WORKING paper

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Why Fiscal Regimes Matter for Fiscal Sustainability

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Abstract

This paper introduces a Regime-Switching Model-Based Sustainability test allowing for periodic (or local) violations of Bohn (1998, QJE)'s sustainability condition. We assume a Markov-switching fiscal policy rule whose parameters stochastically switch between sustainable and unsustainable regimes. We demonstrate that long-run (or global) fiscal sustainability not only depends on regime-specific feedback coefficients of the fiscal policy rule but also on the average durations of fiscal regimes. Evidence on French data suggests that both the No-Ponzi Game condition and the Debt-stabilizing condition hold in the long run, when accounting for fiscal regimes, contrary to standard MBS tests. Drawing on former evidence about the characteristics of monetary policy in France, we discuss about the proper specification of the monetary-fiscal policy mix since 1965.

Keywords: Fiscal Rules, Fiscal Regimes, Public Debt Sustainability, Time-Varying Parameters, Markov-Switching Models

JEL classification: E6, H6

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Non-Technical Summary

Fiscal policy rules describing the reaction of primary balance to the initial level of public debt have long been used to analyze fiscal sustainability. According to Bohn's seminal contribution (1998, QJE), primary public balance must increase after an increase of the public debt-to-GDP ratio to ensure sustainability, as defined by the respect of the government intertemporal budget constraint. This paper is motivated by the empirical evidence of fiscal episodes during which public debt-to-GDP is non-stationary and generates no improvement in primary public balance. Under these episodes, fiscal policy *periodically* violates Bohn's sustainability condition and thus raises critical questions on the long-run fiscal sustainability: is a periodically unsustainable fiscal policy a threat to long-run sustainability of public finance? How long can fiscal policy be periodically unsustainable without violating its sustainability constraints in the long-run?

The paper introduces a *Regime-Switching Model-Based Sustainability* (RS-MBS) test for fiscal policy, building on Bohn's *Model-Based Sustainability* (MBS) framework and on the literature on Markov-switching fiscal policy rules. Bohn's MBS approach relies on two key ingredients: an equilibrium demand for public bonds (i.e. the Euler equation) that pins down the stochastic discount factor and a behavioral equation for fiscal policy (i.e. a fiscal policy rule).

In this paper, we assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes. We define unsustainable regimes by periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violating Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes. We demonstrate how fiscal regimes matter for *global* (in opposition with local) fiscal sustainability analysis.

We consider the two usual concepts of long-run fiscal sustainability: the No-Ponzi game condition (related to the transversality condition) and the debt-stabilizing condition (related to the stationarity of the debt-to-GDP ratio). For each concept of fiscal sustainability, we derive sufficient conditions for long-run (or global) fiscal sustainability which depend on regime-specific feedback coefficients of the Markov-switching fiscal policy rule and on expected durations (or persistence) of fiscal regimes. We show that fiscal policy can be *locally* unsustainable, with a periodically explosive public-debt-to-GDP ratio, and still be *globally* sustainable.

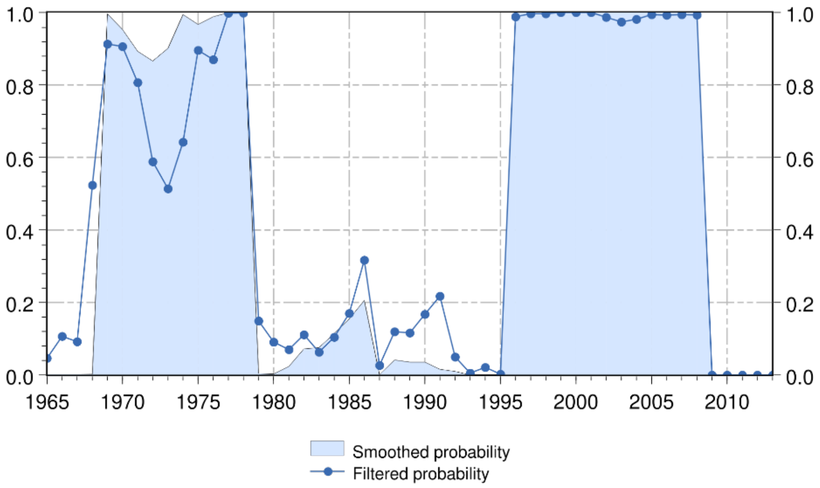
We apply our test to France's fiscal policy. Our empirical results are threefold.

First, we estimate different specifications of Bohn's constant parameter fiscal policy rule. These estimates do not allow rejecting unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests.

Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on the primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. The existence of two fiscal regimes in France helps reconcile the contrasting results achieved in the literature where some papers conclude that fiscal policy has not been sustainable whereas some others show the opposite. In addition, identified fiscal regimes are found to be highly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable despite being under an Excessive Deficit Procedure from 2003 to 2007.

Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. We reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio and conclude that France has achieved global sustainability since 1965, despite a prolonged period of unsustainability from the early 80s to the mid-90s.

Figure 1: Estimated sustainable regime, baseline model, France (1965-2013)



Note: This graph reports the filtered and smoothed probabilities of the sustainable fiscal regime in France (i.e. when fiscal policy stabilizes its debt-to-GDP ratio by increasing primary surplus). Filtered probability at year t is conditional on information from t_0 = 1965 to t and smoothed probability at year t is conditional on information from t_0 = 1965 to T = 2013.

Des implications des régimes budgétaires pour juger de la soutenabilité de la dette publique

RÉSUMÉ

Cet article présente un test de soutenabilité de la dette publique fondé sur un modèle à changement de régime markovien, dans lequel la politique budgétaire peut dévier périodiquement (ou localement) de la condition de soutenabilité de Bohn (1998, QJE). Nous supposons une règle de politique budgétaire à changement de régime markovien, dont les paramètres alternent entre un régime soutenable et un régime insoutenable, de façon stochastique. Nous démontrons que la soutenabilité budgétaire à long terme (ou globale) ne dépend pas seulement des coefficients de réaction de la règle de politique budgétaire propres à chaque régime, mais aussi de la durée moyenne des régimes budgétaires. Les données françaises suggèrent que les conditions de No-Ponzi et de la stabilisation de la dette sont satisfaites à long terme lorsque l'on tient compte des régimes budgétaires et contrairement aux conclusions des tests Model-Based Sustainability (MBS) classiques. En s'appuyant sur la littérature concernant les caractéristiques de la politique monétaire en France, nous caractérisons le *policy mix* monétaire et budgétaire depuis 1965.

Mots-clés : règles budgétaires, régimes budgétaires, soutenabilité de la dette publique, modèles à changement de régime markovien

1 Introduction

Fiscal policy rules describing the reaction of primary balance to the initial level of public debt have long been used to analyze fiscal sustainability. According to Bohn (1998)'s seminal contribution, primary public balance must increase after an increase of the public debt-to-GDP ratio to ensure sustainability, as defined by the respect of the government intertemporal budget constraint. This paper is motivated by the empirical evidence of fiscal episodes during which public debt-to-GDP is non-stationary and generates no improvement in primary public balance. Under these episodes, fiscal policy *periodically* violates Bohn's sustainability condition and thus raises critical questions on the long-run fiscal sustainability: is a periodically unsustainable fiscal policy a threat to long-run sustainability of public finance? How long can fiscal policy be periodically unsustainable without violating its sustainability constraints in the long-run?

To our knowledge, only a few papers have addressed a regime-switching (or time-varying) fiscal policy rule *while also* proposing a testing framework for long-run sustainability. In their seminal contribution Canzoneri et al. (2001) consider a time-varying fiscal policy rule and derive a necessary and sufficient condition such that the government intertemporal budget constraint holds in the long-run. Davig (2005) extends Wilcox (1989)'s unit-root testing procedure to a Markov-switching framework in which discounted debt can be periodically expanding. Finally, there is a literature on regime-switching monetary and fiscal policy rules that has successfully identified *local* equilibria in the data where fiscal policy (or monetary policy) is either "active" or "passive", following Leeper (1991). Still, these papers do not test whether fiscal policy *globally* satisfies the intertemporal budget constraint or the debt-stabilizing criterion in the long-run. Based on a Markov-switching monetary policy rule, Davig and Leeper (2007b) have proposed a long-run Taylor principle such that the price-level is *globally* determined despite periodic violations of the short-run Taylor principle; but there is no equivalent proposition for a *globally* sustainable fiscal policy. In contrast, we derive a formal test of *global* fiscal sustainability which depends on fiscal regimes' transition probabilities and on their respective durations.

The paper introduces a Regime-Switching Model-Based Sustainability (RS-MBS) test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes. We define unsustainable regimes by periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. violating Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes. We demonstrate how fiscal regimes matter for *global* (in opposition with local) fiscal sustainability analysis.

The paper addresses the two usual concepts of long-run fiscal sustainability: the No-Ponzi game condition (related to the transversality condition) and the debt-stabilizing condition (related to the stationarity of the debt-to-GDP ratio). For each concept of fiscal sustainability, we derive sufficient conditions for long-run (or global) fiscal sustainability which depend on regime-specific feedback coefficients of the Markov-switching fiscal policy rule and on expected durations (or persistence) of fiscal regimes. We show that fiscal policy can be *locally* unsustainable, with a periodically explosive public-debt-to-GDP ratio, and still be *globally* sustainable.[1]

[1] Episodes of a locally-explosive debt which does not lead to global unsustainability or default are theoretically

We apply the formal test to France using a single-equation estimation approach without simultaneously identifying and controlling for monetary policy.[2] This choice is motivated by former empirical studies which find that monetary policy in France has been active and not supportive of fiscal policy, at least since 1979 but also before 1973. Empirical evidence suggests France's monetary policy was passive between 1973 and 1979. We review and discuss evidence on French monetary policy in a dedicated section. Drawing on existing empirical research which concludes that monetary policy has most likely been active during most of the sample, a single-equation approach seems sufficient to identify a sustainable regime.

Our empirical results are threefold. First, we estimate different specifications of Bohn's constant parameter fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on the primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. The existence of two fiscal regimes in France helps reconcile the contrasting results achieved in the literature where some papers conclude that fiscal policy has not been sustainable whereas some others show the opposite.[3] In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. We reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio and conclude that France has achieved global sustainability since 1965.

The rest of the paper is organized as follows. Section 2 reviews the literature on fiscal sustainability. Section 3 presents the extension of the Model-based approach of sustainability to regime switches and develops new conditions for fiscal sustainability. Section 4 deals with an application of the empirical methodology to French data. In Section 5, we discuss the monetary-fiscal policy mix in France. Section 6 concludes.

2 Related literature

Bohn (1998) builds a Model-Based Sustainability (MBS) framework to analyze fiscal sustainability through the lens of fiscal policy rules (or fiscal reaction functions) in a simple general equilibrium model, as an alternative to the econometric analyses *à la* Hamilton-Flavin.[4]. Basically, Bohn as-

investigated in Blot et al. (2016).

[2]According to the Fiscal Theory of the Price Level, this choice may expose us to a possible observational equivalence and to an endogeneity bias if monetary policy has behaved passively, i.e. under-reacting to inflation deviations, see Cochrane (1999), Leeper and Li (2017).

[3]The papers are discussed in section 4.

[4]Seminal empirical investigations on fiscal sustainability proposed a testing framework based on the present-value budget constraint and the transversality condition, drawing on stationarity or cointegration properties of fiscal data (Hamilton and Flavin, 1986, Trehan and Walsh, 1988, 1991, Wilcox, 1989, Wickens and Uctum, 1993, Quintos, 1995). Still, the econometric analysis of fiscal sustainability has raised a number of issues and led to important criticisms by Bohn (1995, 1998, 2007).

sumes the following framework composed of a linear fiscal rule (1)

$$s_t = \gamma b_{t-1} + \mu_t \tag{1}$$

where s_t is the primary surplus-to-GDP ratio, b_t is the end-of-period public debt-to-GDP ratio and finally μ_t is a vector including all cyclical components of primary surplus (e.g. output gap, temporary public spending), plus a constant and an error term. Bohn finds that a strictly positive feedback effect $\gamma > 0$ satisfies the No-Ponzi Game (NPG) condition.[5]

Under a stricter sustainability condition, like a debt-stabilizing fiscal policy rule[6], the feedback effect should be larger than the growth-adjusted real average interest rate on public debt, that is $\gamma > (r - y)/(1 + y)$.[7] MBS analysis has been shown to be empirically powerful in the case of US fiscal policy on long-run data (Bohn, 1998, 2008). On international panel data, Mendoza and Ostry (2008) find evidence that fiscal policy is "responsible" (i.e. there is evidence of a strictly positive feedback rule). In the same vein, Collignon (2012) estimates country-specific fiscal policy rules in Europe. His empirical specification follows closely the SGP which imposes an improvement of the cyclically-adjusted primary balance if the country deviates from the reference values for deficit (3% of GDP) and debt (60% of GDP). He finds that up to 2008 the majority of public debts in Europe were sustainable.

Two types of nonlinear specifications of the fiscal rules exist in the literature. On the one hand, fiscal rules are polynomial functions of public debt-to-GDP ratio, i.e. include quadratic and cubic terms (Bohn, 1998). This specification is motivated by the idea that primary surplus may either react more to lagged public debt or on the contrary may become "flatter" at higher public debt levels. This approach has been followed by Ghosh et al. (2013a,b) to account for "fiscal fatigue" where they derive debt limits as the maximum level of public debt beyond which primary balance can no longer adjust to stabilize debt. On the other hand, fiscal rules are time-varying. The assumption that simple linear policy rules (either monetary or fiscal) are constant over time is not convincing regarding multiple evidence of "structural breaks" or "regime changes". In particular, empirical literature on regime-switching fiscal rules has produced evidence that fiscal rules may be better described by "fiscal regimes", see Favero and Monacelli (2005), Chung et al. (2007), Davig and Leeper (2007a, 2011), Bianchi (2012b), Burger and Marinkov (2012), Afonso and Toffano (2013).[8] This literature generally identifies sub-periods during which fiscal policy does not stabilize public debt, and sometimes even displays a negative feedback effect of initial public debt on the primary surplus.

The literature on regime-switching monetary and fiscal rules builds on Leeper (1991)'s seminal

[5]The Non-Ponzi Game condition states that the present-value of public debt tends to zero in the long-run, which means that the government must pay back at least a part of the interest charges.

[6]Bohn (2007) acknowledges that an upper bound on primary surplus, i.e a fiscal limit, requires a stationary public debt-to-GDP for fiscal sustainability to hold. Research about the upper-bound of primary surplus has been recently explored by Bi (2012), Bi and Traum (2012), Davig et al. (2011), Daniel and Shiamptanis (2013). Daniel and Shiamptanis show that stationarity and cointegration restrictions are necessary for fiscal sustainability when assuming the existence of a fiscal limit. Existence of a fiscal limit (i.e. an upper bound on primary balance-to-GDP and on public debt-to-GDP) requires a sustainability criterion ensuring that public debt is stable around a long-run value compatible with the fiscal limit.

[7]Under a fiscal rule with variables in absolute levels rather than as shares of GDP, the feedback effect should be larger than the real average interest rate on public debt. This is basically what Leeper (1991) finds when he derives the stability conditions of an active monetary/passive fiscal regime.

[8]For monetary policy, see Clarida et al. (2000), Auerbach (2002), Lubik and Schorfheide (2004), among others.

contribution, which developed a set of formal conditions for *local* equilibrium determinacy stemming from the properties of the monetary and fiscal rules. Fiscal policy is *passive* under the debt-stabilizing condition, *active* otherwise[9]. Recent research on fiscal policy (Bi, 2012, Bi and Leeper, 2013) explores regime-switching fiscal policies to derive an endogenous and stochastic fiscal limit. This literature interprets fiscal sustainability as a sovereign default probability, computed from the fiscal limit distribution, rather than as generalized conditions on the regime-switching fiscal rule properties[10]. Davig and Leeper (2007b) define the long-run Taylor monetary principle, based on a Markov-switching Taylor rule, allowing for periodic (or local) violations of the short-run Taylor principle. But, to our knowledge, there is no contribution which proposes and tests analogous conditions on a regime-switching fiscal rule such that NPG and debt-stabilizing conditions hold in the long-run. In this respect, this paper's motivation is similar to Davig and Leeper (2007b) but it applies to fiscal policy.

Finally, this paper is extending three important contributions in the field of fiscal sustainability analysis. First, Canzoneri et al. (2001) investigate theoretically a particular time-varying fiscal policy rule in which public debt feedback effect on the primary surplus is positive or null. They show that the primary surplus has to react positively to public debt on an infrequent basis but "infinitely often" in order to satisfy the government intertemporal budget constraint. Their analysis is restrictive in at least two respects. Firstly, assuming that the primary surplus does not react negatively to initial public debt is a critical assumption, at odds with some empirical evidence on regime-switching policy rules (Favero and Monacelli, 2005, Davig and Leeper, 2007a, 2011, Afonso and Toffano, 2013). Secondly, their sustainability condition does not ensure a stationary public debt-to-GDP ratio, which is probably the relevant fiscal sustainability condition when the economy faces a fiscal limit. We take both limitations into account in our framework. Second, Davig (2005) proposes a unit-root testing framework using a Markov-switching model which accounts for episodes of periodically expanding discounted public debt. This approach is inherently subject to the criticisms addressed by Bohn (1995, 2007) to the econometric analysis of fiscal sustainability. In particular, unit-root testing does not provide any information about fiscal policy behavior since it does not involve an explicit model of fiscal policy. Third, Ascari et al. (2017) study the determinacy regions of a rational expectation DSGE model where both monetary and fiscal policy rules are subject to regime switches. In contrast with both former contributions, we empirically assess the fiscal behavior of public (French) authorities.

3 Theory: Regime-Switching Model-Based Sustainability

We assume a stochastic real endowment and cashless economy composed of a representative rational household and a government. By assuming a real cashless economy, we implicitly assume that monetary policy has full control over the price level and inflation dynamics. Using the terminology of the Fiscal Theory of the Price Level (Leeper, 1991, Sims, 1994, Woodford, 1995, Cochrane, 2005), we only consider Ricardian equilibria for which the government intertemporal

[9]The condition on monetary policy rests on the Taylor Principle: monetary policy is labeled "active" (A) when it reacts agressively to inflation (i.e. the Taylor principle holds) and "passive" (P) otherwise. From these two conditions, Leeper (1991) identifies four local regimes (with F for fiscal policy and M for monetary policy): Monetary regime (AM/PF), Fiscal regime (PM/AF), Indeterminacy regime (PM/PF) and Explosive regime (AM/AF).

[10]Fiscal limit distributions are obtained by numerical approximation of the decision rule in calibrated or estimated Real business cycle models.

budget constraint must hold for any path of the price level. Thus we assume that fiscal policy is the only game in town, and we study the worst-case scenario in which fiscal authorities are left without monetary support to ensure public debt sustainability: what are then the fiscal sustainability requirements and can we reject the null hypothesis of unsustainability (i.e. violation of these requirements)? Rejection of unsustainability in this "worst-case" scenario may be interpreted as credible evidence of sustainability.

3.1 Model

Stochastic real endowment. Total output Y_t is following a unit-root with drift:

$$Y_t = Y_{t-1}(1 + y + \varepsilon_t^y) \tag{2}$$

where $y > 0$ is the long-run growth rate of output and ε_t^y is an i.i.d random shock to the growth rate.

Representative household. Representative household's preferences are represented by the utility function $u(.)$ which is strictly increasing ($u'(.) > 0$) and concave ($u''(.) < 0$) and a subjective discount factor β. At each period, consumer chooses consumption C_t and buys public bond B_t at a price $(1 + r_t)^{-1}$ in order to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the following budget constraint:

$$C_t + (1 + r_t)^{-1} B_t = B_{t-1} + Y_t - T_t$$

First order conditions of the representative consumer's maximization program yield the standard Euler equation:

$$(1 + r_t)^{-1} = \beta \mathbb{E}_t \frac{u'(C_{t+1})}{u'(C_t)} \tag{3}$$

Equation (3) evaluates the stochastic discount factor $Q_{t,1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)}$ at the optimal solution of the representative consumer's program, which is the common pricing kernel of any asset in the economy. Hence, a $j-$period public bond has a price $(1 + r_{t,j})^{-1} = \mathbb{E}_t Q_{t,j}$ with $Q_{t,j} = \beta^j \frac{u'(C_{t+j})}{u'(C_t)}$.

Government. Government spends G_t and collects lump-sum taxes T_t. At each start of period t, government carries one-period public bonds B_{t-1} and it will issue B_t at a price $(1 + r_t)^{-1}$ at the end of period. The government faces the following one-period budget constraint:

$$(1 + r_t)^{-1} B_t = (G_t - T_t) + B_{t-1} \tag{4}$$

with $S_t \equiv T_t - G_t$ representing the primary budget balance. Under balanced growth, all variables in level grow at rate y_t, thus we rewrite the government budget constraint in terms of output ratios:

$$b_t = \frac{1+r_t}{1+y_t} b_{t-1} - (1+r_t)s_t \tag{5}$$

where b_t is the end-of-period debt-output ratio, s_t is the primary surplus-output ratio, r_t and y_t are respectively the real interest rate and the growth rate of real output.

Preventing the government from running a Ponzi scheme against its creditors implies the Present-Value Budget Constraint (PVBC) in equation (6). Following Bohn (1995), we write the PVBC using the stochastic discount factor in order to account for uncertainty and consumer's risk-aversion:

$$B_{t-1} = \sum_{i=0}^{+\infty} \mathbb{E}_t \left[Q_{t,i} S_{t+i} \right] \tag{6}$$

which is equivalent to the following transversality condition (TC):

$$\lim_{T \to +\infty} \mathbb{E}_t \left[Q_{t,T+1} B_{t+T} \right] = 0 \tag{7}$$

Both the PVBC and TC must hold with equality since the representative consumer cannot run a Ponzi Scheme against government (Bohn, 1995).

We assume the following Markov-switching fiscal policy rule:

$$s_t = \gamma(z_t) b_{t-1} + \mu_t(z_t) \tag{8}$$

Regime-switching parameter $\gamma(z_t)$ represents the feedback effect of the initial public debt-output ratio b_{t-1} on primary surplus-output ratio *conditional on* a two-state fiscal regime $z_t \in \{0,1\}$. Fiscal regimes are then defined as:

$$\gamma(z_t) = \begin{cases} \gamma_1 & \text{if} \quad z_t = 1 \\ \gamma_2 & \text{if} \quad z_t = 0 \end{cases} \quad \text{with} \quad \gamma_1 > \gamma_2 \tag{9}$$

During a sustainable regimes primary balance improves following a debt increase, i.e. $\gamma_i > 0$, while it does not improve or even worsens during unsustainable regimes, i.e. $\gamma_i \leq 0$, for $i \in \{1,2\}$[11]. Finally, we define $\mu_t(z_t)$ by:

$$\mu_t(z_t) = \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon_t^s \tag{10}$$

where $\hat{y}_t$ is the output gap, $\hat{g}_t$ is temporary public spending, $\alpha(z_t)$ is a regime-switching constant, $\sigma(z_t)$ is the regime switching standard-error associated to an i.i.d distributed shock $\varepsilon_t^s \sim \mathcal{N}(0,1)$. We assume regime-switches to be stochastic and exogenous, following a hidden two-state Markov process z_t describing fiscal regimes. The use of a Markov-switching model rather than endogenous or threshold-switching models represents an agnostic way of modelling regime changes of fiscal policy: it does not require critical assumptions about what drives fiscal regime shifts. Given

[11] Canzoneri et al. (2010, p.959) discuss empirical results of Davig and Leeper (2007a, 2011) and note that a negative coefficient on lagged debt in the fiscal rule may be difficult to interpret since "regardless of whether the fiscal rule is Ricardian or non-Ricardian, we would expect a positive estimated coefficient". Indeed, Cochrane (2001) shows that there exists a positive correlation between primary surplus and initial debt at *equilibrium*, even when fiscal policy is active (with primary surplus following an AR(1) process). Still, empirical research on regime-switching fiscal policy rules provides some evidence of periodic *negative* feedback effect, see Davig and Leeper (2011) and Afonso and Toffano (2013) for instance; these empirical results motivate our specification of unsustainable fiscal regimes by $\gamma_i \leq 0$.

our economy is Ricardian, we assume that fiscal regime z_t is independent of real output's growth rate.

Define $\gamma = (\gamma_1\ \gamma_2)$ a row-vector containing regime-specific parameters and $Z_t = (z_t\ 1 - z_t)^T$ a column-vector associated to the Markov process z_t. Hence, we can define the scalar $\gamma(z_t)$ by:

$$\gamma(z_t) \equiv \gamma Z_t = \begin{pmatrix} \gamma_1 & \gamma_2 \end{pmatrix} \times \begin{pmatrix} z_t \\ 1 - z_t \end{pmatrix} \tag{11}$$

Markov process z_t is associated to a transition matrix P

$$P = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} \tag{12}$$

whose diagonal elements are transition probabilities p_{ii} for $i \in \{1, 2\}$. Finally, the state vector Z_t evolves as:

$$Z_t = PZ_{t-1} + v_t \quad \text{with} \quad v_t \equiv Z_t - \mathbb{E}_{t-1}[Z_t] \tag{13}$$

We assume z_t to be an ergodic Markov process[12] implying that $\mathbb{E}_t Z_{t+j} = P^j Z_t$ converges to a unique ergodic distribution π:

$$P^j Z_t \underset{j \to +\infty}{\longrightarrow} \pi \tag{14}$$

where $\pi = (\pi_1\ \pi_2)^T$ is the column-vector of ergodic probabilities associated to each fiscal regime. Ergodic probabilities are defined by:

$$\pi_i = \frac{1 - p_{jj}}{(1 - p_{ii}) + (1 - p_{jj})} \tag{15}$$

for all $(i, j) \in \{1, 2\}$. Hence, using equations (11) and (14), the conditional expectation at time t of feedback parameter $\gamma(z_t)$ converges toward its *unconditional* expectation, i.e. ergodic (or long-run) value:

$$\mathbb{E}_t[\gamma(z_{t+j})] = \gamma P^j Z_t \underset{j \to +\infty}{\longrightarrow} \gamma\pi \tag{16}$$

The ergodic property of the Markov chain driving fiscal regimes has implications on the way we model households' expectations. It implies that creditors *expect* that fiscal policy will eventually switch back to a sustainable regime, or equivalently that unsustainable fiscal regimes have a finite expected duration. Thus sustainability conditions derived in the following subsections depend on the assumption that agents observe fiscal regimes and expect that fiscal policy will not be unsustainable forever.

3.2 No-Ponzi Game condition

Following Bohn (1998), we derive a sufficient condition on the sequence $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$ such that the Present-Value Budget Constraint (6) and Transversality condition (7) hold. Denoting the j-period

[12] A Markov process is ergodic as long as $p_{ii} < 1$ and $p_{ii} + p_{jj} > 0$ for all $(i, j) \in \{1, 2\}$ (Hamilton, 1994, Chap. 22), meaning there is no absorbing state.

ahead growth-adjusted stochastic discount factor by

$$\tilde{Q}_{t,j} \equiv Q_{t,j} \prod_{i=0}^{j-1} (1 + y_{t+i}) \tag{17}$$

allows us to rewrite the Transversality condition (7) in terms of debt-output ratio by:

$$\lim_{T \to +\infty} \mathbb{E}_t \left[\tilde{Q}_{t,T+1} b_{t+T} \right] = 0 \tag{18}$$

Then, using the regime-switching fiscal policy rule (8) and iterating on the flow budget constraint of government (5) up to date $t + T$, we obtain an expression for expected present-value debt-output ratio $\mathbb{E}_t \left[\tilde{Q}_{t,T+1} b_{t+T} \right]$ which explicitly depends on $\{\gamma(z_{t+i})\}_{i=0}^{\infty}$. Finally, we find a sufficient condition on the regime-switching fiscal policy rule to satisfy the No-Ponzi Game condition, that allows us to conclude to the following proposition.

Proposition 1 (No-Ponzi Game) *In a dynamically efficient economy, provided that $\mu_t(z_t)$ is bounded and assuming* $\text{Cov}(y_t, z_t) = 0$*, a sufficient condition that transversality condition* (18) *holds is*

$$\gamma\pi > 0 \tag{19}$$

with $\gamma\pi \equiv \gamma_1\pi_1 + \gamma_2\pi_2$ being the unconditional expectation of $\gamma(z_t)$. Using the definition of ergodic probabilities (15) *and denoting expected duration of regimes by $d_i = \frac{1}{1-p_{ii}}$, we can highlight three formal cases:*

1. *If $\gamma_1 > \gamma_2 > 0$, then condition* (19) *is satisfied whatever the durations or ergodic probabilities of regimes, which is equivalent to Bohn (1998), in a Markov-switching framework.*
2. *If $\gamma_1 > 0$ and $\gamma_2 = 0$, then condition* (19) *is satisfied as long as $d_1 > 0$ (i.e. $\pi_1 > 0$) which is equivalent to Canzoneri et al. (2001).*
3. *If $\gamma_1 > 0 > \gamma_2$, then condition* (19) *is satisfied as long as*

$$\gamma_1 > |\gamma_2| \frac{d_2}{d_1} \tag{20}$$

provided that the sustainable regime average duration is strictly positive ($d_1 > 0$).

Proof 1 *See appendix A.1.*

To understand this condition, let us consider the following approximation of the tranversality condition when $T \to +\infty$:

$$\mathbb{E}_t \left[\tilde{Q}_{t,T+1} b_T \right] \approx (1 - (1 + y)\gamma\pi)^T b_t \tag{21}$$

Following Bohn (2008), consider a Ponzi Scheme such that $\{s_t\}_{t=0}^{\infty} = 0$. This Ponzi Scheme implies that the debt-output ratio grows at rate $\frac{r_t - y_t}{1+y_t}$. As a consequence the limit value of future discounted debt-output ratio is equal to initial debt-output ratio (which violates the transversality condition):

$$\mathbb{E}_t \left[\tilde{Q}_{t,T+1} b_{t+T} \right] = b_t \tag{22}$$

Thus, $\gamma\pi > 0$ implies the reduction of $\mathbb{E}_t \left[\tilde{Q}_{t,T+1} b_{t+T} \right]$ by a factor $(1 - (1 + y)\gamma\pi)^T$ relative to a Ponzi Scheme. Saying it differently: the average growth rate of debt-output ratio is reduced by a factor $(1 - (1 + y)\gamma\pi) > 0$.

Condition (19) states that a regime-switching fiscal policy has to satisfy the NPG condition *on average*, that is, sustainable regimes have to be frequent enough to balance unsustainable regimes in the long-run. To rule out a Ponzi Scheme requires that the longer unsustainable regimes *vis-à-vis* duration of sustainable regimes, the larger the reaction of primary surplus to debt during sustainable regimes *vis-à-vis* the reaction during unsustainable regimes. Provided (20) holds, fiscal policy can thus be periodically unsustainable *while* satisfying its PVBC.

3.3 Debt-stabilizing condition

The NPG condition requires no upper bound on the primary balance-to-GDP ratio and consequently does not dismiss "non-stationary" policies, *i.e.* ever-increasing primary balance and public debt-to-output ratios. However, the NPG condition may appear too loose as regards the actual use of fiscal instruments by governments, which leads us to consider a stricter sustainability condition implying a stationary debt-to-output ratio.

Indeed, there are several arguments that support the existence of an upper-bound on the primary surplus-to-output ratio, such that $s_t \leq s^{max}$. Such an assumption can be justified by tax evasion, following Daniel (2014), or by the political inability and/or unwillingness to reduce public spending and increase taxes, following Daniel and Shiamptanis (2013)[13]. Second, political incentives to strategic defaults are likely to increase with the level of debt, e.g. defaulting to reduce the debt service. A debt-stabilizing rule would help to offset these incentives and limit the sovereign default.[14]

Therefore, in our model, a regime-switching fiscal rule implies that debt-output ratio follows a Markov-switching autoregressive process, defined by equations (5) and (8):

$$b_t = \phi(z_t)b_{t-1} + u_t(z_t) \tag{23}$$

where

$$\phi(z_t) = \frac{1+r_t}{1+y_t}(1-(1+y_t)\gamma(z_t)) \quad \text{and} \quad u_t(z_t) = -(1+r_t)\mu_t(z_t).$$

A sufficient condition for (strict) stationarity of stochastic processes like (23) is given by Kesten (1973), from which we deduce the following proposition.

Proposition 2 (Debt-stabilizing condition) *Drawing on log approximation, a sufficient condition for a (strictly) stationary debt-output ratio is*

$$\gamma\pi > \frac{r-y}{1+y} \tag{24}$$

Focusing on cases with $\gamma_1 > 0 \geq \gamma_2$, condition (24) can be expressed in terms of expected durations:

$$\gamma_1 > |\gamma_2|\frac{d_2}{d_1} + \frac{r-y}{1+y}\frac{d_1+d_2}{d_1} \tag{25}$$

[13] In a framework with distortionary taxation, the fiscal limit would arise endogenously from the existence of a dynamic Laffer curve, see Bi (2012), Bi and Leeper (2013).

[14] The model formally abstracts from default but yet, we could consider that the real interest rate r_t is equal to the risk-free real rate plus a default-risk premium, depending on the level of debt-to-output ratio and study a sustainability condition ensuring the stability of debt-to-output ratio to reduce incentives for a government to default. Under the Debt-Stabilizing condition, this would imply that the government must raise (on average) the primary surplus-to-GDP ratio by more than the average risk-free real interest rate *plus* the average default risk premium.

Proof 2 *See appendix A.2.*

Provided conditions (24) or (25) hold, then the public debt-output ratio has an ergodic mean:

$$\mathbb{E}[b_t] = \frac{-\mathbb{E}[(1+r_t)\alpha(z_t)] + \text{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1-\phi(z_t)]} \tag{26}$$

where $\mathbb{E}[\alpha(z_t)] < 0$ is the ergodic value of $\alpha(z_t)$.

3.4 Discussion

As long as the growth-adjusted real interest rate is positive, a debt-stabilizing condition is stricter than the NPG condition. During sustainable regimes, the required reaction of primary surplus to initial debt must be large enough to compensate for both primary deficits during unsustainable regimes, weighted by the ratio of expected durations, and the growth-adjusted real interest rate, weighted by the inverse fraction of (expected) time spent in sustainable regimes. On the contrary, when $r < y$, condition (25) could imply government is violating NPG condition (20) which is the minimum requirement for fiscal sustainability. As Blanchard (2019) recently emphasized, the US economy as well as many OECD economies has historically experienced long lasting episodes of $r < y$, especially before the late 1970s –and the current low real interest rates with respect to growth could allow them to benefit from negative growth-adjusted real interest rate to stabilize debt almost without fiscal cost[15].

More generally, the Model-Based Sustainability approach illustrates why testing stationarity of debt-output ratio may sometimes be misleading as a test of fiscal sustainability. Abel et al. (1989) have shown that a dynamic efficient economy requires that the risky real rate –not the safe real rate– should be greater than the growth rate. And given the difficulty to measure precisely the risky real rate in the economy, they advocate to test for dynamic (in)efficiency by looking whether aggregate interest plus dividends are greater than investment in percentage of total economy output, which is a sufficient condition for dynamic efficiency[16]. As a result, evidence of $r < y$ does not imply that Ponzi schemes are feasible and optimal (Diamond, 1965, Bohn, 1995). On the contrary, in an dynamically efficient economy with $r < y$, NPG condition and debt-stabilizing condition can be viewed as complements rather than substitutes: a stationary public debt-output ratio does not always rule out Ponzi Schemes, which are non-feasible.

Sustainability conditions presented in propositions 1 and 2 are different from determinacy conditions in Ascari et al. (2017).[17] First, both propositions 1 and 2 are derived directly from the non-linear model, and not from a log-linearized model . It is an important distinction since, given the regime-switching nature of fiscal policy, the economy admits a steady-state debt-income ratio only during sustainable regimes. Second, we do not study determinacy, i.e. conditions for a unique, stable, rational expectation equilibrium but rather *equilibria* for which fiscal policy satisfies the NPG condition and/or stabilizes the debt-income ratio in the long run.

The assumption of the existence of different fiscal regimes may, in general, imply that the public debt-output ratio can periodically follow an explosive path. To see why, let us consider

[15] The "fiscal cost" is the required primary surplus to stabilize debt, which is negative as long as $r < y$.

[16] See Geerolf (2018) for a recent reassessment of dynamic (in)efficiency in OECD economies, in the context of global saving gluts and low interest rates.

[17] Ascari et al. (2017) apply the perturbation methods for Markov-Switching DSGE models proposed by Foerster et al. (2016) which requires a log-linearization around an ergodic steady state.

the example of Canzoneri et al. (2001) and assume $\gamma_2 = 0$. We find exactly the same proposition: based on equation (19), any infrequent $\gamma_1 > 0$ would be sufficient to rule out Ponzi Schemes (See Proposition A.1). Yet, this equilibrium does not ensure a stable debt-output ratio, that is public debt is $I(1)$. For a stable debt-output ratio, assuming $r - y > 0$ and $\gamma_2 = 0$, a regime-switching fiscal policy must satisfy the following condition, from equation (25):

$$\gamma_1 > \frac{r-y}{1+y}\frac{d_1+d_2}{d_1} \tag{27}$$

Under a regime-switching debt-stabilizing fiscal policy, the debt-output ratio becomes periodically explosive, and explosive regimes can be really frequent without necessarily implying that the debt-output ratio is globally non-stationary.

Periodic explosive dynamics of public debt has critical consequences on regime-switching policy rules, not only on $\gamma(z_t)$ but also on the constant $\alpha(z_t)$. Rewriting equation (8) in terms of deviations of primary balance and public debt from their respective steady-state values $s^*(z_t) = (s_1^*, s_2^*)$ and $b^*(z_t) = (b_1^*, b_2^*)$ yields:

$$s_t - s^*(z_t) = \gamma(z_t)\big(b_{t-1} - b^*(z_t)\big) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + \sigma(z_t)\varepsilon_t^s \tag{28}$$

from which we deduce that $\alpha(z_t)$ is equal to:

$$\alpha(z_t) = s^*(z_t) - \gamma(z_t)b^*(z_t) \tag{29}$$

In a sustainable regime, primary surplus-output ratio s_t and debt-output ratio b_t must admit steady-state values[18]. Provided condition (25) holds, we would expect s_1^* to be equal to the debt-stabilizing primary surplus ratio, for a stationary debt-output target ratio b_1^*:

$$s_1^* = \frac{r-y}{1+y}b_1^* \tag{30}$$

which implies:

$$\alpha_1 = (\frac{r-y}{1+y} - \gamma_1)b_1^* < 0 \tag{31}$$

provided that condition (24) holds, which would account for negative estimates of α_1 but also $\mathbb{E}[\alpha(z_t)] = \pi_1\alpha_1 < 0$ if $\gamma_2 < 0$. As a consequence, insofar as $b_t < b_1^*$ fiscal policy can run primary deficits without necessarily jeopardizing fiscal sustainability.

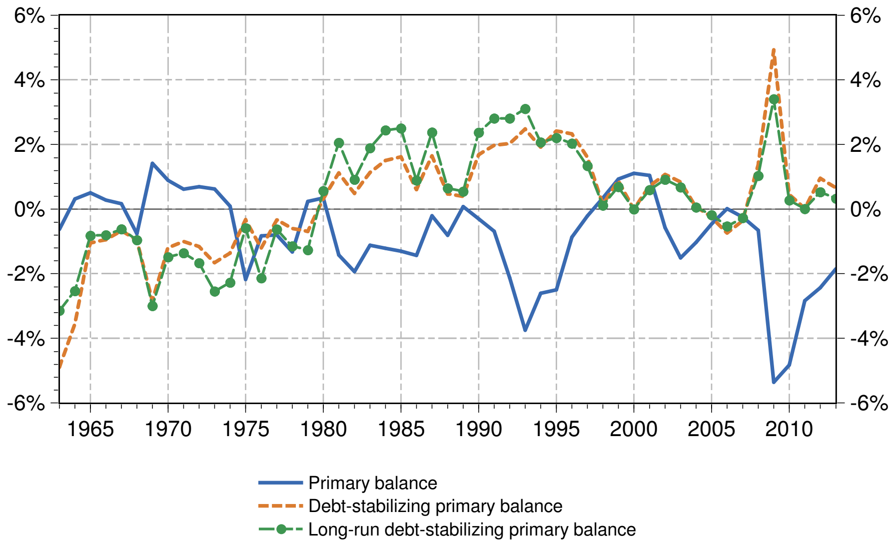
4 Empirical analysis

We apply Regime-Switching MBS analysis to French data. Empirical investigation on French debt sustainability has given rise to contradictory outcomes: Afonso (2005), Lamé et al. (2014), Schoder (2014) did not find evidence of a sustainable fiscal policy in France whereas Afonso and

[18] Under the unsustainable regime and periodic explosive dynamics of public debt, the time series properties of s_t can be twofold, depending on the value of γ_2. When $\gamma_2 < 0$ we expect α_2 to be equal to zero. Explosive debt-output ratio dynamics are not compatible with any steady-state debt-output level, hence $b_2^* = 0$. Then, primary balance would be necessarily non-stationary since the two variables would be negatively cointegrated with $\{b_t\}$ being non-stationary, implying $s_2^* = 0$. Otherwise, if $\gamma_2 = 0$, $\{s_t\}$ could be stationary and then s_2^* would be significantly different from zero.

Jalles (2016), Chen (2014), Fincke and Greiner (2012), Weichenrieder and Zimmer (2014) reached mixed evidence; in contrast, Greiner et al. (2007) found that French fiscal policy was sustainable.

Figure 1: Primary balance and debt-stabilizing primary balance in France (1965 – 2013)



Source: OECD, authors' calculations.
Note: the debt-stabilizing primary balance is calculated as $(r_t - y_t)b_{t-1}/(1 + y_t)$ where r_t, y_t and b_{t-1} respectively denote the real interest rate, the real growth rate and the gross debt-to-GDP ratio. The long-run debt-stabilizing primary balance is calculated similarly but using the empirical mean of gross debt-to-GDP ratio (54.7% of GDP, sample 1963-2013).

Figure 1 plots the primary balance and the debt-stabilizing primary balance, which is not a measure of fiscal sustainability: a government *does not need* to stabilize its debt-to-GDP ratio at each period. Nevertheless, we can observe different sub-periods during which primary surplus and debt-stabilizing primary surplus converged or diverged. From 1965 to the end of the 1970s, the primary surplus was most of the time larger than or equal to the debt-stabilizing primary surplus. From the beginning of the 1980s to the mid-1990s, the primary surplus was constantly lower than its debt-stabilizing level. Finally, the period between the mid-1990s and the Great Recession in 2009 displays relative convergence. These different sub-periods motivate the use of non-linear econometric models, e.g. regime-switching models, to investigate fiscal sustainability.

We develop a two-stage empirical strategy. First, we estimate fiscal rules in France following Bohn's MBS tests. From these tests, we conclude that French public debt is not sustainable. This seems at odds with the mere observation of French sovereign interest rates which have been historically low during the European sovereign debt crisis and convey information on lenders' seemingly expectations that fiscal policy is on a sustainable path. Second, we estimate a Markov-switching fiscal rule and perform a Regime-Switching MBS test. The outcomes challenge the former results obtained with standard techniques: the existence of a locally unsustainable regime cannot be automatically interpreted as global unsustainability. We conclude that omitting fiscal regime switches may lead to reject mistakenly French sustainability. Another advantage of the RS-MBS approach is that it dates sub-periods of sustainability and unsustainability. In retrospect, it permits to check whether these sub-periods fit the history of French public finances.

4.1 Dataset

The choice of annual data is guided by two arguments: availability on a long time span and consistency with the fiscal institutional process. First, fiscal sustainability can only be appreciated in the long-run: PVBC or stationarity might only be satisfied in the long-run –over half a century, or more. Regarding data availability for France, we are forced to renounce using *true* quarterly data which are only available after 1995-Q4 for public debt[19]. Still, a second argument prevents us from using quarterly data: fiscal decisions are taken on an annual basis in the law of finance, despite some infra-annual adjustments. Using quarterly data may result in spurious results as it may add noise to the *true* response of primary balance to the initial stock of debt.

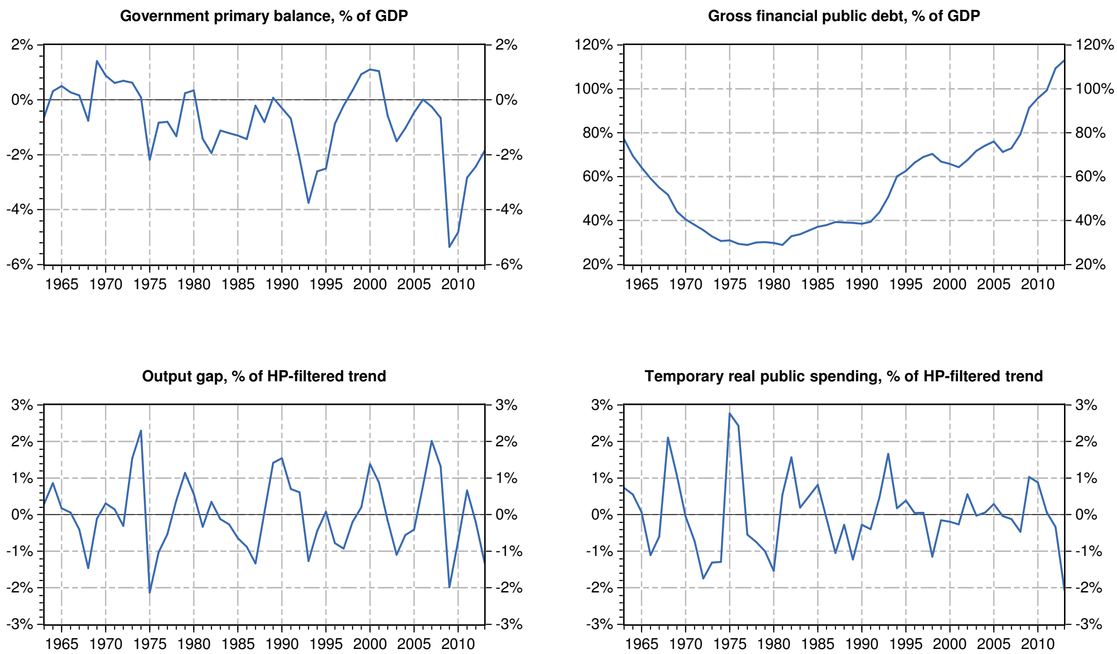
This paper uses the longest time series available for French public debt. Indeed, because of changes in national accounts systems, it is relatively hard to find historical data on French public debt. Most of available time series (in particular, those using Maastricht debt definitions) start by 1978. The IMF Historical Public Debt Database (HPDD) proposes a long-run time series for public debt, but still with missing observations for years 1978 and 1979, because of national accounting issues. Regarding public debt, we use the OECD government total gross financial liabilities rather than the Maastricht definition of gross public debt since the OECD series goes back to 1969. As in Auerbach and Gorodnichenko (2017), we complete this series by backward interpolation using the budget identity $B_t = B_{t-1} + DEF_t$ where B_t is the nominal stock of debt and DEF_t is the nominal with-interests deficit. As a result, for $t < 1969$, public debt at time $t-1$ is equal to public debt at time t minus the government overall budget balance at time t. This backward interpolation implicitly assumes that there were no stock-flow adjustments between 1963 and 1968. This is not a strong assumption on this period. Stock-flow adjustments are more substantial under large financialisation of public assets and liabilities and when public debts can be denominated in a foreign currency. Financialisation in France has started in the 1980s and public debt remains almost entirely denominated in the domestic currency. Regarding time convention in national accounts, public debt stock is the *end-of-period* stock of debt.

Overall budget balance and primary budget balance (budget balance *minus* interests paid) are taken from OECD database for years 1977-2013; observations for years 1963 to 1977 are completed using data collected by Creel and Le Bihan (2006) from French National Institute for Statistics and Economic Studies (INSEE). We build time series for output gap and temporary government spending by detrending and removing the cyclical component of real GDP and real government spending using the HP filter. Regarding the estimation of output gaps, many competing techniques are available and their relative strengths and weaknesses still discussed (see Cotis et al. (2005) for a survey of estimation methods). Our choice of the HP-filtered method has been motivated by its easiness, fastness and recent use by Fincke and Greiner (2012) on debt sustainability and, with more sophistication, by Borio et al. (2014). We follow Ravn and Uhlig (2002)'s rule and use a smoothing parameter $\lambda = 6.25$. To address the end point bias problem of the HP filter, we add univariate 3-year ahead forecasts for each series, using ARIMA models, prior to filtering and then dropping the last 3 observations.[20] Such a "mechanic" correction of the end point bias is

[19] It is possible to build a quarterly measure for public debt using interpolation methods and quarterly government budget balance. Indeed Lamé et al. (2014) report the use of recalculated quarterly data of net French public debt, though on a shorter time span (1980Q1-2007Q4) than the one used in this paper.

[20] We also drop the first 3 observations at the beginning of filtered series which are affected by the end point bias.

Figure 2: Dataset overview, France (1962-2013)



Source: OECD, authors' calculations.
Note: for HP filtered temporary public spending and output gap, we follow Ravn and Uhlig (2002)'s rule and use a smoothing parameter $\lambda = 6.25$ on annual data.

applied, for instance by the European Commission (Havik et al., 2014), while using the HP filter. Finally, our dataset covers 51 years of annual data, from 1963 (1962, for gross public debt) to 2013. Data are reported in Figure 2.

4.2 Model-Based Sustainability tests

We estimate different specifications of a standard fiscal policy rule and use constant-parameter estimates as a benchmark for comparison with Regime-Switching estimates. We specify the following fiscal rule, based on equation (1):

$$s_t = \gamma b_{t-1} + X_t' \beta + \varepsilon_t \tag{32}$$

where the dependent variable s_t is the primary balance-to-GDP ratio, b_{t-1} is the public debt-to-GDP ratio at end of period $t-1$ and X_t is a vector of control variables. It includes a constant, output gap $\hat{y}_t$, and cyclical government spending $\hat{g}_t$ as suggested by Bohn (1998). Then we include a dummy variable FinCrisis_t equal to one for years 2008–2013 in order to account for severe crisis years. To account for potential non-linearities regarding the level of debt, we also estimate fiscal rules as polynomial functions of debt-to-GDP ratio following Bohn (1998) and Mendoza and Ostry (2008). Finally, we account for a potential deterministic time trend, as suggested by unit-root and stationarity tests (available in the technical appendix). We correct for serially correlated residuals of order one or two, depending on the estimated model.

Table 1 presents the results. Based on these estimates of constant-parameter fiscal policy rules,

Table 1: Constant-parameter fiscal policy rules

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Lagged Debt b_{t-1}	-0.0121	-0.0058	0.0283	0.0300*	0.0962	0.0547	0.0735
	(-0.71)	(-0.35)	(0.93)	(-1.72)	(1.50)	(0.86)	(1.38)
Quadratic debt b_{t-1}^2	–	–	–	–	-0.0555	-0.0429	-0.0367
					(-1.18)	(-0.87)	(-0.86)
Constant	-0.0025	-0.0052	0.0014	-0.0065	-0.0190	-0.0219	-0.0179
	(-0.24)	(-0.57)	(0.16)	(-0.96)	(-1.01)	(-1.15)	(-1.19)
Output gap $\hat{y}_t$	0.4190***	0.3807***	0.4800***	0.4527***	0.4565***	0.4163***	0.4360***
	(3 .38)	(3.23)	(3.91)	(3.56)	(3.64)	(3.21)	(3.38)
Temporary spending $\hat{g}_t$	-0.4053***	-0.3667***	-0.3448***	-0.3763***	-0.3754***	-0.4147***	-0.3982***
	(-3.18)	(-3.09)	(-2.73)	(-2.91)	(-2.85)	(-3.08)	(-2.98)
FinCrisis$_t$	–	-0.0179***	–	-0.0160**	–	-0.0112	-0.0131
		(-2.95)		(-2.25)		(-1.37)	(-1.70)
Trend	–	–	-0.0009	-0.0006**	-0.0008	–	-0.0006**
			(-1.55)	(-2.16)	(-1.61)		(-2.06)
DW	1.98	1.99	1.70	1.87	1.68	1.81	1.83
Adj. R^2	0.70	0.75	0.72	0.73	0.72	0.70	0.72
Observations	49	49	50	50	50	50	50

Notes: t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). Models (1)–(2) are controlling for second-order serial correlation in the residuals. Model (3)-(7) control for first-order serial correlation in the residuals.

we find no evidence of fiscal sustainability[21]. Models (1)–(2) give no positive feedback effect, but rather negative though non-significant estimates for γ. We do not find evidence of a polynomial specification of the fiscal policy rule, since coefficients on debt b_{t-1} and quadratic debt b_{t-1}^2 are never significant. Still, point estimates for polynomial specifications would imply a "flattening" of the fiscal policy rule for high debt-output ratios.

Unit-root and stationarity tests conclude to the potential presence of deterministic time trends respectively negative in s_t and positive in b_t. Thus we control for a deterministic trend in equation (32), in models (3)-(5) and (7) of Table 1. With a time trend, the feedback coefficient on initial debt turns out to be positive, but never significant at 5% level. Only model (4) shows a positive but weakly significant (at 10% level) feedback response of primary surplus to initial debt. Moreover, deterministic trends enter negatively in all equations, which is difficult to reconcile with bounded debt-to-output and primary balance-to-output ratio.

4.3 Regime-Switching Model-Based Sustainability test

We estimate the following Markov-switching fiscal rule by direct maximisation of the log likelihood (Hamilton, 1989):

$$s_t = \gamma(z_t)b_{t-1} + \alpha(z_t) + \alpha_y(z_t)\hat{y}_t + \alpha_g(z_t)\hat{g}_t + u_t \tag{33}$$

[21] This result contrasts with Fincke and Greiner (2012) who find a significant positive reaction of the primary surplus to debt. Two differences with our approach are worth mentioning. First, Fincke and Greiner do not strictly reproduce Bohn's fiscal rule: they limit cyclical public spending to spending related to the social insurance system though some of these expenditures may be structural; second, their sample is shorter (1970-2008).

where except the autoregressive residuals and the error variance[22], all remaining parameters can periodically shift between two values, according to a hidden two-state Markov-process z_t.

Numerical optimization of the log likelihood function is raising identification issues, so we choose the following estimation strategy. We randomize the estimation algorithm by drawing 500 starting values and running initial ML estimations with 100 iterations on each draw, in order to reduce the dependence of the ML algorithm on starting values and thus the risk of reaching a local maximum of the log likelihood function; the main estimation algorithm begins using the starting values for which the maximization algorithm reached the highest value of the log likelihood function among the 500 initial random draws. Regarding model specification, we start estimating the most general model, allowing *all parameters, including error variance* to switch between regimes 1 and 2, thus being agnostic on the true structural form of the regime-dependent fiscal rule. At this stage, if the maximization algorithm converges, we can already appreciate how precise the resulting estimates are, both across regimes and in the long-run through the computation of the ergodic value of each parameter. This can be achieved through basic t-statistics and F-statistics analysis. We also look carefully at estimated regimes' properties: transition probabilities associated to the Markov process and filtered and smoothed regime probabilities. We check, in particular, if they are consistent with historical knowledge on fiscal policy shifts, and if they are sufficiently persistent, regarding the timing of fiscal policy.

If any subset of parameters were non-significantly different from zero in *both regimes* or if they were not taking significantly different values across regimes it would be a strong motivation to estimate a restricted model in which this subset of parameters would be regime-invariant. Thus, if any restricted model can be successfully estimated, that is, if the maximization algorithm successfully converges, then the same procedure as described before can be applied.

As a result of our estimation strategy, equation (33), without regime heteroskedasticity, seems to be the best specification of the Markov-switching fiscal policy rule.[23] We also estimated a model with a regime-invariant deterministic trend. We conclude to a non-significant (at 5% level) deterministic trend, while other parameters' estimates do not change with respect to the baseline model, so we choose to exclude the deterministic trend from our baseline specification; results are presented in section 4.4.

Given the short length of the sample, we acknowledge that ML estimates must be considered with caution. Yet, given the potential presence of unit-root in the debt-to-GDP ratio, with stationary primary balance-to-GDP ratio, estimates of a *constant-parameter* fiscal policy rule would be equally dubious. Yet this paper builds on the idea that a non-linear fiscal policy behavior implies periodical explosive dynamics of the public debt-to-GDP ratio, without necessarily implying either instability of public debt-to-GDP ratio or Ponzi schemes in the *long run*.

Table 2 presents estimation results of equation (33). We report estimated parameters for each regime and we also compute implied long-run estimates of regime-switching parameters using ergodic probabilities. Standard deviations of long-run estimates are obtained using standard de-

[22]To account for first-order serial correlation in the data, we assume: $(1-\rho L)u_t = \sigma\varepsilon_t$ with an i.i.d. error term $\varepsilon_t \sim \mathcal{N}(0,1)$

[23]We have also estimated an alternative specification with regime heteroskedasticity. While the MLE successfully converged, our results appeared a posteriori to be highly dependent on initial values for estimation algorithm and they might be a local maximum of the log likelihood function. That is the main reason why we increased the number of random draws at the start of the estimation process. After having randomized the estimation algorithm, we no longer obtain successful convergent ML estimates of an equation with regime heteroskedasticity.

viations and covariance of regime-specific parameters: for any regime-switching parameter $\alpha(z_t)$ which takes two values (α_1, α_2), with associated standard deviations $(\sigma_{\alpha_1}, \sigma_{\alpha_2})$ and covariance $\text{Cov}(\alpha_1, \alpha_2)$, we compute the long-run (ergodic) estimate α using ergodic probabilities (π_1, π_2) by:

$$\alpha \equiv \alpha_1\pi_1 + \alpha_2\pi_2$$

and with standard deviation:

$$\sigma_\alpha \equiv \sqrt{(\pi_1\sigma_{\alpha_1})^2 + (\pi_2\sigma_{\alpha_2})^2 + 2\pi_1\pi_2\,\text{Cov}(\alpha_1, \alpha_2)}$$

The results raise some comments. First, France's fiscal policy is well described by a two-state Markov-switching policy rule. One regime is sustainable with a strongly positive and significant correlation between primary balance s_t and initial debt b_{t-1}, implying a stable debt-to-GDP ratio, while the other one shows a non-significant positive correlation. As expected, the constant is significantly negative in the sustainable regime, which is consistent with a debt-stabilizing fiscal policy, while non-significant in the unsustainable regime, as explained in subsection 3.3. Second, both regimes appear to be strongly persistent with expected durations of 8.1 and 11.9 years respectively for sustainable and unsustainable regimes. This would explain why OLS estimates were inconclusive about the long-run correlation between primary surplus and initial debt in Table 1.

Evidence of an unsustainable regime with a non-significant correlation (which should be considered equal to zero) between primary surplus-to-GDP and lagged debt-to-GDP ratios is very interesting in the context of "observational equivalence" discussed by Cochrane (1999) and Creel and Le Bihan (2006). Cochrane argues that a positive correlation between primary surplus and debt may not be credible evidence of a Ricardian fiscal policy. He notably provides a powerful theoretical example of a non-Ricardian policy, described by an exogenous AR(1) process for the primary surplus, which nonetheless produces a positive correlation between primary surplus and lagged public debt *at equilibrium*. More generally, the Fiscal Theory of the Price Level on which Cochrane builds his argument suggests that equations of "fiscal policy rules" are not describing the behavior of fiscal policy but rather equilibrium relationships. The criticism may certainly be strong against constant-parameter models provided a positive correlation exists. It does not (fully) apply to regime-switching models where subperiods with a positive correlation alternate with subperiods over which the positive correlation disappears as is the case in our estimates. The two regimes show different characteristics and the latter is clearly a non-Ricardian regime.

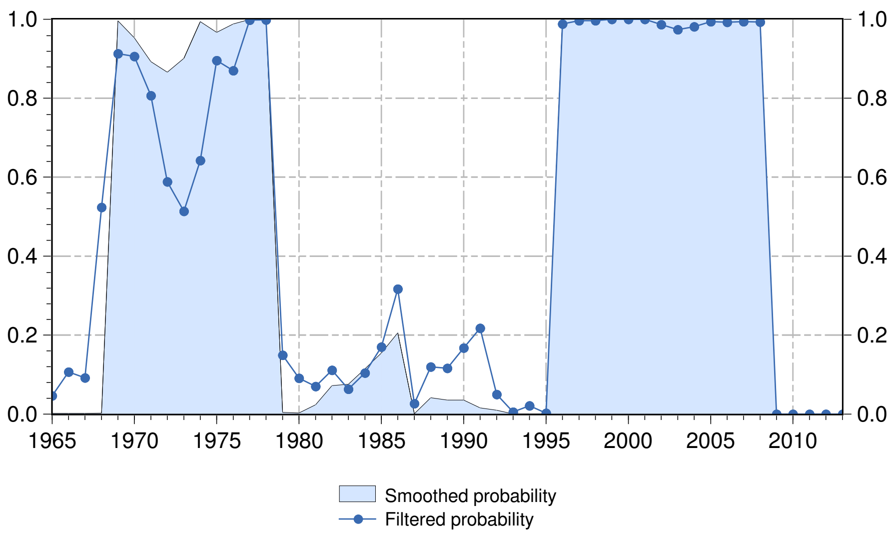
Figure 3 represents estimated smoothed and filtered probabilities for regime 1 which we label "sustainable". Results show a succession of periods of unsustainable or sustainable fiscal policies with marked decades. Public finances were mostly sustainable in the 1970s whereas they were mostly unsustainable in the 1980s. The sharp shift in fiscal regimes between the late 1970s and early 1980s may have been triggered by the change of government. After decades of right-wing governments, the Socialist party won the presidential election of 1981. France then moved from an industry-oriented policy to a mix of social orientations (e.g. adoption of the 39-hour week and of the fifth holiday week for workers) and extension and deregulation of financial markets. These changes provoked an increase in public deficits and reliance of public debt issuances on financial markets, respectively. Filtered probabilities also show a small and transitory increase in the prob-

Table 2: Estimated Markov-switching fiscal rule for France (1965–2013)

Regime-switching parameters	**Regime 1**	**Regime 2**	**Long-run coefficients**
Constant	-0.061	-0.026	-0.040
	(-1.315)	(-0.602)	(-0.907)
Lagged debt b_{t-1}	0.089**	0.002	0.037
	(2.317)	(0.061)	(1.156)
Output gap $\hat{y}_t$	0.421***	0.289**	0.343***
	(2.684)	(2.388)	(4.172)
Temporary spending $\hat{g}_t$	-0.064	-0.549***	-0.352***
	(-0.576)	(-4.301)	(-3.468)
Regime-invariant parameters			
Persistence ρ	0.944***		-
	(9.357)		
Standard-error σ	0.005***		-
	(8.010)		
Regimes properties	**Transition prob.** p_{ii}	**Ergodic prob.** π_i	**Exp. duration** d_i
i=1	0.92	0.59	11.9
i=2	0.88	0.41	8.1
Durbin-Watson stat	1.77	Log likelihood	180.37

Notes: Huber-White robust t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). We control for regime-invariant first-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log \hat{\sigma}$: consequently, standard errors and t-statistics are obtained applying the Delta method. For regime-switching parameters we compute "long-run estimates" as defined earlier.

Figure 3: Estimated sustainable regime, baseline model, France (1965-2013)



Source: OECD, authors' calculations.
Note: Filtered probability of regime 1 (i.e. the sustainable regime) at year t is conditional on information from $t_0 = 1965$ to t and smoothed probability at year t is conditional on information from $t_0 = 1965$ to $T = 2013$.

ability of being in a sustainable regime during the so-called "Tournant de la rigueur" of 1983-1986 when the Socialist government finally turned towards disinflation and deficit-reduction policies. In the 1990s, fiscal policy becomes gradually sustainable (or passive to use Leeper's terminology) and actually so by 1996, until 2008 and the advent of the Great Recession. This finding supports the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually

Table 3: Regime-Switching MBS: unilateral versus bilateral tests

	t-stat	Unilateral test p-value	Bilateral test p-value
No-Ponzi Game condition (20)	2.316804	0.0131	0.0262
Stable long-run debt-to-GDP ratio (25) (r-y)/(1+y) = 0.3%	2.170977	0.0182	0.0364

Notes: Student tests assume γ_2 is equal to 0, based on theoretical analysis in Section 3. Real interest rate is the ex-post real 10-year yield on French public bonds, obtained using the implicit GDP deflator from OECD Economic Outlook database.

made France's fiscal policy more sustainable, despite it being under an Excessive Deficit Procedure from 2003 to 2007. In contrast with Weichenrieder and Zimmer (2014) who show that Euro membership of France has reduced the responsiveness of the primary surplus to debt, results show that the 1999-2011 period (Euro membership years in Weichenrieder and Zimmer) should be separated because it was heterogeneous as regards fiscal responsiveness: it was positive until 2008 and then negative. Overall, results are consistent with a historical analysis of France's fiscal policy.

The long-term estimate of $\gamma\pi$ is positive, equal to 3.7% but non-significant (with a p-value equal to 0.2553). This result raises two comments. First, the long-run estimate of $\gamma\pi$ appears non-significant mainly from the fact that the estimate of γ_2 is strongly non-significant (i.e. with a large estimated standard-error), and thus might be considered equal to 0. Second, significance tests are not appropriate to test for NPG and debt-stabilizing conditions on $\gamma\pi$ since they are *bilateral* tests. On the contrary, Propositions 1 and 2 call for *unilateral* tests for which critical values are lower with respect to bilateral tests.[24] We report the results of these tests in Table 3.

Assuming that γ_2 is equal to 0, we find significant evidence that fiscal policy not only satisfies the No-Ponzi Game condition (Proposition 1) but also the Debt-stabilizing condition (Proposition 2). In other words, given past history of French fiscal policy and fiscal regimes, we find significant evidence that fiscal policy has been sustainable all over the period 1965-2013, despite a prolonged period of unsustainability from 1979 to 1995.

In the vein of Mendoza and Ostry (2008) we use point estimates (reported in Table 2) and historical average for the real interest rate and real GDP growth rate to compute the expected debt-to-GDP ratios, neglecting the covariance terms, under two alternative scenarios (Table 4). In scenario 1, we suppose sustainable regimes last longer and we increase their expected duration (or persistence) while keeping the expected duration of unsustainable regimes constant and equal to its estimated value. In scenario 2, we suppose unsustainable regimes are shorter and we decrease their expected duration while keeping the expected duration of sustainable regimes constant and equal to its estimated value. Computations indicate that gross public debt-to-GDP ratio would reach an average value of 121% across fiscal regimes, which may be interpreted high and able to cause solvency problems. However, this approach does not pretend solvency problems would be ruled out with certainty by a debt-stabilizing fiscal policy rule.[25] Using regime-switching mod-

[24] For instance, a bilateral test of the NPG condition on the parameter $\gamma\pi$ is built upon the null hypothesis $\gamma\pi = 0$ against the alternative $\gamma\pi \neq 0$, whereas the unilateral test is built upon the null hypothesis $\gamma\pi = 0$ against the alternative $\gamma\pi > 0$ which is an adequate testing hypothesis in the context of sustainability.

[25] Daniel and Shiamptanis (2013, p.2308) argue that "*a country following a responsible fiscal rule could still encounter solvency problems due to negative shocks or due to future plans which are insolvent. However, a country following a fiscal rule which is not responsible will encounter solvency problems with certainty.*"

els, this paper proposes a non-linear test to discriminate between "obviously" unsustainable fiscal policies and "most likely" sustainable ones, given fiscal policy can periodically deviate from sustainability requirements. We do not propose a measure of "fiscal space" or "fiscal vulnerability". Moreover, the computed expected debt-to-GDP ratio cannot be interpreted as a long-run steady-state ratio in the usual sense. It represents a long-run average between a regime where public debt follows stable dynamics and a regime with explosive public debt. In particular, assuming $d_1 \to +\infty$ or equivalently $d_2 = 0$, we obtain the underlying debt-to-GDP target ratio $b_1^* = 71\%$ towards which public debt converges during sustainable regimes.[26]

Finally, we show how the debt-to-GDP ratio varies with the level of the growth-adjusted real interest rate given our point estimates, in table 5. A modest increase (resp. decrease) in the growth-adjusted real interest rate results in a significant increase (resp. decrease) of the long-run average public debt-to-GDP ratio.

Table 4: Expected regime durations and Debt-GDP ratios using market long-term interest rate

Scenario 1: Increasing expected duration of sustainable regime						
d_1	π_1	π_2	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2	0.14	0.86	1.27%	Satisfied	Yes	313%
4	0.25	0.75	2.23%	Satisfied	Yes	178%
7	0.37	0.63	3.28%	Satisfied	Yes	129%
8.1	0.40	0.60	3.59%	Satisfied	Yes	121%
15	0.56	0.44	4.94%	Satisfied	Yes	97%
30	0.72	0.28	6.35%	Satisfied	Yes	84%
60	0.83	0.17	7.41%	Satisfied	Yes	77%
∞	1.00	0.00	8.88%	Satisfied	Yes	71%
Scenario 2: Decreasing expected duration of unsustainable regime						
d_2	π_1	π_2	$\gamma\pi$	NPG condition	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
50	0.14	0.86	1.24%	Satisfied	Yes	322%
30	0.21	0.79	1.89%	Satisfied	Yes	207%
15	0.35	0.65	3.12%	Satisfied	Yes	134%
11.9	0.41	0.59	3.60%	Satisfied	Yes	121%
6	0.58	0.42	5.11%	Satisfied	Yes	95%
3	0.73	0.27	6.49%	Satisfied	Yes	83%
1	0.89	0.11	7.91%	Satisfied	Yes	75%
0	1.00	0.00	8.88%	Satisfied	Yes	71%

Notes: Debt-output ratios are computed from equation (26) neglecting covariance terms. For scenarios 1 and 2, we use average market long-term interest rate $r = 3\%$, average real growth rate $y = 2.68\%$ and $r - y = 0.32\%$ (sample: 1963-2013). In scenario 1, we compute expected debt-output ratios under various values of d_1 and for $d_2 = 11.9$. In scenario 2, we compute expected debt-output ratios under various values of d_2 and for $d_1 = 8.1$. All others parameters are constant and equal to point estimates obtained in table 2, except γ_2 which is set to 0.

4.4 Alternative specification

In former estimates of constant-parameter fiscal policy rules, the (rare) significance of a negative deterministic trend, coupled with a non-significant positive reaction of primary surplus to public debt, raised concerns about fiscal sustainability. We check whether this finding remains in

[26]This level cannot be compared to the Maastricht criterion of 60% of gross public debt. Indeed, we use the OECD's gross government financial liabilities in our estimates rather than Maastricht gross public debt, for data availability reasons. These two measures of gross public debt differ in terms of debt instruments and valuation methods. As a result, Maastricht debt is generally much lower than gross government financial liabilities.

Table 5: Growth-adjusted real rates and Debt-GDP ratios

$\frac{r-y}{1+y}$	Stable debt-GDP ratio	$\mathbb{E}[b_t]$
2.5%	Yes	334%
2.0%	Yes	235%
1.5%	Yes	183%
1.0%	Yes	150%
0.5%	Yes	127%
0.0%	Yes	111%
-0.5%	Yes	98%
-1.0%	Yes	89%
-1.4%	Yes	81%
-1.9%	Yes	74%
-2.4%	Yes	69%
-2.8%	Yes	64%

Notes: Debt-output ratios are computed from equation (26) neglecting covariance terms. We use point estimates of γ_1, α_1, α_2, except for γ_2 which is set to 0, and we use expected durations of regime d_1 and d_2 from table 2. Then, we set $r = 3\%$ and compute expected debt-output ratios for various real GDP growth rate.

Markov-switching estimates. We re-estimate the Markov-switching fiscal rule (33) allowing for a time-invariant deterministic trend.[27] Estimates are shown in Table 6 and regime probabilities in Figure 4.

Table 6: Estimated Markov-switching fiscal rule for France with regime-invariant deterministic trend (1965-2013)

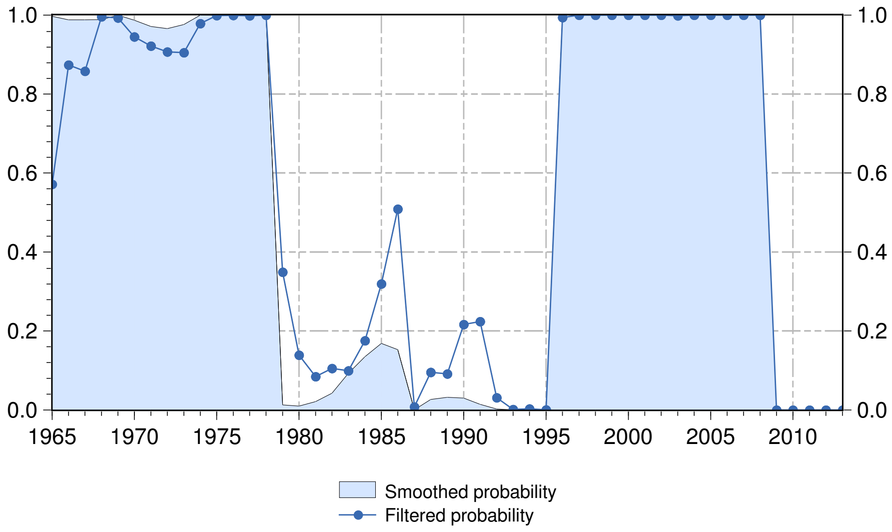
Regime-switching parameters	**Regime 1**	**Regime 2**	**Long-run coefficients**
Constant	-0.017*	0.012	-0.004
	(-1.848)	(1.288)	(-0.495)
Lagged debt b_{t-1}	0.091***	0.015	0.057**
	(2.892)	(0.676)	(2.145)
Output gap $\hat{y}_t$	0.482***	0.341***	0.420***
	(3.464)	(2.881)	(4.530)
Temporary spending $\hat{g}_t$	-0.059	-0.753***	-0.366***
	(-0.556)	(-8.162)	(-5.235)
Regime-invariant parameters			
Deterministic trend	-0.001*		-
	(-1.958)		
Persistence ρ	0.724***		-
	(7.009)		
Standard-error σ	0.004***		-
	(7.583)		
Regimes properties	**Transition prob. p_{ii}**	**Ergodic prob. π_i**	**Exp. duration d_i**
i=1	0.93	0.56	14.9
i=2	0.92	0.44	11.8
Durbin-Watson stat	1.76	Log likelihood	186.52

Notes: Huber-White robust t-stats are in parentheses. Results are significant at 1% level ('***'), 5% level ('**') and 10% level ('*'). We control for regime-invariant first-order serial correlation in the residuals. Basically, estimates for $\hat{\sigma}$ were obtained as $\log \hat{\sigma}$: consequently, standard errors and t-statistics are obtained applying the Delta method. For regime-switching parameters we compute "long-run estimates" as defined earlier.

First, regarding parameters common to each specification, estimates are not significantly different from the baseline estimates in table 2. This is particularly true regarding the estimated

[27] The estimation of a model with regime-switching AR(1) residuals was not successful, due to identification difficulties.

Figure 4: Estimated sustainable regime, model with deterministic trend, France (1965-2013)



Source: OECD, authors' calculations.
Note: Filtered probability of regime 1 (i.e. the sustainable regime) at year t is conditional on information from $t_0 = 1965$ to t and smoothed probability at year t is conditional on information from $t_0 = 1965$ to $T = 2013$.

feedback parameter of public debt in regime 1 –the sustainable regime. Yet the feedback parameter of public debt in the unsustainable regime is far from being significant, and should probably be considered as equal to zero, as in the baseline. Consequently, accounting for a potential deterministic trend *does neither* overturn the finding of a strongly sustainable regime *nor* change the point estimate of the feedback parameter in the sustainable regime. One exception is the constant term in the sustainable regime which is significantly lower in the baseline estimates; we will discuss this point below. Regarding estimated regimes, we observe only a few changes after accounting for a deterministic trend with respect to the baseline estimates. The sustainable regime is more persistent than in the baseline model and fiscal policy in the second-half of the 1960s is identified as sustainable.

Second, the deterministic trend is barely significant at 5% level and negative. This result is linked to the difference in point-estimates of the constant in the sustainable regime. Recall from equation (26) that the constant term determines *ceteris paribus* the level of the long-run expected debt-to-GDP ratio. Hence we interpret both results –weakly significant negative deterministic trend and higher estimated constant in the sustainable regime– as the inability for the estimated model to capture the possible structural change in the level of steady-state debt-to-GDP ratio b_1^* between the late 1970s and the 1990s.

To sum up, the weak significance of the deterministic trend coupled with relatively similar point-estimates between the two models as well as similar fiscal policy estimated regimes lead us to conclude that the baseline regime-switching model is an acceptable representation of France's fiscal policy. A richer specification with a time-varying (stationary) steady-state debt-to-GDP ratio would probably account for the weakly significant deterministic time trend. Yet, this could not be achieved endogenously using Markov-switching dynamic regressions[28].

[28] It would rather require to use a nonlinear Kalman filter and would be much more data-intensive.

5 Discussion on the policy mix

We investigate the stability of the monetary-fiscal policy mix in France since 1965. It is well-known that the estimation of a Bohn-type fiscal rule is not informative on the monetary-fiscal policy mix (Bai and Leeper, 2017, Leeper and Li, 2017). Following Leeper's (1991) terminology, fiscal and monetary policies can be either active or passive. Consequently, a Bohn-type fiscal rule where the reaction of the fiscal instrument towards public debt is positive –the result we achieved overall– will ultimately be a stable regime of monetary dominance if and only if monetary policy is actively targeting inflation, otherwise the monetary-fiscal regime is indeterminate. Alternatively, episodes of active fiscal policies —when fiscal policy's reaction to debt is low or null– lead to a stable regime of fiscal dominance if and only if monetary policy does not actively target inflation, otherwise the monetary-fiscal regime is unstable.

Drawing inferences on the monetary-fiscal regime thus requires studying fiscal and monetary reaction functions. In contrast with Bianchi (2012a) and Chen et al. (2015) who estimate both reaction functions simultaneously on US data, we confront our results for the French fiscal rules with former estimations of the French monetary reaction function. With the adoption of the Euro by France in 1999, it is no longer appropriate to estimate a domestic monetary rule for France, even a non-linear one, on a long horizon.

The design of French monetary policy between 1965 and 2013 has not been invariant and evaluations of the French monetary policy rule gave rise to contrasting results. Three different periods emerge.

The first one, between 1948 and 1979, reveals quite a few changes in the design of monetary policy, hence different fiscal-monetary regimes. Monnet (2014) shows that between 1948 and 1973, the main instrument of monetary policy by Banque de France was not the interest rate but a mix of quantitative controls on liquidity (rediscount ceilings) and on bank credit (credit ceilings). He identifies monetary policy shocks with a narrative approach and shows that restrictive episodes of monetary policy produced decreases in industrial production and inflation. Although Monnet does not estimate a monetary policy rule per se, these restrictive episodes nicely fit episodes of monetary policy aiming at limiting inflation; he notably shows that quantitative controls had negative effects on inflation and GDP growth. After 1973 and until 1979, the design of monetary policy in France has progressively moved towards inflation targeting. Monnet (2015) recalls that the use of monetary policy by Banque de France to fight (double-digit) inflation officially started in 1977 with the explicit use of monetary targets (M2), but was experimented since 1973. However, monetary targets were very often exceeded; between 1973 and 1979 quantitative controls were not binding, which signals a passive monetary policy. To sum up, Monnet (2014, 2015)'s research indicates that France's monetary policy was most likely active during the 1960s until 1973, and passive between 1973 and 1979.

France joined the Exchange Rate Mechanism (ERM) in 1979 and adopted and participated in the European Currency Unit (ECU). The asymmetry of this regime has long been studied and corroborated in the empirical literature. The conclusion has been that France's monetary policy became anchored to Germany's monetary policy under the ERM. Smets (1997) shows that between 1979 and 1996, French monetary policy depended on the ECU exchange rate: monetary policy was driven by the requirement of stabilising the French Franc in the ERM. Unsurprisingly, he does not find any impact of the ECU exchange rate on German monetary policy. Artus et al. (1991) also found evidence of asymmetry in monetary policymaking between the different member states of the ERM. German short-run interest rate acted as an anchor for French monetary policy. Bec et al. (2002) report a high sensitiveness of the French policy rate towards inflation and the German policy rate. The reaction towards real output is not statistically significant. They also show some non-linearities in the French policy rule. The sensitiveness of the policy rate towards the German rate (resp. domestic inflation) is higher during expansions (resp. recessions). It has two implications. First, the period of "competitive disinflation" –sharp policy aimed to fight inflation– that started at the end of the 1980s is clearly visible in the reaction function. Second, whatever the period, expansion or recession, monetary policy was actively fighting inflation, either directly or indirectly by applying the German disinflation preference. Consequently, monetary dominance nicely depicts the monetary-fiscal interactions in France between 1979 and 1998.

In 1999, France adopted the Euro. Linear specifications of the ECB monetary policy are usually consistent with the Taylor-rule principle (Castro, 2011, Gorter et al., 2008, Surico, 2007, among others). Fendel et al. (2011) use interest rate, inflation rate and growth rate forecasts by financial markets participants to evaluate monetary policy rules in G-7 countries. They show that the Taylor principle holds in France between 1989 and 2008, hence confirming former results. Consequently, France has gone through two different monetary-fiscal regimes since 1999: before 2009, monetary dominance prevailed, whereas after 2009, an unstable regime emerged.

This conclusion draws on our identification of different *local* fiscal regimes. However, making use of our main result on *global* debt sustainability in France, we can argue that France has gone through a monetary dominance since the late 1970s without exception: public finances were globally sustainable (or passive) whereas monetary policy was inflation-oriented (or active).

6 Conclusions

This paper introduces a Regime-Switching Model-Based Sustainability test for fiscal policy, building on Bohn's Model-Based Sustainability (MBS) framework and on the literature on Markov-switching fiscal policy rules. We assume a Markov-switching fiscal policy rule that stochastically switches between sustainable and unsustainable regimes, where by unsustainable regime we mean a periodic and persistent *negative or null* feedback effect of initial public debt on primary surplus, i.e. a violation of Bohn's sustainability condition. Consequently, the public debt-to-GDP ratio becomes periodically and persistently explosive during unsustainable regimes, and the existence of fiscal regimes thus matters for the analysis of fiscal sustainability in the long run.

We prove formally that global fiscal sustainability differs from local sustainability. The former depends on the relative sensitiveness of the fiscal rule to the debt-to-GDP ratio from one regime to another, and also on the relative duration and persistence of both regimes.

The Regime-Switching MBS test is then applied to French data over a 51-year horizon and compared to standard MBS tests. Our results are threefold. First, we estimate different specifications of Bohn's constant-parameter fiscal policy rule. These estimates do not allow to reject unsustainability: the feedback coefficient on public debt-to-GDP is rarely positive and never significant, according to standard MBS tests. Second, we estimate a Markov-switching fiscal policy rule. We identify two different fiscal regimes over the period: one regime is sustainable, with a strong positive and significant feedback effect of lagged public debt-to-GDP on primary surplus-to-GDP, while the second one is unsustainable with no significant feedback effect. In addition, identified fiscal regimes are found to be strongly persistent. In particular, our findings support the view that the Maastricht Treaty and the Stability and Growth Pact (SGP) actually made France's fiscal policy more sustainable, despite being under an Excessive Deficit Procedure from 2003 to 2007. Third, we perform RS-MBS tests for No-Ponzi Game and Stationary debt-output ratio. They reject the null hypothesis of a Ponzi Scheme as well as the null of an explosive public debt-to-GDP ratio. Despite periodic violations of sustainaibility conditions, global debt sustainability has been achieved. Finally, drawing on former evidence about the characteristics of monetary policy in France, we conclude that the fiscal-monetary policy mix has been characterized by monetary dominance since 1979.

Future research may now move towards the proper analysis of the interactions between monetary and fiscal policies, in presence of regime-switching policy rules, and their consequences on fiscal sustainability. In contrast with early attempts (see Davig and Leeper (2011) for example), a Euro-area country like France can no longer be described by a domestic monetary policy since 1999. Theoretical research thus requires to match *domestic* fiscal policy with a *federal* monetary policy. Beyond that, the question of fiscal sustainability in a Regime-switching MBS framework could be embedded in a monetary union model. It would introduce another determinant of fiscal sustainability, namely cooperative or non-cooperative fiscal behaviors.

Regarding empirical application, this research could be extended in at least two ways. Empirical research on linear fiscal policy rules usually ignore potential endogeneity problems: reverse causality between primary balance and output gaps through fiscal multipliers or simultaneity bias as argued recently by Leeper and Li (2017) based on the Fiscal Theory of the Price Level. A first way of improving the RS-MBS framework could be to adopt the Control Function approach for Markov-switching dynamic regressions developed by Kim (2010). Given the multiple evidence on time-varying fiscal multipliers (e.g. Auerbach and Gorodnichenko (2012), Riera-Crichton et al. (2015)), effects of fiscal policy on economic activity can be neglected during expansions but cannot be ignored during recessions, when fiscal multipliers are likely positive. Hence, estimates of primary surplus response to public debt would likely be biased downward during recessions. In this respect, our empirical test can be interpreted as a lower bound for fiscal sustainability. Another way of dealing with endogeneity and simultaneity biases is to estimate regime-switching policy rules in empirical DSGE or VAR models following the suggestion of Leeper and Li (2017) and allowing to impose cross-equation restrictions to correctly identify policy behaviors.

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A Appendix

A.1 Proof of Proposition 1 (No-Ponzi Game)

We show that a strictly positive long-run feedback effect, i.e. (19)

$$\gamma\pi > 0$$

is a *sufficient* condition for the NPG (18) to hold, in a dynamically efficient economy and a bounded innovation process $\mu(z_t)$, following Bohn (1998, see online appendix). Using (8) and iterating of (5) yields:

$$\begin{aligned} b_{t+T} = \prod_{i=0}^{T} \frac{1+r_{t+i}}{1+y_{t+i}} \big(1-(1+y_{t+i})\gamma(z_{t+i})\big) b_{t-1} \\ - \sum_{k=0}^{T} (1+r_{t+k}) \Bigg(\prod_{j=k+1}^{T} \frac{1+r_{t+j}}{1+y_{t+j}} \big(1-(1+y_{t+j})\gamma(z_{t+j})\big) \Bigg) \mu_{t+k}(z_{t+k}) \end{aligned} \tag{34}$$

Then, multiplying by (17), one gets an expression for the discounted debt-output ratio at time $t+T$:

$$\begin{aligned} \mathbb{E}_t \, \tilde{Q}_{t,T+1} b_{t+T} = \mathbb{E}_t \prod_{i=0}^{T} \big(1-(1+y_{t+i})\gamma(z_{t+i})\big) b_{t-1} \\ - \mathbb{E}_t \sum_{k=0}^{T} \Bigg(\prod_{j=k+1}^{T} \big(1-(1+y_{t+j})\gamma(z_{t+j})\big) \Bigg) a_{t,k} \end{aligned} \tag{35}$$

with $a_{t,k} = (1+y_{t+k})\tilde{Q}_{t,k}\mu_{t+k}(z_{t+k})$. Taking the absolute value[29] of (35) and using triangle inequality yields:

$$\begin{aligned} |\mathbb{E}_t \, \tilde{Q}_{t,T+1} b_{t+T}| \leq \mathbb{E}_t \Big| \prod_{i=0}^{T} \big(1-(1+y_{t+i})\gamma(z_{t+i})\big) b_{t-1} \Big| \\ + \underbrace{\mathbb{E}_t \Big| \sum_{k=0}^{T} \Bigg(\prod_{j=k+1}^{T} \big(1-(1+y_{t+j})\gamma(z_{t+j})\big) \Bigg) a_{t,k} \Big|}_{W_t} \end{aligned} \tag{36}$$

and applying the triangle inequality on W_t allow us to give an upper bound to the absolute value of (35):

$$|\mathbb{E}_t \, \tilde{Q}_{t,T+1} b_{t+T}| \leq \mathbb{E}_t \prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| |b_{t-1}| + \mathbb{E}_t \sum_{k=0}^{T} \Big| \prod_{j=k+1}^{T} \big(1-(1+y_{t+j})\gamma(z_{t+j})\big) \Big| |a_{t,k}| \tag{37}$$

[29] Note that $f(x) = |x|$ is convex, then Jensen inequality yields for any random variable X:

$$|\mathbb{E}[X]| \leq \mathbb{E}[|X|]$$

An important step is to give a tractable expression for

$$\mathbb{E}_t \prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| \tag{38}$$

in order to study the limit property of equation (35). Thus remark that:

$$\begin{aligned} \mathbb{E}_t \textstyle\prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| &= \mathbb{E}_t \left[\exp\left(\ln \textstyle\prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| \right) \right] \\ &= \mathbb{E}_t \left[\exp\left(T \times \tfrac{1}{T} \textstyle\sum_{i=0}^{T} \ln|1-(1+y_{t+i})\gamma(z_{t+i})| \right) \right] \end{aligned} \tag{39}$$

where $\frac{1}{T}\sum_{i=0}^{T} \ln|1-(1+y_{t+i})\gamma(z_{t+i})|$ is the Lyapunov exponent associated to the present-value debt-output ratio. Since both $(1+y_t)$ and z_t are stationary-ergodic, then we know that:

$$\lim_{T\to+\infty} \frac{1}{T} \sum_{i=0}^{T} \ln|1-(1+y_{t+i})\gamma(z_{t+i})| = \mathbb{E}\left[\ln|1-(1+y_t)\gamma(z_t)| \right] \tag{40}$$

which is measurable at time t. If one assumes $(1+y_t)\gamma(z_t) < 1$[30] then it yields

$$\ln|1-(1+y_t)\gamma(z_t)| = \ln(1-(1+y_t)\gamma(z_t))$$

Applying Jensen's inequality on the logarithm function and the expectation operator yields an upper-bound for

$$\mathbb{E}\ln\left(1-(1+y_t)\gamma(z_t)\right) \le \ln\left(1-\mathbb{E}(1+y_t)\gamma(z_t)\right) \tag{41}$$

From what precedes[31], we deduce it exists an arbitrarily high $N \in \mathbb{N}$ such that:

$$\forall T \ge N, \quad \mathbb{E}_t \prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| \le \exp\left[\ln\left(1-\mathbb{E}(1+y_t)\gamma(z_t)\right)^T \right] \tag{42}$$

which allows us to conclude

$$\mathbb{E}_t \prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| \le \left(1-\mathbb{E}(1+y_t)\gamma(z_t)\right)^T \tag{43}$$

Finally, we define the following upper bound for equation (38):

$$\mathbb{E}_t \prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})| \le \left(1-(1+y)\gamma\pi - (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)\right)^T \tag{44}$$

where $\mathrm{Cov}(y_t, z_t)$ is the unconditional covariance of y_t and z_t.

At this stage, we need two assumptions to proceed further.

Assumption 1 *Following Bohn (1998), we assume dynamic efficiency which implies present-value of*

[30] This assumption is actually purely technical, since it mainly relies on the assumption $|\gamma(z_t)|$ is close to zero, about the size of a small interest rate and $(1+y_t)$ is close to 1.

[31] In particular, Jensen inequality implies that:

$$\frac{1}{T}\sum_{i=0}^{T} \ln|1-(1+y_{t+i})\gamma(z_{t+i})| \le \ln\left(\frac{1}{T}\sum_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})|\right)$$

and allows to define an upper-bound for $\mathbb{E}_t \prod_{i=0}^{T} |1-(1+y_{t+i})\gamma(z_{t+i})|$.

income is finite:

$$\lim_{T \to +\infty} Y_t \sum_{i=1}^{T} \mathbb{E}_t \tilde{Q}_{t,i} = \overline{Y}$$

implying $\lim_{T\to+\infty} \mathbb{E}_t \tilde{Q}_{t,T} = 0$, *by convergence of the serie* $\sum_{i=1}^{T} \mathbb{E}_t \tilde{Q}_{t,i}$.

Assumption 2 *Following Bohn (1998), we assume the innovation process* $\mu_t(z_t)$ *is bounded* $|\mu_t(z_t)| \leq M$.

Assumptions 1-2 jointly imply $\lim_{T\to+\infty} \mathbb{E}_t a_{t,k} = 0$[32] that is:

$$\forall \delta > 0\,, \quad \exists K \in \mathbb{N} \quad / \quad \forall k > K\,, \quad |\mathbb{E}_t a_{t,k}| \leq \delta \tag{45}$$

Then, using assumptions 1-2 along with equation (44) yields:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| \leq \big(1-(1+y)\gamma\pi - (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)\big)^T |b_{t-1}| \\ + \Omega\big(1-(1+y)\gamma\pi - (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)\big)^{T-K} \\ + \sum_{k=K}^{T} \big(1-(1+y)\gamma\pi - (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)\big)^{T-k} \delta \end{aligned} \tag{46}$$

where $\Omega = \sum_{k=0}^{K-1} E_t \prod_{j=k+1}^{K-1} |1-(1+y_{t+i})\gamma(z_{t+i})||\mathbb{E}_t\, a_{t,k}|$ is finite. Finally, rearranging the last expression allows us to write:

$$\begin{aligned} |\mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T}| \leq \big(1-(1+y)\gamma\pi - (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)\big)^T |b_{t-1}| \\ + \Omega\big(1-(1+y)\gamma\pi - (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)\big)^{T-K} \\ + \frac{\delta}{(1+y)\gamma\pi + (\gamma_1-\gamma_2)\,\mathrm{Cov}(y_t,z_t)} \end{aligned} \tag{47}$$

Assumption 3 *In our Ricardian economy, we assume the fiscal regime* z_t *is independent of the real growth rate of the economy* y_t, *i.e.* $\mathrm{Cov}(y_t,z_t) = 0$.

Therefore, under assumption 3, a sufficient condition for the NPG condition only requires:

$$\gamma\pi > 0 \tag{48}$$

which implies $(1+y)\gamma\pi > 0$. Therefore, we find that

$$\forall \hat{\epsilon} > 0\,, \quad \exists K \in \mathbb{N} \quad / \quad \forall T \geq K \quad |\mathbb{E}_t \tilde{Q}_{t,T+1} b_T| < \hat{\epsilon}$$

provided one sets $\hat{\epsilon} = \frac{\delta}{|(1+y)\gamma\pi|}$, from which we conclude that:

$$\lim_{T\to+\infty} \mathbb{E}_t \tilde{Q}_{t,T+1} b_{t+T} = 0 \tag{49}$$

Discussion. In a more general framework with $\mathrm{Cov}(y_t,z_t) \neq 0$, a sufficient condition to rule out Ponzi schemes, given a Markov-switching fiscal rule such as (8), would be:

$$\gamma\pi > -\frac{(\gamma_1-\gamma_2)}{1+y}\,\mathrm{Cov}(y_t,z_t) \tag{50}$$

[32] Given that $\lim_{T\to+\infty} \mathbb{E}_t \tilde{Q}_{t,T} = 0$ also implies $\lim_{T\to+\infty} \mathbb{E}_t (1+y_T)\tilde{Q}_{t,T} = 0$

and would critically depend on the covariance term $\text{Cov}(y_t, z_t)$. If positive (i.e. if sustainable regimes are positively correlated to higher growth), it implies that a strictly positive $\gamma\pi$ would not be required to rule out Ponzi schemes; if negative, on the contrary, it would not be sufficient. Still, our empirical results provide an *ex post* validation for assuming $\text{Cov}(y_t, z_t) = 0$, since the estimated unconditional covariance between smoothed probabilities of a sustainable regime (i.e. the empirical counterpart of z_t) and the growth rate of real GDP is non-significantly different from zero, with a positive point estimate.

A.2 Proof of Proposition 2 (Debt-stabilizing condition)

Using the sufficient condition for a *strictly* stationary Markov-switching autoregressive process of order one, we show a strictly larger feedback effect than the average growth-adjusted real interest rate, i.e. (24), is a sufficient condition for the debt-output ratio process (23) to be strictly stationary and fluctuate around its ergodic mean (26).

Considering stochastic processes $\{x_t\}$ described by:

$$x_t = \phi_0 + \phi(z_t)x_{t-1} + \varepsilon_t \tag{51}$$

where z_t is a discrete-time Markov process, defined on the state-space $z(\Omega)$. We know from Kesten (1973) that a sufficient condition for *strict stationarity* is:

$$\mathbb{E}\left[\ln|\phi(z_t)|\right] \equiv \sum_{i \in z(\Omega)} \ln|\phi(i)|\pi(i) < 0 \tag{52}$$

which means that a globally stationary process $\{x_t\}$ can be locally (or periodically) non-stationary. This condition ensures that $\{x_t\}$ is strictly (or strongly) stationary implying its joint-probability distribution does not change over time. Strict stationarity only implies $\{x_t\}$ has a finite mean but does not imply necessarily a finite variance. Since weak stationarity requires finite variance, this condition is not sufficient for weak stationarity. For a finite variance, this process must verify a stronger condition. Define $\Phi \equiv \text{diag}\Big(\phi(i), \forall i \in z(\Omega)\Big)$ and $\rho(M)$ the spectral radius of any square-matrix M. Then, for this stricly stationary process to admit a unique stationary solution at second-order, it must satisfy the following condition:

$$\rho(\Phi^2 P) < 1 \tag{53}$$

where P is the transition matrix of the underlying Markov-chain.

Applying condition (52) to equation (23) yields the following condition:

$$\mathbb{E}[\ln|\phi(z_t)|] = \mathbb{E}\left[\ln|\frac{1+r_t}{1+y_t}| + \ln|1-(1+y_t)\gamma(z_t)|\right] < 0 \tag{54}$$

Hence, using usual approximation $\ln(1+x) \sim x$ when $x \to 0$ and taking unconditional expectations of r_t, y_t and $\gamma(z_t)$, we find a sufficient condition for strict stationarity of process $\{b_t\}$ is:

$$\gamma\pi > \frac{r-y}{1+y} \tag{55}$$

assuming that $\mathrm{Cov}(y_t, z_t) = 0$. [33] Therefore, process $\{b_t\}$ has an ergodic mean equal to

$$\begin{aligned}\mathbb{E}[b_t] &= \frac{-\,\mathbb{E}[(1+r_t)\alpha(z_t)] + \mathrm{Cov}(\phi(z_t), b_{t-1})}{\mathbb{E}[1-\Phi(z_t)]} \\ &= \frac{-(1+r)\,\mathbb{E}\,\alpha(z_t) - (\alpha_1-\alpha_2)\,\mathrm{Cov}(r_t,z_t) + \mathrm{Cov}(\phi(z_t), b_{t-1})}{(1+r)\gamma\pi + (\gamma_1-\gamma_2)\,\mathrm{Cov}(r_t,z_t) - \frac{r-y}{1+y}}\end{aligned} \tag{58}$$

which we approximate by

$$\mathbb{E}[b_t] \simeq \frac{-(1+r)\,\mathbb{E}\,\alpha(z_t)}{(1+r)\gamma\pi - \frac{r-y}{1+y}} \tag{59}$$

neglecting covariance terms, following Bohn (1998, 2008) and Mendoza and Ostry (2008).

A.3 Data on real interest rates and real GDP growth rate

Table 7 presents descriptive statistics on long-run ex-post real interest rate (using the yield on 10-year public bonds) and real GDP growth. Figure 5 plots the growth-adjusted real interest rate and each time series separately.

Table 7: Descriptive statistics on real interest rates and real GDP growth, 1963-2013

	Long-term real rate	Real GDP growth rate
Mean	3.00%	2.68%
Median	2.86%	2.31%
Maximum	6.99%	6.91%
Minimum	-2.94%	-3.11%
Std. Dev.	2.2%	2.1%
Observations	51	51

[33] One could use another approximation using logs:

$$\begin{aligned}\ln\left|\frac{1+r}{1+y}\right| + \ln|1-(1+y)\gamma\pi| &= \ln\left|1+\frac{r-y}{1+y}\right| + \ln|1-(1+y)\gamma\pi| &< 0 \\ &\approx \frac{r-y}{1+y} - (1+y)\gamma\pi &< 0\end{aligned}. \tag{56}$$

and we find:

$$\gamma\pi > \frac{r-y}{(1+y)^2} \tag{57}$$

We choose to keep the first stricter condition, for two reasons. First, approximations are not precise enough to determine scale factors, and second, we remain conservative when testing the debt-stabilizing condition and therefore study the "worst-case" scenario.

Figure 5: Growth-adjusted real interest rate, real interest rates and real GDP growth rate