of firms and thus reduce emissions as soon as it is known and not upon its actual implementation.

In summary:

Result 2. *Announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not at the time of its implementation.*

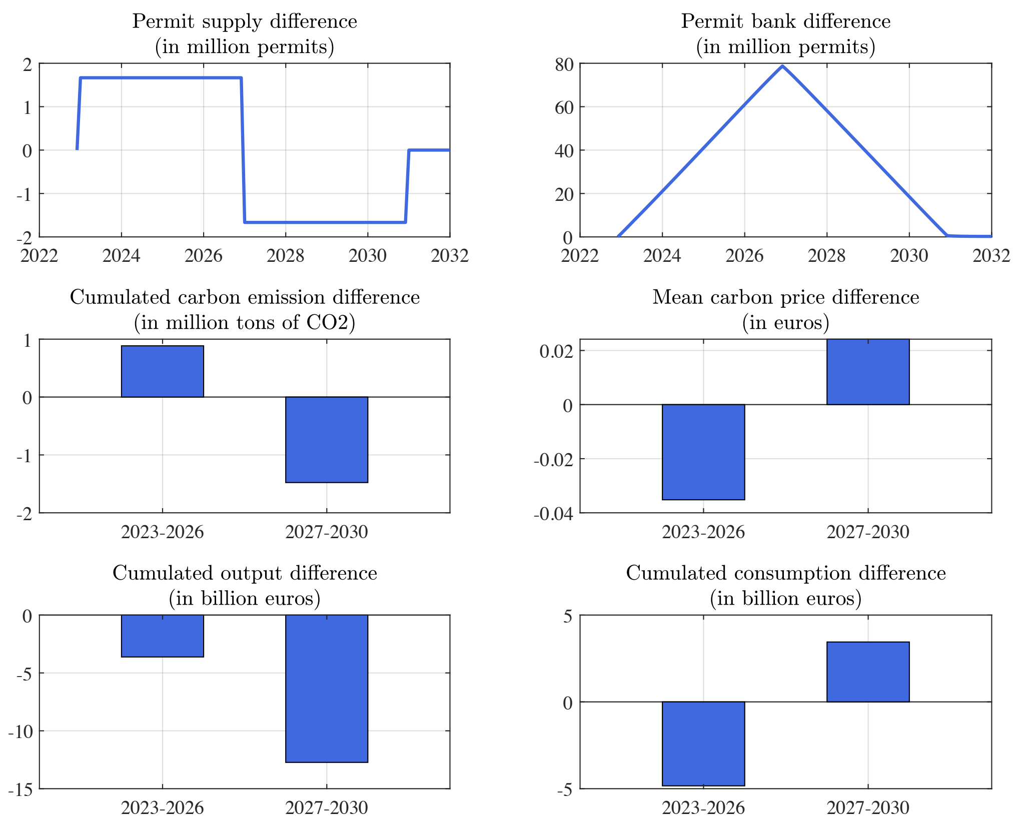
4.3. **Carbon permit supply frontloading.** On February 21, 2023, the European Parliament formally adopted an amendment (Regulation (EU) 2023/435) to include chapters of the European Commission's REPowerEU plan in the Recovery and Resilience Facility.[16] The purpose is to increase the resilience, security and sustainability of the Union's energy system through a decrease in the dependence on fossil fuels and the diversification of energy supplies. This initiative seeks to boost the roll-out of renewables by increasing the bloc's target from 40% to 45% of the total energy supply by 2030. One of the sources to support these measures is the Emission Trading System, with 20 billion euros coming from the auction of ETS allowances.

[15]We also conducted an analogous analysis while using the "Alternative I" scenario. Given that "Alternative I" represents a less stringent emission regulation announcement compared to "Alternative II," the observed effects are inverse to those depicted in Figure 9. However, the mechanisms at work remain the same.

[16]The aim of the Recovery and Resilience Facility is to mitigate the economic and social impact of the coronavirus pandemic and make European economies and societies more sustainable, resilient and better prepared for the challenges and opportunities of green and digital transitions.

Eight of the 20 billion will come from the frontloading of the allowances. Indeed, from 2023 to August 2026, a number of allowances from the quantity that would otherwise be auctioned from January 2027 to December 2030 will be auctioned until the revenue obtained reaches 8 billion euros. In principle, the allowances should be auctioned in equal annual volumes over the 2023-2026 period. To quantitatively assess the economic impact of such frontloading, we explicitly modify the baseline scenario of Subsection 5.1. Figure 10 displays the differences between the baseline scenario and the case with frontloading for the relevant variables.

FIGURE 10. The impacts of fontloading allowances



Note: This figure displays the differences due to a frontloading policy over its implementation phases for relevant variables (frontloading versus baseline scenarios).

Providing more permits at first naturally increases the level of emissions by 0.9 MtCO2e over the period 2023-2026 and then removing it leads to a drop of 1.5 MtCO2e over the 2027-2030 period. We might think that there is a total gain carbon emissions but we remember once again that what accumulates in the atmosphere cannot be removed. In other words, the asymmetric nature of the stock of atmospheric pollution implies that (i) emitting emissions fuels it with certainty, but (ii) a reduction in emissions does not automatically reduce it.

Moreover, the "saved" 0.6 MtCO2e will be emitted anyway after 2030 because the total permit supply remains unchanged. Given the temporary nature of this fontloading, firms increase their bank of permits during the first period to use it in the second period. The overall effect on the carbon price is slightly negative. The price drops when the permit supply increases and vice versa. However the magnitude of these movements is limited because banking opportunities act as stabilizers. The macroeconomic effects are generally negative. Despite a higher permit supply, consumption falls during the first period. This is because firms devote resources to buying extra permits without using all of them to immediately support production. Later, when firms use permits they have already paid for and saved in the bank, consumption is allowed to increase. The net effect on consumption during the two periods is slightly negative. GDP falls in both periods and more sharply in the second period due to a drastic reduction in investment.

Result 3. *Frontloading permit allowances results in (i) a net drop in emissions but after an increase in the stock of atmospheric pollution, (ii) a net negative effect on the carbon price and (iii) a reduction in GDP over both periods of frontloading and withdrawal.*

4.4. **The market stability reserve.** In 2014, amid the built-up of a surplus of allowances in circulation that started in 2009 (see Figure 1), the European Commission postponed the auction of some allowances. The surplus, or bank, amounted to over 2 billion allowances at the start of Phase 3 of the EU-ETS. It is, in part, owed to the financial crisis and remains substantial until this day. A large surplus threatens the functioning of the ETS in several ways. This reduces the short-term demand for newly issued allowances, thus reducing the carbon price and incentives for firms to engage in a green transition. In the long term, this can also affect the ability of the ETS to meet more demanding emission reduction targets cost-effectively (see the European Commission's dedicated webpage). Concerns that the surplus would remain over 2 billion allowances for a decade or more, despite the increase in the LRF, urged the Commission to react. In fact, the bank of allowances was still at almost 1.5 billion in 2021, and our previous subsections predict that, in the absence of an adjustment mechanism, it could eventually amount to over 4 billion.

In an attempt to tackle structural supply-demand imbalances, the European Parliament postponed the auction of 900 million allowances over the 2014-2016 period. This was meant as a short-term solution as Decision (EU) 2015/1814 introduced the market stability reserve (MSR) to be implemented at the beginning of 2019. It functions by triggering adjustments to annual auction volumes if the requirements based on the level of the aggregate bank of allowances are met. For this purpose, the European Commission has begun publishing the total number of allowances in circulation (TNAC) annually. When TNAC exceeds a certain threshold, the quantity of allowances that should have been auctioned during the next 12 months, calculated as a percentage of TNAC, is instead placed in the reserve. By contrast, when TNAC is below a certain threshold, allowances are released from the reserve and auctioned off. Any allowances placed in the MSR above a certain threshold are cancelled. The 900 million allowances postponed in 2014-2016 were placed in the reserve instead of being auctioned in 2019-2020, as initially planned. In its most recent version, after amendments announced in Directive (EU) 2018/410 and Directive (EU) 2023/959 of the European Parliament, the MSR works in the following manner. If TNAC is between 833 million and 1,096 million, the difference between TNAC and 833 million is transferred to the reserve. If TNAC is above 1,096 million, the number of allowances to be placed in the reserve amounts to 24% of TNAC. This percentage should go (back) down to 12% after 2030. If TNAC is less than 400 million, 100 million allowances should be released from the reserve and auctioned off (if there are less than 100 million allowances in the reserve, they should all be released). In addition, any allowances held in the reserve above 400 million are cancelled.[17]

In our framework, TNAC is represented by the firms' bank of permits. To incorporate the adjustments in the supply of permits due to the MSR in our simulations, we modify the aggregated version of Equation (21) accordingly:

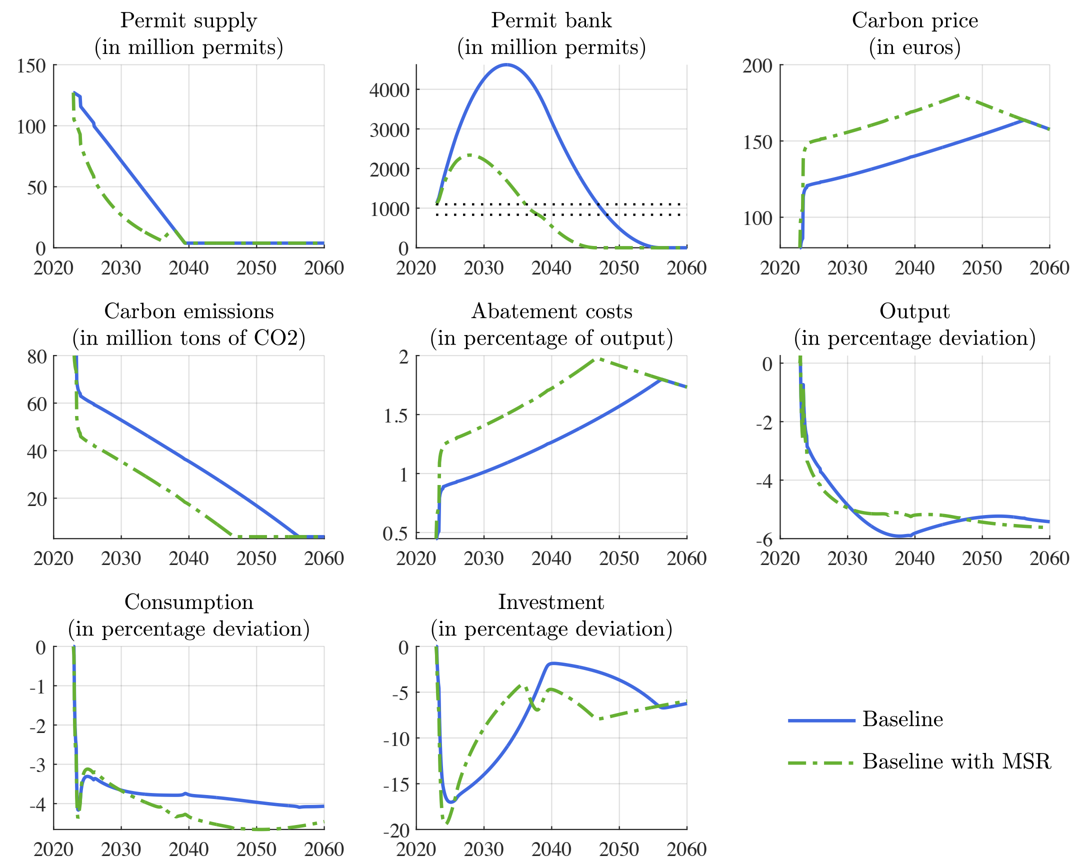
$$\vartheta_t = \bar{\vartheta}\varepsilon_{\vartheta,t} - \mathbb{1}_{\{(b_t > \underline{b}) \cap (b_t < \bar{b})\}} \frac{b_t - \underline{b}}{12} - \mathbb{1}_{\{b_t > \bar{b}\}} \tau \frac{b_t}{12} \tag{23}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function, $\underline{b} = 833$ and $\bar{b} = 1096$ are the first and second thresholds on the bank (in million allowances), respectively; and $\tau = 24\%$ is the percentage of allowances in the bank removed from the supply above the second threshold. We divide both terms related to the adjustments by 12, because our model is at a monthly frequency. Note that for computational reasons, we do not consider allowances released from the MSR when TNAC is below 400 million. However, there is little quantitative difference because in our perfect foresight setup, once the reserve starts to be depleted, it is not filled again. Hence,

[17] We also investigated an alternative approach involving the restriction of permit banking through a reduced intertemporal trading ratio (ITR). To this end, we expanded the law of motion of the bank of permits accordingly: $b_{j,t} = \delta_b b_{j,t-1} + \vartheta_{j,t} - e_{j,t}$, where $0 \leq \delta_b \leq 1$ is the intertemporal trading parameter. We calibated δ_b such that firms experience a 1% annual reduction in their banked permits (i.e., establishing a yearly intertemporal trading ratio of 1–to–0.99). Our findings revealed that the simulated paths under ITR are very close to those obtained with MSR. Nevertheless, despite the similarities between ITR and MSR outcomes, the latter affords regulators the benefit of adjusting permit quantities using new supply as a control variable, instead of depending on permits previously sold and purchased by firms.

introducing this feature would only increase the permit supply by 400 million allowances over the course of four years. Moreover, even with the MSR, TNAC is not expected to return to 400 million in the coming years, leaving time for changes in regulations. Not considering these aspects prevents us from having a state variable that tracks the number of permits in the reserve and requires fewer conditional statements.

FIGURE 11. Permit banking and the market stability reserve



Note: This figure displays the trajectories of the main environmental and macroeconomic variables under alternative cap scenarios. The two dotted horizontal lines correspond to the thresholds associated with the total number of allowances in circulation. Variables are expressed in monthly terms. Permit supply is assumed to be equal to the cap policy.

The results are shown in Figure 11. Introducing the MSR further reduces the supply of permits and increases the price of carbon, which is expected to reach 180 euros by 2046. Consequently, the MSR eats away the bank, and it takes approximately 14 years for banking to fall below the intake threshold $\underline{b}$, in line with Quemin and Trotignon (2021). Once both the new permit supply and the bank of permits reach zero, the carbon price and abatements costs peak before naturally decreasing due to progress in abatement technology efficiency. During the transition, GDP falls on average slightly less in the presence of the MSR than in the baseline case (+0.05pp), due to the faster recovery of investment but also an increased cost component. Consumption is initially slightly higher in the presence of the MSR, despite the decreased permit supply, and increased carbon price and abatement effort. This is because firms subjected to the MSR drastically reduce their bank intake rate from the start, directly using a larger proportion of permits at the time they buy them. By 2030, firms are already using stored (and hence paid) permits. However, by the time firms subjected to the baseline cap begin depleting their own permit banks, consumption in the MSR case falls gradually and durably below. By 2060, consumption has fallen by almost 5% in the MSR case versus 4% in the baseline case. The combination of the MSR and LRF trajectories announced in 2023 makes it possible to achieve net-zero emissions before the date planned by the Paris Agreement (i.e., at the end of 2046 instead of 2050).

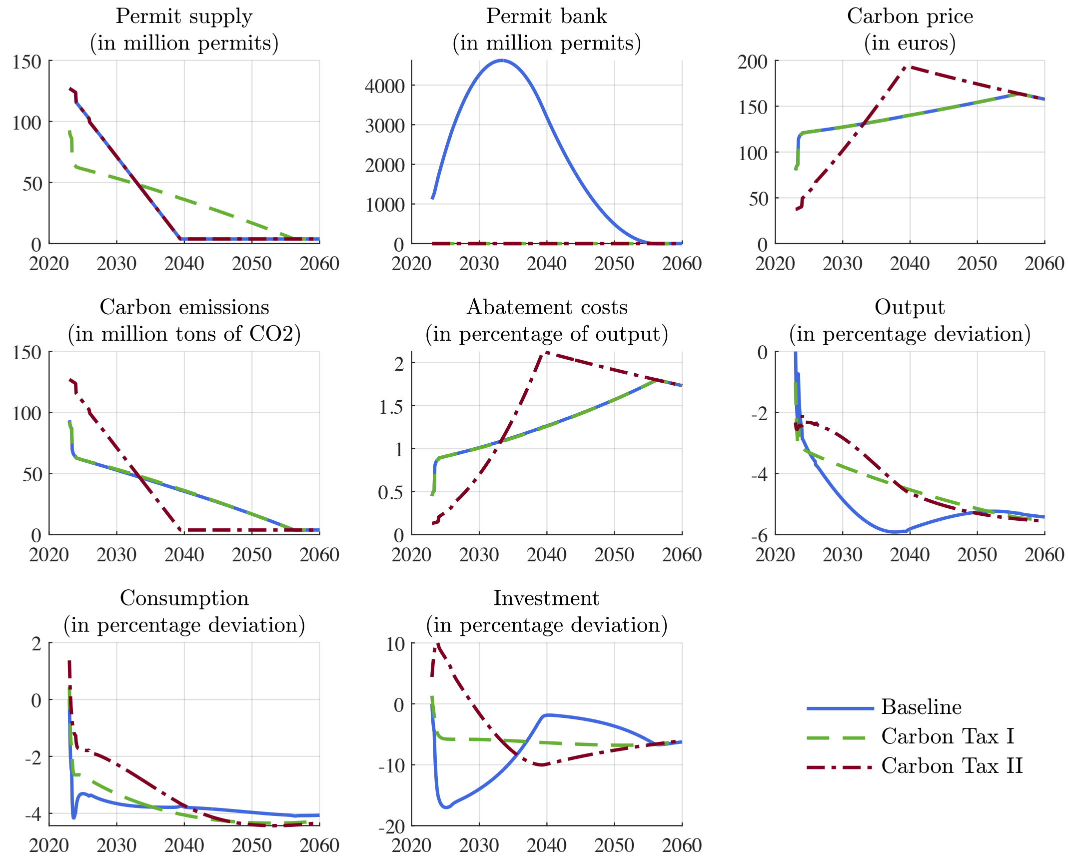
Result 4. *The market stability reserve is a powerful tool to slow down firms' banking of permits, and thus reduce emissions more quickly. This mechanism facilitates achievement of the objective of net-zero emissions prior to 2050 without incurring any additional costs in terms of GDP (relative to the baseline case).*

5. Cap policy versus carbon tax

This section emphasizes the importance of using an appropriate framework to implement the projection exercises. This implies accounting for the permit banking dimension when the current policy allows it. To do so, we compare our baseline cap decrease (i.e., with permit banking) developed in Section 4.1 to two different yet comparable tax scenarios.[18] In a scenario called "Carbon Tax I", we set a tax that replicates the carbon price obtained in our baseline cap scenario. In a scenario called "Carbon Tax II", we set a tax that results in firms choosing to emit as much carbon as they would receive permits in the baseline scenario. In a business cycle setup where a tax sets a fixed carbon price and a cap (without banking) sets a fixed level of emissions, the two have different properties (Fischer and Springborn, 2011). However, it is important to note that under certainty, for each increasing carbon tax, there is an equivalent decreasing cap (without banking), which returns to the result of Weitzman (1974). Carbon tax II can thus be thought of as setting the increasing tax described earlier or as setting a cap at the same level and decreasing at the same rate as in the baseline scenario, but without allowing permit banking. The results are shown in Figure 12.

[18] Comparisons of carbon regulation instruments in general equilibrium have thus far been limited to taxes versus intensity targets versus cap policies without banking (e.g., Fischer and Springborn, 2011, Annicchiarico and Di Dio, 2015, Annicchiarico and Diluiso, 2019). Furthermore, impact assessments of relevant policy actions typically focus on taxes (e.g., Finkelstein Shapiro and Metcalf, 2023).

FIGURE 12. Cap policy versus carbon tax



Note: This figure displays the trajectories of the main environmental and macroeconomic variables under alternative cap/carbon tax scenarios (in Carbon Tax I, a tax is set to replicate the carbon price obtained in the baseline cap scenario; in Carbon Tax II, a tax is set such that firms choose to emit as much carbon as they would receive permits in the baseline scenario). Variables are expressed in monthly terms. Permit supply is assumed to be equal to the cap policy.

The Carbon Tax I scenario yields results close to the baseline for the climate-related variables. In the latter case, the opportunity to store permits increases the demand for new permits early and lowers it after 2033 when firms use their stored permits. This extra demand in turns relatively increases the carbon price early, before decreasing it. This information is stored in the Carbon Tax I's price. Now that firms only pay for the carbon they emit immediately, they will first buy fewer permits than in the baseline case (and then more). In this way, they match the level of carbon emissions of firms subjected to the baseline cap at all times. The same carbon price implies the same abatement dynamics. Because firms buy fewer permits at the same price, Carbon Tax I leaves more room for early consumption and investment at the cost of a stronger future consumption loss. The absence of permit banking does not offer temporal flexibility for the private sector to adjust gradually to changes in the carbon price. Nevertheless, the early gains in investment facilitate the maintenance of reduced output losses throughout the simulation period.

In contrast, the Carbon Tax II scenario implies significant deviations from the baseline for all variables. Information on firms' forward-looking behavior disappears. Firms face a low carbon price early, and emit a large amount of carbon immediately. The carbon price gradually increases so that, at all times, firms emit as much as they would be offered permits under the baseline scenario (equivalently, the cap decreases at the same rate as in the baseline scenario, but does not allow for permit banking). This results in a significant frontloading of emissions, which are much higher during the first years of the simulation, but are already zero by 2040. Firm abatement follows the carbon price signal. A lower carbon price, lower abatement and higher emissions early entail a significant relative gain in consumption and investment until 2035. This gain is more important and lasts longer than under Carbon Tax I, particularly because of the investment amplification mechanism. Therefore, output losses are even smaller in this scenario. This means that projections that forget permit banking would largely underestimate the macroeconomic impacts of the implemented policies.

Two important results emerge from this exercise:

Result 5. *A policymaker can achieve the same emission reduction path as under cap policy by setting a carbon price that accounts for firms' forward-looking behavior implied by the ETS. This choice allows her to reduce GDP losses during the transition period.*

Result 6. *Forgetting permit banking leads to (i) a significant underestimation of the macroeconomic effects of policy tightening, and (ii) an incorrect carbon emission path. The latter misleadingly suggests that achieving net-zero emissions would occur by 2040.*

6. Conclusion

This study investigates the general equilibrium effects of permit banking during the transition to a low-carbon economy. We develop and estimate an E–DSGE model incorporating a stylized emission trading system in which firms can store permits but are not allowed to borrow them. We illustrate the properties of the model by *(i)* conducting policy exercises inspired by recent environmental regulations in the European Union's economy and *(ii)* examining the nonlinear dynamics between environmental and macroeconomic variables through projections up to 2060.

Implementing the EU-ETS 2023 cap reform in the model, defined as a new sequence of linear reduction factors of the cap for stationary installations (4.3% from 2024 to 2027 and 4.4%

from 2028 onward), leads to a more significant reduction in both permit banking and carbon emissions, as well as a 40% to 50% increase in the carbon price (depending on whether the market stability reserve is accounted for) compared to pre-reform projections, without substantial additional GDP loss by 2060. The market stability reserve, which triggers adjustments to annual auction volumes if the requirements based on the level of the aggregate bank of allowances are met, is a powerful tool that slows down firms' permit banking and reduces emissions more quickly. This approach facilitates the achievement of the objective of net zero emissions prior to 2050. Announcing a policy in advance allows agents to modify their behavior accordingly, thus reducing emissions from the day of the announcement and not at the time of its implementation. Importantly, forgetting permit banking when assessing cap policies would lead to a significant underestimation of total macroeconomic effects and an incorrect carbon emission path.

Our new estimated model contributes to the literature by quantifying the role of permit banking and its interaction with cap policies. Nevertheless, its structure can be extended in several dimensions, which represent interesting research avenues. For instance, it was assumed that the EU-ETS market was applied to all companies in the economy. However, it covers approximately 40% of total EU's greenhouse gas emissions and approximately 10,000 companies in the energy sector and manufacturing industry, as well as aircraft operators. Hence, it would be pertinent to categorize companies into two subsets: one subject to regulation and the other not, and examine its impact on GDP. A further extension would be to incorporate negative pollution externalities through the implementation of a damage function (e.g., Heutel, 2012; Golosov et al., 2014). In such a function, temperature (or the stock of carbon) is assumed to have a positive impact on damages, which quantifies the proportion of output or productivity lost due to climate change.

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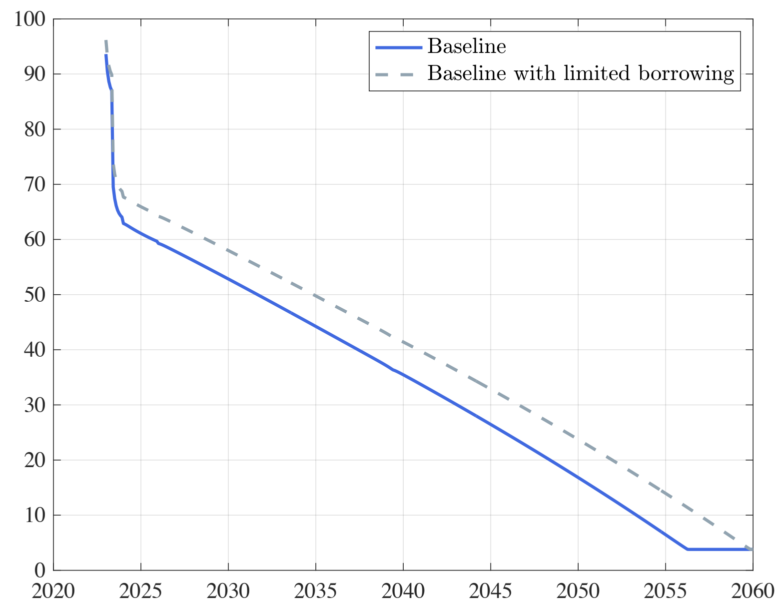
Appendix A. Authorizing a limited amount of permit borrowing

This appendix illustrates the impact on carbon emissions resulting from the authorization of a limited amount of permit borrowing. In the absence of borrowing restrictions, firms would attempt to borrow infinitely, potentially leading to a Ponzi scheme scenario. Therefore, we assume that firms can borrow up to a value $\underline{b}$ per period, such that Constraint 11 is expressed as $b_{j,t} \geq -\underline{b}$. Figure A.1 shows that borrowing postpones the emission reductions (by 4 years in this example where $\underline{b}$ is close to the monthly supply amount) necessary to achieve the net-zero objective. The larger the borrowing amount, the longer the delay is expected to be.

Figure A.1. Carbon emissions with limited borrowing (in million tons of CO2)



Note: This figure displays the trajectories of carbon emissions under (i) the post EU-ETS 2023 cap reform (baseline scenario, plain blue line) and (ii) the same scenario with a limited amount of permit borrowing (dashed gray line). The variable is expressed in monthly terms. Permit supply is assumed to be equal to the cap policy.

Appendix B. Solution method

This appendix provides details of the model resolution method in the presence of occasionally-binding constraints.

B.1. **Background.** The non-negativity constraint on the bank of permits introduces nonlinearity and creates de facto two regimes (see Equation (20)). Consequently, conventional linear methods that provide only a local approximation, cannot be used to solve the model. Thus, we rely on the piecewise linear perturbation approach proposed by Guerrieri and Iacoviello (2015), which is a variant of the extended perfect-foresight path method introduced by Fair and Taylor (1983). In a nutshell, the occasionally binding constraint can be handled as different regimes of the same model: under one regime, the occasionally binding constraint is slack, and under the other regime, the same constraint is binding. The model is first linearized around the non-stochastic steady state of one of the two regimes, chosen to be the "reference regime". This allows us to obtain a linear approximation of the decision rule under this regime. In our context, the reference regime is that in which $\lambda_{f_2,j,t} \geq 0$ in Equation (20). When the constraint is evaluated as binding, the model switches regime. A "guess and verify" method is then used to retrieve the decision rule and determine how long the constraint will bind. The starting guess of the expected durations is based on a linear solution that ignores the constraint. A Newton-like algorithm *(i)* iterates backward until convergence to the reference regime to form a decision and *(ii)* verifies that the resulting decision rule is consistent with the guess. If required, a new guess is formulated and the same procedure is applied.

If the left-hand side of Equation (20) is selected as the reference regime, then Equation (19) leads to $p_e = 0$ in the steady state. To avoid this situation, there should be no permit banking in the equilibrium. In addition, in the absence of shocks, the solution algorithm requires the model to converge back to the reference regime in finite time. This means that permit banking is transitory although it can last for multiple successive periods. It eventually fades if no more shocks occur.

Importantly, this numerical approach generates a nonlinear state-space representation. Indeed, the dynamics in one of the two regimes may crucially depend on how long one expects to remain in that regime. The expected duration of this regime depends on the state vector. This interaction results in a high degree of nonlinearity.

B.2. **Technicalities.** Let us stack the endogenous variables into one vector:

$$z_t = [c_t, y_t, x_t, k_t, n_t, r_{k,t}, w_t, q_t, \mu_t, b_t, p_{e,t}, \lambda_{h,t}, \lambda_{f_1,t}, \lambda_{f_2,t}, \varepsilon_{a,t}, \varepsilon_{\mu,t}, \varepsilon_{\vartheta,t}]', \tag{B.1}$$

and all structural shocks in $\epsilon_t = [\zeta_{a,t}, \zeta_{\mu,t}, \zeta_{\vartheta,t}]'$.

The regime-switching model reads as:

$$\mathbb{1}_{b_t=0}\mathrm{E}_t\{f\left(z_{t+1}, z_t, z_{t-1}, \epsilon_t\right)\} + \mathbb{1}_{b_t\geq 0}\mathrm{E}_t\{f^*\left(z_{t+1}, z_t, z_{t-1}, \epsilon_t\right)\} = 0, \tag{B.2}$$

where $f(\cdot)$ is the system of equations under the normal regime, when all banking has been exhausted, and $f^*(\cdot)$ is the alternative regime when there is positive banking.

Consider a first-order Taylor expansion around the normal regime model (with $\bar{z}$ satisfying $f(\bar{z}, \bar{z}, \bar{z}, 0) = 0$), with endogenous variables denoted as $\hat{z}_t = z_t - \bar{z}$. The Taylor expansion of each regime $f(\cdot)$ and $f^*(\cdot)$ yields :

$$F\mathrm{E}_t\{\hat{z}_{t+1}\} + G\hat{z}_t + H\hat{z}_{t-1} + L\epsilon_t = 0, \tag{B.3}$$

$$F^*\mathrm{E}_t\{\hat{z}_{t+1}\} + G^*\hat{z}_t + H^*\hat{z}_{t-1} + L^*\epsilon_t + \mho^* = 0, \tag{B.4}$$

where F, F^*, G, G^*, H, H^*, L, L^* are Jacobian matrices from each regime, while $\mho^*$ is a constant vector that accounts for the difference in the steady state across the two regimes (i.e., $f^*(\bar{z}, \bar{z}, \bar{z}, 0) \neq 0$).

The normal regime is assumed to be the baseline regime. The recursive solution of the problem around the normal regime is given by:

$$\hat{z}_t = P\hat{z}_{t-1} + Q\epsilon_t. \tag{B.5}$$

Note that if $\hat{b}_t$ is positive, one switches to the other regime. The tricky issue is how to deal with the expectation term $F^*\mathrm{E}_t\{\hat{z}_{t+1}\}$. Under rational expectations, agents take their decisions knowing how long the alternative regime will last, so how long $f^*(\cdot)$ applies. There is no closed-form expression to find the duration. The latter must be determined numerically by iterations.

The general formulation of the solution for any duration d in the alternative regime can be described by the following system:

$$\hat{z}_t = P(d)\,\hat{z}_{t-1} + Q(d)\,\epsilon_t + R(d)\,, \tag{B.6}$$

$$P(d) = [F^* P(d-1) + G^*]\, H^*, \tag{B.7}$$

$$Q(d) = [F^* P(d-1) + G^*]\, L^*, \tag{B.8}$$

$$R(d) = [F^* P(d-1) + G^*]\, \mho^*, \tag{B.9}$$

where $P(0) = P$, $Q(0) = Q$ and $R(0)$ is a vector of zeros.

Therefore, the solution of the model is state dependent. The duration of the alternative regime affects propagation and strongly enriches the dynamics of the model. However, a drawback is that the duration mus be guessed. Guerrieri and Iacoviello (2015) discuss one possible solution that they call the guess-and-try algorithm. Note also that the duration d

must be finite in order to solve the problem numerically, put differently dynamics of the model must go back to the normal regime.