WORKING paper

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Shadow Economy: What Factors Matter in the French Case? *

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ABSTRACT

We build a model based on a structural dynamic approach to assess the Non-Observed Economy (NOE) over the period 1990-2019 in France. Our strategy is focused on a systematic scan of the potential causes of shadow economy. We show that the discrepancy between electricity consumption and real GDP growth rates is the main driver of the NOE. However, factors, such as drug offences and net shipments of banknotes also have significant effects on hidden activities even though their effects do not seem to be as strong. The NOE remains non-negligible in France, but its ratio, relative to the GDP, has decreased considerably in the 2000s. Finally, we observe strong links between the NOE index and the cash demand indicators. Thus, concordance tests show a noticeable synchronization between the NOE indexes (global and legal components) and the net issuance of banknotes, especially the total net issuance and the net issuance of the €50 and €200 denominations. Furthermore, the NOE indexes and GDP as well as self-employment are synchronized. We also observe positive correlations between the cyclical components of the total net issuance of banknotes and the estimated shadow economy indexes. Finally, there are some bi-directional causal relationships between the NOE indexes and the aggregate banknote demand. However, there is only a unidirectional causality between these indexes and the demand for Small denominations (€5, €10, €20).

Keywords: Shadow Economy, Non-Observed Economy, Structural Equations, MIMIC Model

JEL classification: C32, C51, O17

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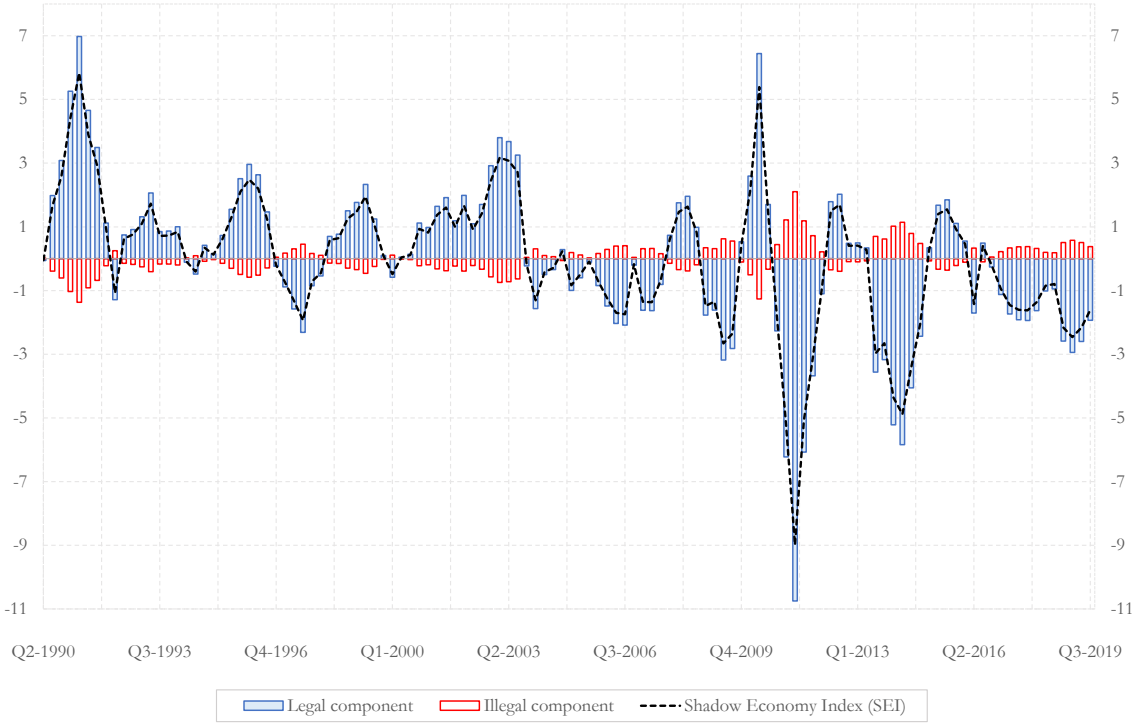
NON-TECHNICAL SUMMARY

The paper examine some solutions to the problem of the measurement of the Shadow Economy (SE) or Non-Observed Economy (NOE) in the case of the French economy. The SE (or NOE) includes all activities that are not declared to the regulators or to the public administration, both legal (mainly motivated by tax avoidance) or illegal (activities that do not comply with the law and may be subject to criminal convictions like drug offences, etc.). Non-monetary transactions are also part of SE (stolen goods, production of drugs for own use, or totally legal do-it-yourself activities etc.). The economists usually employ the label SE to define the unregistered activities that are close to the NOE.

The Benchmark SEI and its Legal and Illegal Components

(Growth rate in %; contributions in percentage points)



The authors employ a structural dynamic approach to assess the SE over the period 1990-2019 on a quarterly basis. More specifically, the Multiple Indicators Multiple Causes (MIMIC) model with a Kalman filter is used to build a Shadow Economy Index (SEI) in France (see Figure). The structural or state equation questions the dynamics of the NOE (latent variable) through the changes in its causes. The empirical strategy is backed by a systematic scan of the potential legal as well as illegal causes of the NOE which are either theoretically grounded or consistent with the economic intuition.

Despite its presence, the NOE's relative size compared to the overall GDP has decreased notably in France during the 2000s. The study finds that the primary evidence of the SE in France is the discrepancy between electricity consumption and real GDP growth rates even though the influence of its illegal components, such as drug offences or net shipments of banknotes also have, to a lesser extent, significant effects on shadow activities.

Finally, the paper shows the existence of strong links between the SEI and cash demand indicators. Thus, concordance tests show a noticeable synchronization between SEIs (global and legal components) and the net issuance of banknotes, especially the total net issuance and the net issuance of the €50 and €200 denominations. Furthermore, SEIs and GDP as well as self-employment are synchronized. There are positive correlations between the cyclical components of net banknote issuance and the estimated SEIs. Above all, there are some bi-directional causal relationships between all SEIs and aggregate banknote demand. However, regarding the different classes of denominations, there is only a unidirectional causality from these indexes to the demand for Small (€5, €10, €20) denominations.

L'économie souterraine : Quels en sont les moteurs en France ?

RÉSUMÉ

Nous utilisons une approche dynamique structurelle pour évaluer l'importance relative de l'économie souterraine en France sur la période 1990-2019. Notre stratégie est fondée sur une analyse systématique des sources potentielles, légales et illégales, de l'économie souterraine. Nous montrons que l'écart entre les taux de croissance de consommation d'électricité et du PIB réel est le principal moteur de l'économie souterraine. Cependant, les infractions liées à la drogue ou les envois nets de billets à l'étranger ont aussi des effets significatifs sur les activités souterraines, même si leurs effets sont de moindre ampleur. L'économie souterraine reste importante en France, mais, rapportée au PIB, son poids a considérablement baissé dans les années 2000. Enfin, on observe de fortes relations entre l'indicateur d'économie souterraine et les dénominations des billets. Ainsi, les tests de concordance montrent une synchronisation forte entre les indicateurs de l'économie souterraine (globale et légale) et les émissions nettes de billets (notamment les émissions nettes totales et les émissions nettes des coupures de 50 et 200 euros). En outre, les indicateurs de l'économie souterraine et de l'activité réelle (PIB réel, travailleurs indépendants) sont synchronisés. Nous observons également des corrélations positives entre les composantes cycliques des émissions nettes totales de billets de banque et des indicateurs estimés de l'économie souterraine. Par ailleurs, il existe des relations de causalité bidirectionnelles entre tous les indices de l'économie souterraine et la demande globale d'espèces. Toutefois, il n'existe qu'une causalité unidirectionnelle entre ces indices et la demande de petites dénominations (5, 10 et 20 euros).

Mots-clés : économie informelle, économie non observée, équations structurelles, modèle MIMIC

I. Introduction

The statisticians use the label "Non-Observed Economy (NOE)" to refer to "all productive activities that may not be captured in the basic data sources used for national accounts compilation" (Dell'Anno, 2016, 2021 a and b; Adair, 2020). The economists usually employ the label "Shadow Economy (SE)" to define the unregistered activities that are close to the NOE. Many other labels, recovering almost similar activities, are available in the literature: informal, hidden economy, black, gray, unrecorded, unregistered, unofficial, underground activities, cash economy and lack of economy (Hart, 2008; Trebicka, 2014; Hassan and Schneider, 2016; Elgin *et al.*, 2021). This wide variety of terms makes it difficult to identify, assess, and interpret the phenomenon, even though each of them has a specific meaning. For example, the cash economy refers to purchases and sales that are paid in cash without any transfers or interventions by banks. The labels "SE" and "NOE" are related to both legal and illegal activities that are not declared to the regulators or to the public administration. Moreover, NOE can be monetary (drug dealing or gambling, etc.) or non-monetary (stolen goods, production of drugs for own use, etc.). It does not comply with legal rules or with labor market and business codes. Here, the main lesson is a need to define exactly what the label "SE" means.

Due to our perspicuous will to build a bridge between the researchers and national account statisticians, our definition of shadow economy is closer to that based on the NOE (see Appendix A, Table A1). For the fluidity purpose of the presentation, in this paper, we have interchangeably used "SE" and "NOE" to designate the hidden and non-observed activities. In this way, the SE broadly corresponds with economic activities that would be taxed if they were reported to the tax authorities: unregistered activities, and the production of legal and illegal goods and services that avoid detection are included in this broad definition (Smith, 1994; Schneider and Enste, 2000 and 2002; Dell'Anno, 2021 a and b).

However, Ihrig and Moe (2004) and Schneider (2005) proposed a narrower definition of the SE as follows: "All market-based production of goods and services that are deliberately concealed from public administration" to avoid the payment of taxes or social security contributions or "to avoid meeting some legal requirements regarding the labor market or complying with certain administrative procedures". This definition only considers the legal part of the SE. Here, we introduce illegal activities in the assessment of the NOE when statistics are publicly available over a long period (Dreher and Schneider, 2006). This is one of our contributions to the research in this area where most empirical studies only include legal contributions in their measurement of the NOE.

Furthermore, Lippert and Walker (1997) set up a taxonomy of shadow activities, grouped into four main segments which correspond to the main types of the NOE that we consider in this paper (see Appendix A, Table A2, see also Schneider and Enste, 2000; Buehn and Schneider, 2012 and 2016; Schneider and Haigner, 2019; Dell'Anno, 2021 a and b).

Furthermore, many papers have investigated alternative identification methods and measurements of the NOE (Schneider, 2005; Schneider *et al.,* 2010; Buehn and Schneider, 2012; Ardizzi *et al.,* 2012; Seitz *et al.,*

2020; Medina and Schneider, 2019; Bartzsch *et al.,* 2019; Reimers *et al.,* 2021; Dell'Anno, 2021 a and b, and Elgin *et al.,* 2021).

Various bodies have also taken an increased interest in this field, notably the International Monetary Fund, IMF, the Organization for Economic Cooperation and Development, OECD, the European Central Bank, ECB, the International Labor Organization, ILO. For example, the OECD (2002) had a crucial contribution in the definition and the assessment of the NOE. The ILO (2021) provided us with the more recent study in which it formalized a harmonized single-definition of informality by introducing three concepts: Informal Production Activities (IPA), Informal Economy (IE), and Informal Market Economy (IME), (see Dell'Anno, 2021 for details)[1]. The ILO looks for reconciling the views of economists and statisticians on informality.

Several empirical papers have proposed assessments of the level of SE relative to the GDP especially in G7 countries (Dreher and Schneider, 2006; Medina and Schneider, 2019; Schneider and Haigner, 2019; Adair, 2020; Schneider, 2022). These are based on various approaches but the assumptions underlying these assessments are often questionable.

Four main reasons have prompted us to build synthetic indexes of NOE (benchmark, augmented, legal and illegal indexes) for the French economy:

1) Assessments of NOE on a panel of countries generally propose measures based on a basket of factors available for the countries and over a long period for most of these countries. The problems of availability of particular data in countries may lead to omit certain specificities of a part of the countries in the evaluation of the NOE;

2) Due to the efforts made in France to put in place the most relevant measures in order to prevent the NOE, it seems rational to construct a synthetic indicator of the SE in accordance with the new tools implemented in this area (see Box in Section II.2);

3) As we have collected information relating in particular to illegal activities, it seems discerning to use this additional information to build a more general indicator in line with a broad definition of the SE and close to that of NOE;

4) Finally, our indicator needs to meet specific requirements linked to the behavior of households and firms regarding means of payment (see Appendix A, Table A1).

In this paper, we have also four objectives that are not necessarily new but we have extended the area of the unregistered activities over a large sample of data and a long period. In addition, these goals are not

[1] According to ILO (2021), IPA is defined as "all productive activities carried out by persons and economic units that are –in law or practice- not recovered by formal arrangements; IME is associated with "all productive activities carried out by workers and economic units for pay or profit that are –in law or practice- not recovered by formal arrangements"; IE is defined as "all productive activities carried out by persons and economic units".

often compiled together in the case of the French economy. Our index can help to fix it. Our objectives are as follows:

(1) To provide relevant information over a recent period by using more factors –which are generally not mobilized in a panel study- regarding the main drivers or triggers of the NOE in France. It is in line with a "one-size-fits-all" vision (Dell'Anno, 2021);

(2) To build some synthetic indicators of NOE and to identify the links between these indicators and some macroeconomic aggregates;

(3) To check the effectiveness of the performance of our indicators by comparing them to those available, especially over the recent periods;

(4) To consider the dimension of the net circulation of banknotes to examine their potential causal links with SE.

The rest of the paper is structured as follows. Section 2 analyses some stylized facts regarding NOE, with a special focus on the French case. Section 3 presents a brief survey of recent literature in this research field, and examines the formal model implemented in this paper. Section 4 describes the data sources and provides us with information regarding the causes of the NOE. Section 5 is devoted to the empirical investigation that leads to the construction of synthetic indexes of SE. It also examines some alternative measurements of the NOE and compares our indexes with those existing in the empirical literature. Section 6 presents some statistical analyses of the links between the indexes and cash demand on one hand, and the indexes and real activities on the other hand. Section 7 concludes.

II. Stylized facts

II.1. General Overview

To benchmark the French case, we examine some available assessments of the ratios of SE relative to GDP for the OECD or Eurozone countries. These ratios depend on the countries, the methodologies and the historical data used, etc. However, their profiles are rather similar. For this reason, any of the available assessments can be employed for illustration purposes. Thus, in this section, we have used those carried out by Medina and Schneider (2019) (see also Schneider, 2005). We have split up the period under review into two phases (SP1 from 1991 to 2004; SP2 from 2005 to 2017), broadly corresponding to two regimes in the dynamic of SE in these countries.

Over SP1, the average ratio of SE in 21 OECD countries was greater than 15% of GDP. However, the picture is highly contrasted. Indeed, in the main Eurozone countries, the ratio of the NOE increased continuously but three main sub-groups could be isolated:

i) The heavy ratio sub-group in which SE represents is between 25 and 30% of GDP (Italy, Greece, Spain);

ii) The moderate ratio sub-group where the NOE is more or less equivalent to 20% of the GDP (Belgium, Portugal, etc.);

iii) The low ratio sub-group in which SE is significantly less than 20% of the GDP (Austria, France, Germany, Ireland, Netherlands, etc.). Over this period, the size of the SE in Germany was significantly greater than in France (on average, 14.1% versus 12.7%) while in Austria and Netherlands, it was smaller than in France.

These figures are also consistent with those previously obtained by Dell'Anno *et al.* (2007) for France, Spain and Greece (13.5%, 27.4% and 26.3% respectively over 1990-2002).

Regarding the main countries in the Anglosphere, there is a noticeable disparity in ratios:

a) The United States has the lowest ratio (less than 10% of GDP versus around, 12% on average for the other countries), followed by the United Kingdom and New Zealand (around 11% at the end of the period) and by Australia and Canada (with ratios close to those observed in France, 14%);
b) For countries in this area, the main result is that the ratio of the NOE remains significant over SP1.

Over SP2, the main findings regarding the ratio of hidden economy to GDP are:

1) Overall, the ratios display a marked decrease in most countries;

2) Concerning the Eurozone, the three main sub-groups mentioned above remain relevant. However, Germany has moved considerably closer to Austria and Netherlands, with a ratio of 10% of GDP on average, while SE has slightly stabilized at on average 11.6% in France. Regarding the moderate and heavy ratios groups of countries in the Eurozone, we observe a drop in the ratio of between 1 and 2 percentage points after 2005;

3) For the Anglo-Saxon countries, from 2005 to 2017, the previous "hierarchy" remains relevant with a very remarkable performance in the United States where the NOE ratio reached less than 9% of GDP;

In conclusion, the NOE has still a significant impact in the OECD countries but the picture depends on the definition of SE.

II.2. What measures have been undertaken to limit the NOE activities in France?

We do not have a preconceived idea on the links between the NOE and the measures taken by the French authorities to thwart hidden activities. Nonetheless, it is relevant to assess the consistency of these measures. Thus, we assume that drivers of the frauds could be the triggers for the NOE.

French Public Authorities have launched initiatives and set up various bodies to thwart corruption, or financial and fiscal malfeasance (see Box below). These bodies monitor and control financial crimes throughout France. Their remit covers acts carried out in both the private and public sectors.

The examples of measures set up in the 2000s to tackle fraud, and financial and fiscal delinquencies include:

- The creation of the *Haute Autorité pour la Transparence de la Vie Publique* (HATVP, October 2013) and the *Office Central de Lutte contre la Corruption et les Infractions Financières et Fiscales* (OCLCIFF). HATVP collects and checks declarations of interest and assets from elected officials and senior officials, while OCLCIFP launches legal investigations into tax fraud and breach of probity. OCLCIFP contains the *Brigade Nationale de Répression de la Délinquance Fiscale* (BNRDF);
- The establishment of the *Parquet National Financier* (PNF, December 2013). PNF is responsible for tracking down tax fraud and serious financial crimes. Its jurisdiction extends to the entire national territory but is limited to the most serious economic and financial offences;
- The Sapin 2 law and the creation of the *Agence Française Anticorruption* (AFA, December 2016). The Sapin 2 law is dedicated to transparency, the fight against corruption and the promotion of economic modernization. It applies to French companies with more than 500 employees and a turnover of more than €100 million, as well as to all companies meeting these criteria that belong to a group whose parent company has its head office in France[2]. AFA performs advisory, assistance and control missions; it also centralizes, disseminates information to help prevent and detect acts of corruption, influence peddling, misappropriation, illegal taking of interest, embezzlement of public funds, favoritism, etc.

The creation and independence of these organizations have been widely welcomed. However, some reservations have been voiced about their effectiveness, mainly due to a lack of resources. Thus, the *Cour des Comptes* (2019) underlined weaknesses in their organization and use in their human and financial resources.

More specifically, the *Cour des Comptes* strengthened the needs to:

- Improve the definition of priorities and consultations between the *Ministère de l'Intérieur* and the *Ministère de la Justice*. The shortening of procedural delays requires a more precise definition of the investigative acts to be carried out;
- Clarify the split of responsibilities between specialized and versatile services or jurisdictions. Prosecutors should have better visibility over the availability of investigative services. Thus, the priority must be to use and coordinate the existing means better;
- Adjust the human resources policy of the *Ministère de l'Intérieur* and the *Ministère de la Justice* to meet the needs for specialized skills.

Furthermore, the French Government, the Parliament and INSEE are in the process of completing the arsenal of tools for measuring various criminal acts (fraudulent debits on bank accounts, etc.). In addition, significant efforts have been made at the global level. Thus, in 1989, the G7 countries created the Financial

[2] Measure and arrangements to comply with the Sapin 2 law under the due diligence obligation:
1) To establish and adopt a code of conduct; 2) To set up an internal ethics alert system; 3) To map your risks; 4) To establish procedures for evaluating business partners; 5) To fix accounting control procedures; 6) To deploy an internal training and awareness-raising program for teams; 7) To impose a disciplinary regime; 8) To monitor and evaluate the implementation of measures.

Action Task Force which aims to develop and promote appropriate actions for coping with financial offences, especially money laundering.

The links between criminal acts and a part of the NOE are obvious. As the procedures regarding illegal activities are progressing, re-examining the different indicators of the NOE by introducing its illegal causes into the framework is relevant for economic policies.

Finally, the Digital Shadow Economy (DSE) has appeared with the growing use of big data, machine learning and new means of payments in economic analyses (Gaspariene *et al.*, 2016). Indeed, cybercrime would be one of the three greatest scourges in the world: in 2018, the global cost of cyberattacks was $600 billion; the average cost per company was €8.6 million in France (Sénat, 2021). The DSE should have been included in our analysis, but we do not have data on the DSE over a long period.

Box: Inventory of some crime measures: a permanent evolution

- **Key milestones**

INSEE has drawn up an inventory of the evolution of crime measurement over time in France. Here is a summary of the main dates that marked the evolution of the measurement of the number of crimes:

- 1972: Creation of a statistical tool measuring crime through the forces of law and order reports;
- 1985: First national victim study to complete crime measurements;
- 2007: Systematization of annual victim studies;
- 2012: Progressive transition from global indexes to disaggregated measures of crime;
- 2015: Systematization of the disaggregated measurement of crimes through several indicators after the creation of the *Service Statistique Ministériel de la Sécurité Intérieure* (SSMSI) in 2014;
- 2016: First annual statistical report provided by SSMSI and INSEE with harmonized crime data;
- 2018: Geolocation of crimes in order to create a territorial analysis for crime statistics.

- **Other control and monitoring bodies**

We have selected a number of institutions set up at various periods to illustrate the efforts made by France to fight against financial delinquency and embezzlement.

- 1990: Central Office for the Suppression of Major Financial Crime (*Office Central pour la Répression de la Grande Délinquance Financière*, OCRGDF);
- 2006: Central Office for the Fight against Organized Crime (*Office Central de Lutte contre le Crime Organisé*, OCLCO);
- 2011 : National Division of Financial and Tax Investigations (*Division Nationale d'Investigations Financière et Fiscales*, DNIFF);
- 2013 : - High Authority for Transparency in Public Life (*Haute Autorité pour la Transparence de la Vie Publique,* HATVP);
  - National Financial Prosecutor's Office (*Parquet National Financier, PNF*);
- 2014 : Cybercrime sub-directorate (*Sous-direction de lutte contre la cybercriminalité*);
- 2016 : French Anti-Corruption Agency (*Agence Française Anticorruption, AFA*).

III. A short review of the main approaches and driving forces of the NOE

As highlighted by Dell'Anno (2021), there are three interconnected issues in the literature on unregistered activities: a) a definitional issue that consists of providing us with the appropriate definition of the NOE; b) a measurement issue, which allows estimating the size of the NOE; c) a theoretical explanation issue that gives us theoretical foundation of the NOE. The last issue provides us with the theoretical bases regarding the causes of the NOE; the explanation about its persistence or the existence of the potential links between the NOE and the recorded activities. We focus on the measurement issue that leads to an empirical contribution. We do not delve into the theoretical approaches but we lean on the existing models to build our framework. There is not a universal method for evaluating the NOE (Adam and Ginsburgh, 1985; Breusch, 2005; Schneider and Haigner, 2019; Dell'Anno, 2007, 2016, 2021 a and b; and Schneider, 2022).

Dell'Anno (2021 a and b) has provided us with a detailed review on the conceptual methods of NOE by distinguishing three baseline approaches: the neoclassical approach, the macro econometric approach and the conceptual approaches. In the neoclassical approach -that includes micro and macro-economic theories, there are rational agents who have decided to work in the unregistered economic activities to maximize their objective function. In the macro econometric approach that fundamentally focused on econometric analyses, the unregistered or hidden activities are defined in terms of value added (as a percentage of GDP). Finally, the conceptual approach is theoretical and rather multidisciplinary. It also deeply scrutinizes the links between formal and informal sectors. Our paper is related to the macro econometric approach in which Dell'Anno distinguishes six main categories of drivers: taxation system; regulatory system; labor force composition; enforcement system; tax morale and institution (Schneider and Enste, 2000). These categories are consistent with our set of causes (see Appendix D for expected signs for these causes).

Besides, Medina and Schneider (2019) and Dell'Anno (2021 a and b) (see also, de Paula and Scheinkman, 2011, Orsi *et al.,* 2014) have introduced SE into their theoretical macroeconomic models. In addition, the findings by Schneider (2004) and ILO (2013 and 2018) showed the existence of a positive relationship between the NOE and registered activities in the neoclassical approach. In contrast, other papers have shown that there is a negative correlation between these variables. Thus, on both theoretical and empirical bases, the effects of the SE on registered activities is ambiguous. Moreover, it is not clear that a bi-directional causality exists between the NOE and the official GDP. Giles *et al.* (2002) found significant evidence of Granger causality between GDP and the SE for Canada, but the inverse relation seems less robust. However, Breusch (2005) criticized their approaches and results.

Other categories of papers have measured NOE using structural micro–grounded models (for instance, the dynamic stochastic general equilibrium models, Orsi *et al.*, 2014) to derive the equilibrium conditions to evaluate unobservable variables. Due to its complexity, these models remain in the minority in the field of the NOE's studies. Furthermore, approaches based on micro data or firms and households' surveys have been also implemented (Putniņš and Sauka, 2015; Reilly and Kristic, 2019; Schneider and Haigner, 2019; Elgin *et al.*, 2021). They are promising because they propose a micro-foundation to behaviors of agents.

They could also provide us with initial values for calibrating the parameters of micro-founded models. Unfortunately, data corresponding to these methods are not necessarily available over a large sample.

III.1. Methods for evaluating the NOE: A synthesis

In the macro-econometric perspective, there are four main approaches to assess the NOE indexes (Schneider and Enste, 2000; Medina and Schneider, 2019; Adair, 2020; Dell'Anno, 2021 a and b):

i) The direct approach (DA) which consists of using some specific data and surveys (tax audit, labor force and consumption surveys) to investigate SE (Adair, 2020). Its main drawback is the possible existence of a bias linked to the choice of the indicator (representativeness, relevance of the survey, etc.);

ii) The indirect approach (IA) provides us with a proxy of the NOE (unemployment rate, self-employment, electricity consumption and currency demand, etc.). For example, the physical input method (Kaufmann and Kaliberda, 1996; Lackó, 2000) suggested that electric consumption is the best physical indicator of both official and shadow economic activities. The difference between the growth rates of electricity consumption and real registered GDP determines the variations in the SE production. Nevertheless, some hidden activities may not require a huge consumption of electricity. Therefore, electricity consumption could exhibit a limited explanatory power as far as SE is concerned. These two examples show that the indirect approaches only focus on a partial view of SE;

iii) In the Currency Demand Approach (CDA), the NOE results from the estimation of a money demand function (Cagan, 1958; Ahumada *et al.*, 2009). In this approach, it is also possible to test the suitability of a wide range of indicators of hidden activities and to distinguish the effects of legal and illegal components of SE (Breusch, 2005; Ardizzi *et al.*, 2012; Bartzsch *et al.*, 2019). However, Breusch (2005) showed that the estimated parameters drawn from this approach are unstable. Furthermore, the CDA assumes that transactions related to the SE are mostly paid for in cash and the velocity of circulation is assumed to be the same as regards to both legal and illegal transactions. This strong hypothesis can lead to an underestimation of SE;

iv) The Multiple Indicators Multiple Causes (MIMIC) model allows deriving (from covariance structures) the relationship between the NOE, a latent variable, and its causes (Frey and Weck-Hanneman, 1984). The MIMIC is related to the *Linear Structural Relation,* LISREL, described by structural and measurement equations (Cziraky, 2004). The MIMIC model is more general than the previous ones and probably supplements them.

Schneider (2005) proposed a MIMIC model in first differences, the dynamic MIMIC, DYMIMIC, which allows for: a) Estimating the long- and short-run dynamics of the variables under review; b) Avoiding spurious regressions. In addition, Medina and Schneider (2019) propounded a hybrid method combining CDA and MIMIC approaches to estimate a shadow economy index ("SEI"). For instance, Medina and

Schneider (2019) suggested implementing the Rubin's predictive means matching method to overcome the calibration problem (Rubin, 2004). However, this approach is not fairly validated in the applied studies. For this reason, we favor a more pragmatic approach consisting of imposing then relaxing constraints on some parameters and running the regressions many times to stabilize the results. Helberger and Knepel (1988) highlighted the instability of the results of MIMIC models when minor changes affect the data or the period of estimation. The set of the indicators describing the measurement equations could also be questionable. Sometimes, two additional drawbacks of the MIMIC approach are mentioned: i) Some variables are introduced as indicators as well as causes; ii) The way to calibrate MIMIC estimates is questionable. Therefore, the MIMIC approach could also lead to a discriminatory benchmark of SE in absolute terms (Jovanovic, 2015). To overcome these drawbacks, Jovanovic (2015) suggested implementing a Kalman filter to assess the latent variable (Aigner *et al.*, 1988).

The MIMIC approach and its extensions have substantial advantages relative to the first three approaches: It allows for many causes of the SE; It differentiates between causes and indicators related to the NOE; It relies on fewer assumptions and is probably more precise and less restrictive (Schneider *et al.*, 2010; Trebicka, 2014). Due to its flexibility and relative completeness and despite their drawbacks, the MIMIC and DYMIMIC approaches seem good candidates for evaluating SE (Seitz *et al.*, 2018; Hassan and Schneider, 2016; Schneider and Haigner, 2019; Adair, 2020; Reimers *et al.*, 2021; Elgin *et al.*, 2021). In this paper, we have preferred the DYMIMIC model in which we have also introduced illegal and legal factors. Their choices are based on their economic relevance and their availability. Our specification allows us to avoid using a factor as cause as well as an indicator.

III.2. A formal framework.

The benchmark model belongs to the family of LISREL combined with a Kalman filter. We estimate the NOE index resulting from a combination of a structural equation (Equation 1, latent variable) and a system of measurement relationships (System of Equations 2). The structural or state equation questions the dynamics of the latent variable (NOE) through the changes in its causes (Equation 1). The causes of the NOE are either theoretically grounded or consistent with the economic intuition. The system of measurement equations is devoted to the examination of the dynamics of the indicators, which depend on the NOE.

The structural equation is as follows:

$$\eta_t = X_t \Gamma' + \varsigma_t \; ; \; t = 1, \dots, T \qquad \text{(Equation 1)}$$

Where:

η_t is the latent variable (the NOE index); $X_t = (x_{1t}, x_{2t}, \dots, x_{qt})$ is a matrix of its causes; one of these causes can be the lagged latent variable; $\Gamma = (\gamma_1, \gamma_2, \dots, \gamma_q)$ is the vector of the unknown parameters; ς_t is the error term of the structural equation; T is the size of the sample.

The system of the measurement equations is as follows:

$$\mathrm{Y}_t = \eta_t \Lambda' + Z_t \mathrm{M}' + \mathrm{E}_t \; ; \; \mathrm{t} = 1, \ldots, \mathrm{T} \qquad \text{(Sys. of Equations 2)}$$

Where:

$Y_t = (y_{1t}, y_{2t}, \ldots, y_{pt})$ is a matrix of p economic indicators, depending on the latent variable;

$Z_t = (z_{1t}, z_{2t}, \ldots, z_{rt})$ is a matrix of r exogenous variables;

$\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p)$ is a vector of the unknown parameters describing the effect of the SE in the measurement equations;

$M = (M_1, M_2, \ldots, M_r)$ is a vector of the unknown parameters of exogenous variables in the measurement equations;

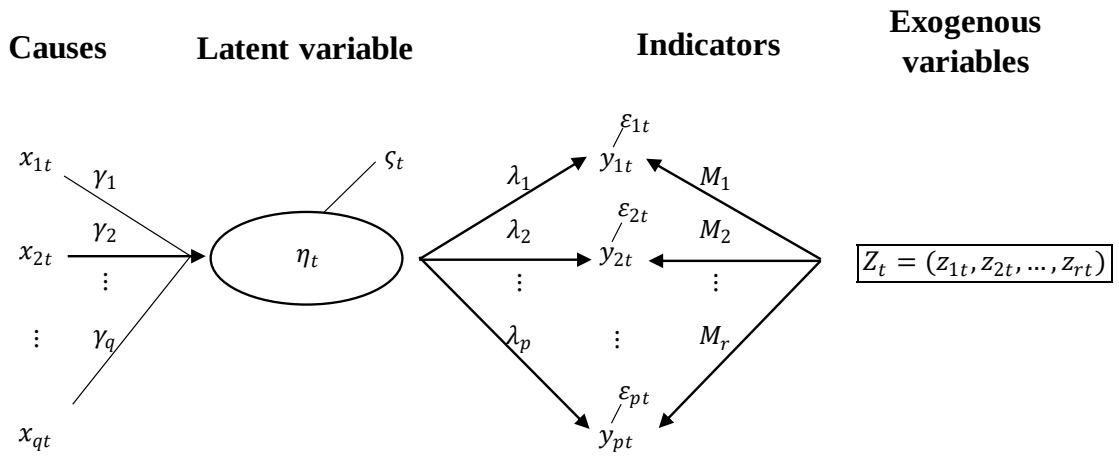
$\mathcal{E}_t = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{pt})$ is the vector of the error terms of the measurement equations.

The system of the measurement equations could be extended to some additional indicators. However, due to the size of the sample and for parsimony purposes, we have decided to limit our investigation to two measurement equations. Our set of these equations is close to those available in the empirical literature (Hassan and Schneider, 2016). Chart 1 is a simplified structure of diagram of the LISREL that describes the general structure of the MIMIC models. It provides us with a picture highlighting the link between:

- The NOE and its causes (left side of the diagram);
- The relations between the real and monetary registered activities and the NOE (right side of the diagram).

The NOE is a "bridge" between its own causes and registered real and monetary activities.

Chart 1: A Generic Structure of a MIMIC Model



Sources: Schneider, Buehn and Montenegro (2010); Hassan and Schneider (2016); Medina and Schneider (2019).

We distinguish two types of causes or sources of the NOE: the monetary and non–monetary transactions of a legal nature (legal factors) and the recorded criminal activities (illegal factors).

Legal factors consist of productive economic activities that are deliberately concealed from public authorities to avoid payment of taxes, safety standards or administrative procedures. Among them, we include the discrepancy between electricity consumption and GDP growth rates, the strengthening of laws on illegal immigration and public expenditures. The discrepancy between electricity consumption and GDP growth rates, introduced commonly as an indicator, is utilized as a cause to evaluate its direct contribution to the dynamics of the NOE (see IV.1.1 below for more details). This hypothesis is consistent with the results of Granger causality tests between this factor and well-known proxies of the NOE[3].

Illegal factors are associated with activities that do not comply with the law and may be subject to criminal convictions or fines (drug offences, fraud, cheating, rackets, etc., Gyomai and van de Ven, 2014; Medina and Schneider, 2019). We include in this class of sources, crimes or delinquencies recorded by the courts and the security and citizen protection authorities: the number of drug offences; the unusual net shipments of Euro banknotes; the gross net income of casinos.

IV. The data

The empirical analysis is carried out on a quarterly basis over the period 1990-2019 (see Appendix B, Table B). We draw real indicators and money aggregates from INSEE and *Banque de France* databases. Legal and illegal factors of the NOE are taken from French Administrations datasets (*Ministère de l'Intérieur*, *Sénat*, *Legifrance*). Demographic and fiscal indicators are collected from the World Bank and the European Commission databases. Data are on an annual basis with the exception of GDP, GDP deflator, and the unemployment rate, which are on a quarterly basis. We interpolate annual data to a quarterly basis (Chow and Lin, 1971). The GDP deflator deflates nominal data. In addition, a few variables are available over a shorter period and have been retropolated and seasonally adjusted (see Appendix B, Table B).

Prostitution and drug consumption drawn from wastewater data could be relevant in this analysis, but they are not available over long periods. They have been cancelled from our framework. Moreover, we are aware that a risk of double counting may occur but the multifactorial aspects of causes behind the NOE need to be tackled with many independent factors. It is also possible that the overestimation can affect the estimates of the NOE in terms of GDP (Hassan and Schneider, 2016; Schneider and Haigner, 2019). The crosschecking of the results with the available assessments of the NOE could help to justify our evaluation. The usual unit root tests, the legal as well as the illegal causes are integrated to an order 1. We build a set of

[3] The choice of *SECGDP* as a cause is also justified with the fact that this factor Granger causes the shadow economy index estimated by Medina and Schneider and self-employment at 5% level. However, there is no evidence when considering the unemployment rate.

data with the stationary variables (first order differences) to run the regressions.[4] The drivers of the NOE and the economic indicators are displayed in Appendix C.

IV.1. Causes of the NOE

IV.1.1. The legal causes

The legal causes of the NOE drive the incentive to hide activities from control authorities. Based on the recent findings, we assume that the difference between electricity consumption and GDP growth rates, immigration law, gross net income of casinos and the tax burden are the main causes of SE.

- The discrepancy between electricity consumption and real GDP growth rates

The part of growth of electricity consumption which is not explained by the growth of GDP is assumed to be a trigger to NOE. Thus, the gap between the electricity (in gigawatt hours) and real GDP (registered activity) growth rates ("*SECGDP*") is assumed to drive the NOE.

We notice that the discrepancy between electricity consumption and real GDP growth rates is volatile between 1990 and 2019 (see Chart C1). A sharp increase followed by a significant decrease appears around 2010 and 2011. These fluctuations derive from energy consumption and should be related to the hidden activities. Indeed, the changes in GDP are less volatile than those in this discrepancy.

- Immigration law

Immigration legislation reflects the willingness of the public authorities to reduce illegal immigration that could be associated with the unregistered economic activities. Thus, if regulation is tighter, NOE should reduce, but, illegal immigration could also be higher when immigration legislation is tighter. So, the effects of immigration policies on NOE could be positive or negative.

The immigration indicator ("*dImmig*") is set at 1 when legislation becomes tighter and 0 otherwise. Immigration legislation was tightened in 2003, 2006 and 2011 with the objective of reducing migratory flows. The indicator displays a picture corresponding to a more repressive policy, especially in the 2000s (see Chart C2).

- Public expenditures

An increase in public expenditures should reflect an increase in taxation for the next generation and it could lead to a rise in the NOE (Giles and Caragata, 2000 and 2001). So, to capture the tax burden effect, we examine the public expenditures to the GDP ratio. The first differences of this factor ("Δ*PublicExp*") fluctuate around zero and shows the highest peak in 2008 (see Chart C3). Registered activities do not display a similar pattern. A diversion of public expenditure towards unrecorded activities is then possible.

[4] The results are available upon request.

Compulsory levies on GDP and private sector taxes have also been taken into consideration but they do not lead to any convincing results.

IV.1.2. The illegal causes

It is not easy to measure the NOE even when considering its legal causes. The exercise is more complicated for illegal factors since statistical and econometric applications require large samples of data. In addition, there are many heterogeneous sources of data regarding the illegal causes of the NOE and information drawn from these sources is not necessarily coherent. Thus, the measurement of hidden activities is still a controversial issue. Therefore, we only select the factors (drug offences, residual net shipments, gross net income of casinos and corruption index) which are publicly available over a long period.

- Drug offences

"τ*Drugs"* represents the annual number of registered offences related to drug arrests and collected by the police departments and the Gendarmerie. It has been interpolated on a quarterly basis. As it only describes registered offences, it corresponds to a more or less significant part of drug offences. We notice an upward trend in the profile of this factor since Q2 1990: it reached 56,000 in Q4 2019 versus 20,000 in Q2 1990. Regarding the variations in τ*Drugs*, Chart C4 shows a more contrasted picture. Large fluctuations appear between 1990 and 2001; they are followed by less volatile changes.

- Unusual net shipments of banknotes

The net shipments of banknotes ("$\Delta NetShip^{AT}$") refer to registered imports and exports of euro banknotes between France and the Rest of the World (non-resident credit institutions, central banks, etc.). The total net shipments represent the cumulated exported banknotes minus the cumulated imported banknotes (Politronacci *et al.*, 2017). The total net shipments of banknotes should capture some of the foreign unregistered activities. Real net shipments correspond with net banknotes shipments deflated by the GDP price. They are quarterly interpolated. The unusual or detrended (cyclical and irregular components) real net shipments are associated with NOE. The unusual net shipments are stationary in first difference and broadly stable before the euro changeover in Q1 2000. The introduction of the euro led to a break followed by some significant fluctuations (see Chart C5).

- Gross net income of casinos

The gross net income of casinos gives insight into the activities of casinos. Its hidden part is included in the NOE. Formally, we examine the growth rate of GNI deflated by the GDP price ("τ*CasinosGNI"*), on a quarterly basis. Chart C6 shows that real GNI followed an upward trend before the 2008 global turmoil and a slight decrease followed by weak fluctuations during and after the financial and sovereign debt crises in Europe.

IV.2. Other drivers

We have added to the previous set of causes demographic factors, uncertainty, corruption, fiscal rules and digitalization of payments indexes. They could generate hidden activities (see Chart C7) and be introduced in the evaluation of an augmented indicator of the NOE.

With the exception of the digitalization of payments index ("ΔDPI^{dT}"), the data providers make the direct indicators of these causes available. We define the ΔDPI^{dT} as the ratio of non-cash payments to total payments that we have quarterly interpolated and detrended. We have calculated its first difference for the purposes of the acceptance of the stationary hypothesis.

The choice of the causes is consolidated through a principal component analysis (PCA) which helps to identify the factors that influence the dynamics of the NOE. PCA confirms the relevance of our selected causes of hidden activities[5].

IV.3. Indicators

In this paper, we introduce two main indicators: electricity consumption and net issuance of banknotes. The selection of these indicators is partially based on the usual practices in this research field and our need to highlight the links between SE and cash demand.

- Electricity consumption

We assume that the NOE activities are reflected in the state of the registered economy. Kaufmann and Kaliberda (1996) demonstrated that electricity consumption is one of the best indicators of SE. Lackó (2000) also showed that a certain part of SE is strongly linked with the consumption of electricity in households. Despite the fact that all the unregistered activities do not request a high input of electricity, the rise of the technical progress and the electricity production efficiency give more importance to this indicator. Therefore, we impose the growth rate of the electricity consumption ("$\tau Elec$") as an indicator in our framework (see Chart C8).

- Net issuance of banknotes

An assumption commonly made is that people engaged in the unregistered activities primarily use cash. In the context of a rapid innovation and technological progress, an exclusive use of cash in the NOE must be called into question. However, It could be useful to introduce a cash demand indicator in the framework to

[5] The normalized PCA shows that all factors are well represented. $\Delta PublicExp$ and $SECGDP$ are highly correlated with the first dimension (*Dim1*). $\Delta NetShip^{dT}$, $\tau CasinosGNI$ and $\tau Drugs$ are highly linked to *Dim2*. Thus, *Dim1* and *Dim2* can be respectively interpreted as the legal and illegal dimensions of the factors. In addition, a K-Means clustering is applied to the PCA results and two clusters are identified: the first one for illegal factors and the second one for legal factors. The results are available upon request.

catch the intensity of the link between cash demand and the NOE in France. We consider here the cumulated net issuance of banknotes growth rate ("τ*NetIssuance*"; see Chart C9).

V. Empirical results

V.1. The benchmark model

The empirical framework consists of specific cases of the structural and measurement representations. Equation 3 is the structural relationship that describes the dynamics of the NOE (latent variable, "τ*Shadow" in first difference*). Relations 4a and 4b, the measurement equations, are devoted to the analysis of the dynamics of real and monetary indicators linked to the NOE. Equation 4a is based on the physical input method by considering the relationship between growth in electricity consumption and economic registered and unregistered growth. Equation 4b investigates the dynamics of cumulated net issuance of banknotes, exclusively explained by the NOE[6].

Electricity consumption is one of the usual indicators introduced into the system of measurement equations (Lackó, 2000). The growth rate of GDP ("τ*GDP*") and τ*Shadow* are its main explanatory factors. Thus, τ*GDP* explains the growth rate of electricity consumption of registered activities whereas τ*Shadow* drives consumption related to the unregistered activities.

Monetary aggregates and indicators of real activity are usual dependent indicators. We aim to identify the links between NOE and cash demand. Therefore, we privilege the money aggregate as the additional endogenous variable of the system of the measurement equations.

For the benchmark model, we test in Equation 1 the effects of three legal causes (immigration, discrepancy between the growth rates of electricity consumption and GDP, public expenditures) and three illegal factors (drug offences, gross net income of casinos, net shipments of banknotes). In the augmented model, we add to the previous set of factors other explanatory variables, *Add* (demographic indicator, fiscal factor, financial market uncertainty, digitalization of payments and corruption indexes). Electricity consumption and net issuance of banknotes are not included in the set of causes of the NOE which are only economic indicators (see Appendix D).

[6] We also run additional regressions by changing measurement equations. The first with only Equation 4a; The second with the velocity of banknotes in circulation instead of the current Equation 4b. We obtain similar results, but the quality of the estimate is not as good as that of the current estimate. The results are available upon request.

The benchmark model is as follows:

$$\tau Shadow_t = \alpha_{11} + \alpha_{12}\tau Shadow_{t-1} + \alpha_{13}\Delta PublicExp_t + \alpha_{14}\tau Drug_t + \alpha_{15}\tau CasinosGNI_t + \alpha_{16}\Delta NetShip_t^{dT} + \alpha_{17}SECGDP_t + \alpha_{18}dImmig_t + \alpha_{19}Add_t + \varepsilon_t \quad \text{(Equation 3)}$$

$$\tau Elec_t = \alpha_{21} + \alpha_{22}\tau GDP_t + \alpha_{23}\tau Shadow_t + \kappa_t \quad \text{(Equation 4}a\text{)}$$

$$\tau NetIssuance_t = \alpha_{31} + \alpha_{32}\tau Shadow_t + \sigma_t \quad \text{(Equation 4}b\text{)}$$

As the variables are integrated of order 1, the equations are formulated with the first order differences of the variables. Thus, the variables "X"are either expressed in growth rates (τX) or first differences (ΔX); the dummy variable *dImmig* is in its absolute terms.

There is not an autoregressive term in the benchmark model ($\alpha_{12} = 0$ in Equation 3). In addition, initial estimates have shown that the intercept is not significantly different from zero in the structural equation ($\alpha_{11} = 0$). *Add* is the matrix that allows for the extension of the benchmark model ($\alpha_{19} = 0$) to the augmented model ($\alpha_{19} \neq 0$). According to Medina and Schneider (2019), SE represented around 16% of GDP in France in recent years. We draw initial values of parameters from preliminary regressions in which the indicator of NOE is defined as a fixed proportion of GDP. An alternative way of fixing the initial parameters consists of imposing some restrictions on certain parameters. In the empirical literature, two constraints are usually set up:

i) The first constraint ($\alpha_{22} = \alpha_{23}$) means that electricity consumption reacts with the same intensity to τGDP and $\tau Shadow$. This hypothesis means that registered and hidden activities drive the consumption of electricity with the same intensity. Due to the fast diffusion of new technology in the economy, this assumption is not necessarily too strong;

ii) The assumption $\alpha_{12} = 0.5$ allows us to accept the stationary hypothesis for the latent variable. Here, this assumption is not severe as the latent variable is a linear combination of stationary variables. It is also possible to iterate over the possible values of this coefficient.

The lagged $\tau Shadow$ is not an explanatory factor in our benchmark equation since it does not significantly improve the quality of the regression. As a result, the hypothesis regarding α_{12} is not helpful. Here, we consider the previous estimated initial values and impose the first constraint ($\alpha_{22} = \alpha_{23}$). Our starting values of the unknown parameters are set according to OLS estimates of equations[7] where the growth of GDP is a proxy for the NOE. We have conducted robustness checks by initializing the starting values differently and we obtain similar results[8]. The calibration procedure is based on the Kalman filtering algorithm and the Marquardt optimization of the log-likelihood. We calibrate prior mean and variance with

[7] The results of OLS regressions are available upon request.

[8] We also use coefficients different from those given by the OLS equations. Only the number of iterations to reach convergence differs. The results are available upon request.

the recent estimates by Schneider for France (2019). Thus, we use -0.17 and 3.36 respectively to initialize the starting values of the mean and variance of the model.

V.2. The estimate of the benchmark model

We estimate the model over the period Q2 1990 - Q4 2019 (see Table 1). We do not introduce 2020 in the estimation period as: a) It allows for performing an out of sample simulation; b) Due to the pandemic effects it could bias the results; c) It could be useful to evaluate the effects of the pandemic through the tools implemented before the occurrence of the global pandemic. The results are presented in Table 1. Overall, the magnitudes of the estimated coefficients are relatively high and similar to those available in the empirical literature. , it is the case of the most significant parameters (those of *SECGDP*, $\Delta NetShip^{IT}$). The *SECGDP* is the main driver of the NOE (see Table 1, Equation 3): its coefficient is different from zero at 1% level. Unusual net shipments and, to a lesser extent, the gross income of casinos and the factor describing the immigration policy, have also a relatively strong impact on the dynamics of SE (their coefficients are significant at around 10%). The negative sign of the immigration coefficient confirms that the regulation of illegal immigration tends to reduce the NOE. Finally, even though the levels of significance of their parameters are not usual (between 20% and 25%), the impacts of public expenditures and registered drug offences on the NOE are not negligible. Contrary to some previous findings, unemployment does not influence the changes in the NOE in France. The estimated parameters of the measurement equations are significantly different from zero and consistent with the economic intuition. Moreover, the negative sign of the coefficient of SE in Equation 4b shows that an increase in NOE does not lead to an increase in issuances of banknotes. Thus, the two variables display an evolution in opposite directions. That confirms the intuition regarding the use of alternative means of payment in the unregistered activities.

The regression statistics (Log-Likelihood, RMSE, etc.) are in line with usual econometric practices. Furthermore, regarding the profile of the NOE, we notice a decrease in the indicator; especially over the second half of the period under review (see Chart 2). For instance, from the beginning of 2012 to the end of 2019, the growth rate of SE is always negative except for a very short sub-period (on average -1% over this period against 0.34% before). Thus, the growth rate of SE fluctuated weakly around 0 from 1990 to 2010; there was then a sharp increase followed by a strong slump in 2011. An opposite trend appeared in 2012 before another sharp fall in 2014. Another downward trend has taken shape in recent quarters of the period under review. The legal (illegal) component of the NOE is the combination of legal (illegal) factors in which the coefficient of each factor is its estimated value (see Chart 2). The legal component is more volatile. The impacts of the two components on the NOE are significant even though the contributions of the legal component are higher. The two components are often of opposite signs and the profile of the index is close to that of the legal component.

Table 1: The Estimates of the Benchmark Model

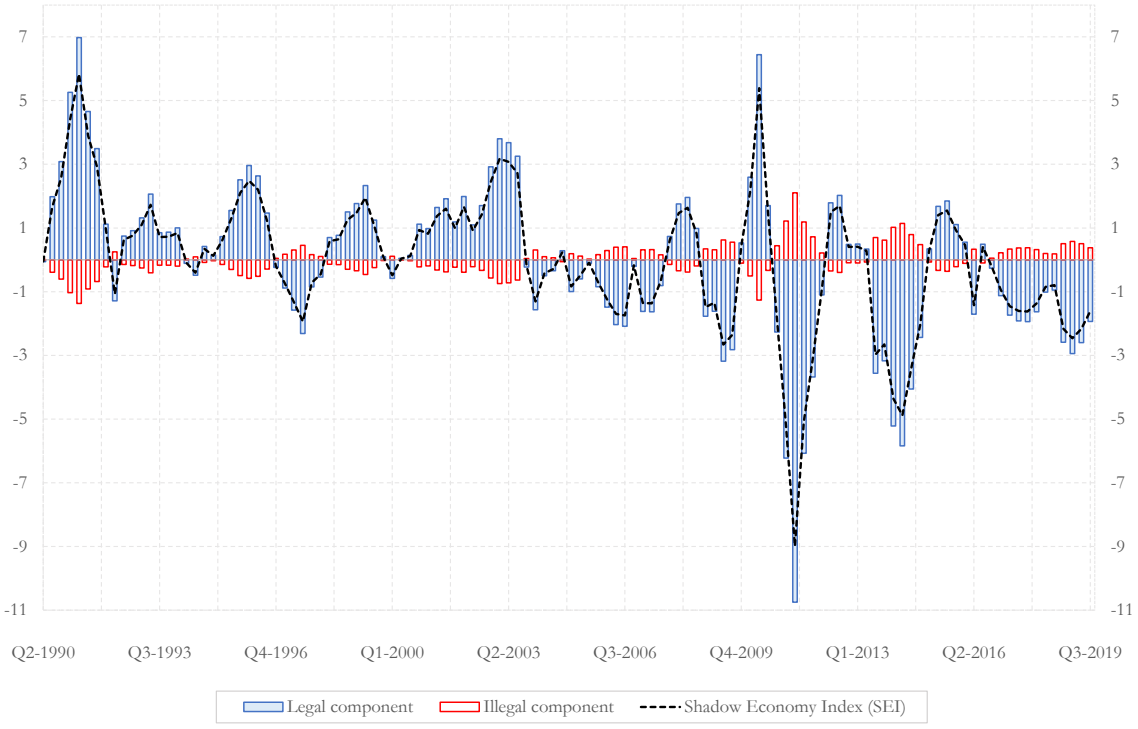
	Parameters	*Std. Error*	*P-values*
Structural Equation			
$\tau Shadow$ (Equation 3)			
$\Delta PublicExp$	0.57	0.44	0.20
$\tau Drugs$	0.11	0.09	0.26
$\tau Casinos$GNI	0.18	0.11	0.11
$\Delta NetShip^{dT}$	-0.54	0.29	0.06
$SECGDP$	1.77	0.65	0.01
$dImmig$	- 0.84	0.54	0.12
Measurement Equations			
$\tau Elec$ (Equation 4a)			
$Constant$	0.16	0.15	0.29
$\tau Shadow + \tau GDP$	0.33	0.12	0.01
κ_t	0.37	0.05	0.00
$\tau NetIssuance$ (Equation 4*b*)			
$Constant$	0.91	0.13	0.00
$\tau Shadow$	-0.24	0.11	0.02
σ_t	0.95	0.10	0.00

	Final state	R MSE	P-value	
$\tau Shadow$	-0.08	0.02	0.00	Log-likelihood: -274.99 Number of parameters: 12 Number of iterations: 18

Source: Authors' calculations.

Chart 2: The Benchmark Shadow Economy Index and its Legal and Illegal Components

(Growth rate in %; contributions in percentage points)



Source: Authors' calculations.

It is interesting to examine the impact of the COVID-19 pandemic on the dynamics of the NOE, based on the benchmark model. We perform some out-of-sample simulations (see Chart 3) by assuming that:

i) The forecasts of the exogenous factors realized before the occurrence of the COVID-19 are still valid (Scenario 1, *S1*);

ii) We use the observed values of the exogenous considering the occurrence of the COVID-19 pandemic, which leads to a sharp decrease in real activity (Scenario 2, *S2*).

In *S1*, the NOE decreases gradually while it strongly rises before a sharper drop below the level reached at the end of 2019 in *S2*. So, the pandemic initially induced an increase in shadow activities caused by the general lockdown. The decisions made by French governments to preserve activity and employment as far as possible undoubtedly led to a strong correction going beyond the intensity of the effects of the initial shock. These findings are consistent with of those of Schneider (2022).

Chart 3: Out-of-Sample Simulation (OoS) of the (Benchmark) NOE

(Growth rate in %)

Source: Authors' calculations.

V.3. Has the NOE sharply moved in France?

Now, we compare our index to those available in the literature. In France, according to Schneider (2005), in 1991, the SE represented 14% of GDP while INSEE estimated its ratio at 4% of GDP in 1988 (INSEE, 1989). We assume that these evaluations are the highest and lowest bounds for the inference on the starting point. We have simulated the ratios of the NOE with starting points varying between 4% and 14% (see Appendix E, Chart E). Based on these simulations, a "sensible" starting point seems to be 8% of GDP in Q1 1990. By combining the previous assumption with our estimated growth rates for the NOE, we simulate the ratio of the SE by inferring on the previous starting point[9]. Then, we examine our index and that provided by Schneider. These indexes are based to 100 in Q1 1991.

In 1993, the NOE based on our benchmark model represented 10% of GDP in France. This ratio gradually increases to 14.5% in 2003 and finally decreases after 2010 to reach 7.5% in 2019 (see Chart 3). Broadly, our index displays a strong upward trend followed by a marked downward trend. Its turning point coincides with the establishments of laws and controls or regulation institutions, making it possible to tighten up the existing rules. The ratio by Medina and Schneider (2019), Reimers *et al.* (2021) and Schneider (2022) fluctuated weakly at the beginning of the period under review. This was followed by a marked downward trend and then a slight upward trend at the end of the sample (see Chart 4). By comparing our indicator with those of Medina and Schneider (2019), we notice that, overall, our ratios are weaker. Sometimes, the spread between these indicators is significantly pronounced: around 8 percentage points at the peak (even if this gap is smaller between Q1 2003 and Q4 2010). The differences come mainly from the legal components of NOE.

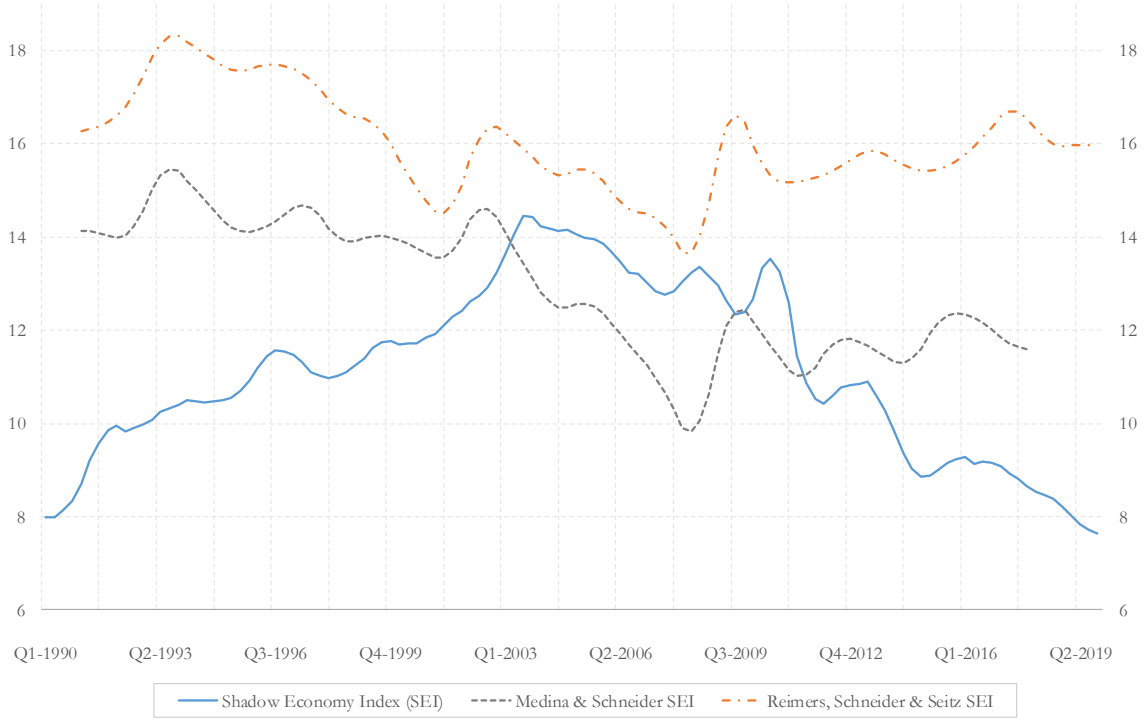
A caveat to the previous conclusion is that the gaps between our estimations and evaluations of the OECD, INSEE and Schneider *et al.* could be, at least partially, explained by the approaches implemented or the fields covered by NOE (OECD, 2012; Gyomai and van de Ven*.,* 2014; Adair, 2020). Indeed, even in the national accounts system, the NOE can cover more or less similar realities (Dell'Anno, 2021 a and b).

Besides, according to Medina and Schneider (2019) and Reimers *et al.* (2021), the ratio of shadow economy literally melted over the period in Germany while in France it does not display the same trend. However, similar policies have been set up on both sides of the Rhine. Our estimate seems to be more consistent with the measures implemented in France. Indeed, the profile of our index appears in line with the French policies than those of Reimers *et al.* which do not seem to be sensitive to institutional and regulatory changes in France. Now, when we look at the univariate indicator and those drawn from a PCA, some similarities with our index also appear in different sub-periods. Then, whatever the indicator, a downturn trend is visible especially at the end of the period under review.

[9]According to the OECD (2012), the non-observed economy in France in 2008 was about 7.0% (see also Gyomai and van de Ven, 2014; and Adair, 2020).

Chart 4: The Ratio of the SE Relative to GDP in France

(% of GDP)



Sources: Authors' calculations; Medina and Schneider (2019); Reimers et al. (2021).

V.4. Do alternative specifications for the NOE index matter in France?

Now, we extend the set of causes by adding demographic factors, financial market uncertainty, the corruption, and the digitalization of payments indexes (see Table 2). As mentioned before, due to a lack of data, the DSE and cyber-criminality indexes are discarded in this paper. With this new set of factors, we run regressions over the period Q2 1990-Q4 2019 to obtain the augmented indexes.

The effects of demographic factors, financial market uncertainty, and the corruption indexes are not statistically significant in the NOE equation. Consequently, we have dropped them from the regression. The regressions performed by adding ΔDPI^{dT} in the benchmark model, lead to more satisfactory results (see Table 2): The coefficient of ΔDPI^{dT} is significant and the significance of the main causes (*SECGDP*, $\Delta NetShip^{dT}$, etc.) have been improved. The discrepancy between electricity consumption and the real GDP growth rates remains the main driver of the NOE. The usual statistics also display better performances.

To sum up: Firstly, in the augmented framework, the measurement equations are also well estimated. The parameters are rather significantly different from zero and consistent with the economic intuition; Secondly, the coefficients of the variables common to the two specifications are similar; Thirdly, the augmented and benchmark indexes are strongly collinear (see Chart 4): Their coefficient of correlation is close to one. Nevertheless, we notice slight qualitative differences between these indexes. For example, the sharp slumps

observed in 2011 and 2014 are more marked for the augmented index. Even though the performance of the augmented model is slightly better than that of the benchmark model, the latter is more parsimonious. It could be useful for forecasting exercises. Then, we privilege the benchmark model in this paper.

Table 2: The Estimates of the Augmented Model

	Coefficients	*Std. Error*	*P-values*
Structural Equation			
$\boldsymbol{\tau Shadow}$ (Equation 3)			
$\Delta PublicExp$	0.63	0.39	0.11
$\tau Drugs$	0.11	0.08	0.19
$\tau Casinos GNI$	0.19	0.11	0.08
$\Delta NetShip^{dT}$	-0.39	0.22	0.08
$SECGDP$	1.59	0.52	0.00
$dImmig$	-0.78	0.47	0.10
ΔDPI^{dT}	0.53	0.38	0.16
Measurement Equations			
$\boldsymbol{\tau Elec}$ (Equation 4a)			
$Constant$	0.10	0.16	0.54
$\tau Shadow + \tau GDP$	0.37	0.12	0.00
κ_t	0.36	0.04	0.00
$\boldsymbol{\tau NetIssuance}$ (Equation 4b)			
$Constant$	0.94	0.13	0.00
$\tau Shadow$	-0.26	0.11	0.02
σ_t	0.97	0.10	0.00
$\tau Shadow$	Final state: -0.07	RMSE: 0.02	P-value: 0.00
			Log likelihood: -273.36
			Number of parameters: 13
			Number of iterations: 22

Source: Authors' calculations.

VI. Is the NOE significantly related to cash demand and real activity?

In this paper, in addition to the usual denominations (€5, €10, €20, €100, €200, €500), we introduce three subgroups of denominations ("*Small*", "*Medium*", and "*Large*") to distinguish between different cash uses:

- *Small* refers to cumulative low denominations (€5, €10, €20);
- *Medium* which is associated to the €50 denomination;
- *Large* represents cumulative high denominations (€100, €200, €500).

Here, the €50 denomination is not included in *Small* or *Large* denominations because it is used for both hoarding purposes and a means of payment[10].

To check the influence of some legal decisions regarding cash payment limitations, we also build specific dummies (1 when the decision is made and 0 elsewhere) that we introduce in the analysis. Most of these

[10] €50 banknotes are the most issued denominations in value (with 36% of the total cumulated outflows and 35% of the inflows) followed by €20 banknotes (respectively 51% and 45% of the total net cumulated issuance in Q4 2019).

decisions have been set to gauge anonymous cash payments and prevent tax evasion: A threshold was fixed at €3,000 in mid-June 2010; it was reduced five years later to €1,000[11]; The ECB decided to halt the issuance and the production of the €500 banknotes[12]. It is worth noting that the NOE decreases after the establishment of these decisions (see Appendix F, Chart F). Besides, based on the Christiano and Fitzgerald (2003) filter, we split in frequency domain some variables into two components: The cyclical (irregular component) and the trend (smooth component). It allows for discriminating between these components in terms of concordance or correlation as these components are driven by different dynamics.

In this section, we analyze statistical relationships between i) the NOE and cash demand aggregated and disaggregated net issuances ; ii) the NOE and real GDP and Self-employment; iii) the NOE and some legal or institutional decisions regarding cash demand. We perform concordance tests and calculate dynamic correlations between those variables and their components in frequency domains. Finally, we run causality tests, especially between the NOE indexes and the banknotes issuances (or their different denominations).

VI.1. NOE, cash demand, and real activity: a contrasted picture through concordance tests results

The concordance tests indicate the average number of periods in which two variables coincide in the same phase of the cycle (Harding and Pagan, 2002; Avouyi-Dovi and Matheron, 2005; Avouyi-Dovi *et al.*, 2006). It allows for checking whether two variables move in the same phase of the cycle (synchronization or concordance) or do not move in the same phase (desynchronization or anti-concordance). The decision rule is as follows: if the concordance statistic is equal to 1, then the variables x and y are always in the same phase and if this statistic equals 0, then x and y are always in opposite phases. A value of 0.50 indicates the absence of any systematic relationship between the dynamics of the two variables.

However, it is not always possible to compute its properties. Thus, the analysis of synchronization or desynchronization could be inaccurate. These tests are interpreted like a first-round illustration of the similarity between phases of the cycles of two variables. We assume that if the empirical statistic of the test is significantly higher than 0.50 then the hypothesis of concordance is accepted. Conversely, if this statistic is significantly lower than 0.50, the hypothesis of anti-concordance prevails.

We run concordance tests between the SEIs and monetary aggregates (see Table 3). The net issuance of the €50 (*Medium*) and €200 are significantly in concordance with our SEIs and its legal component while the illegal component is significantly in anti-concordance with the net issuance of the €50 and €200. However, the total net issuance are desynchronized with the SEIs and its legal component whereas a synchronization with the illegal component appears. The inversion of the relationships between the SEIs and the *Medium* and €200 denominations could mean that these denominations are consistent with the unregistered activities. Furthermore, we notice that there is a similar pattern between the *Large* and the €50 and €200

[11] See Décret n° 2010-662 of 16th June 2010 and Th10-15, see Décret n° 2015-741 of 24th June 2015.
[12] On 4 May 2016 the ECB Governing Council decided to halt the production of the €500 banknotes. In practice, their issuance was stopped on 27 January 2019 for France and most countries of the Eurosystem.

denominations. We do not observe a clear concordance between the SEIs and *Small* denominations contrary to Medina and Schneider who showed a strong concordance between their SEIs and the €10 and total net issuance.

Besides, we observe that the cyclical component of the total net issuance is synchronized with the cyclical component of the SEIs (and its legal component) whereas a desynchronization with the cyclical component of the illegal part appears. We notice similar relations between the cyclical components of the €200 denomination and the SEIs. This difference is due to the fact that the cyclical component of the *Medium* are desynchronized with the cyclical component of the SEIs and its legal component whereas a synchronization with the cyclical component of the illegal part exists. The synchronization with the cyclical components of the *Large* denominations is not significant. Concerning the trend component, the results are close to those of the total net issuances. The trend component of €200 denomination are significantly in concordance with the illegal component but it is in anti-concordance with the SEIs and its legal component. This may reflect a link between illegal activities and high denomination.

Furthermore, the real GDP is synchronized with the SEIs and its legal component, but it is desynchronized with the illegal component of the NOE. The concordance between the global and the legal SEIs and self-employment is even stronger than that previously noticed for the SEIs and GDP.

In conclusion, the concordance tests show a noticeable synchronization between the SEIs and net issuance of banknotes (especially total net issuance and net issuance of the €50 and €200 denominations), on one hand, and between the SEIs and the real activity indicators on the other hand. The results regarding the different components in frequency domain are not homogenous.

Table 3: Pairwise Concordance Tests on the SEIs, Net Issuance, Real Activity and Legal Measures

		Net Issuance										Real Activity		Legal Measures		
	x/y	€5 (⊥)	€10 (⊥)	€20 (⊥)	Medium or €50 (⊥)	€100 (⊥)	€200 (⊥)	€500 (⊥)	Total (Δ)	Small (⊥)	Large (⊥)	RGDP (q/q%)	Self-employment (Δ)	Th10	Th10-15	D500
Series	SEI (Δ)	ns	ns	ns	0.63 (***)	ns	0.67 (***)	ns	0.41 (*)	ns	0.63 (**)	0.59 (**)	0.61 (***)	ns	ns	ns
	Legal component (⊥)	ns	ns	ns	0.61 (***)	ns	0.69 (***)	ns	0.40 (*)	ns	0.64 (***)	0.60 (**)	0.61 (***)	ns	ns	ns
	Illegal component (⊥)	ns	ns	ns	0.36 (***)	ns	0.31 (***)	ns	0.58 (*)	ns	0.36 (***)	0.42 (*)	0.38 (***)	ns	ns	ns
	Augmented SEI (Δ)	ns	ns	ns	0.61 (**)	ns	0.66 (***)	ns	0.40 (*)	ns	0.64 (***)	0.58 (*)	0.62 (***)	ns	ns	ns
	M.S SEI (Δ)	ns	0.34 (***)	ns	ns	ns	ns	ns	0.38 (***)	ns	ns	ns	ns	ns	ns	ns
Cycle	SEI (Δ)	ns	ns	ns	0.34 (***)	ns	0.68 (***)	0.61 (*)	0.61 (*)	ns	ns	ns	ns	ns	ns	ns
	Legal component (⊥)	ns	ns	ns	0.34 (***)	ns	0.68 (***)	0.61 (*)	0.61 (*)	ns	ns	ns	ns	ns	ns	ns
	Illegal component (⊥)	ns	ns	ns	0.65 (***)	ns	0.31 (***)	ns	0.37 (**)	ns	ns	ns	ns	ns	ns	0.47 (*)
	Augmented SEI (Δ)	ns	ns	ns	0.37 (**)	ns	0.71 (***)	ns	0.65 (**)	ns	ns	ns	ns	ns	ns	0.52 (*)
	M.S SEI (Δ)	0.69 (***)	ns	ns	ns	ns	ns	ns	ns	ns	0.39 (*)	ns	ns	ns	ns	ns
Trend	SEI (Δ)	0.35 (**)	ns	ns	0.40 (**)	ns	0.18 (***)	ns	0.27 (***)	ns	ns	0.66 (**)	0.60 (**)	0.32 (***)	0.32 (***)	0.37 (**)
	Legal component (⊥)	0.35 (**)	ns	ns	0.40 (**)	ns	0.18 (***)	ns	0.27 (***)	ns	ns	0.66 (**)	0.60 (**)	0.32 (***)	0.32 (***)	0.37 (**)
	Illegal component (⊥)	0.73 (***)	ns	ns	0.58 (**)	ns	0.77 (***)	ns	0.65 (***)	ns	ns	0.26 (***)	ns	0.76 (***)	0.76 (***)	0.71 (***)
	Augmented SEI (Δ)	0.32 (***)	ns	ns	ns	ns	0.31 (***)	0.39 (*)	ns	ns	ns	0.66 (**)	ns	0.32 (***)	0.32 (***)	0.37 (**)
	M.S SEI (Δ)	ns	ns	ns	ns	ns	ns	ns	ns	ns	ns	ns	ns	0.42 (**)	ns	ns

*Notes: ***, **, * means p-values less than 1%, 5%, and 10%, respectively;" ns" means non-significant; "M.S SEI" means Medina & Schneider SEI; The tests for denominational breakdown (from €5 to €500 banknotes), Small, Large denominations and Legal measures are conducted from Q2 2002 to Q4 2019.*

Source: Authors' calculations.

VI.2. Cross-correlations between the NOE, monetary aggregates, and real activity: certain marked links

We display in Table 4 cross-correlations between net issuance of banknotes and the NOE indexes in the frequency domain. We do not notice strong correlations between the SEIs and the macro aggregates with exception of the self-employment and the benchmark SEI to a lesser extent; this relation is also validated for Medina and Schneider's SEIs.

However, there is a positive correlation between cyclical components of net issuance and benchmark and augmented SEIs. The benchmark and augmented indexes seem highly correlated with aggregate net issuance (around 0.5 with a lag of one quarter). In contrast, this relation is the opposite for the Medina and Schneider SEI. Moreover, the correlations between the "*Medium"* denominations and SEIs reach their maximum (0.25 to 0.35) with a lag of three quarters. The link is even more pronounced with the "*Large"* denominations (more than 0.5): A positive variation in the cyclical component of aggregate net issuance jointly follows an increase in the cyclical component of the benchmark and augmented SEIs (with a lag of 2 or 3 quarters). For the "*Small"* denominations there is no significant correlation with the benchmark and augmented index on this step.

Table 4: Cross-Correlations between the NOE, the Net issuance and the Real Activity Indicators

$Corr[a(t), r(t+k)]$			r								
			Series			*Cycle*			*Trend*		
		k	SEI (Δ)	Aug SEI (Δ)	M.S SEI (Δ)	SEI (Δ)	Aug SEI (Δ)	M.S SEI (Δ)	SEI (Δ)	Aug SEI (Δ)	M.S SEI (Δ)
a	Total (Δ)	-4	ns	ns	ns	ns	ns	0.24 (*)	ns	ns	ns
		-3	ns	ns	ns	-0.47 (***)	-0.45 (***)	0.30 (**)	ns	ns	ns
		-2	ns	ns	ns	-0.57 (***)	-0.54 (***)	0.30 (**)	0.20 (*)	0.19 (*)	ns
		-1	ns	ns	ns	-0.39 (***)	-0.36 (***)	0.22 (*)	ns	ns	ns
		0	ns	ns	ns	ns	ns	ns	ns	ns	ns
		1	ns	ns	ns	0.33 (***)	0.33 (***)	ns	ns	ns	ns
		2	ns	ns	ns	0.49 (***)	0.48 (***)	-0.26 (**)	ns	ns	ns
		3	ns	ns	ns	0.43 (***)	0.42 (***)	-0.32 (***)	ns	ns	ns
		4	ns	ns	ns	0.23 (*)	0.23 (*)	-0.26 (**)	ns	ns	ns
	Medium or €50 (⊥)	-4	ns	ns	ns	ns	ns	0.20 (*)	0.19 (*)	0.22 (*)	0.28 (**)
		-3	ns	ns	ns	-0.20 (*)	ns	0.27 (**)	0.22 (*)	0.24 (*)	0.29 (**)
		-2	ns	ns	ns	-0.32 (***)	-0.29 (**)	0.22 (*)	0.29 (**)	0.32 (***)	0.33 (***)
		-1	ns	ns	ns	-0.29 (**)	-0.28 (**)	ns	ns	ns	0.43 (***)
		0	ns	ns	ns	ns	ns	ns	ns	ns	0.22 (*)
		1	ns	ns	ns	ns	ns	ns	ns	ns	0.31 (***)
		2	ns	ns	ns	0.26 (**)	0.26 (**)	ns	0.29 (**)	0.33 (***)	0.33 (***)
		3	ns	ns	ns	0.33 (***)	0.34 (***)	ns	ns	ns	0.30 (**)
		4	ns	ns	ns	0.30 (**)	0.31 (**)	ns	ns	ns	0.40 (***)
	Large (⊥)	-4	ns	ns	ns	ns	ns	0.31 (**)	-0.22 (*)	-0.24 (*)	ns
		-3	ns	ns	ns	-0.43 (***)	-0.40 (***)	0.31 (**)	ns	ns	-0.25 (**)
		-2	ns	ns	ns	-0.49 (***)	-0.46 (***)	0.22 (*)	-0.22 (*)	-0.23 (*)	-0.26 (**)
		-1	ns	ns	ns	-0.33 (***)	-0.31 (***)	ns	-0.37 (***)	-0.40 (***)	-0.23 (*)
		0	ns	ns	ns	ns	ns	ns	-0.28 (**)	-0.30 (**)	-0.44 (***)
		1	ns	ns	ns	0.30 (**)	0.28 (**)	-0.30 (**)	ns	ns	-0.29 (**)
		2	ns	ns	ns	0.51 (***)	0.49 (***)	-0.39 (***)	-0.23 (*)	-0.26 (**)	-0.33 (***)
		3	ns	ns	ns	0.56 (***)	0.55 (***)	-0.38 (***)	ns	ns	-0.46 (***)
		4	ns	ns	ns	0.47 (***)	0.46 (***)	-0.26 (**)	-0.19 (*)	-0.22 (*)	-0.35 (***)
	Small (⊥)	-4	ns	ns	ns	ns	ns	ns	0.33 (***)	0.35 (***)	0.43 (***)
		-3	ns	ns	ns	ns	ns	ns	0.44 (***)	0.46 (***)	0.48 (***)
		-2	ns	ns	ns	ns	ns	ns	0.40 (***)	0.43 (***)	0.50 (***)
		-1	ns	ns	ns	-0.22 (*)	-0.21 (*)	ns	0.25 (**)	0.28 (**)	0.53 (***)
		0	ns	ns	ns	ns	ns	ns	0.29 (**)	0.32 (***)	0.58 (***)
		1	ns	ns	ns	ns	ns	ns	0.26 (**)	0.29 (**)	0.56 (***)
		2	ns	ns	ns	ns	ns	ns	0.39 (***)	0.44 (***)	0.59 (***)
		3	ns	ns	ns	ns	ns	ns	ns	0.21 (*)	0.62 (***)
		4	ns	ns	ns	ns	ns	ns	ns	ns	0.62 (***)
	RGDP (q/q%)	-4	ns	ns	ns	0.43 (***)	0.38 (***)	-0.36 (***)	ns	ns	ns
		-3	ns	ns	ns	0.20 (*)	ns	-0.22 (*)	ns	ns	ns
		-2	ns	ns	ns	ns	ns	ns	ns	ns	ns
		-1	ns	ns	ns	ns	ns	0.36 (***)	-0.32 (***)	-0.33 (***)	ns
		0	ns	ns	ns	ns	ns	0.55 (***)	0.46 (***)	0.47 (***)	ns
		1	ns	ns	ns	ns	ns	0.55 (***)	-0.29 (**)	-0.31 (***)	ns
		2	ns	ns	ns	ns	ns	0.33 (***)	ns	ns	ns
		3	ns	ns	ns	ns	ns	ns	ns	ns	ns
		4	ns	ns	ns	ns	ns	-0.29 (**)	ns	ns	ns
	Self-employment (Δ)	-4	ns	ns	0.25 (**)	ns	ns	0.48 (***)	ns	ns	ns
		-3	ns	ns	0.31 (***)	ns	ns	0.45 (***)	ns	ns	ns
		-2	-0.20 (*)	ns	ns	ns	ns	0.34 (***)	ns	ns	ns
		-1	-0.22 (*)	ns	ns	ns	ns	ns	ns	ns	ns
		0	ns	ns	ns	ns	ns	ns	ns	ns	ns
		1	ns	ns	ns	ns	ns	-0.25 (**)	ns	ns	ns
		2	ns	ns	-0.19 (*)	ns	ns	-0.42 (***)	ns	ns	ns
		3	ns	ns	-0.22 (*)	ns	ns	-0.49 (***)	0.20 (*)	0.21 (*)	ns
		4	ns	ns	-0.22 (*)	ns	ns	-0.45 (***)	ns	ns	ns

*Notes: The correlations are calculated from Q3 2002 to Q4 2017; ***, **, * means p-values less than 1%, 5%, and 10%, respectively; "ns" means non-significant; "M.S SEI" means Medina & Schneider SEI; For parsimonious purposes, we limit to the comparison between Large, Medium, Small, Activity indicators and SEI, Aug SEI and M.S SEI.*

Source: Authors' calculations.

Regarding the trend component, the correlations between the SEIs are overall significant. For example, in the case of the *Small* denominations, we observe strong correlations with the SEIs (0.45 for our augmented index and 0.60 for the Medina & Schneider SEI, see Table 4). However, the link highlighted by Medina and Schneider SEI seems stronger in the case of the aggregate net issuance.

VI.3. Is there a strong causality between cash demand and the NOE?

We run Granger causality tests for net issuance of banknotes (total and by denomination or by categories, "*Small*", "*Medium* "and "*Large*", see Table 5) and the SEIs (benchmark, legal and illegal components, augmented index. Over the full period (Q2 1990 - Q4 2019), the SEIs significantly drive aggregate net issuance as well as the demand for €20 banknotes: The inverse causal relationship also exists (Giles, 1997; Giles *et al.*, 2002; and Breusch, 2005). Bi-directional causal relationships are validated for the couples "NOE / aggregate net issuance" and "NOE / €20 banknotes". Besides, a causal relationship between "*Small*" denominations and the SEIs also prevails, but there is no causal relationship between the SEI and higher denominations. In addition, a causality appears between a) GDP and the benchmark index, on one hand; b) GDP and both the legal component of the NOE and the augmented index, on the other hand. It could mean that the registered production is a driver of the NOE, but the inverse relationship does not work. There is no causal relationship between self-employment and our indexes whereas it prevails for the Medina and Schneider index. For legal or institutional measures (Giammatteo *et al.*, 2021), there is a significant causality between the first threshold of €3,000 (Th10), and the benchmark indexes (global and components, see Table 5). We reach a similar conclusion with the second threshold indicator (Th10-15). Finally, a causal relationship exists between the discontinuation of the €500 (D500) and the global and legal component indexes. It could mean that the discontinuation of the €500 has made it more difficult to transport large sums of cash, which tends to penalize illegal activities to a lesser extent.

Table 5: Granger Causality Tests on the SEIs, Net Issuance, Real Activity and Legal Measures

			Net Issuance										Real Activity		Legal Measures		
		x/y	€5 (⊥)	€10 (⊥)	€20 (⊥)	Medium or €50 (⊥)	€100 (⊥)	€200 (⊥)	€500 (⊥)	Total (Δ)	Small (⊥)	Large (⊥)	RGDP (q/q%)	Self-employment (Δ)	Th10	Th10-15	D500
$x \xrightarrow[Granger\ cause]{} y$	SEI (Δ)		ns	ns	***	ns	ns	ns	ns	***	***	ns	ns	ns	***	**	ns
	Legal component (⊥)		ns	ns	***	ns	ns	ns	ns	***	***	ns	ns	ns	***	ns	ns
	Illegal component (⊥)		ns	ns	***	ns	ns	ns	ns	***	***	ns	ns	ns	***	*	ns
	Augmented SEI (Δ)		ns	ns	***	ns	ns	ns	ns	***	***	ns	ns	ns	***	**	ns
	M.S SEI (Δ)		ns	ns	ns	ns	ns	***	**	ns	***	ns	***	***	***	ns	ns
$y \xrightarrow[Granger\ cause]{} x$	SEI (Δ)		**	ns	ns	ns	ns	**	**	***	ns	ns	***	ns	***	***	*
	Legal component (⊥)		**	ns	ns	ns	ns	**	**	***	ns	ns	***	ns	***	***	*
	Illegal component (⊥)		*	ns	ns	ns	ns	*	**	***	ns	*	***	ns	***	**	**
	Augmented SEI (Δ)		*	ns	ns	ns	ns	*	**	***	ns	*	***	ns	***	***	ns
	M.S SEI (Δ)		ns	ns	ns	ns	ns	ns	***	ns	ns	ns	ns	***	ns	ns	ns

*Notes: ***, **, and * mean p-values less than 1%, 5%, and 10%, respectively; "ns" means non-significant; "The tests for denominational breakdown (from €5 to €500 banknotes), Small, Medium and Large denominations and Legal measures are conducted from Q2 2002 to Q4 2019; For parsimonious purposes, the results for cyclical and trend components are not reported here but are available upon request.*

Source: Authors' calculations.

VII. Conclusion

We combine the MIMIC model with a Kalman filter to build an index of the NOE activities in France. In the benchmark model, we test the significance of the effects of the legal and illegal causes of the unregistered activities. The spread between the electricity consumption and the real GDP growth rates, and to a lesser extent, the tax burden and immigration, are the main drivers of SE. Drug offences, gross net income of casinos and unusual net shipments of banknotes also play a non-negligible role in the changes in the dynamic of the NOE.

In the augmented version of the model, the set of causes in the benchmark model is completed with the digitalization payments index. The augmented specification improves the performance of the model but the benchmark model is more parcimonious. We privilege the benchmark framework that is still relevant and in which the projections of the causes seem easier.

Our indexes highlight the effects of the measures taken by the public authorities to prevent financial delinquencies. In addition, the profiles of the NOE relative to GDP are comparable in France and Germany, especially in recent years. These results are different from those published in recent papers which do not consider the similarity of decisions adopted in these countries. They are consistent with the fact that these decisions should more or less have the comparable effects in these countries.

Finally, we observe strong links between the NOE index and cash demand indicators. Thus, concordance tests show a noticeable synchronization between SEIs (global and legal) and net issuance of banknotes (especially total net issuance and net issuance of the €50 and €200 denominations). Furthermore, the SEIs, and the real activity indicators (GDP, self-employment) are synchronized.

We also notice positive correlations between the cyclical components of the total net issuance of banknotes and the estimated SEIs. Above all, there are some bi-directional causal relationships between the NOE indexes and aggregate banknotes demand. However, regarding the different classes of denominations, there is only one unidirectional causality between these indexes and demand for "*Small*" denominations.

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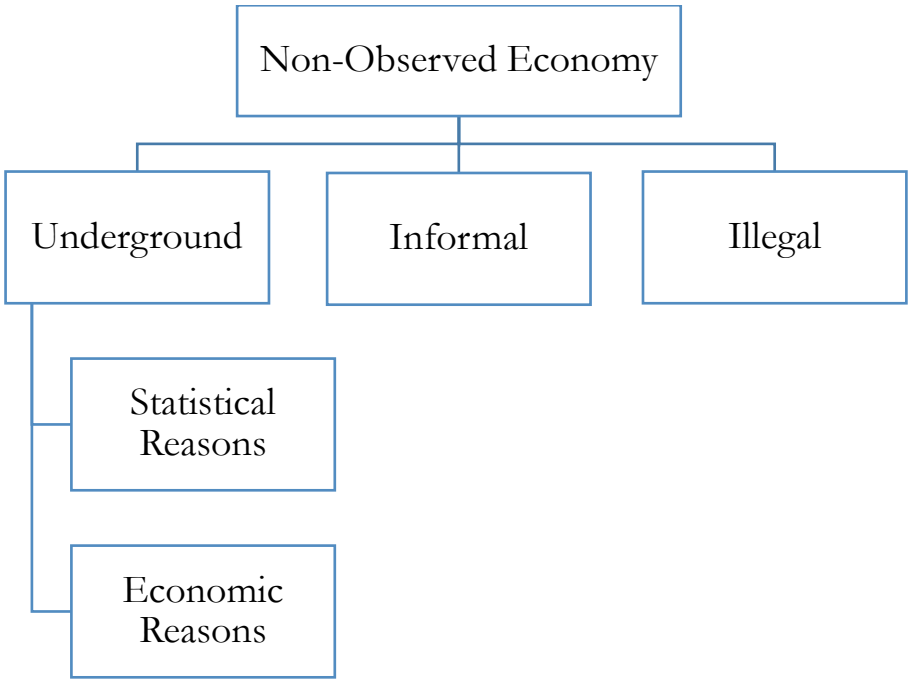
APPENDICES

A: Definitions

Table A1: The NOE based on Istituto Nazionale di Statistica (ISTAT) Framework



Source: Dell'Anno (2021 a).

Table A2: A Taxonomy of Shadow Activities

Illegal Activities	**Monetary Transactions:** Trade in stolen goods; drug dealing and manufacturing; prostitution; gambling; smuggling and fraud.		**Non-Monetary Transactions:** Barter: drugs; stolen goods, etc.; production or growing of drugs for own use.	
Legal Activities	*Tax evasion* Unreported income from self-employment; wages, salaries and assets from unreported work related to legal services and goods	*Tax avoidance* Employee discounts, fringe benefits	*Tax evasion* Barter or legal goods and services	Tax avoidance All do-it-yourself and neighbor help

Sources: Lippert and Walker (1997); Schneider and Enste (2000); Buehn and Schneider (2012).

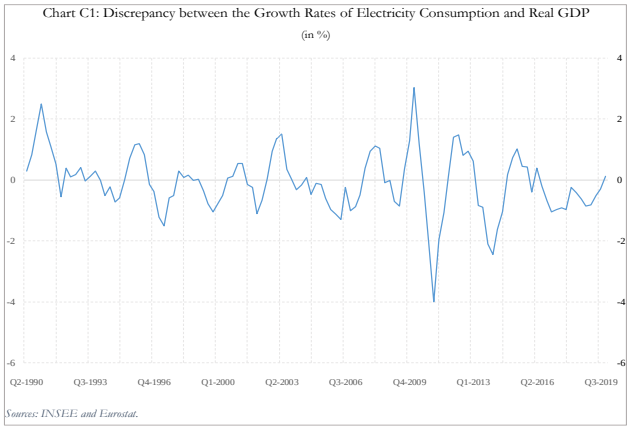
B: Data and Sources

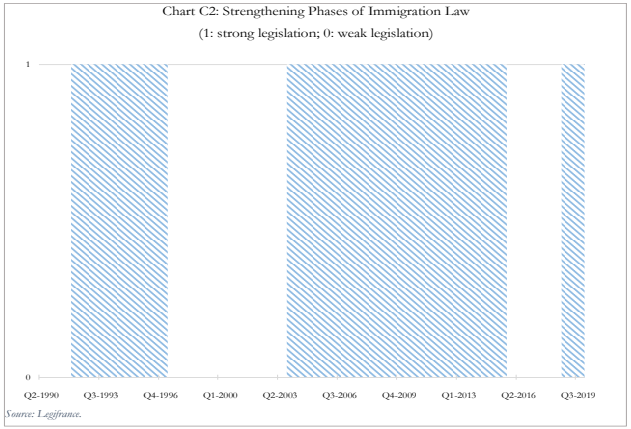
Table B: Indicators and Causes of the NOE

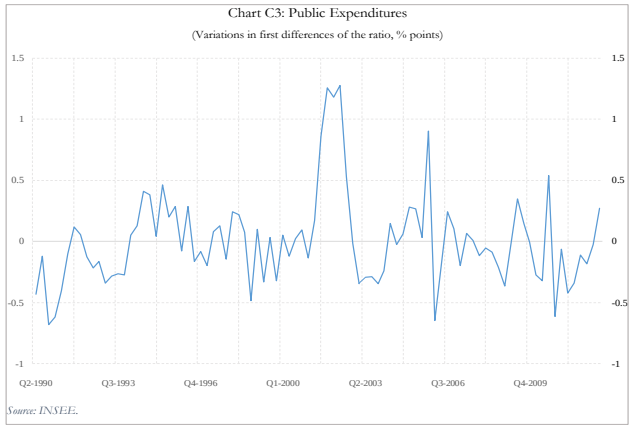
Variables	*Frequencies*	*Data availability*	*Sources*
Gross domestic product	*Q*	*1975 - 2020*	*INSEE[a,d,e]*
Total French cumulated net issuance of banknotes	*M*	*1968 - 2020*	*Banque de France[a,c,d,e,f]*
Harmonized unemployment rate	*Q*	*1975 - 2020*	*INSEE[a]*
Total French net shipments of banknotes	*A*	*1990 - 2020*	*Banque de France[b,c,d,g]*
Drug offences	*A*	*1990 - 2020*	*Ministère de l'Intérieur and Observatoire des Drogues et des Toxicomanies[b]*
Gross net income of casinos	*A*	*1987 - 2020*	*Rapport Sénatorial (Trucy, 2010) and INSEE[b,d]*
Illegal immigration	*Q*	*1980 - 2020*	*Legifrance*
Electricity consumption	*A*	*1990 - 2020*	*Eurostat[b]*
Public expenditures (% of GDP)	*Q*	*1980 - 2020*	*INSEE[a]*
Urban population and population density	*A*	*1960 - 2020*	*World Bank[b]*
Self-employment	*A*	*1982 - 2019*	*INSEE[b]*
World Governance Indicators: Control of corruption, government effectiveness, rule of law	*A*	*1995 - 2020*	*World Bank[b]*
Transparency international index	*A*	*1995 - 2020*	*Transparency International[b]*
Fiscal rules index	*A*	*1990 - 2020*	*European Commission (Directorate-General for Economic and Financial Affairs)[b]*
Digitalization index: share of non-cash payments (% of total payments)	*A*	*1990 - 2020*	*Eurostat[b,g]*
Equity market-related economic uncertainty Index	*D*	*1985 - 2020*	*Baker, Bloom and Davis (2016)[f,b]*

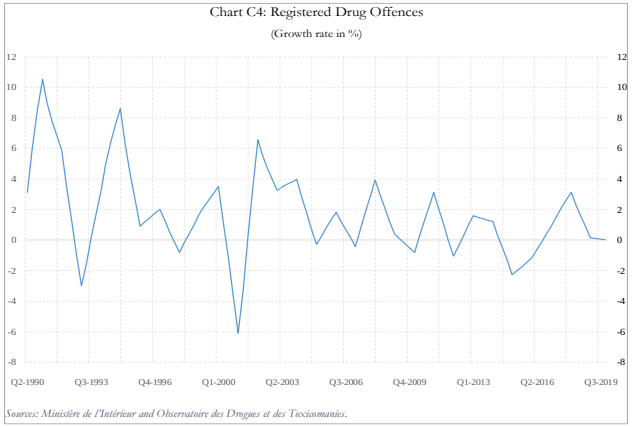
Notes: A: Annual; Q: Quarterly; M: Monthly; D: Daily. a) Seasonally adjusted using "RJDemetra"; b) Quarterly disaggregation; c) Retropolation based on French Franc currency in circulation; d) Inreal terms; e) Chain linked volumes; f) Quarterly aggregation; g) Detrended by Hoddrick-Prescott filter; h) Recurrent updates by the authors. Data are considered until 2019 for estimations and until 2020 is for out-of-samples simulations.

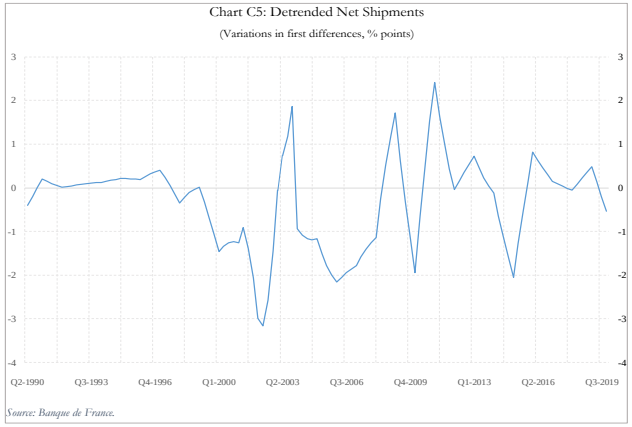
C: Legal, Illegal Causes and Indicators of the NOE (1990-2019)

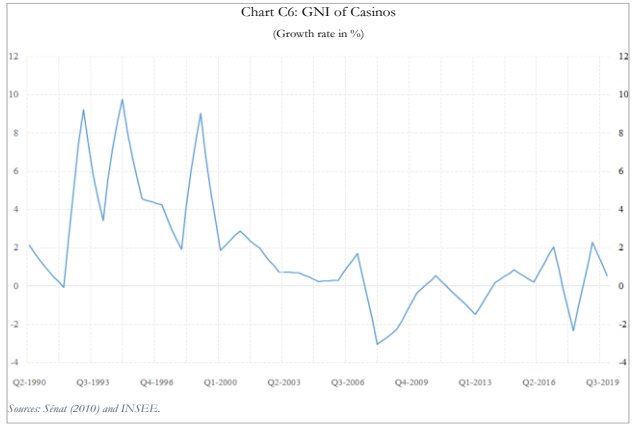


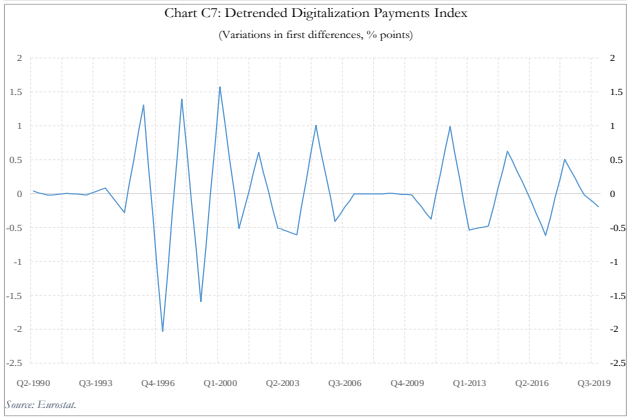


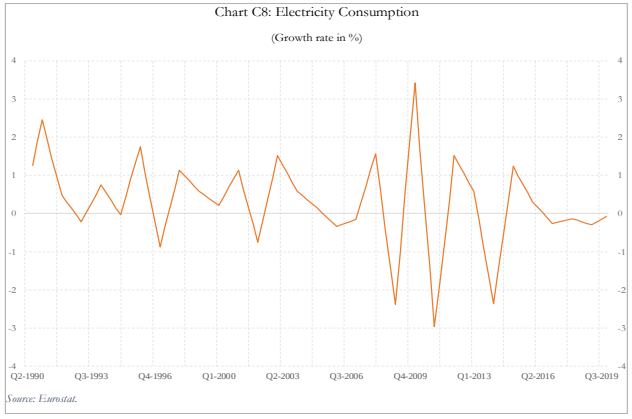


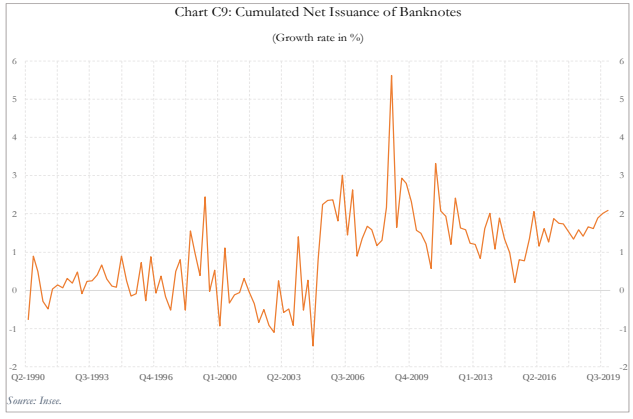












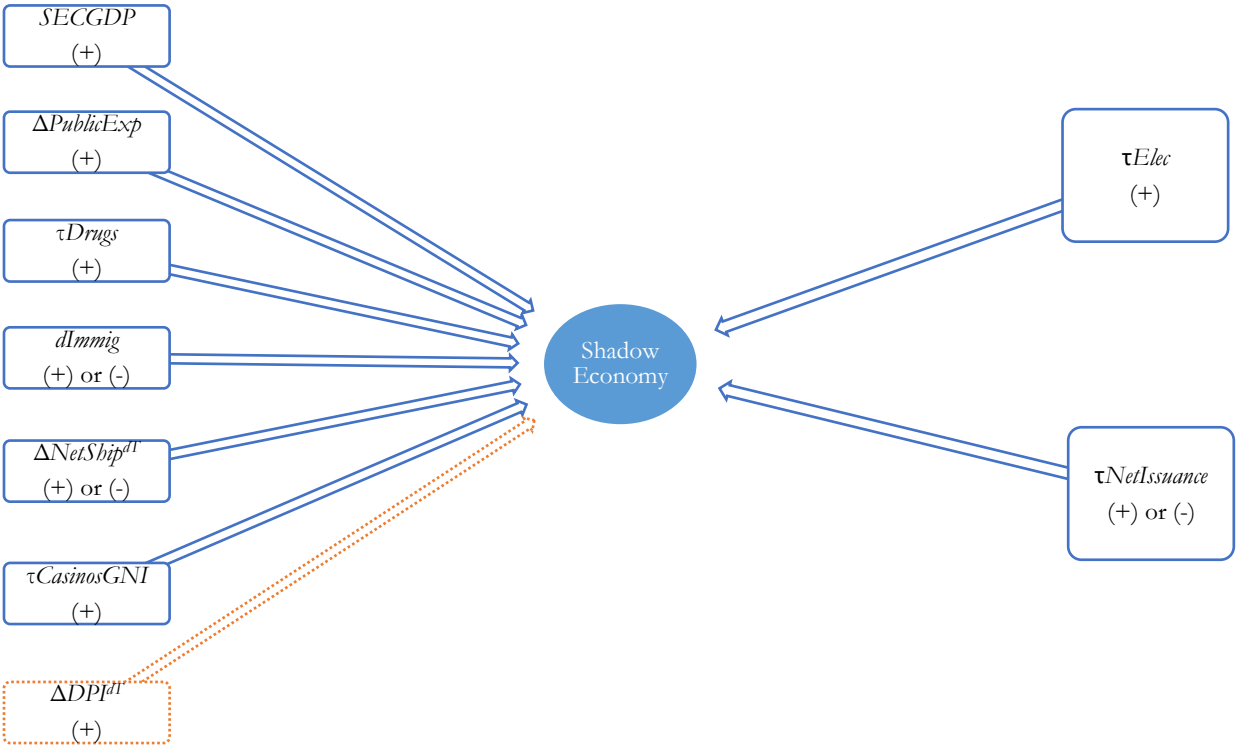
Notes: Charts C8 and C9 (orange curves) refer to the indicators used.

Source: Authors' calculations.

D: Drivers of the NOE in a MIMIC approach

Here, we propose to summarize the theoretical discussions regarding the relationships between the NOE and its main factors. Economic theory allows for establishing the links between the NOE and its causes and building the relation between the NOE and economic indicators (see Chart D). For instance, *SECGDP*, Δ*PublicExp*, τ*CasinosGNI,* τ*Drugs* and ΔDPI^{dT} are theoretically positively correlated to the NOE whereas the effects of *dImmig* and Δ$NetShip^{dT}$ are more ambiguous. Regarding indicators, τ*Elec* is supposed to have a positive effect whereas τ*NetIssuance* is also ambiguous.

Chart D: Expected Signs of Causes and Indicators on the NOE

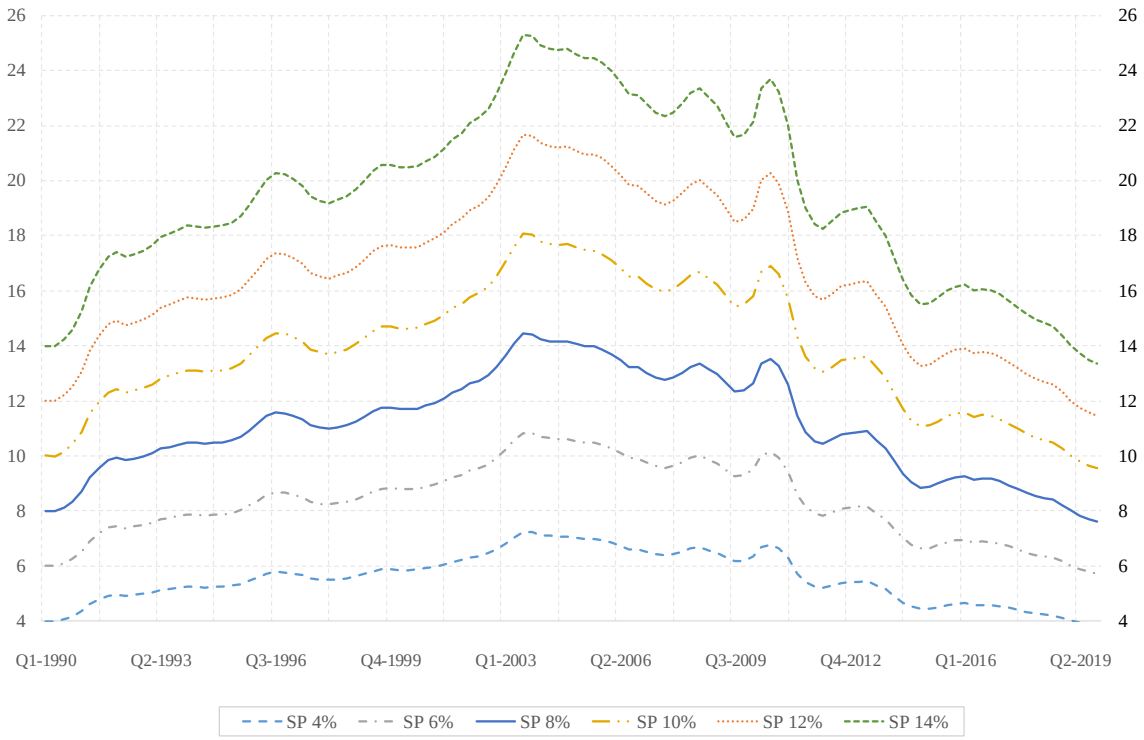


Notes: the dotted line refers to the augmented model.

Source: Authors.

E: Simulations based on Starting Points

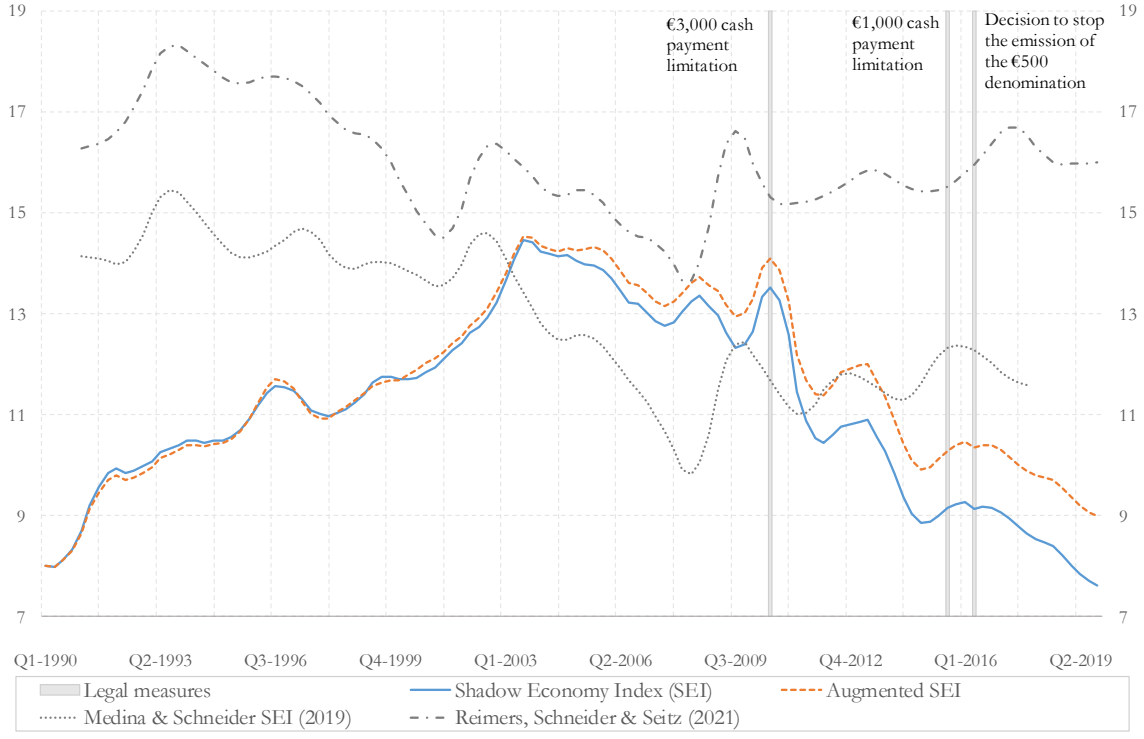
Chart E: Inference on Starting Points
(in % of GDP)



Source: Authors' calculations.

F: Benchmark and Augmented indexes versus Medina and Schneider's Indexes

Chart F: The Ratio of the NOE
(in % of GDP)



Sources: Authors' calculations, Medina and Schneider (2019), Reimers et al., (2021).