WORKING paper

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Welfare-Based Optimal Macroprudential Policy with Shadow Banks

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Abstract

In this paper, I show that the existence of non-bank financial institutions (NBFIs) has implications for the optimal regulation of the traditional banking sector. I develop a New Keynesian DSGE model for the euro area featuring a heterogeneous financial sector allowing for potential credit leakage towards unregulated NBFIs. Introducing NBFIs raises the importance of credit stabilization relative to other policy objectives in the welfare-based loss function of the regulator. The resulting optimal policy rule indicates that regulators adjust dynamic capital requirements more strongly in response to macroeconomic shocks due to credit leakage. Furthermore, introducing non-bank finance not only alters the cyclicality of optimal regulation, but also has implications for the optimal steady-state level of capital requirements and loan-to-value ratios. Sector-specific characteristics such as bank market power and risk affect welfare gains from traditional and NBFI credit.[2]

Keywords: Macroprudential Regulation, Monetary Policy, Optimal Policy, Non-Bank Finance, Shadow Banking, Financial Frictions

JEL classification: E44, E61, G18, G23, G28

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Non-Technical Summary

The relevance of non-bank financial institutions (NBFIs) for financial stability has recently been addressed by financial regulators. For instance, imbalances in the non-bank financial sector have been identified as a main risk to financial stability in the euro area during the Covid-19 pandemic. Furthermore, the importance of NBFIs has been acknowledged in recent discussions on a "Capital Markets Union (CMU)" in Europe. However, designing a macroprudential framework for the non-bank financial sector similar to the approach applied to commercial banks is barely feasible. While traditional banks directly intermediate funds between borrowers and savers, a multitude of specialized financial corporations operating in complex intermediation chains are usually involved in non-bank credit intermediation.

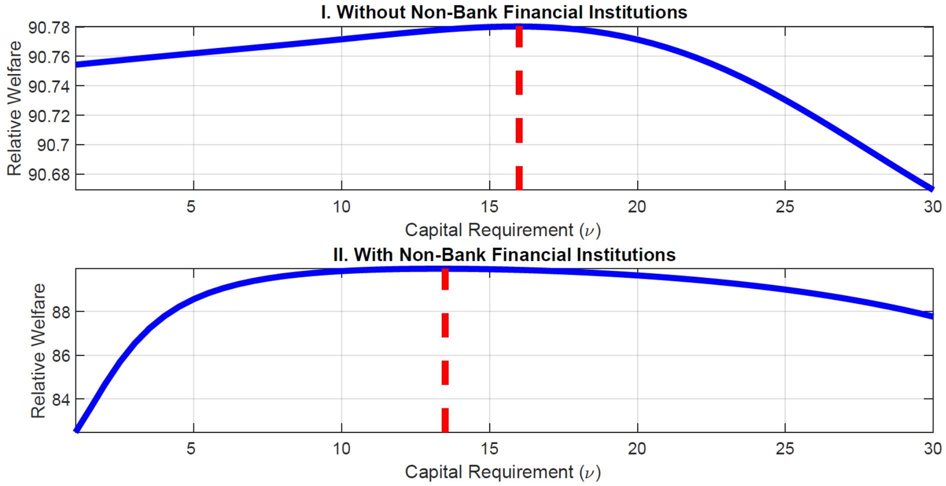
Nevertheless, changes in macroprudential regulation for the commercial banking sector can shift credit intermediation towards less regulated parts of the financial system. For instance, higher capital requirements for traditional banks potentially lead to credit leakage towards unregulated NBFIs: As tighter banking regulation does not initially affect credit demand, higher regulation for commercial banks may incentivize borrowers to switch to NBFIs as commercial bank credit becomes relatively costly. Consequently, prudential authorities need to decide on an optimal level of regulation such that on the one side, banks' equity buffers are sufficiently high, but on the other side credit leakage to non-banks is limited.

In this paper, I study the optimal design of bank capital requirements and loan-to-value (LTV) ratios in the presence of a non-bank financial sector. I base the analysis on a New-Keynesian dynamic stochastic general equilibrium (DSGE) model featuring a heterogeneous financial sector calibrated to match economic and financial conditions in the euro area. The findings on optimal policy reveal that in the presence of NBFIs, the welfare-optimal level of static capital requirements is lower (13.5 percent) than in a counterfactual scenario where credit is intermediated only by traditional banks (16 percent). I highlight that the difference in optimal regulation can be attributed to an additional trade-off the regulator has to take into account, which relates to the composition of credit provided by commercial banks and NBFIs. Furthermore, NBFI presence affects the optimal dynamic response of macroprudential regulation to fluctuations in output and credit. Whenever macroeconomic disturbances imply credit leakage towards NBFIs, regulatory adjustments are more pronounced as in an economy without non-bank finance.

I then show that the additional policy trade-off is shaped by structural characteristics of financial institutions. For instance, empirical evidence suggests a significant degree of market power in the euro area commercial banking sector. In contrast, some studies find that non-bank finance can increase efficiency in financial markets by providing alternative financing sources and due to the involvement of highly specialized institutions in the intermediation process. However, NBFI intermediation can increase systemic risk, as structural characteristics, economic motivations, and regulatory constraints within the diverse non-bank financial sector can accelerate financial stress and macroeconomic disturbances, and finally pose a threat to financial stability.

In summary, the findings indicate that neglecting NBFIs potentially impairs the efficiency of macroprudential policies, as regulators do not internalize credit leakage and an additional trade-off related to the composition of credit. Thus, they should consider developments in the non-bank financial sector, even if their policies only apply to traditional banks. Furthermore, the lack of macroprudential tools for NBFIs raises potential gains from coordinating the implementation of different macroprudential policy measures. In addition, coordination with monetary policy can play a role, as NBFIs' activity is also related to the overall price of credit in the economy. Thus, credit leakage may be aggravated when the effective lower bound (ELB) on nominal interest rates is reached.

Welfare for Different Levels of Permanent Capital Requirements



Note: Relative welfare levels under optimal policies for different values of the permanent capital requirement (percentage points). Maximum indicated by red lines.

Politique macroprudentielle optimale et shadow banking

RÉSUMÉ

Cet article examine les implications des institutions financières non bancaires (IFNB) sur la régulation optimale du secteur bancaire traditionnel, à l'aide d'un modèle DSGE néo-keynésien pour la zone euro avec un secteur financier hétérogène permettant une fuite potentielle de crédit vers les IFNB non régulées. L'introduction d'IFNB accroît l'importance relative de la stabilisation du crédit dans la fonction de bien-être du régulateur. La règle de politique optimale qui en résulte conduit à un renforcement des exigences cycliques en fonds propres en réponse aux chocs macroéconomiques en raison de la fuite de crédit. En outre, l'introduction de la finance non bancaire ne modifie pas seulement le caractère cyclique de la réglementation optimale, mais a également des implications sur le niveau optimal en régime permanent des exigences de capital et des ratios LTV (loan to value). Les caractéristiques spécifiques aux secteurs, telles que le pouvoir de marché au secteur bancaire et le niveau de risque au secteur IFNB, affectent les gains en bien-être provenant du crédit traditionnel et des IFNB.

Mots-clés : Réglementation macroprudentielle, Politique monétaire, Politique optimale, Finance non bancaire, Shadow banking, Frictions financières.

1 Introduction

The financial crisis of 2007/2008 triggered a substantial debate about the optimal stance of financial regulation. As of today, a broad consensus on the necessity of a *macroprudential* approach to target systemic developments in financial markets has been reached among scholars and policy makers.[1] Contemporaneously, the neglected treatment or complete absence of financial intermediaries and frictions in canonical pre-crisis dynamic stochastic general equilibrium (DSGE) models has widely been criticized. In response, banking-augmented macro models have been developed and employed to assess, inter alia, the effectiveness of different macroprudential tools in the presence of financial frictions. In particular, significant progress has been made with respect to the consideration of commercial banking at the macro level, both in theoretical models and in the field of financial regulation.

In comparison, the role of non-bank financial intermediation[2] has for a long time been understated in both areas. Only recently, the introduction of heterogeneous financial sectors in macro models has been initiated. On the policy side, the importance of non-banks has been acknowledged in the recent and ongoing debate on the optimal design of a "Capital Markets Union (CMU)" in Europe.[3] Also, imbalances in the non-bank financial sector have been identified as main risks for financial stability in the euro area during the current Covid-19 pandemic.[4]

The shift in attention towards NBFIs finally reflects the fact that non-bank finance has substantially gained importance in the euro area over the last two decades. Figure 1 shows the evolution of the total amount of outstanding credit to non-financial corporations, provided by traditional banks and non-bank financial intermediaries in the euro area.[5] Whereas commercial banks provide the largest share

[1] See Borio (2011, 2009) or Borio and Shim (2007) for a detailed description of the macroprudential approach. For a review of the pre-crisis microprudential approach, see Kroszner (2010), Borio (2003), or Allen and Gale (2000).

[2] In this paper, the terms "non-bank financial intermediation" and "shadow banking" will be used interchangeably to describe credit intermediation outside the regulated traditional commercial banking sector. See for instance Adrian and Jones (2018) for a discussion on terminology.

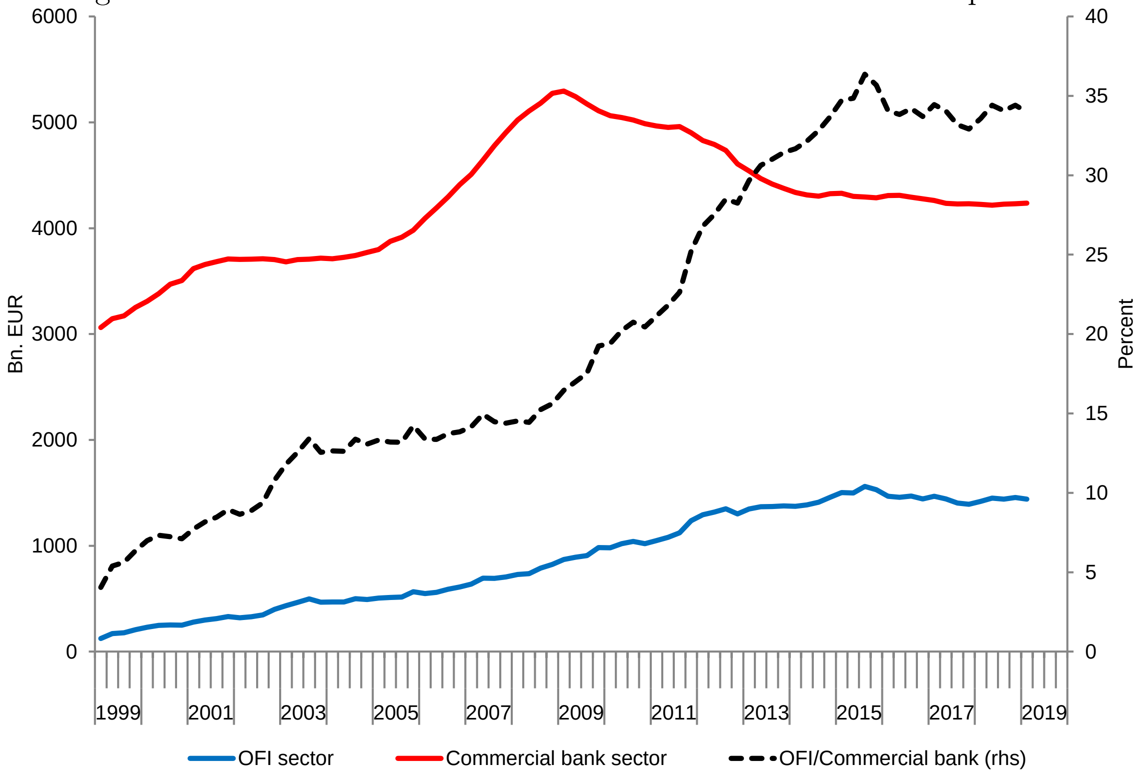
[3] In a CMU, non-bank finance could play an important role to mitigate bank-dependency in the European financial sector, but would require further strengthening of regulatory measures. See for instance Pires (2019).

[4] See for instance ECB Financial Stability Review, May 2021.

[5] Non-bank credit is defined as the aggregate loans provided by "Other Financial Intermediaries", a composite of different financial corporations other than commercial banks or institutions

of lending to corporates, non-bank lending has steadily increased since the implementation of the euro and has currently reached more than 35 percent of traditional lending.

Figure 1: Commercial and Non-Bank Loans to Non-Financial Corporates



Note: Outstanding amount of loans of commercial banks and non-banks (OFI) to non-financial corporates (billions of euro). Source: Euro Area Accounts and Monetary Statistics (ECB).

In this paper, I discuss optimal macroprudential policies while allowing credit to be intermediated by both commercial and non-bank financial intermediaries (NBFIs). I base the analysis on a New Keynesian DSGE model featuring a heterogeneous financial sector similar to the one derived in Gebauer and Mazelis (2020). NBFIs and commercial banks differ in the degree of competitiveness and risk and are affected to a different degree by regulation. Methodologically, the framework combines elements of two leading strands of the literature on financial frictions in DSGE models that appear well-suited to model these structural and regulatory differences. For the commercial banking sector, a financial framework similar to the

belonging to the Eurosystem. However, alternative measures of non-bank credit can straightforwardly be derived by marginal adjustments of OFI aggregates. See for instance Gebauer and Mazelis (2020), Doyle et al. (2016) or Bakk-Simon et al. (2012).

one derived in Gerali et al. (2010) is introduced which allows explicitly for commercial bank capital regulation. Furthermore, it features structural elements that describe the banking sector in the euro area well. For NBFIs, elements of the banking framework developed in Gertler and Karadi (2011) are introduced. Instead of being affected by banking regulation, non-bank credit is limited by a moral hazard friction between investors and NBFIs that results in an endogenous leverage constraint.

To discuss optimal regulation, I derive welfare loss functions and optimal policies under commitment following a "linear-quadratic (LQ)" approach as introduced in the literature on monetary policy. The approach relies in large part on the derivation of optimal policy under the timeless perspective developed in Giannoni and Woodford (2003a,b), Benigno and Woodford (2005, 2012) and Woodford (2011). I derive optimal policy under commitment to study the design of an optimal policy rule to which a macroprudential policy maker would commit at all future dates. Ultimately, the aim of deriving such an optimal rule under commitment is to base policy decisions on a framework that allows for a systematic adjustment of capital requirements in response to financial market developments.

I find that first, NBFI credit matters for optimal macroprudential regulation as the derived welfare loss function for the model with NBFIs features NBFI credit. The relative weights on both commercial bank and NBFI credit are large compared to the commercial bank credit weight in the loss function dervided from the same model without NBFIs. Furthermore, it turns out to be optimal for the policy maker to take the volatility in nominal interest rates, set by the central bank without coordination, into account as well. This finding provides some indication that coordinating both policy areas to some degree might be welfare-improving, even when no coordination is assumed a priori. Finally, and in line with the "revealed-preferences" literature on macroprudential regulation, credit and a measure for the output gap enter the welfare loss functions.

Furthermore, not only the variation of target variables, but also deviations of credit *levels* from efficient values have welfare implications. Inefficiencies in commercial bank and NBFI credit markets cause permanent distortions in steady state and provide scope for time-invariant policies that close the gaps between actual and efficient steady-state credit levels. I find that resolving distortions in both credit markets requires two separate tools, each one employed to remove inefficiencies in one credit market. I propose that permanent commercial bank capital requirements can be set accordingly to remove inefficiencies stemming from monopolistic competition in the banking sector. As NBFIs cannot be regulated directly, I propose credit demand tools such as borrower loan-to-value (LTV) ratios to account for permanent distortions in NBFI credit markets. The proposed framework implies that such borrower-side regulations are set to levels that mitigate NBFI credit distortions. In return, time-invariant capital requirements are set conditional on these regulations to levels that resolve commercial bank credit inefficiencies.

The main implication from these findings is that optimal macroprudential policies for commercial banks should be designed in coordination with other policies whenever unregulated NBFIs exist. Thereby, borrower-side policies such as LTV ratios can be employed to target the share of credit intermediated by institutions that do not fall under the jurisdiction of credit-supply policies. Furthermore, monetary policy can play a role in the optimal policy mix. Short-term interest rates depict a universal tool to reach through "all the cracks in the economy" (Stein, 2013) and therefore affect both commercial bank and NBFI intermediation.

In addition to the analytic derivations of welfare loss functions and policy rules, I conduct simulation exercises to discuss the optimal design of policies quantitatively. In the model with NBFIs, the optimal permanent level of capital requirements turns out to be lower than in a comparable model without non-bank finance. Due to undesirable credit leakage towards risky NBFIs, regulators optimally set requirements to 13.5 percent in steady state. In a model without non-bank finance, the absence of the credit leakage trade-off results in an optimal level of bank capital requirements of 16 percent.

I finally evaluate dynamic policies by deriving an optimal capital requirement rule and discuss optimal regulatory responses to exogenous disturbances. I show that macroprudential regulators adjust capital requirements countercyclically, i.e. they raise (lower) capital requirements in response to positive (negative) deviations of the output gap and commercial bank credit from their efficient levels. They also try to mitigate credit leakage towards non-bank intermediaries. Consequently, if both credit aggregates move in the same direction after macroeconomic shocks, they adjust requirements less strongly than they would in the absence of NBFIs. In contrast, whenever macroeconomic shocks cause leakage, i.e. credit aggregates to move in opposite directions, regulators will adjust capital requirements more aggressively as in a situation without non-bank finance.

I review the related literature in section 2 and briefly discuss the model and its calibration in sections 3 and 4. In sections 5 to 7, I derive welfare-based loss functions for scenarios with and without NBFIs and discuss both time-invariant and cyclical macroprudential policies in detail. Section 8 concludes.

2 Related Literature

To my knowledge, my paper is the first to discuss the optimal design of macroprudential policies in the presence of non-bank finance in a dynamic general equilibrium framework. In doing so, it strongly connects to three strands of the literature. First, several recent studies use static or partial-equilibrium banking models to discuss how the introduction of shadow banking alters optimal capital regulation for commercial banks (Ordonez, 2018; Farhi and Tirole, 2017; Plantin, 2015; Harris et al., 2014). Despite differences in microfoundations for the interaction between shadow bank and commercial bank lending and assumptions on regulatory coverage, they find that the existence of shadow banks significantly alters the optimal level of capital regulation. However, these studies do not discuss general equilibrium effects and dynamic policy responses to macroeconomic disturbances.

Second, this paper relates to the large literature on the analysis of macroprudential policies with the help of banking-augmented DSGE models. In response to the global financial crisis, the neglection of financial intermediaries in pre-crisis DSGE models has widely been criticized (Christiano et al., 2018). In response, banking-augmented macro models have been developed and used to assess the effectiveness of monetary, fiscal, and macroprudential policies in the presence of financial frictions.[6] One prominent strand of the literature employs models with a moral hazard problem located between depositors and intermediaries that implies an endogenous leverage constraint for banks (Kiyotaki and Moore, 2012; Gertler and Kiyotaki, 2011; Gertler and Karadi, 2011). In contrast, some studies feature models with frictions in the intermediation of funds between borrowers and banks, and emphasize on the role of collateral borrowers have to place with lenders in return for funding (Iacoviello and Guerrieri, 2017; Gambacorta and Signoretti, 2014; Gerali et al., 2010; Iacoviello, 2005). Finally, some studies incorporate agency problems on both sides of the credit intermediation market (Silvo, 2015; Christensen et al., 2011; Meh and Moran, 2010;

[6] Such models have also been used to assess financial frictions and their implications for (unconventional) monetary policy transmission (Gertler and Karadi, 2011; Cúrdia and Woodford, 2010a,b, 2011), or in studies on bank runs (Gertler et al., 2016; Gertler and Kiyotaki, 2015).

Chen, 2001; Holström and Tirole, 1997).

However, only few studies derive optimal macroprudential policies on welfare-theoretic grounds in models with financial frictions: Cúrdia and Woodford (2010b) and De Paoli and Paustian (2013) find that credit frictions enter welfare-based loss functions for macroprudential policy. Ferrero et al. (2018) discuss coordination between macroprudential and monetary policy and derive a welfare-based loss function that provides scope for active macroprudential policy to overcome imperfect risk-sharing in their model due to household heterogeneity. Aguilar et al. (2019) derive welfare loss functions in a model featuring endogenous bank default as in Clerc et al. (2015) and study different macroprudential rules for the euro area. More often, optimal macroprudential policy analyses rely on a "revealed preferences" approach to define macroprudential objectives (Binder et al., 2018; Silvo, 2015; Angelini et al., 2014; Collard et al., 2014; Gelain and Ilbas, 2014; Angeloni and Faia, 2013; Bean et al., 2010). Based on real-world discussions among policy makers and statements of macroprudential authorities, it is usually assumed that these institutions are primarily concerned with the stabilization of credit and business cycles. Therefore, credit measures as well as measures of economic activity usually enter ad-hoc loss or policy functions used for welfare analyses in these studies, whereas such functions are not derived from first principles. Furthermore, these studies do not take the existence of NBFIs explicitly into account.

In this paper, NBFIs are at the core of the financial sector setup of the model. Therefore, this paper is in close connection to a third strand of the literature that evaluates implications from shadow bank existence with DSGE models. Acknowledging the critique on the absence of NBFI intermediation in canonical DSGE models prior to the financial crisis and thereafter (Christiano et al., 2018), recent studies proposed different approaches to incorporate shadow banking (Gebauer and Mazelis, 2020; Poeschl, 2020; Aikman et al., 2018; Fève and Pierrard, 2017; Meeks and Nelson, 2017; Begenau and Landvoigt, 2016; Gertler et al., 2016; Mazelis, 2016; Verona et al., 2013). These studies evaluate different aspects of the NBFI sector, rely to a different degree on calibration and estimation techniques to match time-series data for the US and the euro area with model-implied dynamics, and discuss the interaction of the NBFI sector with the traditional banking sector and the rest of the economy in different ways. However, all of these studies lack a welfare-based discussion of optimal capital regulation for commercial banks whenever NBFIs are present.

3 A New Keynesian DSGE Model

In the following, I employ a heterogeneous financial sector model closely related to the model in Gebauer and Mazelis (2020).[7] Patient households provide funds to impatient entrepreneurs[8] which are intermediated via financial institutions. Final goods producers buy output produced by entrepreneurs on competitive markets and resell the retail good with a markup to households. The model features price stickiness which is modelled as in Calvo (1983) and implies a New-Keynesian Phillips curve. The financial sector of the model features two representative agents, commercial banks and NBFIs. These financial sector agents are based on different microfoundations, and those differences have welfare implications.

First, financial institutions are differently affected by regulation. Commercial banks, on the one side, have to fulfill capital requirements, and borrowing from these institutions requires compliance with regulatory loan-to-value (LTV) ratios. Therefore, both credit supply and demand policies directly affect commercial bank credit intermediation. The NBFI sector, in contrast, is assumed to consist of a multitude of specialized institutions which intermediate funds through a prolonged intermediation chain. Thus, on aggregate, they provide the same intermediation services as traditional banks, but are not covered by macroprudential regulation. Absent regulatory oversight, NBFIs can default on their obligations and divert funds without reimbursing investors. They will do so whenever the present value of future returns from intermediation is lower than the share of funds they can retain after default. This moral hazard problem between NBFIs and investors implies an endogenous constraint on NBFI leverage, as investors are only willing to provide funding as long as NBFIs behave honestly.

The limit on funding provided to NBFIs implies that the risk-adjusted return NBFIs earn over the deposit rate paid to investors can be positive.[9] However, due to NBFI risk, investors demand a higher return on NBFI investments.[10] Thus, the

[7] The complete set of the nonlinear model equations is provided in appendix B.

[8] Different values in the discount factors determine the borrower-lender relationship between entrepreneurs and households.

[9] See Gertler and Karadi (2011).

[10] Several studies have highlighted that higher non-bank/shadow banking activity can increase overall risk in financial markets and undermine financial stability, for instance if investors neglect tail-risks in unregulated credit markets, see Adrian and Ashcraft (2016), Adrian and Liang (2016) or Gennaioli et al. (2013). Furthermore, default in the shadow banking sector has been identified as a key driver of the global financial crisis of 2007/2008, see for instance Christiano et al. (2018).

spread between NBFI and commercial bank loan rates is positive. Higher returns on NBFIs cause welfare costs as resulting NBFI profits are not transferred to households. The permanent spread can therefore be interpreted as an additional per-unit default cost paid every period.

Finally, the market structure differs in both sectors. In line with empirical evidence on the euro area banking sector, commercial banks exert market power and act under monopolistic competition. NBFIs, on the contrary, act under perfect competition. In reality, the non-bank intermediation sector includes specialized institutions such as money market mutual funds, hedge funds, bond funds, investment funds or special purpose vehicles, and specialization of these institutions implies a high degree of intermediation efficiency in the non-bank sector.

Consequently, the model framework implies that non-bank finance can increase efficiency in the financial system, as long as intermediation outside the regulated banking sector does not pose a threat to financial stability.[11] Furthermore, tighter commercial bank regulation fosters leakage of credit intermediation towards the unregulated part of the financial system. Changes in capital requirements for commercial banks increase intermediation costs and result in reduced intermediation by these institutions. As credit demand by real economic agents is not initially affected by changes in banking regulation, the leverage constraint MBFIs face becomes less binding and intermediation via NBFIs more attractive.

3.1 Households

The representative patient household i maximizes the expected utility

$$\max_{C_t^P(i),\, L_t^P(i),\, D_t^{P,C}(i),\, D_t^{P,S}(i)} E_0 \sum_{t=0}^{\infty} \beta_P^t \Big[\tilde{u}^P(C_t^P) - \int_0^1 \tilde{\nu}^P(L_t(j))dj \Big] \tag{1}$$

where

$$\tilde{u}^P(C_t^P) \equiv \frac{C_t^{P\,1-\sigma}}{1-\sigma} = \ln(C_t^P) \text{ if } \sigma \to 1 \tag{2}$$

$$\tilde{\nu}^P(L_t^P) \equiv \frac{L_t^{P\,1+\phi^P}}{1+\phi^P}. \tag{3}$$

[11] See for instance Acharya et al. (2013).

Each household (i) consumes the composite consumption good C_t^P which is given by a Dixit-Stiglitz aggregate consumption good

$$C_t^P \equiv \left[\int_0^1 c_t^P(i)^{\frac{\theta^P - 1}{\theta^P}} di \right]^{\frac{\theta^P}{\theta^P - 1}} \tag{4}$$

with $\theta^P > 1$.[12] Each type of the differentiated goods $c_t^P(i)$ is supplied by one monopolistic competitive entrepreneur. I assume $\sigma \to 1$ such that utility from consumption in equation 2 can be expressed as log-utility. Entrepreneurs in industry j use a differentiated type of labor specific to the respective industry, whereas prices for each class of differentiated goods produced in sector j are identically set across firms in that sector. I assume that each household supplies all types of labor and consumes all types of goods. The representative household maximizes utility subject to the budget constraint

$$C_t^P(i) + D_t^{P,C}(i) + D_t^{P,S}(i) \leq w_t L_t^P(i) + (1 + r_{t-1}^{dC}) D_{t-1}^{P,C}(i) + (1 + r_{t-1}^{dS}) D_{t-1}^{P,S}(i) + T_t^P(i) \tag{5}$$

where $C_t^P(i)$ depicts current total consumption. Total working hours (allotted to the different sectors j) are given by L_t^P and labor disutility is parameterized by ϕ^P. The flow of expenses includes current consumption and real deposits and investments placed with both commercial banks and NBFIs, $D_t^{P,C}(i)$ and $D_t^{P,S}(i)$. Resources consist of wage earnings $w_t^P L_t^P(i)$ (where w_t is the real wage rate for the labor input of each household), gross interest income on last period investments $(1+r_{t-1}^{dC}) D_{t-1}^{P,C}(i)$ and $(1+r_{t-1}^{dS}) D_{t-1}^{P,S}(i)$, and lump-sum transfers T_t^P that include dividends from firms and commercial banks (of which patient households are the ultimate owners).

3.2 Entrepreneurs

Entrepreneurs engaged in a certain sector j use the respective labor type provided by households as well as capital to produce intermediate goods that retailers purchase in a competitive market. Each entrepreneur i derives utility from consumption $C_t^E(i)$, and maximizes expected utility

$$\max_{C_t^E(i),\, L_t^P(i),\, B_t^{E,C}(i),\, B_t^{E,S}(i)} E_0 \sum_{t=0}^{\infty} \beta_E^t \frac{C_t^{E\,1-\sigma}}{1-\sigma} \tag{6}$$

[12] In the simulation exercises, I calibrate $\theta^P = 1.1$.

subject to the budget constraint

$$C_t^E(i) + w_t l_t^P(i) + (1 + r_{t-1}^{bC})B_{t-1}^{E,C}(i) + (1 + r_{t-1}^{bS})B_{t-1}^{E,S}(i) \\ \leq \frac{y_t^E(i)}{x_t} + B_t^{E,C}(i) + B_t^{E,S}(i) \quad (7)$$

with x_t determining the price markup in the retail sector. Entrepreneurs' expenses, consisting of period-t consumption $C_t^E(i)$, wage payments $w_t l_t^P(i)$, and gross repayments of loans taken on in the previous period from commercial banks and NBFIs $((1 + r_{t-1}^{bC})B_{t-1}^{E,C}(i)$ and $(1 + r_{t-1}^{bS})B_{t-1}^{E,S}(i))$ are financed by production output $\frac{y_t^E(i)}{x_t}$ and period-t borrowing.

Entrepreneurs face a constraint on the amount of loans $B_t^{E,C}(i)$ they can borrow from commercial banks depending on the fixed stock of capital K they hold as collateral.[13] Whereas a regulatory loan-to-value (LTV) ratio m_t^E applies for funds borrowed from commercial banks, NBFI funding is not prone to regulation. Due to a positive spread between interest rates charged for NBFI and commercial bank loans, entrepreneurs have an incentive to borrow from commercial banks first and turn to NBFI lending only whenever the possible amount of commercial bank funds, determined by $m_t^E K$, is reached. Further borrowing can be obtained from shadow banks by using capital holdings not reserved for commercial bank funds, $(1 - m_t^E)K$.[14] As the stock of physical capital is assumed to be fixed, the two respective borrowing constraints are given by

$$(1 + r_t^{bC})B_t^{E,C} \leq m_t^E K \quad (8)$$

$$(1 + r_t^{bS})B_t^{E,S} \leq (1 - m_t^E)K \quad (9)$$

where the LTV ratio for commercial banks m_t^E is set by a separate regulator in an exogenous way. In contrast, the LTV ratio applying to NBFI lending, $m_t^{E,S} = 1 - m_t^E$, depicts an endogenous variable in the model. As borrowers use the share of capital not reserved as collateral for commercial bank credit for funding from NBFIs, non-bank credit may rise if either LTV ratios for commercial bank credit are tightened, or if the borrowing constraint 8 does not bind. In appendix D, I show how the introduction of commercial bank market power and resulting commercial bank credit rationing result in a shift of credit towards NBFIs compared to the efficient steady

[13] In Iacoviello (2005), entrepreneurs use commercial real estate as collateral. However, I follow Gerali et al. (2010) by assuming that creditworthiness of a firm is judged by its overall balance sheet condition where real estate housing only depicts a sub-component of assets.

[14] See the online appendix of Gebauer and Mazelis (2020) for a detailed analysis.

state without market power, resulting in a permanent deviation of credit by both intermediaries from efficient levels.

As in Iacoviello (2005), entrepreneurs face binding borrowing constraints in equilibrium, such that equations 8 and 9 hold with equality.[15] One can furthermore derive an expression for firm net worth as in Gambacorta and Signoretti (2014)

$$NW_t^E = \alpha \frac{y_t^e}{x_t} + K - (1 + r_{t-1}^{bC}) B_{t-1}^{E,C} - (1 + r_{t-1}^{bS}) B_{t-1}^{E,S} \tag{10}$$

where firm net worth in period t is given by net revenues minus wage and interest expenses. Finally, as in Gambacorta and Signoretti (2014), entrepreneur consumption C_t^E is dependent on firm net worth:

$$C_t^E = (1 - \beta_E) NW_t^E. \tag{11}$$

3.3 Financial Intermediaries

The financial sector consists of two types of banks, regulated commercial banks and unregulated NBFIs. Furthermore, commercial banks act under monopolistic competition in the loan market, whereas NBFIs are perfectly competitive entities, but constrained by a moral hazard friction arising with the investing household.

3.3.1 Commercial Banks

Following Gebauer and Mazelis (2020) and Gambacorta and Signoretti (2014), commercial banks consist of two agents: A wholesale unit managing the bank's capital position and taking deposits from households, and a retail loan entity lending funds managed by the wholesale unit to entrepreneurs, charging an interest rate markup.[16]

The *wholesale branches* of commercial banks operate under perfect competition and are responsible for the capital position of the respective commercial bank. On the asset side, they hold funds they provide to the retail loan branch, $B_t^{E,C}$, earning the wholesale loan rate r_t^C. On the liability side, they combine commercial bank net worth, or capital, K_t^C, with household deposits, $D_t^{P,C}$ which earn the policy rate r_t.

[15] Iacoviello (2005) discusses the deviation from certainty equivalence in appendix C of his paper.

[16] In contrast to Gebauer and Mazelis (2020), I do not consider market power in deposit markets in the model, as monopolistic competition in loan markets is sufficient to derive the key findings. However, the model could straightforwardly be extended by introducing a monopolistically competitive deposit entity and deposit rate markdowns as in Gebauer and Mazelis (2020).

Furthermore, the capital position of the wholesale branch is prone to a regulatory capital requirement, ν_t. Moving away from the regulatory requirement imposes a quadratic cost to the bank, which is proportional to the outstanding amount of bank capital and parameterized by κ_k^C.

The wholesale branch maximization problem can be expressed as

$$\max_{B_t^{E,C}, D_t^{P,C}} \quad r_t^C B_t^{E,C} - r_t D_t^{P,C} - \frac{\kappa_k^C}{2} \left(\frac{K_t^C}{B_t^{E,C}} - \nu_t \right)^2 K_t^C \tag{12}$$

subject to the the balance sheet condition

$$B_t^{E,C} = K_t^C + D_t^{P,C}. \tag{13}$$

The first-order conditions yield the following expression:

$$r_t^C = r_t - \kappa_k^C \left(\frac{K_t^C}{B_t^{E,C}} - \nu_t \right) \left(\frac{K_t^C}{B_t^{E,C}} \right)^2. \tag{14}$$

Aggregate bank capital K_t^C is accumulated from retained earnings only:

$$K_t^C = K_{t-1}^C (1 - \delta^C) + J_t^C \tag{15}$$

where J_t^C depicts aggregate commercial bank profits from the two bank branches, see equation B.26 in appendix B. Capital management costs are captured by δ^C.

Finally, *retail loan branches* act under monopolistic competition. They buy wholesale loans, differentiate them at no cost, and resell them to borrowing entrepreneurs. In doing so, the retail loan branch charges a markup μ_t over the wholesale loan rate, and the retail loan rate is thus given by

$$r_t^{bC} = r_t - \kappa_k^C \left(\frac{K_t^C}{B_t^{E,C}} - \nu_t \right) \left(\frac{K_t^C}{B_t^{E,C}} \right)^2 + \mu_t. \tag{16}$$

3.3.2 Non-Bank Financial Institutions

In contrast to the commercial banking sector, NBFIs are not regulated and do not operate under monopolistic competition. Furthermore, NBFIs' ability to acquire external funds is constrained by a moral hazard problem as in Gebauer and Mazelis (2020) and Gertler and Karadi (2011) that limits the creditors' willingness to provide external funds.

NBFIs are assumed to have a finite lifetime: they disappear from the market after some years, whereas the point of exit is unknown a priori. Each NBFI faces an i.i.d. survival probability σ^S with which he will be operating in the next period, so his exit probability in period t is $1-\sigma^S$. Every period new NBFIs enter with an endowment of w^S they receive in the first period of existence, but not thereafter. The number of NBFIs in the system is constant.

For NBFI j, as long as the real return on lending, $(r_t^{bS} - r_t^{dS})$ is positive, it is profitable to accumulate capital until he exits the non-bank finance sector. The NBFI's objective to maximize expected terminal wealth, $v_t(j)$, is given by

$$v_t(j) = \max E_t \sum_{i=0}^{\infty} (1-\sigma^S) {\sigma^S}^i \beta_S^{i+1} K_{t+1+i}^S(j). \tag{17}$$

As I assume some NBFIs to exit each period and new bankers to enter the market, aggregate capital K_t^S is determined by capital of continuing NBFIs, $K_t^{S,c}$, and capital of new bankers that enter, $K_t^{S,n}$

$$K_t^S = K_t^{S,c} + K_t^{S,n}. \tag{18}$$

Following Gebauer and Mazelis (2020) yields the following law of motion for NBFI capital:

$$K_t^S = \sigma^S[(r_{t-1}^{bS} - r_{t-1}^{dS})\phi_{t-1}^S + (1 + r_{t-1}^{dS})]K_{t-1}^S + \omega^S B_{t-1}^{E,S} \tag{19}$$

and the aggregate NBFI balance sheet condition is given by

$$B_t^{E,S} = D_t^{P,S} + K_t^S. \tag{20}$$

Finally, I assume a non-negative spread between the interest rates earned on NBFI investments, r_t^{dS}, and on the deposits households can place with commercial banks, r_t^{dC}, which is determined by the parameter τ^S, with $0 \leq \tau^S \leq 1$:[17]

$$1 + r_t^{dS} = \frac{1 + r_t^{dC}}{1 - \tau^S \varepsilon_t^{\tau}}. \tag{21}$$

3.4 Monetary Policy and Market Clearing

The central bank is assumed to follow a Taylor-type policy rule given by

$$1 + R_t = (1+R)^{1-\rho^r}(1+R_{t-1})^{\rho^r}\left[\pi_t^{\phi^\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi^y}\right]^{1-\rho^r}(1+\epsilon_t^R) \tag{22}$$

[17] In the online appendix to Gebauer and Mazelis (2020), a microfoundation for the existence of a positive spread is provided.

where ρ^r is equal to zero in the analytic derivations of appendix E. The model features sticky prices à la Calvo (1983), which are introduced following Benigno and Woodford (2005). The aggregate resource constraint is given by

$$Y_t = C_t + K + \frac{K_{t-1}^C \delta^C}{\pi_t}. \tag{23}$$

Market clearing implies

$$Y_t = \gamma_y y_t^E \tag{24}$$
$$C_t = C_t^P \gamma_p + C_t^E \gamma_e \tag{25}$$
$$B_t = B_t^{E,C} + B_t^{E,S}. \tag{26}$$

NBFI and commercial bank credit-to-GDP ratios are defined as:

$$Z_t = \frac{B_t^{E,C}}{Y_t} \tag{27}$$
$$Z_t^{SB} = \frac{B_t^{E,S}}{Y_t}. \tag{28}$$

Loan and deposit rate spreads paid by commercial bank and NBFIs are given by

$$\Delta_t^{loan} = r_t^{bS} - r_t^{bC} \tag{29}$$
$$\Delta_t^{deposit} = r_t^{dS} - R_t \tag{30}$$

and the spreads earned on intermediation by commercial banks and NBFIs by

$$\Delta_t^C = r_t^{bC} - R_t \tag{31}$$
$$\Delta_t^S = r_t^{bS} - r_t^{dS}. \tag{32}$$

4 Calibration

Calibrated parameters are largely based on the estimated parameter values in Gebauer and Mazelis (2020) and shown in table 1.[18] In the baseline calibration, the steady-state commercial bank capital requirement is set to 10.5 percent, in line with the proposed level in the Basel III framework. The discount factors for households and firms are calibrated in line with Gerali et al. (2010) and allow for distinguishing between patient households as savers and impatient entrepreneurs as borrowers.

[18] I compare dynamic simulations under this parameterization with an estimated version of the actual model described in section 3 in appendix C.

The commercial bank steady-state LTV-ratio is set to 0.3, in line with empirical estimates derived in Gerali et al. (2010). Firms can therefore acquire 30 percent of lending relative to collateral they pledge, and can furthermore use the remaining 70 percent of their collateral to borrow from NBFIs. In the following, the parameters

Table 1: Calibration

Parameter	**Description**	**Value**
ν	Steady-State Capital Requirement	0.105
β_P	Discount Factor Households	0.9943
β_E, β_S	Discount Factor Entrepreneurs and NBFIs	0.975
m^E	Steady-State LTV Ratio vs. Commercial Banks	0.3
γ^S	Steady-State Share of NBFI Lending	0.33
τ^S	Deposit/Investment Rate Spread Parameter	0.05
θ^S	SB Share of Divertible Funds	0.2
σ^S	SB Survival Probability	0.9
α	Capital Share in Production Function	0.2
δ^C	Bank Capital Management Cost	0.1049
θ^p	Calvo Parameter	0.87
ϕ^π	Taylor-Rule Coefficient π	1.87
ϕ^y	Taylor-Rule Coefficient y	0.24
ϕ^r	Interest Rate Smoothing Parameter	0.88
$\gamma_y, \gamma_p, \gamma_e$	Population Weights	1

Note: Calibration in part based on Gebauer and Mazelis (2020), Gerali et al. (2010) and Gertler and Karadi (2011).

governing commercial bank market power and NBFI risk will have significant welfare implications. The steady-state commercial bank loan rate markup μ is set to 200 basis points, such that it closely matches with the average annualized commercial bank loan rate spread with respect to the EONIA rate in the empirical sample of Gebauer and Mazelis (2020). Furthermore, as discussed in this study, finding an empirical estimate for the spread parameter τ^S is difficult. Under the baseline calibration, the parameter is set such that the implied default probability of NBFIs is approximately five percent per quarter and the resulting annualized spread between NBFI investment and commercial bank deposit rates is approximately two percentage points in steady state. When discussing welfare implications of steady-state NBFI risk in section 6 and appendix D, I evaluate the sensitivity of results with respect to different values of τ^S, thereby acknowledging that the empirical variation in actual returns and resulting spreads can be large on the micro-level.

Remaining parameters are calibrated such that basic empirical relationships observed in the euro area data on commercial banking and non-bank finance are matched.[19] NBFI leverage is equal to one third in the baseline calibration, in line with the share of corporate lending-related activities of shadow-bank type financial firms relative to their net worth in the data. The overall share of NBFIs in total lending activity is also set to 33 percent, in line with estimates derived on the grounds of the empirical data used in the introduction. The remaining parameters are set as discussed in Gebauer and Mazelis (2020).

5 Welfare Analysis: Loss Functions

In the following, I summarize the derivations of welfare loss functions for the cases with and without non-bank finance described in detail in appendix E and discuss welfare-optimal macroprudential regulation both from a static and a dynamic perspective. In the iterative substitution of the terms in the utility functions sketched below, I make use of the Taylor rule as an additional model equation linking the nominal interest rate to output growth and inflation. Thus, I assume that macroprudential policy takes the central bank's actions as given, and sets policy by assuming these actions to be conducted in a Taylor-type fashion. Therefore, no coordination among policy makers is assumed at this point.[20]

5.1 No Non-Bank Finance

In each case, the welfare function is derived following Benigno and Woodford (2005, 2012) from a second-order approximation of aggregate utility. Following Lambertini et al. (2013) and Rubio (2011), the social welfare measure is given by a weighted

[19]See Gebauer and Mazelis (2020).

[20]Several papers recently deviated from this strict assumption by discussing the case of policy coordination, either by assuming perfect coordination or in the form of strategic-interaction games, see for instance Bodenstein et al. (2019), Binder et al. (2018), Gelain and Ilbas (2014), or Beau et al. (2012). The analysis here could be extended in the same direction, by deriving optimal monetary and macroprudential policies jointly. However, as I will show in the following, my analysis will provide scope for policy coordination even without the assumption of jointly-optimal policy coordination of some form in the first place.

sum of patient households' and impatient firms' welfare functions:[21]

$$\mathscr{W}_{t_0} = (1-\beta_P)\mathscr{W}^P_{t_0} + (1-\beta_E)\mathscr{W}^E_{t_0}. \tag{33}$$

For patient household and entrepreneurs, the respective welfare function is given by the conditional expectation of lifetime utility at date t_0,

$$\mathscr{W}^P_{t_0} \approx E_{t_0} \sum_{t=t_0}^{\infty} \beta_P^{t-t_0} [U(C^P_t, L^P_t)] \tag{34}$$

and

$$\mathscr{W}^E_{t_0} \approx E_{t_0} \sum_{t=t_0}^{\infty} \beta_E^{t-t_0} [U(C^E_t)]. \tag{35}$$

Starting from a second-order approximation of the patient household's utility function in equation 1, one can derive an approximated period welfare measure $\widehat{W}^P_t$:

$$\begin{aligned} \widehat{W}^P_t = & \tfrac{1}{2}\psi^{Y^2}_{(8)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(4)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(3)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z^2}_{(4)}\widehat{Z}^2_t + \\ & + \psi^{Y}_{(7)}\widehat{Y}_t + \psi^{\pi}_{(4)}\widehat{\pi}_t + \psi^{\nu}\widehat{\nu}_t + \psi^{z}_{(2)}\widehat{Z}_t + \\ & + covars + t.i.p. + O^3 \end{aligned} \tag{36}$$

where $\widehat{W}^P_t \equiv \frac{U^P_t - U^P}{U^P_{C^P} C^P}$. Hats denote percentage deviations from steady state and the parameters are given in appendix E.1. The terms $covars$ summarizes the sum of covariances in equation 36. As in Benigno and Woodford (2005, 2012), $t.i.p.$ covers terms independent of policy decisions and O^3 terms of higher order.

Similarly, a period welfare term for entrepreneurs

$$\widehat{W}^E_t = \widehat{C}^E_t + (1-\sigma)\frac{1}{2}(\widehat{C}^E_t)^2 \tag{37}$$

can be derived from the second-order approximation of the firm utility function (equation 6). Finally, the terms for $\widehat{W}^P_t$ and $\widehat{W}^E_t$ can be used in the approximation of the period joint welfare function

$$W_t = (1-\beta_P)W^P_t + (1-\beta_E)W^E_t. \tag{38}$$

Using second-order approximations of structural relations in the model, the resulting loss function can be expressed as

$$\widehat{L}_t = \tfrac{1}{2}\lambda^{y^2}\widetilde{Y}^2_t + \tfrac{1}{2}\lambda^{r^2}\widetilde{r}^2_t + \tfrac{1}{2}\lambda^{z,cb^2}\widetilde{Z}^2_t + \tfrac{1}{2}\lambda^{\nu^2}\widehat{\nu}^2_t + \lambda^{z,cb}\widehat{Z}_t. \tag{39}$$

[21] Under such a definition, households and firms derive the same level of utility from a constant consumption stream.

The period welfare loss depends on the variation of the efficient output gap $\widetilde{Y}_t = \widehat{Y}_t - \widehat{Y}_t^*$,[22] the variation in the efficient policy rate gap $\widetilde{r}_t = \widehat{r}_t - \widehat{r}_t^*$, the efficient commercial bank credit-to-GDP ratio gap $\widetilde{Z}_t = \widehat{Z}_t - \widehat{Z}_t^*$, and the capital requirement $\widehat{\nu}_t$. In addition, deviations from the steady-state level of the credit-to-GDP ratio Z_t affect period welfare. The parameters λ^{y^2}, λ^{r^2}, λ^{ν^2}, λ^{z,cb^2}, and $\lambda^{z,cb}$ are determined by steady-state relationships and the structural parameters.

The derived welfare loss function generally resembles the functions employed under the "revealed preferences approach" (Binder et al., 2018; Angelini et al., 2014) in that welfare depends on variations in the output gap, credit-to-GDP, and the macroprudential policy tool ν_t. However, even without an explicit a-priori mandate for policy coordination, the monetary policy tool enters the welfare objective of the regulator.[23] Furthermore, the derived loss function features a level term and therefore does not only contain purely quadratic terms. In section 6.1, I describe the role of level terms in period loss functions as an indication of distortionary effects arising from inefficiencies in the economy related to credit.

5.2 Non-Bank Finance

Whereas the broad structure of the derivation is the same for the model with NBFIs, I briefly highlight how these institutions enter the welfare analysis.[24] The derivation of the second-order approximation of the patient household's welfare criterion $\widehat{W^P}_t$ does not change once NBFIs are allowed for in the model. NBFIs enters the overall welfare criterion via entrepreneurs, as entrepreneur net worth now depends on borrowing from both intermediaries (equation B.19). By including NBFI credit via firm net worth, one can derive a respective loss function for the model with NBFIs which is given by

$$\widehat{L}_t' = \tfrac{1}{2}\lambda^{y^2\prime}\widetilde{Y}_t^2 + \tfrac{1}{2}\lambda^{r^2\prime}\widetilde{r}_t^2 + \tfrac{1}{2}\lambda^{z,cb^2\prime}\widetilde{Z}_t^2 + \tfrac{1}{2}\lambda^{z,sb^2\prime}(\widetilde{Z}_t^{SB})^2 + \tfrac{1}{2}\lambda^{\nu^2}\widehat{\nu}_t^2 + \\ + \lambda^{z,cb\prime}\widehat{Z}_t + \lambda^{z,sb\prime}\widehat{Z}_t^{SB} \quad (40)$$

where $\widetilde{Z}_t^{SB} = \widehat{Z}_t^{SB} - \widehat{Z}_t^{SB*}$ depicts the efficient NBFI credit-to-GDP gap, based on the NBFI credit-to-GDP ratio Z_t^{SB}. Due to the inclusion of non-bank finance, the

[22]Deviations from steady state in the efficient economy absent any frictions are indicated with asterisks. In such an economy, variations are only determined by exogenous shocks.

[23]By substituting the approximated Taylor rule, the inflation rate instead of the nominal interest rate would appear in the loss function, indicating that the policy objectives of both the central bank and the macroprudential regulator are similar.

[24]See appendix E.2 for the derivation of the loss function with NBFIs.

composite parameters in equation 40 take different values compared to the parameters in equation 39. Furthermore, the level terms with respect to credit-to-GDP ratios indicate that both commercial bank and NBFI credit relative to GDP deviate permanently from the optimal level whenever $\lambda^{z,cb'}$ and $\lambda^{z,sb'}$ are different from zero; even when no variations in the objective variables are observed. In section 6.1, I discuss potential reasons for distortionary credit levels and evaluate how these distortions can be corrected.

5.3 Static Evaluation

Analytic derivations of the coefficients in equations 39 and 40 allow for a computation of parameter values under the baseline calibration. Table 2 depicts the respective parameter values on the quadratic terms in the form of "sacrifice ratios": The parameters on the quadratic terms related to the capital requirement, the output gap, the NBFI credit-to-GDP ratio, and the interest rate are expressed relative to the coefficient on the commercial bank credit-to-GDP ratio. Thus, the relative importance of other policy objectives vis-à-vis commercial bank credit stabilization in the welfare criterion can be evaluated. The level term parameters $\lambda^{z,cb}$, $\lambda^{z,cb'}$ and $\lambda^{z,sb'}$ are reported in absolute terms.

Table 2: Loss Function Parameters

		No Non-Bank Finance	**Non-Bank Finance**
$\lambda^{y^2}/\lambda^{z,cb^2}$	Output Gap	2.72	0.76
$\lambda^{z,sb^2}/\lambda^{z,cb^2}$	SB Credit/GDP	-	0.92
$\lambda^{r^2}/\lambda^{z,cb^2}$	Interest Rate	34.25	12.90
$\lambda^{\nu^2}/\lambda^{z,cb^2}$	Capital Requirement	0.009	0.002
$\lambda^{z,cb}$	CB Credit/GDP level	-0.16	-1.33
$\lambda^{z,sb}$	SB Credit/GDP level	-	1.52

Note: Values of coefficients in equations 39 and 40 under baseline parameterization. See appendix E for derivations.

Strikingly, the importance of credit stabilization relative to interest rate and output gap stabilization increases substantially once NBFIs enter the model. Whereas the weight on output gap stabilization is almost three times larger than the weight on commercial bank credit stabilization in the model without NBFIs, the latter exceeds the output gap weight in the loss function of the model including non-bank finance. Also, the weight on commercial bank credit stabilization increases substantially relative to the weight on the interest rate objective in the model with non-bank finance. Furthermore, even though the regulator cannot directly stabilize NBFI credit, he puts a relatively high weight on its variation when setting policy: Stabilization of credit in the non-bank financial sector enters with almost the same weight as commercial bank credit variations. Thus, total credit stabilization plays a much larger role in the model with non-bank finance compared to the case of perfectly implementable financial regulation without NBFIs.

Finally, the parameters on commercial bank credit level terms, $\lambda^{z,cb}$ and $\lambda^{z,cb'}$ are negative in both model versions, whereas the parameter for the NBFI credit level term $\lambda^{z,sb'}$ is positive under reasonable parameter values. As discussed in more detail in section 6.1 and appendix D, due to market power and NBFI inefficiencies, steady-state levels of commercial (shadow) bank credit are below (above) efficient levels that would prevail in a frictionless economy. Due to these deviations, a marginal increase (decrease) in commercial (shadow) bank credit has a positive welfare effect (as losses are reduced). I discuss the existence of level terms in the loss functions and implications for policy in the following section.

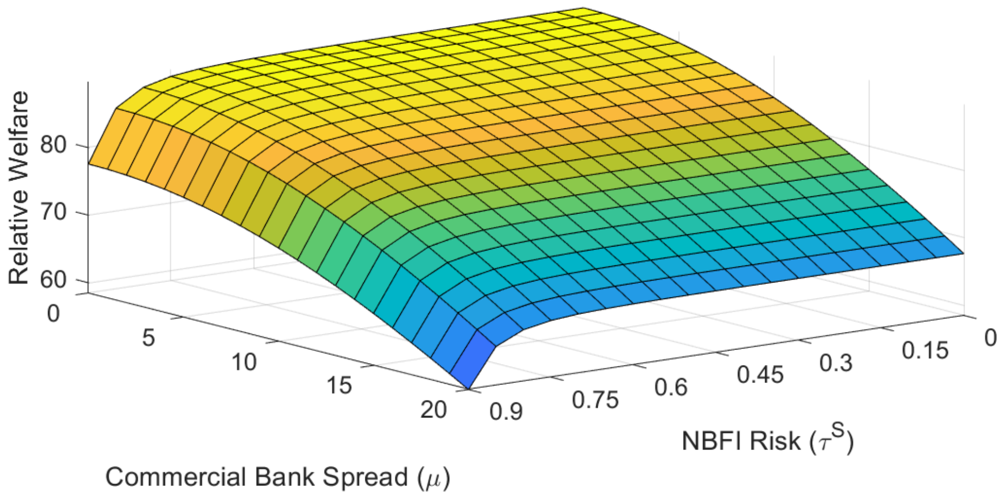
6 Welfare Analysis: Optimal Level Policy

The above loss functions indicate that social welfare not only depends on the ability of policy makers to stabilize *cyclical* fluctuations in the target variables. Also, the permanent *levels* of commercial bank and NBFI credit have welfare implications. Thus, the model provides scope that both time-invariant and cyclical macroprudential policies can be welfare-enhancing. In the following, I discuss how financial frictions induce permanent steady-state distortions that provide scope for time-invariant macroprudential policies. Furthermore, I evaluate how different permanent regulatory tools can be employed to resolve the resulting policy trade-off.

6.1 Distortionary Effects of Bank Market Power and NBFI Inefficiencies

As I discuss in detail in the steady-state analysis of appendix D, financial frictions in both the commercial banking and NBFI sector result in permanent deviations of shadow and commercial bank credit from their efficient levels. Due to market power, commercial banks charge a steady-state markup μ on the credit they pro-

Figure 2: Welfare Implications of Steady-State Distortions



Note: Relative welfare levels under Ramsey-optimal policies based on objective 41 for different values of the commercial bank loan markup μ (percentage points) and NBFI risk τ^S. Welfare levels are in relation to levels obtained in the decentralized economy presented in section D.2.

vide to borrowing entrepreneurs, and the amount of credit intermediated by these institutions is below the efficient level. To accommodate their demand for funding, entrepreneurs turn to perfectly competitive but risky NBFIs, using a larger share of their collateral capital stock K to pledge against borrowing from these institutions. Thus, both monopolistic competition in the commercial banking sector and the default risk of NBFIs – where the frictions are governed by μ and τ^S, respectively – imply welfare losses.

Figure 2 reports welfare implications of increases in both friction parameters. Relative welfare levels are expressed in terms of consumption equivalents given by

$$1 - \xi \equiv (1 - \xi^P)^{1-\beta_E} (1 - \xi^E)^{1-\beta_P} = \exp[(\mathscr{W}_{t_0} - \mathscr{W}^*_{t_0})(1 - \beta_P)]^{1-\beta_E} \tag{41}$$

derived from the welfare criterion 33 in appendix F. Cost parameters ξ^P and ξ^E determine the loss in consumption by households and entrepreneurs in the economy with financial, real and nominal frictions, compared to the decentralized economy presented in appendix section D.2. In the decentralized economy, both shadow and commercial banks exist. They intermediate funds equally efficient since no financial frictions such as market power and risk (and no real frictions or nominal rigidities from sticky prices) are present in this scenario. Welfare in the friction economy ($\mathscr{W}_{t_0}$) relative to welfare in the decentralized frictionless economy ($\mathscr{W}^*_{t_0}$) is compared in terms of composite consumption equivalents, i.e. by the maximum fraction ξ of consumption that both households and entrepreneurs would be willing to forego in the economy featuring financial, nominal and real frictions to join the decentralized economy of appendix D.2. The composite cost ξ is defined such that an increase in the welfare share of one agent in equation 33 results in a lower contribution of the other agent's consumption losses to overall losses, given that $0 < \beta_P, \beta_E < 1$.

An increase in the friction parameters results in a reduction of overall welfare in the model with NBFIs, whereas the amplification of the welfare losses increases for high levels of distortions in both cases. Particularly for high levels of default risk, welfare drops sharply. Furthermore, as shown in appendix D.5.1, both frictions imply that the market-clearing level of time-invariant capital requirements is different in NBFI and commercial bank credit markets. While the efficient level of capital requirements in the decentralized economy absent financial frictions

$$\nu^* = \frac{K^C}{\beta_P m^E K} \tag{42}$$

results in clearing of both markets, the levels of steady-state capital requirements implied by clearing in each credit market – ν^C and ν^S, respectively – are given by

$$\begin{aligned} \nu^C &= \frac{K^C(1+\beta_P\mu)}{\beta_P m^E K} \\ \nu^S &= K^C\left[\beta_P K - (1-\tau^S)\beta_P\left(1 - \frac{1}{1+\beta_P\mu} m^E\right)K\right]^{-1} \end{aligned} \tag{43}$$

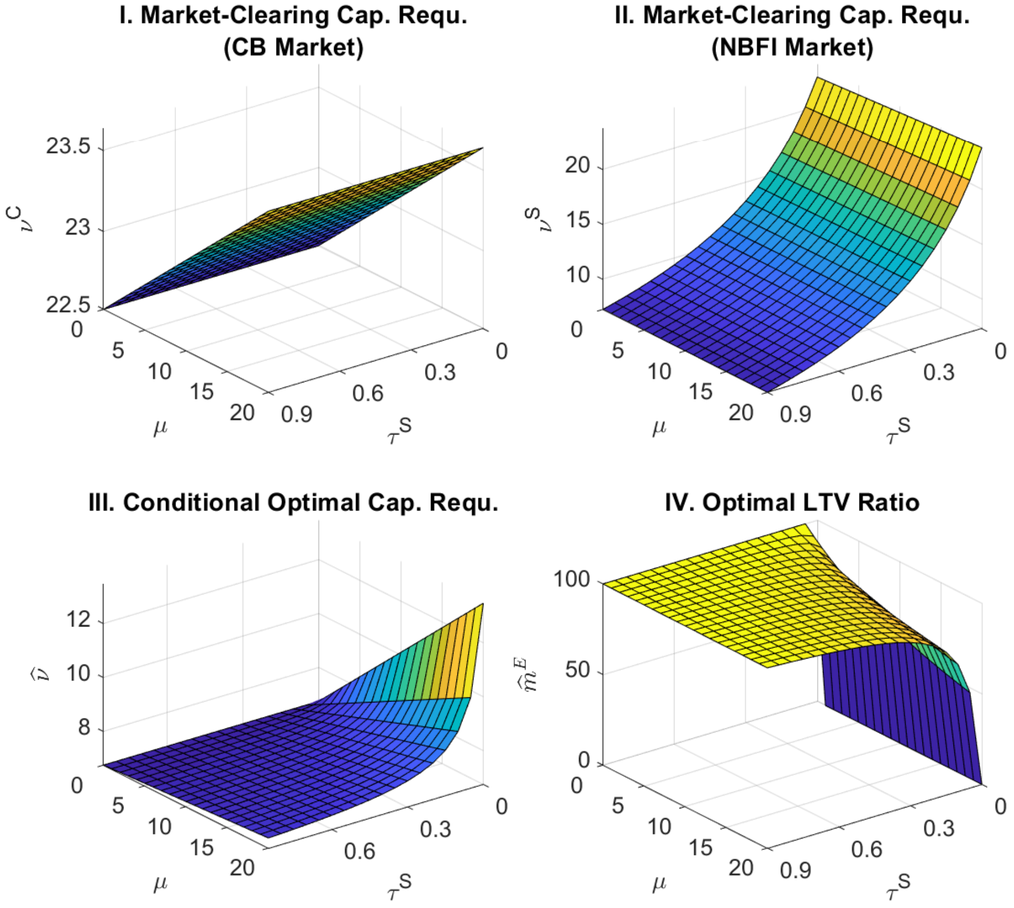
in the steady state featuring financial distortions. As discussed in proposition 6 in the appendix and shown in the upper part of figure 3, these requirements

1. differ from the efficient level ν^* in the decentralized frictionless economy
2. increase (decrease) in commercial bank market power (NBFI risk)

The discrepancy in market-clearing levels of permanent capital requirements due to steady-state distortions has implications for optimal time-invariant macroprudential regulation. As a consequence, it is not feasible to account for *both* origins of steady-state distortions with only one macroprudential tool. However, in line with the Tinbergen (1952) principle, policy makers can pursue a strategy of targeting credit deviations from socially optimal levels in each lending market separately by applying one tool to one distortion.

In section D.5.2 of the appendix, I propose that a mix of both supply- and demand-side oriented time-invariant macroprudential policy tools can lead to allocations where steady-state levels of both commercial bank and NBFI credit are at

Figure 3: Time-Invariant Levels of Macroprudential Policies



Note: Levels of steady-state capital requirements (ν^C, ν^S, $\widehat{\nu}$) and LTV ratios ($\widehat{m}^E$) for different values of the commercial bank loan markup μ (percentage points) and NBFI risk τ^S.

their efficient levels. Capital requirements, targeting credit supply of commercial banks directly, appear suited to account for distortions stemming from commercial bank market power. Additionally, whenever NBFI credit supply cannot be regulated directly, borrower-side tools such as LTV ratios present a means for taking account of distortions in this market.

In the strategy outlined, the authority responsible for permanent LTV ratios sets regulation such that the efficiency gap in the NBFI credit market, i.e. the difference in NBFI credit levels in the distorted steady state and the steady state of the decentralized economy absent financial frictions given by

$$\widehat{B}^{E,S} = B^{E,S} - B^{E,S*} = \left[\left(1 - \frac{1-\tau^S}{1+\beta_P\mu}\right)m^E - \tau^S\right]K\beta_P \tag{44}$$

is zero. The implied optimal level of steady-state LTV ratios is then given by

$$\widehat{m}^E = \tau^S \frac{1+\beta_P\mu}{\tau^S + \beta_P\mu}. \tag{45}$$

Conditional on the gap-closing level $\widehat{m}^E$ set by the LTV authority, steady-state capital requirements are chosen such that the commercial bank efficiency gap given by

$$\widehat{B}^{E,C} = B^{E,C} - B^{E,C*} = \left(\frac{\beta_P}{1+\beta_P\mu} - \beta_P\right)m^E K \tag{46}$$

is closed. The resulting optimal capital requirement is equal to

$$\widehat{\nu} = \frac{K^C(\tau^S + \beta_P\mu)}{\beta_P\tau^S K}. \tag{47}$$

The lower part of figure 3 shows the implied optimal levels of capital requirements and LTV ratios that close credit gaps stemming from steady-state distortions in the economy with financial frictions.[25] Whenever NBFI risk is almost absent in the economy ($\tau^S \to 0$), it is optimal for the regulator to set permanent LTV ratios close to zero, independent of the degree of commercial bank market power (quadrant IV.). In this case, limiting credit intermediation of monopolistic-competitive commercial banks and enforcing a shift towards (almost) risk-free NBFIs which act under perfect competition is beneficial. Similarly, an increase in commercial bank market power leads to the relative superiority of NBFI credit.

In contrast, higher levels of NBFI risk and lower levels of bank market power induce an increase in the optimal LTV ratio level, as a welfare-optimal lending

[25] $\widehat{\nu}$ is only defined whenever the NBFI risk parameter τ^S is positive. Whenever $\tau^S = 0$, $\nu^C = \nu^S$ and the efficient level coincides with these expressions (if $\mu > 0$), or with ν^* (if $\mu = 0$).

mix features a larger share of commercial bank credit in these cases. Therefore, lowering borrowing standards with respect to commercial bank lending becomes beneficial, and in the boundary case of no commercial bank market power, it is optimal to set LTV ratios to 100 percent, such that all intermediation is conducted by commercial banks. Similarly, the optimal level of steady-state capital requirements increases whenever bank market power increases and NBFI risk is low. Again, tighter regulation for commercial bank is welfare-enhancing whenever NBFI credit becomes relative more attractive.

6.2 Welfare-Optimal Permanent Capital Requirements

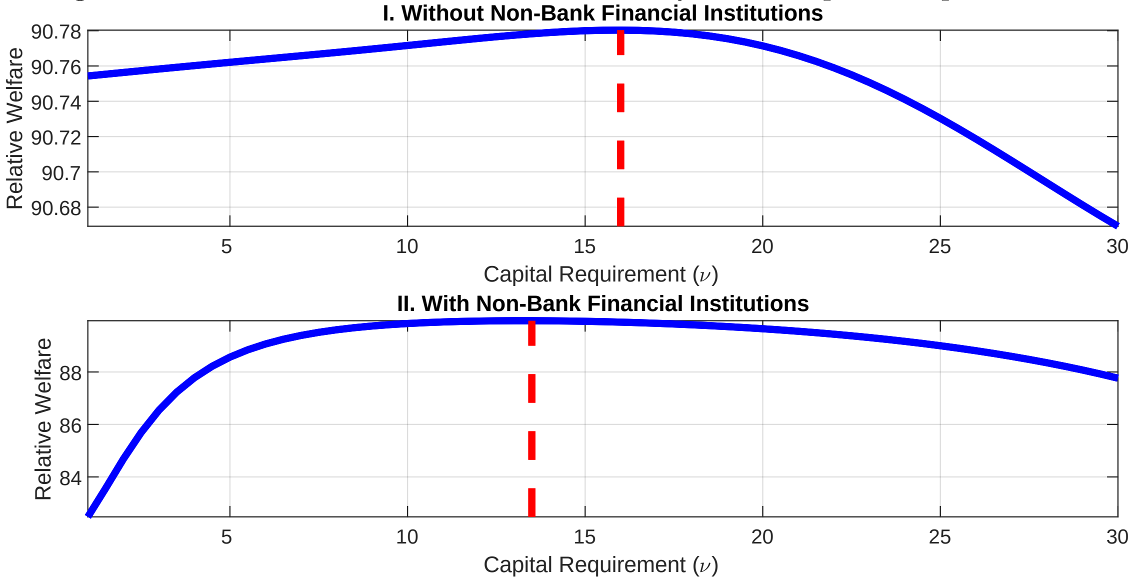
In the previous section, I showed analytically that the existence of commercial bank market power and NBFI default risk implies a trade-off for policy makers deciding on the adequate level of commercial bank capital requirements. Quadrant III in figure 3 indicates that it is optimal for regulators to set capital charges to a high level in the presence of commercial bank market power to shift intermediation to the perfectly competitive NBFI sector. However, the presence of NBFI risk induces welfare losses[26] that limit the optimal amount of credit intermediation by these institutions. Due to the implied trade-off, the optimal level of steady-state capital requirements is unclear a-priori. Figure 4 shows relative welfare according to equation 41 under the baseline calibration of μ and τ^S for different levels of ν. The optimal level of capital requirements is given by approximately 13.5 percent for the model with NBFIs, which coincides with the computed value of $\widehat{\nu}$ under the baseline calibration.

Furthermore, independent of the level of capital requirements, welfare levels are universally lower once NBFI intermediation is taken into account (panel II.), compared to an economy with only commercial bank intermediation (panel I.). Thus, NBFI risk has adverse welfare implications in the model economy, which are not compensated by efficiency gains related to non-monopolistic intermediation in the NBFI sector. Instead, NBFI lending introduces an additional trade-off which complicates welfare-optimal policy making.

Finally, the shape of the welfare profile in figure 4 depends on the presence of NBFIs. In the absence of NBFIs (panel I.), welfare is relatively high for capi-

[26] In the model, the actual losses stem from the fact that NBFI profits – which increase in response to higher intermediation as the leverage constraint of NBFIs is loosened – are not transferred to households.

Figure 4: Welfare for Different Levels of Steady-State Capital Requirements



Note: Relative welfare levels under Ramsey-optimal policies based on objective 41 for different values of the steady-state capital requirement ν (percentage points). Welfare levels are in relation to levels obtained in the decentralized economy presented in section D.2, when NBFIs are absent (I.) or present (II.).

tal requirements below the optimum level of approximately 16 percent, but drops significantly for higher levels. Commercial banks are the only intermediaries and therefore the financial sector as a whole is affected by regulation. Whenever capital requirements are above the optimal level, subdued intermediation adversely affects real economic activity, and ultimately household and firm consumption. In contrast, the drop in welfare associated with steady-state capital requirements above the optimal level is only moderate in the model with NBFIs (panel II.), compared to welfare losses for lower-than-optimal requirement levels. In response to excessive regulation, the decline in commercial bank lending is partly compensated by NBFI intermediation, and adverse effects for the real economy due to higher-than-optimal requirements are mitigated.

7 Welfare Analysis: Optimal Dynamic Policy

In the previous section, I discussed the importance of time-invariant macroprudential policies and the adequate permanent *level* of capital requirements. Under Basel III, regulators have the opportunity to adjust bank capital charges in a dynamic fashion within bands around such permanent levels,[27] depending on movements in business and credit cycles. In principle, policy makers agreed that these cyclical buffers should be adjusted in a *countercyclical* fashion, i.e. raised (lowered) whenever lending and potentially real economic activity are "excessively" high (low). However, the discussion on the definition of excessive lending and the optimal design of dynamic policy rules for setting countercyclical capital requirements is still ongoing.[28]

In the following, I discuss the *cyclical* component of optimal regulation by deriving the optimal policy from a timeless perspective as in Benigno and Woodford (2005, 2012). First, I derive the welfare-optimal rule analytically in section 7.1.1 and discuss its properties. As the rule relates the adjustment of capital requirements to both contemporaneous and lagged values of a variety of target variables, less complex rules might be desirable from a practical perspective. Therefore, I evaluate the performance of more simple rules that only feature a subset of variables in comparison to the welfare-optimal rule in section 7.1.2. Finally, I discuss optimal dynamic responses to exogenous disturbances in a simulation exercise in section 7.2.

7.1 Optimal Policy Rules with Non-Bank Finance

7.1.1 The Welfare-Optimal Policy Rule

Based on the derivations in section 5, I derive an optimal macroprudential policy rule. To do so, I minimize the quadratic loss function subject to the linearized model constraints and initial conditions related to the timeless-perspective approach. However, the linear-quadratic approach requires the welfare (loss) function to contain purely quadratic terms only, such that linear approximations to equilibrium conditions are sufficient to evaluate the second-order welfare criterion.[29] To pursue with a purely quadratic loss function, I rely on the findings in the previous section and calibrate steady-state capital requirements and LTV ratios to 13.5 and 91.4 percent, the levels implied by equations 45 and 47 under the baseline calibration of section 4. As shown in appendix D, the permanent gap between steady-state commercial bank and NBFI credit and the respective efficient levels is closed when time-invariant

[27]The regulatory bands for countercyclical capital requirements allow for symmetric deviations of up to 2.5 percentage points from permanent levels under Basel III.

[28]See for instance Binder et al. (2018), Angelini et al. (2014), Cúrdia and Woodford (2010b), or De Paoli and Paustian (2013).

[29]See Benigno and Woodford (2012).

macroprudential policies are set to these values, such that the distortionary level terms $\widetilde{Z}_t$ and $\widetilde{Z}_t^{SB}$ in loss function 40 disappear. This allows for the evaluation of a purely quadratic welfare objective and the derivation of an optimal policy rule following the LQ-approach of Benigno and Woodford (2005, 2012) and Giannoni and Woodford (2003a,b). The welfare loss function to be minimized subject to the log-linearized structural model equations therefore only includes purely quadratic terms and is given by

$$\widehat{L}_t' = \tfrac{1}{2}\lambda^{y^{2\prime}}\widetilde{Y}_t^2 + \tfrac{1}{2}\lambda^{r^{2\prime}}\widetilde{r}_t^2 + \tfrac{1}{2}\lambda^{z,cb^{2\prime}}\widetilde{Z}_t^2 + \tfrac{1}{2}\lambda^{z,sb^{2\prime}}(\widetilde{Z}_t^{SB})^2 + \tfrac{1}{2}\lambda^{\nu^{2\prime}}\widehat{\nu}_t^2. \tag{48}$$

Furthermore, as outlined in appendix G, the rule is derived such that Lagrange multipliers on lagged terms in the first-order conditions of the Ramsey planner (equations F.2 to F.43 in appendix G) are treated as parameters. Thus, initial conditions are honoured and not automatically set equal to zero in the minimization problem of the Ramsey planner. Thus, the time-dependence problem arising in the implementation of policy in period t_0 is taken into account. Therefore, optimal policy is derived from a timeless perspective,[30] and the policy rule describes the optimal response of the policy maker to random disturbances in all periods $t \geq 0$.[31]

Minimizing loss function 48 subject to the linearized structural equations given in appendix B and following the iterative approach outlined in appendix G yields the macroprudential policy rule

$$\begin{aligned}\widehat{\nu}_t = {} & \rho^{\nu} + \rho_1^{\nu}\widehat{\nu}_{t-1} + \rho_2^{\nu}\widehat{\nu}_{t-2} + \rho_3^{\nu}\widehat{\nu}_{t-3} + \\ & + \phi_1^{r}\widetilde{r}_t + \phi_2^{r}\widetilde{r}_{t-1} + \phi_3^{r}\widetilde{r}_{t-2} + \phi_4^{r}\widetilde{r}_{t-3} + \\ & + \phi_1^{y}\widetilde{Y}_t + \phi_2^{y}\widetilde{Y}_{t-1} + \phi_3^{y}\widetilde{Y}_{t-2} + \phi_4^{y}\widetilde{Y}_{t-3} + \\ & + \phi_1^{z,cb}\widetilde{Z}_t + \phi_2^{z,cb}\widetilde{Z}_{t-1} + \phi_3^{z,cb}\widetilde{Z}_{t-2} + \phi_4^{z,cb}\widetilde{Z}_{t-3} + \\ & + \phi_1^{z,sb}\widetilde{Z}_t^{SB} + \phi_2^{z,sb}\widetilde{Z}_{t-1}^{SB} + \phi_3^{z,sb}\widetilde{Z}_{t-2}^{SB} + \phi_4^{z,sb}\widetilde{Z}_{t-3}^{SB}\end{aligned} \tag{49}$$

where the policy parameters ρ^{ν}, $\rho_k^{\nu}, k \in \{1,2,3\}$ and $\Phi_n^m, m \in \{r; y; z, cb; z, sb\}; n \in \{1,2,3,4\}$ are composite parameters consisting of structural parameters and steady-

[30] By treating initial multiplier conditions as parameters being equal to zero or steady-state values, I derive optimal policy from a timeless perspective as referred to in Schmitt-Grohé and Uribe (2005) when the initial multipliers are set to steady-state.

[31] See for instance Bodenstein et al. (2019), Benigno and Woodford (2005, 2012), Giannoni and Woodford (2003a,b), or Schmitt-Grohé and Uribe (2005) for extensive discussions on the time-inconsistency problems arising from neglecting initial conditions and on the derivations of optimal policy from a timeless perspective for the cases of optimal monetary and fiscal policies.

state relations.[32] In the terminology of Giannoni and Woodford (2003a,b), the rule given by equation 49 depicts a *robustly optimal* rule, as none of the derivations outlined in appendix G depends on the structural form of the disturbance processes of the model.[33] It is also a robustly optimal *direct* policy rule, as it does not involve direct response to exogenous shocks, but to observed target variables only. It is furthermore an *implicit* policy rule, as contemporaneous values of the target variables in addition to lagged (predetermined) values enter equation 49, for which contemporaneous projections have to be formed implicitly. Table 3 reports parameter values under the baseline calibration reported in table 1.

Several observations can be drawn from rule 49 and the parameter values under baseline calibration in table 3. First, macroprudential regulators raise capital requirements under optimal policy whenever the output gap and the commercial bank credit-to-GDP ratio increase above their efficient levels. Therefore, the optimal rule features countercylcical elements usually incorporated in ad-hoc rules in the "revealed preferences" literature. Whereas the optimal response to output gap deviations shows some inertia, macroprudential regulators put a high weight on contemporaneous variations in commercial bank credit-to-GDP. Cumulatively, the weights associated to these variables are the largest, followed by the cumulative weight on NBFI credit in absolute terms. Quantitatively, the response to the nominal interest rate is relative moderate in the derived rule, even if the interest rate weight in loss function 40 turned out to be relatively large (table 2).

Second, the regulator attaches negative weights to deviations in NBFI credit-to-GDP from efficient levels under optimal policy. Whenever NBFI lending increases over the efficient level, the macroprudential regulator, ceteris paribus, has a motive to *lower* capital requirements for commercial banks to counteract credit leakage. Thus, the additional trade-off stemming from credit leakage already highlighted in the evaluation of optimal steady-state levels in section 5.3 is reflected in the policy rule. Without NBFIs, this trade-off would be absent, and optimal regulation would unambiguously prescribe higher capital requirements in response to exogenous shocks that increase credit intermediation – which would then be conducted by commercial banks only. However, the optimal reaction with NBFIs depends on the nature of the shock and its relative effect on both credit aggregates, and on the relative size of the credit coefficients.

[32]See appendix G where auxiliary parameters defined in the calculations are reported. A full set of parameters defined in the derivation is available upon request.

[33]See section B.6 for a description of the assumed shock processes.

Table 3: Policy Rule Parameters

Parameter		$\Upsilon = 0$	$\Upsilon = \overline{\Upsilon}$
Inertia Parameter	ρ^{ν}	0.000	0.092
	ρ_1^{ν}	0.562	0.562
	ρ_2^{ν}	$<$0.000	$<$0.000
	ρ_3^{ν}	$<$0.000	$<$0.000
Nominal Interest Rate	Φ_1^{r}	-0.030	-0.030
	Φ_2^{r}	-0.027	-0.027
	Φ_3^{r}	-0.059	-0.059
	Φ_4^{r}	-0.031	-0.031
Output Gap	Φ_1^{y}	1.729	1.729
	Φ_2^{y}	1.909	1.909
	Φ_3^{y}	0.156	0.156
	Φ_4^{y}	-0.082	-0.082
CB Credit-to-GDP	$\Phi_1^{z,cb}$	6.103	6.103
	$\Phi_2^{z,cb}$	$<$0.000	$<$0.000
	$\Phi_3^{z,cb}$	$<$0.000	$<$0.000
	$\Phi_4^{z,cb}$	$<$0.000	$<$0.000
SB Credit-to-GDP	$\Phi_1^{z,sb}$	-0.100	-0.100
	$\Phi_2^{z,sb}$	-0.122	-0.122
	$\Phi_3^{z,sb}$	-0.256	-0.256
	$\Phi_4^{z,sb}$	-0.135	-0.135

Note: Values of policy parameters in rule 49 under the baseline calibration. $\Upsilon = 0$ ($\Upsilon = \overline{\Upsilon}$) when initial conditions given by vector F.44 are equal to zero (equal to steady state values).

Third, macroprudential policy responds to movements in the nominal interest rate, indicating scope for coordination among policy makers. In the model, optimal macroprudential policy operates to mitigate adverse effects on credit and output, as capital requirements are loosened whenever the policy rate is raised by the central bank. As discussed in Gebauer and Mazelis (2020), higher interest rates induce credit leakage to NBFIs in the model, which provides an additional rationale for the macroprudential regulator to lower capital requirements in response to tighter monetary policy. Under optimal policy coordination, these adverse effects would be considered in the monetary-macroprudential policy trade-off.

Finally, optimal capital regulation for commercial banks appears to be described by some degree of time-dependence, as both lagged values of the capital requirement itself and the target variables enter the optimal rule. In some circumstances, parameter values indicate a stronger weight on past values instead of contemporaneous projections of target variables. For instance, the response to the output efficiency gap in $t-1$ should be slightly larger than the contemporaneous response. For the nominal interest rate and NBFI credit, the largest weight is attached to observations in $t-2$. Only in the case of commercial bank credit, the optimal rule indicates a strong contemporaneous response.

7.1.2 Optimal Simple Rules

In the following, I study whether the complex optimal policy rule 49 can be approximated by simple implementable rules without substantial welfare losses. Following the "revealed preferences" literature, the generic simple rule is given by:

$$\widehat{\nu}_t = \rho^{\nu}\widehat{\nu}_{t-1} + \Phi'\mathbf{X_t} \tag{50}$$

$$\Phi = \begin{bmatrix} \phi_S^{y} \\ \phi_S^{z,cb} \\ \phi_S^{z,sb} \end{bmatrix} \mathbf{X_t} = \begin{bmatrix} \widetilde{Y}_t \\ \widetilde{Z}_t \\ \widetilde{Z}_t^{SB} \end{bmatrix} \tag{51}$$

The macroprudential authority sets the capital requirement $\widehat{\nu}_t$ by considering an autoregressive component as well as deviations of output and credit-to-GDP gaps from efficient steady-state levels. Thus, the authority minimizes the loss function 48 by choosing the parameters in Φ, such that the optimization problem is given by:

$$\min_{\Phi} \widehat{L}_t' = \tfrac{1}{2}\lambda^{y^2\prime}\widetilde{Y}_t^2 + \tfrac{1}{2}\lambda^{r^2\prime}\widetilde{r}_t^2 + \tfrac{1}{2}\lambda^{z,cb^2\prime}\widetilde{Z}_t^2 + \tfrac{1}{2}\lambda^{z,sb^2\prime}(\widetilde{Z}_t^{SB})^2 + \tfrac{1}{2}\lambda^{\nu^2\prime}\widehat{\nu}_t^2 \tag{52}$$

$$\text{s.t. } \widehat{\nu}_t = \rho^{\nu}\widehat{\nu}_{t-1} + \Phi'\mathbf{X_t} \tag{53}$$

$$0 = E_t[f(\mathbf{x_t}, \mathbf{x_{t+1}}, \mathbf{x_{t-1}}, \theta^{\mathbf{m}})] \tag{54}$$

where the last line represents constraints arising from the model structure. The function $f(\bullet)$ refers to the model equations, $\mathbf{x_t}$ to the vector of endogenous variables, and $\theta^{\mathbf{m}}$ to the vector of model parameters. Table 4 summarizes the optimized parameters for different variants of the generic rule 50 which are given by:

$$\text{OSR/CR 1: } \widehat{\nu}_t = \phi_S^{z,cb}\widetilde{Z}_t \tag{55}$$

$$\text{OSR/CR 2: } \widehat{\nu}_t = \phi_S^{y}\widetilde{Y}_t + \phi_S^{z,cb}\widetilde{Z}_t \tag{56}$$

$$\text{OSR/CR 3: } \widehat{\nu}_t = \phi_S^{y}\widetilde{Y}_t + \phi_S^{z,cb}\widetilde{Z}_t + \phi_S^{z,sb}\widetilde{Z}_t^{SB} \tag{57}$$

$$\text{OSR/CR 4: } \widehat{\nu}_t = \rho^{\nu}\widehat{\nu}_{t-1} + \phi_S^{y}\widetilde{Y}_t + \phi_S^{z,cb}\widetilde{Z}_t + \phi_S^{z,sb}\widetilde{Z}_t^{SB} \tag{58}$$

The simplest rule given by equation 55 indicates that the regulator only adjusts capital requirements in response to a contemporaneous deviation of the commercial bank credit-to-GDP gap from the efficient steady state. In the rules given by equations 56 to 58, contemporaneous deviations of the output and the NBFI credit-to-GDP gap as well as an autoregressive term are iteratively introduced.

Table 4: Simple Rule Parameters

	Optimal Simple Rules (OSR)				Constrained Rules (CR)			
Parameter	OSR 1	OSR 2	OSR 3	OSR 4	CR 1	CR 2	CR 3	CR 4
ρ_S^{ν}				0.562				0.562
ϕ_S^{y}		0.133	0.241	0.243		1.729	1.729	1.729
$\phi_S^{z,cb}$	12.231	33.169	50.564	52.337	6.103	6.103	6.103	6.103
$\phi_S^{z,sb}$			-11.103	-38.424			-0.100	-0.100
Relative Loss	0.0005	0.0005	0.0003	0.0002	0.0014	1.9915	1.9915	5.5584

Note: Values of policy parameters in rules 55 to 58. Optimal simple rules (OSR) refer to rules with optimized parameters, while constrained rules (CR) indicate rules with parameters directly taken from the fully optimal rule 49 under the baseline calibration. Welfare losses under each rule are expressed relative to welfare losses obtained under the fully optimal policy regime.

The left column of table 4 indicates that for all variants, parameters can be chosen by the regulator such that the welfare loss relative to the losses obtained under the welfare-optimal rule 49 is small. However, achieving the same level of welfare losses with simple rules requires large parameter values in absolute terms. Neglecting lags and additional variables such as short-term interest rates enforces stronger reactions to the contemporaneous variables under consideration. Strikingly, considering credit on a disaggregated level (OSR 3 given by equation 57) results in a strong increase in the parameter on commercial bank credit compared to simpler rules, as the sizeable negative coefficient on the NBFI credit-to-GDP gap counteracts the effect of changes in commercial bank credit.

The last four columns of table 4 report constrained rules (CR) designed according to equations 55 to 58, but without optimized coefficients. Instead, coefficients on contemporaneous variables are fixed at the respective coefficient values derived for the fully optimal rule 49 reported in table 3. By incorporating additional contemporaneous variables (moving from CR 1 to CR 4) in the constrained rule, the relative welfare loss increases. Thus, even by incorporating more information in policy rules, welfare losses can increase if simple rule parameters are not separately optimized.

7.2 Simulation Analysis

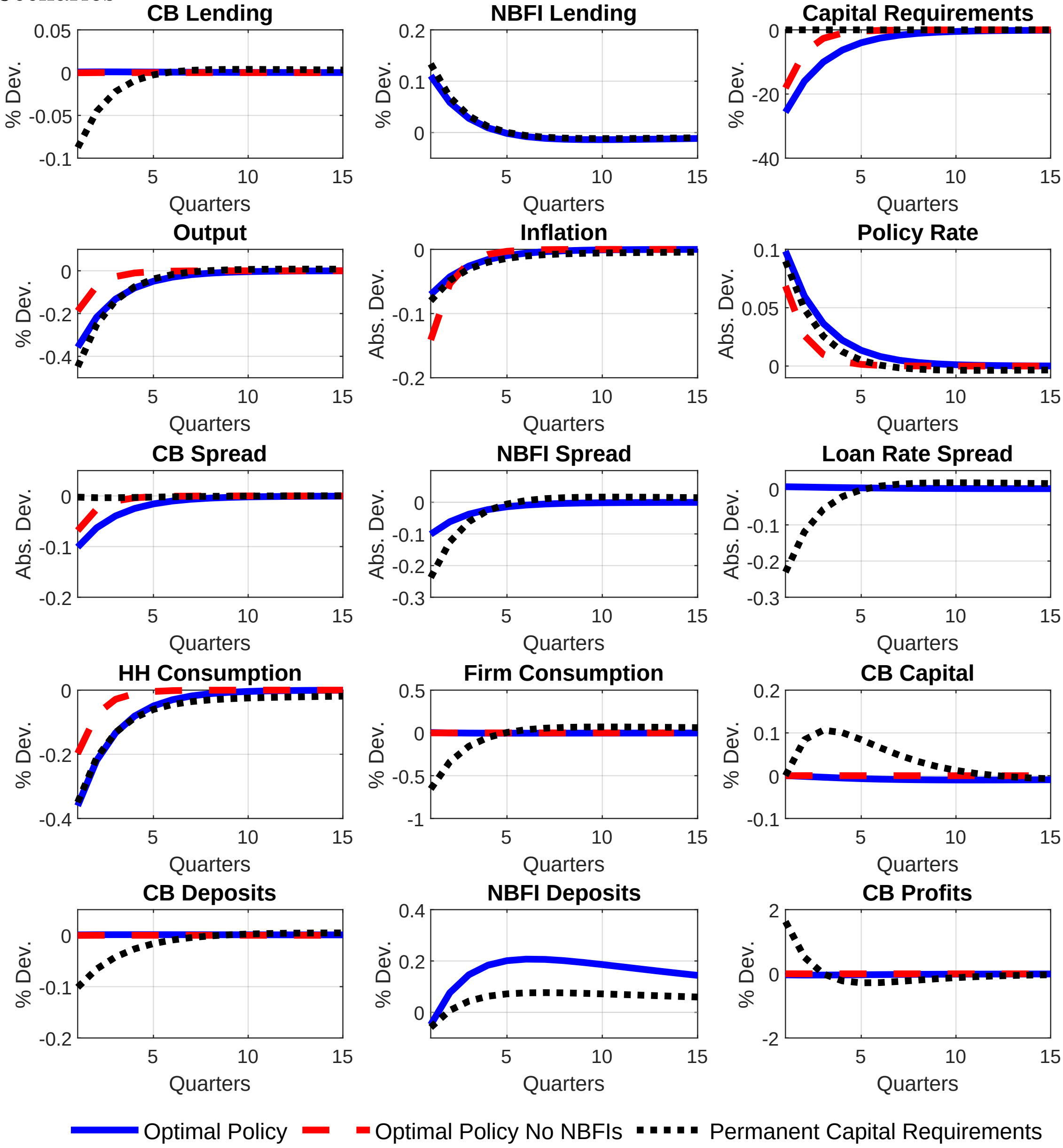
As indicated in the previous section, the optimal dynamic policy response to exogenous disturbances particularly depends on movements in both NBFI and commercial bank credit. In the following simulation exercise, I evaluate how the introduction of NBFIs alters the policy makers' ability to stabilize both the financial sector and real economic activity in response to exogenous macroeconomic shocks. Figures 5 and 6 show welfare-optimal dynamic responses to an unexpected tightening in monetary policy (aggregate demand shock) and to an exogenous improvement of firms' production technology (aggregate supply shock).[34] I simulate these responses under optimal policy for the cases with (blue lines) and without non-bank finance (red dashed lines). Furthermore, I consider a scenario with NBFIs where capital requirements are not dynamically adjusted, but kept at the optimal steady-state level of 13.5 percent (black dotted lines).

The impulse responses allow for several observations. First, optimal dynamic macroprudential regulation is effective in stabilizing commercial bank credit, both in the presence and absence of NBFIs. However, even under optimal policy, the regulator is not able to completely neutralize credit leakage to NBFIs in response to macroeconomic shocks. As in Gebauer and Mazelis (2020), unexpected monetary policy tightening induces a shift of credit intermediation towards NBFIs.[35] However, the quantitative effects of credit leakage are smaller compared to the response under the ad-hoc policy rules discussed in the previous section. Similarly, an unexpected positive technology shock increases entrepreneurs' production income and ultimately induces borrowing constraint 8 to be less binding. Lower credit constraints with

[34] In appendix section C, I provide the same set of optimal impulse responses for an estimated version of the model for comparison.

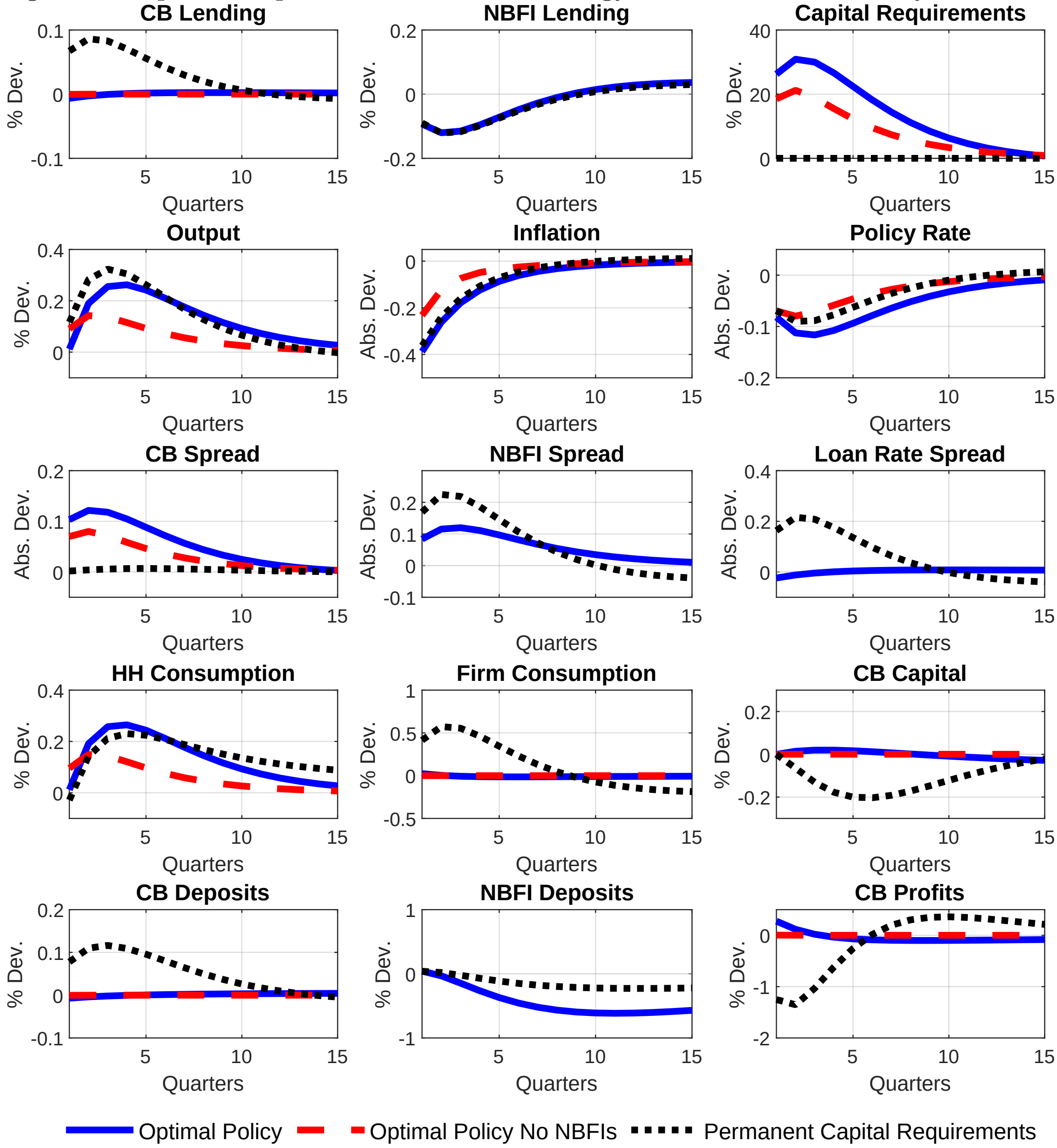
[35] Several studies found empirical evidence for credit leakage towards non-bank institutions in response to monetary policy shocks. See Gebauer and Mazelis (2020).

Figure 5: Impulse Response Functions Monetary Policy Shock: Different Policy Scenarios



Note: Impulse responses to a one-standard-deviation monetary policy shock with welfare-optimal response by macroprudential regulator. Rates in absolute deviations from steady state, all other variables as percentage deviations from steady state.

Figure 6: Impulse Response Functions Technology Shock: Different Policy Scenarios



Note: Impulse responses to a one-standard-deviation technology shock with welfare-optimal response by macroprudential regulator. Rates in absolute deviations from steady state, all other variables as percentage deviations from steady state.

respect to commercial bank credit in turn reduce entrepreneurs necessity to turn to NBFI creditors, such that the share of credit intermediated by commercial banks increases.

Second, capital requirements are adjusted countercyclically in response to macroeconomic shocks. In the case of an adverse demand shock (a monetary policy tightening), regulators lower capital requirements to stabilize commercial bank credit. Equally, an accommodative supply shock (positive technology shock) induces regulators to tighten commercial credit requirements to stabilize credit.

Third, and in line with policy rule 49, disturbances resulting in credit leakage, i.e. in *inverse* responses of commercial bank and NBFI credit, induce regulators to adjust capital requirements more aggressively in the presence of non-bank finance. In response to an unexpected monetary policy tightening, the regulator immediately decreases capital requirements by approximately 25 percent – which implies a decrease from 13.5 percent in the optimal steady state to 10.1 percent – whenever NBFIs are present. In the scenario with commercial banks only, capital requirements decrease by only 18 percent – from 13.5 to 11.1 percent – on impact.

Consequently, implications of non-bank finance for cyclical macroprudential policy crucially depend on the direction in which commercial bank and NBFI credit move in response to disturbances. As discussed in the previous section, macroeconomic disturbances leading to the *same* direction of commercial bank and NBFI credit responses provide a motive for mitigating the regulatory response to commercial bank credit.[36] In contrast, the presence of credit leakage leading to inverse credit responses provides a rationale for a stronger policy response.

Fourth, the results for both monetary policy and technology shocks indicate that optimal capital regulation – while suited to stabilize commercial bank credit intermediation – fails to stabilize the output gap efficiently in response to macroeconomic shocks. Even more, the additional policy trade-off between bank market power and NBFI risk mitigates the ability of regulators to stabilize the output gap in the presence of NBFIs, compared to the case where they can fully reach a homoge-

[36]The finding is also in line with results from the counterfactual simulation in Gebauer and Mazelis (2020). However, while the ad-hoc rules employed there do consider movements in *overall* credit, they do not feature the credit leakage motive of optimal policy. Still, as shown in Gebauer and Mazelis (2020), regulators concerned with overall credit would have tightened requirements less strongly in the years preceding the financial crisis – a period of growth in both commercial bank and NBFI credit (figure 1) – compared to regulators that would have only considered commercial bank credit.

neous financial sector with their policies. In both scenarios, the direct link between macroprudential regulation and commercial bank credit allows regulators to stabilize commercial bank activity efficiently, while NBFI intermediation and real economic activity are only partly stabilized. Therefore, while capital requirements might be suited to directly target volatility in commercial bank intermediation, additional policies targeting business cycle fluctuations or non-bank finance more directly are likely to increase economic and financial stability and to provide even further welfare improvements.

Fifth, regulators are particularly efficient in stabilizing commercial bank credit under dynamic optimal policy. Under the fixed-requirement scenario (black dotted line), an unexpected increase in the policy rate leads to a rise in deposit and commercial bank loan rates. In turn, higher commercial bank credit costs reduce lending by commercial banks (figure 5). NBFI lending increases slightly more compared to the optimal policy scenario, as the spread between NBFI and commercial bank loan rates decreases. Furthermore, the drop in output and inflation is stronger under fixed capital requirements, even though the difference in the responses is relatively small in both scenarios. Welfare-optimal adjustments of capital requirements therefore provide only limited additional stabilization of business cycles, confirming the above findings. Again, the adjustment of capital requirements has a particular impact on commercial bank activities, as these institutions are directly affected.

Similarly, the unexpected productivity shock depicted in figure 6 results in an increase in commercial bank lending whenever capital requirements are fixed, while commercial bank credit is almost completely stabilized under the welfare-optimal policy. Again, an increase in capital requirements by 26 percent – from 13.5 to 17 percent – only mildly affects business cycle dynamics but has substantial impact on commercial banks' activity.

8 Conclusion

In this paper, I study optimal macroprudential regulation for commercial banks in the presence of unregulated non-bank financial intermediaries (NBFIs). I analytically derive welfare-optimal policies under commitment in a New Keynesian DSGE model featuring both intermediaries based on different microfoundations. I compare my findings to a scenario where the financial sector only consists of regulated commercial banks.

The derived period loss functions resemble ad-hoc welfare criteria usually employed in the "revealed preferences" approach towards optimal macroprudential policy. However, in addition to output- and credit-related terms, they also include a stabilization criterion with respect to nominal short-term interest rates. Thus, even without an a-priori assumption on policy coordination, I find potential welfare gains from cooperation between monetary and macroprudential authorities.

Due to commercial bank market power and NBFI riskiness, steady-state lending by both intermediaries permanently deviates from efficient levels: Commercial bank lending is below the optimal level, and NBFI intermediation is higher in the distorted steady state. While bank capital regulation alone cannot mitigate inefficiencies in both credit markets, I show that a combination of static capital requirements and LTV ratios can resolve both steady-state distortions. The welfare-optimal level of permanent capital requirements is 13.5 percent in the model including NBFIs, compared to 16 percent in a model where commercial banks are the only lenders. Raising capital requirements induces a shift of intermediation towards risky NBFIs, as the relative cost of commercial bank credit increases with tighter capital regulation. Thus, by neglecting credit leakage to NBFIs, the costs from tightening regulation are not fully internalized by regulators.

Finally, non-bank finance affects the optimal dynamic response of macroprudential regulation to fluctuations in output and credit. Whenever macroeconomic disturbances imply credit leakage towards NBFIs, regulatory adjustments are larger than in a model without NBFIs. For instance, after an unexpected increase in the policy rate by annualized 40 basis points, capital requirements decrease from 13.5 percent to 10.1 percent in the presence of non-bank finance. In the scenario without NBFIs, capital requirements decrease to only 11.1 percent.

My findings indicate that neglecting NBFIs potentially impairs the efficiency of macroprudential policies, as regulators do not internalize credit leakage and the trade-off related to the *composition* of credit. Thus, they should consider developments in the non-bank financial sector, even if their policies only apply to traditional banks. Furthermore, the lack of macroprudential tools for NBFIs raises potential gains from coordinating different macroprudential measures. In addition, coordination with monetary policy can play a role, as NBFIs' activity is also related to the overall price of credit in the economy. Thus, nominal interest rate levels matter, and credit leakage may be aggravated when the effective lower bound (ELB) on nominal interest rates is reached.

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A The Role of Non-Banks for Regulation

The increasing importance of non-bank financial intermediation and the resulting relevance for financial stability has recently been recognized by supervisors. However, designing a macroprudential framework for the non-bank financial sector similar to the approach introduced for commercial banks is barely feasible. While traditional banks usually intermediate funds between borrowers and savers in a universal fashion, a multitude of specialized financial corporations operating in a complex intermediation chain are usually involved in non-bank credit intermediation.[37] Therefore, NBFI regulation is largely limited to microprudential approaches or special regulative measures that can be introduced for a set of institutions involved in credit intermediation.[38]

Nevertheless, changes in regulation for the *commercial* banking sector can trigger a shift of credit intermediation towards less regulated parts of the financial system. In a scenario with only commercial banks, the trade-off the regulator faces arises from the contemporaneous stabilization of credit and economic activity (figure 7):[39] Since the regulator's policy applies to the whole financial system in such a (counterfactual) scenario, changes in capital requirements affect total credit intermediation. Therefore, higher capital requirements can directly result – given that bank capital barely adjusts in the short run – in a reduction of credit intermediation, as all financial intermediaries in the economy have to reduce their assets to oblige with the regulatory requirement.[40] Lower credit intermediation potentially comes at the expense of lower economic activity, and the regulator has to decide on the optimal capital requirement level to balance the benefits of reduced lending activity and thus (potentially) higher financial stability with the cost of lower output growth.

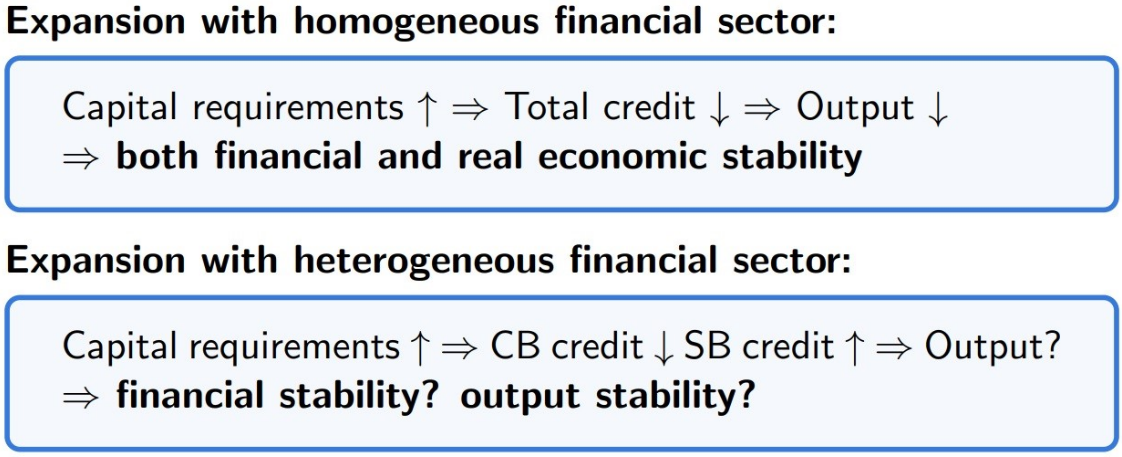
[37] See for instance Adrian (2014), Adrian and Liang (2016), or Pozsar et al. (2010) for a discussion of the shadow bank intermediation chain.

[38] In Europe, the updated Markets in Financial Instruments Directive (MiFID II/MiFIR) aims at increasing transparency and investor protection in market-based finance, thereby applying to a subset of institutions under the broad definition of NBFIs used here. However, the approach primarily focuses on the harmonization of reporting and conduct of business standards and authorization requirements. Explicit capital requirements, affecting the non-bank financial sector as a whole, are not part of the regulatory package.

[39] See for instance Angelini et al. (2014) or Binder et al. (2018).

[40] There is ample empirical evidence that a tightening of capital regulation is usually associated with a decline in lending by financial intermediaries. See for instance De Jonghe et al. (2020), Meeks (2017), or Aiyar et al. (2016).

Figure 7: Stylized Exercise on Policy Trade-Off



Note: Introduction of NBFIs and resulting credit leakage add an additional trade-off macroprudential policy makers face.

However, the existence of NBFIs introduces a further dimension to the trade-off the macroprudential policy maker, concerned with the regulation of traditional banking, faces. Higher capital requirements potentially lead to *credit leakage* towards unregulated NBFIs: As tighter banking regulation does not initially affect credit demand by real economic agents, higher regulation for commercial banks incentivizes borrowers to switch to NBFIs as commercial banking becomes relatively costly.

The additional policy trade-off caused by credit leakage is furthermore shaped by structural characteristics of financial institutions. For instance, empirical evidence suggests a significant degree of market power in the euro area commercial banking sector.[41] In contrast, empirical evidence on NBFI competition is hard to obtain, as the sector consists of highly diverse institutions operating in different market environments. However, some studies find that non-bank finance can increase efficiency in financial markets by providing alternative financing sources and due to the involvement of highly specialized institutions in the intermediation process.[42] At the same time, NBFI intermediation can increase systemic risk, as structural characteristics, economic motivations, and regulatory constraints within the diverse non-bank financial sector can accelerate financial stress and macroeconomic disturbances and

[41] See for instance Gerali et al. (2010), Berger et al. (2004), Degryse and Ongena (2008), Claessens and Laeven (2004), or De Bandt and Davis (2000).

[42] See for instance Adrian and Ashcraft (2016, 2012) or Bundesbank (2014) for evidence how shadow banking can increase efficiency in financial markets.

finally pose a threat to financial stability.[43]

Against this background, the degree to which activities in the non-bank financial sector should be taken into account in the design of optimal regulation for traditional banks is not clear a priori. As macroprudential tools towards the aggregate non-bank financial sector are not implementable, it appears even more important to study the adequate design of commercial bank regulation in the presence of potential spillovers towards non-bank intermediation.

B Appendix: The Full Non-Linear DSGE Model

B.1 Households

The representative patient household i maximizes the expected utility

$$\max_{C_t^P(i), L_t^P(i), D_t^{P,C}(i), D_t^{P,S}(i)} E_0 \sum_{t=0}^{\infty} \beta_P^t \left[\tilde{u}^P(C_t^P; \varepsilon_t) - \int_0^1 \tilde{\nu}^P(L_t(j); \varepsilon_t) dj \right] \tag{B.1}$$

where

$$\tilde{u}^P(C_t^P; \varepsilon_t) \equiv \frac{C_t^{P\,1-\sigma}}{1-\sigma} = \ln(C_t^P) \text{ if } \sigma \to 1 \tag{B.2}$$

$$\tilde{\nu}^P(L_t^P; \varepsilon_t) \equiv \frac{L_t^{P\,1+\phi^P}}{1+\phi^P}. \tag{B.3}$$

Each household (i) consumes the composite consumption good C_t^P which is given by a Dixit-Stiglitz aggregate consumption good

$$C_t^P \equiv \left[\int_0^1 c_t^P(i)^{\frac{\theta^P-1}{\theta^P}} di \right]^{\frac{\theta^P}{\theta^P-1}} \tag{B.4}$$

with $\theta^P > 1$. Each type of the differentiated goods is supplied by one monopolistic competitive entrepreneur. Entrepreneurs in industry j use a differentiated type of labor specific to the respective industry, whereas prices for each class of differentiated goods produced in sector j are identically set across firms in that sector. I assume that each household supplies all types of labor and consumes all types of goods. The representative household maximizes utility subject to the budget constraint

$$C_t^P(i) + D_t^{P,C}(i) + D_t^{P,S}(i) \leq w_t L_t^P(i) + (1 + r_{t-1}^{dC}) D_{t-1}^{P,C}(i) + (1 + r_{t-1}^{dS}) D_{t-1}^{P,S}(i) + T_t^P(i) \tag{B.5}$$

[43]See for instance Adrian and Jones (2018) and the large body of references therein.

where $C_t^P(i)$ depicts current total consumption. Total working hours (allotted to the different sectors j) are given by L_t^P and labor disutility is parameterized by ϕ^P. The flow of expenses includes current consumption and real deposits and investments to be placed with both commercial banks and NBFIs, $D_t^{P,C}(i)$ and $D_t^{P,S}(i)$. Resources consist of wage earnings $w_t^P L_t^P(i)$ (where w_t is the real wage rate for the labor input of each household), gross interest income on last period investments $(1+r_{t-1}^{dC})D_{t-1}^{P,C}(i)$ and $(1+r_{t-1}^{dS})D_{t-1}^{P,S}(i)$, and lump-sum transfers T_t^P that include dividends from firms and banks (of which patient households are the ultimate owners).

First-order conditions of the household maximization problem gives the intertemporal Euler equation

$$\frac{1}{C_t^P} = \beta_P E_t \Big[\frac{1+r_t}{C_{t+1}^P} \Big] \tag{B.6}$$

and the labor supply condition

$$w_t = C_t^P L_t^{\phi^P}. \tag{B.7}$$

B.2 Entrepreneurs

Entrepreneurs engaged in a certain sector j use the respective labor type provided by households as well as capital to produce intermediate goods that retailers purchase in a competitive market. Each entrepreneur i derives utility from consumption $C_t^E(i)$, and finances consumption with production returns and with loans from financial intermediaries. They maximize expected utility

$$\max_{C_t^E(i),\, L_t^P(i),\, B_t^{E,C}(i),\, B_t^{E,S}(i)} \quad E_0 \sum_{t=0}^{\infty} \beta_E^t \frac{{C_t^E}^{1-\sigma}}{1-\sigma} \tag{B.8}$$

subject to the budget constraint

$$\begin{aligned} C_t^E(i) + w_t l_t^P(i) + (1+r_{t-1}^{bC})B_{t-1}^{E,C}(i) + (1+r_{t-1}^{bS})B_{t-1}^{E,S}(i) & \\ \leq \frac{y_t^E(i)}{x_t} + B_t^{E,C}(i) + B_t^{E,S}(i) & \end{aligned} \tag{B.9}$$

with x_t determining the price markup in the retail sector. I thus express output y_t^E produced by the entrepreneur in terms of the relative competitive price of the wholesale good, given by $\frac{1}{x_t}$. Output is produced according to the Cobb-Douglas technology

$$y_t^E(i) = a_t K^{\alpha} L_t(i)^{1-\alpha} \tag{B.10}$$

where the (stochastic) total factor productivity (TFP) is given by a_t.

Entrepreneurs face a constraint on the amount they can borrow from commercial banks depending on the fixed stock of capital they hold as collateral.[44] Whereas a regulatory loan-to-value (LTV) ratio m_t^E applies for funds borrowed from commercial banks, NBFI funding is not prone to regulation. Due to a positive spread between interest rates charged for NBFI and commercial bank loans, entrepreneurs have an incentive to borrow from commercial banks first and turn to NBFI lending only whenever the possible amount of commercial bank funds, determined by $m_t^E K$, is reached. Further borrowing can be obtained from NBFIs by using capital holdings not reserved for commercial bank funds, $(1 - m_t^E)K$. As physical capital is assumed to be fixed, the two respective borrowing constraints are given by

$$(1 + r_t^{bC})B_t^{E,C} \leq m_t^E K \tag{B.11}$$

$$(1 + r_t^{bS})B_t^{E,S} \leq (1 - m_t^E)K \tag{B.12}$$

where the LTV ratio for commercial banks m_t^E is set exogenously by the regulator and follows an exogenous AR(1) process with mean m^E.

As in Iacoviello (2005) the borrowing constraints is assumed to bind around the steady state such that uncertainty is absent in the model.[45] Thus, in equilibrium, entrepreneurs face binding borrowing constraints, such that equations B.11 and B.12 hold with equality. Based on the maximization problem of the entrepreneur, entrepreneurs consumption Euler equation and labor demand are given by

$$\frac{1}{C_t^E} = \beta_E E_t \Big[\frac{1 + r^{bC}}{C_{t+1}^E} \Big] \tag{B.13}$$

$$w_t = \frac{(1 - \alpha)y_t^E}{L_t x_t} \tag{B.14}$$

where x_t is the retail sector markup to which marginal costs are inversely related:

$$MC_t = \frac{1}{x_t}. \tag{B.15}$$

Entrepreneurs' leverage with respect to commercial and central banks, χ_t^C and χ_t^S is determined by the borrowing constraints the entrepreneur faces when acquiring

[44] In Iacoviello (2005), entrepreneurs use commercial real estate as collateral. However, I follow Gerali et al. (2010) by assuming that creditworthiness of a firm is judged by its overall balance sheet condition where real estate housing only depicts a sub-component of assets.

[45] Iacoviello (2005) discusses the deviation from certainty equivalence in appendix C of his paper.

funds from each intermediary:

$$\chi_t^C = \frac{m_t^E}{1 + r_t^{bC}} \tag{B.16}$$

$$\chi_t^S = \frac{1 - m_t^E}{1 + r_t^{bS}}. \tag{B.17}$$

Entrepreneur consumption is linked to net worth

$$C_t^E = (1 - \beta_E) NW_t^E \tag{B.18}$$

which is given by

$$NW_t^E = \alpha \frac{y_t^e}{x_t} + K - (1 + r_{t-1}^{bC}) b_{t-1}^{eC} - (1 + r_{t-1}^{bS}) b_{t-1}^{eS} \tag{B.19}$$

or, expressed in terms of leverage, as

$$NW_t^E = \frac{K(1 - \chi_t^C - \chi_t^S)}{\beta_E}. \tag{B.20}$$

The aggregate production technology entrepreneurs employ is given by:

$$y_t^E = a_t K^\alpha L_t^{1-\alpha} \tag{B.21}$$

As physical capital, which entrepreneurs use as collateral for borrowing from both intermediaries, is fixed, loans from commercial banks and NBFIs are given by

$$B_t^{E,C} = K \chi_t^C \tag{B.22}$$

$$B_t^{E,S} = K \chi_t^S. \tag{B.23}$$

B.3 Commercial Banks

The commercial bank balance sheet is given by

$$B_t^{E,C} = K_t^C + D_t^{P,C} \tag{B.24}$$

where bank capital K_t^C is accumulated from bank profits J_t^C:

$$K_t^C = K_{t-1}^C (1 - \delta^C) + J_t^C. \tag{B.25}$$

Aggregate bank profits are given by

$$J_t^C = r_t^{bC} B_t^{E,C} - r_t D_t^{P,C} - K_t^C \frac{\kappa_k^C}{2} \left(\frac{K_t^C}{B_t^{E,C}} - \nu_t \right)^2. \tag{B.26}$$

As described above, the retail loan rate is given by

$$r_t^{bC} = r_t - \kappa_k^C \left(\frac{K_t^C}{B_t^{E,C}} - \nu_t.)(\frac{K_t^C}{B_t^{E,C}} \right)^2 + \mu_t \tag{B.27}$$

B.4 Non-Bank Financial Institutions

The aggregate NBFI balance sheet is given by

$$B_t^{E,S} = D_t^{P,S} + K_t^S. \tag{B.28}$$

Following the derivations in section 3.3.2 and Gebauer and Mazelis (2020), NBFI capital is given by

$$K_t^S = \sigma^S[(r_{t-1}^{bS} - r_{t-1}^{dS})\phi_{t-1}^S + (1 + r_{t-1}^{dS})]K_{t-1}^S + \omega^S B_{t-1}^{E,S} \tag{B.29}$$

where, following Gertler and Karadi (2011), NBFI loans are given by

$$B_t^{E,S} = \frac{\eta_t^S}{\theta^S - \nu_t^S} K_t^S \tag{B.30}$$

with

$$\eta_t^S = E_t[(1 - \sigma^S) + \beta_S \sigma^S \Psi_{t,t+1}^S \eta_{t,t+1}^S] \tag{B.31}$$

$$\nu_t^S = E_t[(1 - \sigma^S)\beta_S(r_t^{bS} - r_t) + \beta_S \sigma^S \Xi_{t,t+1}^S \nu_{t,t+1}^S] \tag{B.32}$$

$$\Psi_{t,t+1}^S = \frac{K_{t+1}^S}{K_t^S} = (r_{t+1}^{bS} - r_t)\phi_t^S + r_t \tag{B.33}$$

$$\Xi_{t,t+1}^S = (\phi_{t+1}^S / \phi_t^S)\Psi_{t,t+1}^S \tag{B.34}$$

and where NBFI leverage ϕ_t^S is given by

$$\phi_t^S = \frac{B_t^{E,S}}{K_t^S}. \tag{B.35}$$

As in Gebauer and Mazelis (2020), I assume the spread on commercial bank deposit and NBFI investment rates to be given by:

$$1 + r_t^{dS} = \frac{1 + r_t^{dC}}{1 - \tau^s \varepsilon_t^\tau}. \tag{B.36}$$

B.5 Monetary Policy and Market Clearing

The central bank is assumed to follow a Taylor-type policy rule given by

$$1 + R_t = (1 + R)^{1-\rho^r}(1 + R_{t-1})^{\rho^r}\left[\pi_t^{\phi^\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi^y}\right]^{1-\rho^r}(1 + \epsilon_t^R) \tag{B.37}$$

where ρ^r is equal to zero in the analytic derivations of in appendix E. The aggregate resource constraint is given by

$$Y_t = C_t + K + \frac{K_{t-1}^C \delta^C}{\pi_t}. \tag{B.38}$$

Market clearing implies

$$Y_t = \gamma_y y_t^E \tag{B.39}$$

$$C_t = C_t^P \gamma_p + C_t^E \gamma_e \tag{B.40}$$

$$B_t = B_t^{E,C} + B_t^{E,S} \tag{B.41}$$

NBFI and commercial bank credit-to-GDP ratios are defined as:

$$Z_t = \frac{B_t^{E,C}}{Y_t} \tag{B.42}$$

$$Z_t^{SB} = \frac{B_t^{E,S}}{Y_t} \tag{B.43}$$

Loan and deposit rate spreads paid by commercial banks and NBFIs are given by

$$\Delta_t^{loan} = r_t^{bS} - r_t^{bC} \tag{B.44}$$

$$\Delta_t^{deposit} = r_t^{dS} - R_t \tag{B.45}$$

and the spreads earned on intermediation by commercial banks and NBFIs by

$$\Delta_t^C = r_t^{bC} - R_t \tag{B.46}$$

$$\Delta_t^S = r_t^{bS} - r_t^{dS} \tag{B.47}$$

B.6 Shock Processes

Deposit Spread Shock:

$$\varepsilon_t^\tau = 1 - \rho^\tau + \rho^\tau \varepsilon_{t-1}^\tau + \epsilon_t^\tau \tag{B.48}$$

Productivity Shock:

$$a_t = (1 - \rho^a)a + \rho^a a_{t-1} + \epsilon_t^a \tag{B.49}$$

Entrepreneur LTV Shock:

$$m_t^E = (1 - \rho^{m^E})m^E + \rho^{m^E} m_{t-1}^E + \epsilon_t^{m^E} \tag{B.50}$$

Loan Rate Markup Shock:

$$\mu_t = (1 - \rho^\mu)\mu + \rho^\mu \mu_{t-1} + \epsilon_t^\mu \tag{B.51}$$

C Appendix: Estimation

In the main part of the paper, I rely on the parameters estimated with the quantitative model developed in Gebauer and Mazelis (2020) which features investment, household habit formation, and bank market power in deposit markets. I abstract from these characteristics in the model of this study for the sake of tractability of analytic derivations. In this section, I report estimation results for my model. For comparability, I apply the same full-information Bayesian estimation approach as Gebauer and Mazelis (2020).[46] For estimation purposes, I incorporate all shock processes reported in Gebauer and Mazelis (2020) into the model, except for a deposit markdown shock μ_t^d, and an investment efficiency shock $\varepsilon_t^{q^k}$. For remaining shock processes, I estimate standard deviations and autoregressive parameters relying on the same prior distributions as in Gebauer and Mazelis (2020). I also draw on the same data series, but exclude data on investment and deposit rates. I estimate the same set of structural parameters, only excluding the parameters governing bank market power, κ^{bE} and κ^d, and investment adjustment costs, κ^i. I also exclude the parameter governing habit formation, a^P, as this feature is absent in my model. Table 5 reports the posterior distribution for both the estimated version of the model presented in section 3 and the model in Gebauer and Mazelis (2020).

For comparison, I conduct the same analysis as in figures 5 and 6 with the estimated parameters and report impulse response functions to an unexpected monetary policy tightening and an expansionary technology shock in figures 8 and 9.[47] The impulse responses under optimal policy are qualitatively and quantitatively comparable for the monetary policy shock under both parameterizations (figures 5 and 8). The drop in household consumption and output is less pronounced for the estimated model, and thus the decline in inflation is also more benign. For banking-related variables, differences between the calibrations are minor. For the productivity shock (figures 6 and 9), dynamics are similar qualitatively under both parameterizations, but a few quantitative differences emerge. The expansion in the economy is larger, and thus lending dynamics are more pronounced in the estimated model. In return, interest rate spreads are higher, and swings in bank capital and profits are stronger.

[46] However, in the Metropolis-Hastings algorithm, I conducted 5 chains with only 100,000 draws each, as convergence was reached already at that stage, while Gebauer and Mazelis (2020) relied on 500,000 draws per chain in the estimations.

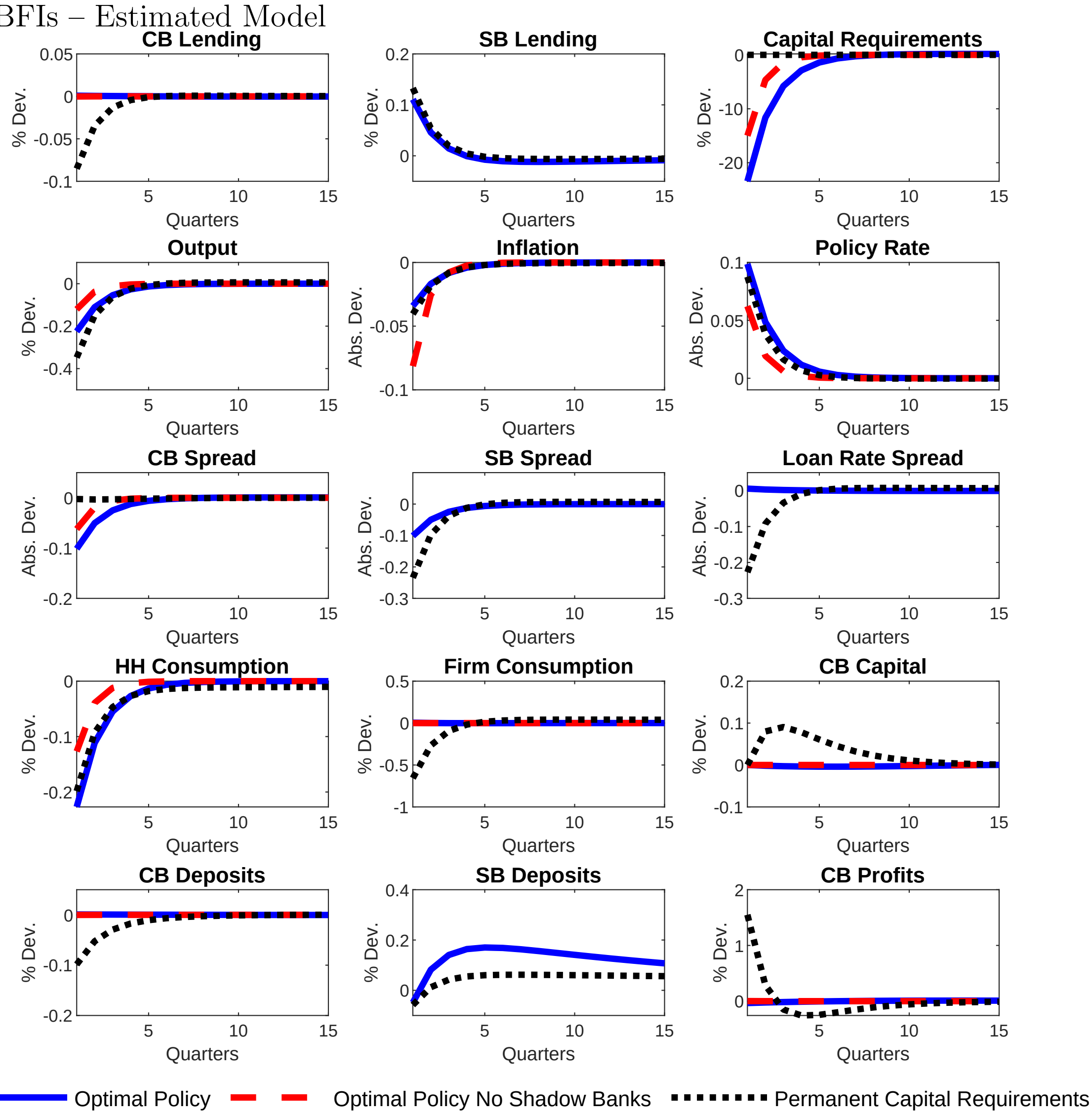
[47] For comparability of the dynamic responses, I set structural parameters to the estimated values, but employ the same shock processes as under the baseline calibration.

Table 5: Posterior Distributions: Full Model vs. Modified Model

		Baseline Model				Gebauer and Mazelis (2020)			
		5 Perc.	Median	95 Perc.	Mode	5 Perc.	Median	95 Perc.	Mode
Structural Parameters									
θ^p	Calvo Parameter	0.80	0.81	0.82	0.80	0.83	0.87	0.90	0.86
κ^i	Investment Adjustment Cost	-	-	-	-	2.98	3.98	5.14	3.67
κ^d	Deposit Rate Adjustment Cost	-	-	-	-	10.00	13.26	16.72	12.62
κ^{bE}	Loan Rate Adjustment Cost	-	-	-	-	4.84	8.34	14.23	7.56
κ_k^C	CCR Deviation Cost	0.03	12.49	22.37	9.25	0.01	10.05	21.32	24.71
ϕ^π	TR Coefficient π	2.29	2.73	3.14	2.71	1.44	1.87	2.30	1.75
ϕ^y	TR Coefficient y	0.10	0.15	0.21	0.15	0.14	0.24	0.34	0.20
ϕ^r	Interest Rate Smoothing	0.63	0.70	0.77	0.70	0.84	0.88	0.91	0.88
a^P, a^E	HH Habit Formation	-	-	-	-	0.70	0.77	0.84	0.77
Exogenous Processes (AR Coeff.)									
ρ^τ	Deposit Rate Spread	0.65	0.81	0.96	0.85	0.62	0.81	0.95	0.85
ρ^z	Consumer Preference	0.82	0.89	0.95	0.88	0.82	0.87	0.92	0.87
ρ^a	Technology	0.70	0.83	0.95	0.85	0.31	0.42	0.52	0.42
ρ^{mE}	Entrepreneur LTV	0.95	0.98	0.99	0.98	0.91	0.94	0.97	0.95
ρ^d	Deposit Rate Markdown	-	-	-	-	0.27	0.36	0.46	0.36
ρ^μ	Loan Rate Markup	0.66	0.82	0.96	0.85	0.51	0.63	0.75	0.64
ρ^{qk}	Investment Efficiency	-	-	-	-	0.33	0.46	0.58	0.49
ρ^y	Price Markup	0.28	0.40	0.52	0.41	0.25	0.36	0.47	0.37
ρ^l	Wage Markup	0.93	0.96	0.99	0.97	0.64	0.71	0.77	0.71
ρ^{Kb}	Commercial Bank Capital	0.95	0.97	0.99	0.98	0.93	0.96	0.99	0.97
Exogenous Processes (Std. Dev.)									
σ^τ	Deposit Rate Spread	0.002	0.007	0.017	0.005	0.002	0.007	0.016	0.005
σ^z	Consumer Preference	0.001	0.002	0.002	0.001	0.008	0.011	0.014	0.011
σ^a	Technology	0.002	0.003	0.004	0.003	0.025	0.029	0.033	0.028
σ^{mE}	Entrepreneur LTV	0.015	0.176	0.204	0.171	0.006	0.008	0.009	0.007
σ^d	Deposit Rate Markdown	-	-	-	-	0.002	0.002	0.002	0.002
σ^μ	Loan Rate Markup	0.000	0.001	0.001	0.001	0.002	0.002	0.003	0.002
σ^{qk}	Investment Efficiency	-	-	-	-	0.001	0.002	0.002	0.002
σ^r	Monetary Policy	0.001	0.002	0.002	0.002	0.001	0.001	0.002	0.001
σ^y	Price Markup	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.001
σ^l	Wage Markup	0.006	0.008	0.009	0.008	0.035	0.041	0.047	0.040
σ^{Kb}	Commercial Bank Capital	0.017	0.019	0.023	0.019	0.003	0.003	0.004	0.003

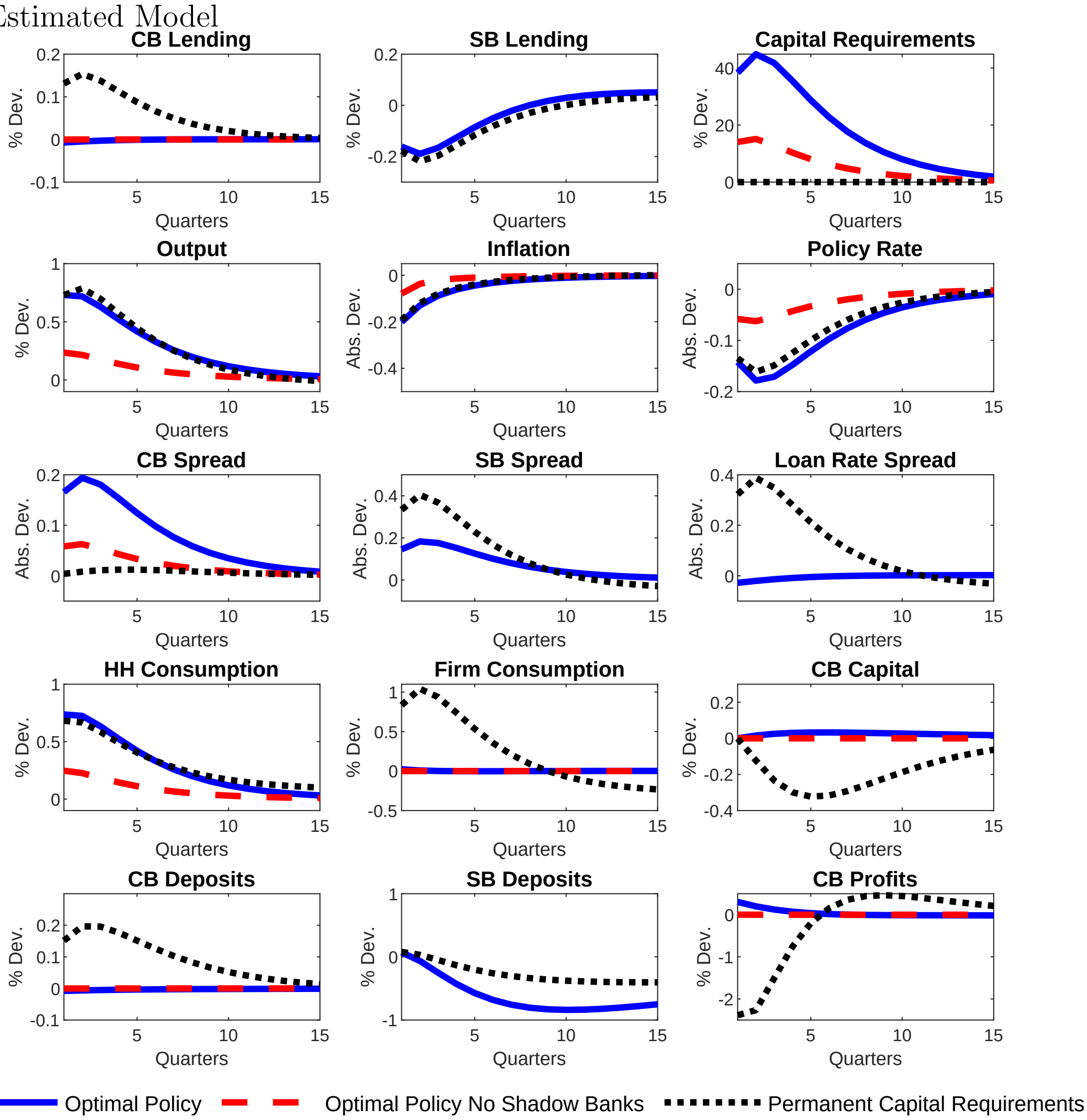
Note: Results are based on 5 chains with 100,000 draws each based on the Metropolis-Hastings algorithm. Columns 3 to 6 report the posterior moments from the estimated version of the model presented in section 3. Columns 7 to 10 report results from the baseline estimation in Gebauer and Mazelis (2020).

Figure 8: Impulse Response Functions Monetary Policy Shock: With and Without NBFIs – Estimated Model



Note: Impulse responses to a one-standard-deviation monetary policy shock with welfare-optimal response by macroprudential regulator. Rates in absolute deviations from steady state, all other variables as percentage deviations from steady state.

Figure 9: Impulse Response Functions Technology Shock: With and Without NBFIs – Estimated Model



Note: Impulse responses to a one-standard-deviation technology shock with welfare-optimal response by macroprudential regulator. Rates in absolute deviations from steady state, all other variables as percentage deviations from steady state.

D Appendix: Efficient Steady State and Financial Sector Distortions

In this section, I derive zero-inflation ($\Pi = 1$) steady state values starting from a perfectly competitive and frictionless financial sector. I then discuss how financial sector inefficiencies result in deviations of credit variables from efficient levels in the decentralized economy. Steady state allocations are *efficient* whenever they are equal to the values determined in a *frictionless economy*, i.e. in a model with

- no price dispersion ($\Delta = 1$)
- no monopolistic competition in the firm sector ($x = 1$)
- no monopolistic competition in the commercial banking sector ($\mu = 0$)
- no moral hazard friction and risk in the non-bank financial sector ($\theta^S = \tau^S = 0$)

I then discuss how different time-invariant macroprudential policies - capital requirements and LTV ratios - can be employed to obtain efficient steady-state allocations in the decentralized economy and in the presence of steady-state distortions.

D.1 Social Planner Economy

As given by equation 33, the social planner maximizes a weighted average of patient household and impatient entrepreneur utility:

$$\mathbb{W} = (1 - \beta_P)U(C^P, L^P) + (1 - \beta_E)U(C^E) \tag{D.1}$$

where the Pareto weights are determined as in Lambertini et al. (2013) and Rubio (2011) and $U(\bullet)$ are the per-period utility functions. In choosing allocations, the social planner is constrained by the aggregate production function B.21 and the goods market clearing condition B.38. However, the social planner is not subject to the borrowing constraints 8 and 9.

Combining the aggregate production function and the goods market clearing condition yields

$$K^\alpha L^{1-\alpha} = \gamma_P C^P + \gamma_E C^E. \tag{D.2}$$

Letting λ depict the Lagrange multiplier on constraint D.2, the first-order conditions yield

$$(1-\beta_P)U^{'}_{C^P} = -\lambda\gamma_P \tag{D.3}$$

$$(1-\beta_E)U^{'}_{C^E} = -\lambda\gamma_E \tag{D.4}$$

$$(1-\beta_P)U^{'}_{L^P} = \lambda(1-\alpha)\frac{Y}{L^P}. \tag{D.5}$$

Assuming unity in consumption weights ($\gamma_P = \gamma_E = 1$), the efficient steady state implies that the patient household's marginal rate of substitution between consumption and labor equals the economy's marginal rate of transformation between output and labor:

$$-\frac{U^{'}_{L^P}}{U^{'}_{C^P}} = (1-\alpha)\frac{Y}{L^P}. \tag{D.6}$$

Using the explicit utility functions of equations 1 and 6 in the first-order conditions, the relation between marginal utilities of borrowers and savers is given by

$$(1-\beta_P){C^P}^{-\sigma} = (1-\beta_E){C^E}^{-\sigma}. \tag{D.7}$$

Solving for C^E and using in the aggregate consumption identity $C = C^P + C^E$ yields

$$C^P = \left[1 + \left(\frac{1-\beta_E}{1-\beta_P}\right)^{\frac{1}{\sigma}}\right]^{-1} C. \tag{D.8}$$

Assuming a subsidy set to remove distortions from monopolistic competition in the firm sector such that $x = 1$, the efficient steady state labor market equilibrium is determined by equations B.7 and B.14

$$C^P L^{\phi^P} = (1-\alpha)\frac{Y}{L}. \tag{D.9}$$

Plugging in the expression for C^P derived above, and substituting the aggregate production function and the social planner constraint D.2, one can derive

$$L = \left[(1-\alpha)\left\{1 + \left(\frac{1-\beta_E}{1-\beta_P}\right)^{\frac{1}{\sigma}}\right\}\right]^{\frac{1}{\alpha(1-\alpha)\phi^L}}. \tag{D.10}$$

Finally, using the efficient steady state level of labor input in the production function determines steady-state output, which is independent of the distribution of debt and credit intermediated in the economy:

$$Y^* = K^\alpha\left[(1-\alpha)\left\{1 + \left(\frac{1-\beta_E}{1-\beta_P}\right)^{\frac{1}{\sigma}}\right\}\right]^{\frac{1}{\alpha\phi^L}}. \tag{D.11}$$

Proposition 1 (Efficient level of output)**.** *In the frictionless economy, the efficient level of output is not affected by the distribution of debt and the relative credit shares from intermediaries.*

In the frictionless planner economy, credit supply by commercial banks is only limited due to regulation and given by

$$B^{E,C} = \frac{K^C}{\nu}. \tag{D.12}$$

Furthermore, one can show that given perfect intermediation by both types of intermediaries, borrowers and savers are indifferent between channeling funds through commercial banks or NBFIs, as the two intermediaries are identical.[48] Formally, I assume that in the frictionless economy, NBFIs are not able to divert funds ($\theta^S = 0$) and are riskless intermediaries ($\tau^S = 0$), such that they are structurally identical to commercial banks. In fact, one can show that steady-state leverage of NBFIs is given by

$$\phi^S = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{D.13}$$

where
$a = \theta^S \beta^S \sigma^S \Delta^S$
$b = -(1 - \sigma^S)(\theta^S - \beta^S \Delta^S)$
$c = 1 - \sigma^S$.

One can straightforwardly see that $\phi^S = 0$ whenever $\theta^S = 0$ and $\Delta^S = 0$, as is the case in the frictionless economy. Therefore, steady-state NBFI lending in the planner economy which is given by

$$B^{E,S} = \phi^S K^S \tag{D.14}$$

is equal to zero and NBFIs are nonexistent in the planner economy.

Proposition 2 (Shadow and commercial bank credit in the planner economy)**.** *In the frictionless economy, the efficient level of NBFI credit is equal to zero, such that NBFIs are nonexistent, as NBFIs and commercial banks are effectively identical institutions. Absent borrowing constraints, credit intermediation is determined by credit supply, which depends on capital regulation.*

[48] See benchmark case in the online appendix of Gebauer and Mazelis (2020).

D.2 Decentralized Economy

As shown above, the frictionless planner economy does not provide scope for non-bank finance, such that the efficient level of NBFI credit is equal to zero. However, whenever borrowers face constraints with respect to lending from commercial banks, as in the decentralized economy studied in the following, the potential for non-bank finance increases as borrowers will try to circumvent credit constraints by turning to NBFIs which determine an additional source of funding. I will discuss how the fact that borrowers face credit constraints in the decentralized economy provides scope for non-zero NBFI activity, even in the absence of bank market power and moral hazard or default risk in the non-bank financial sector. In the decentralized economy, the real interest rate is determined by the patient household's discount rate such that

$$1 + r = \frac{1}{\beta_P}. \tag{D.15}$$

For now, all intermediaries efficiently intermediate funds between borrowers and savers and earn zero profits. Therefore, the interest rate spreads are zero in the decentralized economy's steady state such that

$$r^{bC*} = r^{bS*} = r^{dC*} = r^{dS*} = r. \tag{D.16}$$

Furthermore, borrowing constraints 8 and 9 the entrepreneur faces bind. As financial intermediaries intermediate funds efficiently, equilibrium credit from both intermediaries is determined not only by credit supply but also by credit demand in steady state, which is determined by the borrowing constraints

$$B^{E,C*} = \frac{m^E K}{1 + r^{bC*}} = \beta_P m^E K \Leftrightarrow \chi^{C*} = \beta_P m^E \tag{D.17}$$

$$B^{E,S*} = \frac{(1 - m^E)K}{1 + r^{bS*}} = \beta_P (1 - m^E) K \Leftrightarrow \chi^{S*} = \beta_P (1 - m^E). \tag{D.18}$$

Solving for m^E and combining yields

$$B^{E,S*} = \beta_P K - B^{E,C*}. \tag{D.19}$$

In the frictionless planner economy's steady state discussed in the previous section, macroprudential regulation determined *total* credit supply and intermediation. In the decentralized and in the distorted steady states discussed below, credit demand constraints in combination with financial market distortions furthermore affect the *relative* provision of credit by shadow and commercial banks.

Proposition 3 (Credit leakage in decentralized economy)**.** *Due to credit leakage as in Gebauer and Mazelis (2020), higher levels of credit provided by commercial banks lower the credit demanded from NBFIs and vice versa in the decentralized steady state. Due to borrower constraints on commercial bank credit, scope for NBFI intermediation is present in the decentralized economy.*

D.3 Friction 1: Commercial Bank Market Power

In the following, I introduce financial market frictions and allow for market power in the commercial banking sector. In Gebauer and Mazelis (2020), these frictions were microfounded via monopolistic competition in commercial bank credit markets. In this paper, I economize on the analytic derivations by assuming a permanent additive markup $\mu > 0$ that commercial banks charge over the deposit rate they pay to households. While I assume steady-state distortions due to monopolistic competition in the firm sector to be removed by a subsidy such that $x = 1$, I allow distortions stemming from financial sector inefficiencies such as bank market power to affect steady-state levels of credit. Thus, whenever I refer to the *distorted steady state* in this paper, I assume distortions in the real economy to be compensated with adequate (fiscal) policies, while distortions related to financial markets affect credit aggregates and are not yet compensated.

Due to the markup charged, the commercial bank loan rate is now given by

$$1 + r^{bC} = 1 + r + \mu = \frac{1}{\beta_P} + \mu = \frac{1 + \beta_P \mu}{\beta_P} \tag{D.20}$$

such that $r^{bC} > r^{bC*}$ for $\mu > 0$. Using the steady-state bank loan rates in the steady-state loan demand condition yields

$$B^{E,C} = \frac{m^E K}{1 + r^{bC}} = \frac{\beta_P}{1 + \beta_P \mu} m^E K \tag{D.21}$$

in the inefficient economy such that $B^{E,C} < B^{E,C*}$. The difference between the level of commercial bank credit in the efficient and the distorted steady state is given by

$$\widehat{B}^{E,C} = B^{E,C} - B^{E,C^*} = \left(\frac{\beta_P}{1 + \beta_P \mu} - \beta_P \right) m^E K. \tag{D.22}$$

As perfectly competitive and for now risk-free NBFIs provide the same credit good to borrowers, the introduction of a loan markup in the commercial banking sector, ceteris paribus, increases the demand for NBFI credit by entrepreneurs. Conversely, market power in the commercial bank credit market induces that borrowers demand less credit from commercial banks than determined by a binding borrowing constraint 8. Therefore, a negative value of $\widehat{B}^{E,C}$ implies that the borrowing constraint for commercial bank credit is not binding. As laid out in detail in the online appendix of Gebauer and Mazelis (2020), the borrowing constraint on NBFI credit 9 should not be interpreted as a regulatory constraint. Instead, it is determined by the share of physical capital K pledged by borrowers to receive commercial bank lending. In fact, borrowers use the share of their capital endowment not reserved as collateral for commercial bank credit and pledge it against NBFI borrowing. Thus, $m^{E,S}$ is affected by both the regulatory LTV ratio for commercial banks (if borrowers are able to borrow from these institutions until constraint 8 binds), and by the deviation of commercial bank credit from the efficient level, which is depicted by the level of commercial bank credit when 8 binds:

$$m^{E,S} = 1 - m^E - (1 + r^{bC})\frac{\widehat{B}^{E,C}}{m^E K}. \tag{D.23}$$

The last term depicts the additional amount of NBFI credit that can be received by pledging collateral not used for commercial bank credit whenever commercial bank borrowing deviates from the efficient credit level of the decentralized economy. It is determined by the gross lending that could have been received from commercial banks without credit rationing due to bank market power, relative to the potential level of commercial bank borrowing. With bank market power, NBFI credit is therefore given by

$$B^{E,S} = \frac{m^{E,S} K}{1 + r^{bS}}. \tag{D.24}$$

Using $m^{E,S}$ in this condition and simplifying yields

$$B^{E,S} = \beta_P \left(1 - \frac{1}{1 + \beta_P \mu} m^E\right) K \tag{D.25}$$

implying $B^{E,S} > B^{E,S*}$. Equation D.25 takes account of the fact that higher demand for NBFI credit also affects the relative cost of funding from these institutions. Assuming entrepreneurs to accommodate their frictionless steady-state level of total credit demand $B = B^{E,C} + B^{E,S}$, the shift towards NBFIs raises returns of these institutions. Due to arbitrage, the loan rate efficient NBFIs earn will finally converge towards the commercial bank loan rate, such that $r^{bS} \to r^{bC}$ in the limit. As a consequence, steady-state net worth of entrepreneurs, given by

$$NW^E = \alpha Y + K - (1 + r^{bC})B^{E,C} + (1 + r^{bS})B^{E,S} \tag{D.26}$$

or

$$NW^E = \alpha Y + K - (1 + r^{bC})(B^{E,C} + B^{E,S}) \tag{D.27}$$

will be lower than the efficient level NW^{E^*} as credit costs are larger due to commercial bank market power.

Importantly, the deviation of commercial bank credit leaves the efficient level of output from proposition 1 unaffected. One can therefore express the deviation in credit in the form of steady-state credit-to-GDP ratio

$$\widehat{Z} = Z - Z^* \tag{D.28}$$

$$\widehat{Z}^{SB} = Z^{SB} - Z^{SB^*} \tag{D.29}$$

where $Z = \frac{B^{E,C}}{Y^*}$, $Z^* = \frac{B^{E,C*}}{Y^*}$, $Z^{SB} = \frac{B^{E,S}}{Y^*}$, $Z^{SB^*} = \frac{B^{E,S*}}{Y^*}$ and therefore $Z < Z^*$ and $Z^{SB} > Z^{SB^*}$. In equations 39 and 40, it will exactly be due to this distortion that permanent gaps between the observed and the efficient levels of commercial bank and NBFI credit-to-GDP ratios open up.

Proposition 4 (Credit distortions due to CB market power)**.** *Market power in the commercial banking sector induces steady-state distortions that result in deviations of commercial bank and NBFI credit from efficient levels in the decentralized economy, as commercial bank (NBFI) credit is lower (higher) compared to the level obtained in the frictionless economy. Due to market power, commercial banks provide less credit than in the efficient economy, and borrowers will demand credit from NBFIs to keep total credit received at the efficient level. Higher credit costs due to bank market power increases funding costs from both types of intermediaries for borrowing entrepreneurs. Thus, their net worth is lower than in the frictionless economy.*

D.4 Friction 2: Moral Hazard in the Non-Bank Financial Sector

Introducing monopolistic competition in the commercial banking sector already provided a rationale for permanent deviations of commercial bank and NBFI credit from efficient levels. In the following, I furthermore discuss how introducing moral hazard and risk in the NBFI sector affects the above results and induced an additional trade-off for time-invariant macroprudential level policies.

First, I allow NBFIs to secretly divert a share of investments which opens up the common moral hazard problem developed in Gertler and Karadi (2011) underlying the microfoundations of the non-bank financial sector, implying steady-state NBFI leverage $\theta^S > 0$. Second, due to absence of regulation, NBFIs are risky, such that investors demand a risk premium on the funds provided. According to equation 21, the steady-state deposit rate spread therefore becomes

$$1 + r^{dS} = \frac{1 + r^{dC}}{1 - \tau^S}. \tag{D.30}$$

The risk premium NBFIs are facing on funding markets is expected to also increase the steady-state cost of NBFI loans[49]:

$$1 + r^{bS} = \frac{1 + r^{bC}}{1 - \tau^S}. \tag{D.31}$$

Thus, due to the loan rate risk premium, NBFI loans are relatively unattractive for borrowers, and NBFIs potentially earn a premium on intermediaton in the distorted steady state whenever NBFI intermediation is non-zero. Furthermore, as discussed in Gertler and Karadi (2011), this risk-adjusted premium on credit intermediation is also positive due to the introduction of market imperfections in the form of moral hazard, as NBFIs' ability to obtain funds is limited. Thus, the steady-state spread NBFIs earn on intermediation $\Delta^S > 0$ in the distorted steady state. In this case, the incentive constraint that limits NBFI leverage endogenously binds in steady state, as NBFIs would otherwise indefinitely expand their lending. Therefore, in the distorted steady state with moral hazard and risk in the non-bank financial sector, NBFI leverage given by equation D.13 will be greater than zero and NBFI credit will be above the efficient level. Furthermore, due to the riskiness of NBFIs, non-bank credit in the distorted steady state becomes

$$B^{E,S} = \frac{1 - \tau^S}{1 + r^{bC}} m^{E,S} K = \frac{1 - \tau^S}{1 + r + \mu} m^{E,S} K \tag{D.32}$$

or

$$B^{E,S} = (1 - \tau^S)\beta_P \left(1 - \frac{1}{1 + \beta_P \mu} m^E\right) K. \tag{D.33}$$

Finally, one can express the difference between NBFI credit in the distorted and the efficient steady state as

$$\begin{aligned} \widehat{B}^{E,S} = B^{E,S} - B^{E,S*} &= (1 - \tau^S)\beta_P \left(1 - \frac{1}{1 + \beta_P \mu} m^E\right) K - \beta_P (1 - m^E) K \\ &= \left[\left(1 - \frac{1 - \tau^S}{1 + \beta_P \mu}\right) m^E - \tau^S\right] K \beta_P \end{aligned} \tag{D.34}$$

[49] Assuming a transmission of funding costs to loan rates allows to capture explicit and implicit risk-related costs for borrowers when obtaining NBFI funding, such as funding and screening costs related to market and liquidity risks in NBFI loan markets.

implying $B^{E,S} > B^{E,S*}$ under the baseline calibration. However, high values of τ^S and low values of μ potentially result in a negative value of $\widehat{B}^{E,S}$ as both higher risk in the non-bank financial sector and low market power of commercial banks can induce a reverse shift of credit towards commercial banks.

Proposition 5 (Moral hazard and NBFI risk)**.** *Due to moral hazard in the non-bank financial sector, NBFI leverage is greater than zero which potentially magnifies the deviation of steady-state NBFI credit in the decentralized economy from its efficient level. However, high levels of NBFI risk can mitigate the effect, as the risk premium on NBFI credit investors demand decreases NBFI credit demand compared to the case without NBFI risk ($\tau^S = 0$).*

D.5 Implications for Permanent Macroprudential Policy

D.5.1 Market-Clearing Levels of Macroprudential Policies

As shown in section D.1, the first-best allocation in a frictionless economy features zero intermediation by NBFIs. Given that both bank types intermediate funds in an identical and perfectly competitive manner in this economy, welfare-costless commercial bank intermediation induces that it is optimal to reduce regulatory constraints to zero, such that $\nu = 0$ is optimal in this environment.

However, bank market power and inefficiencies in the non-bank financial sector as introduced in sections D.3 and D.4 induce a policy trade-off that affects the optimal long-term level of time-invariant capital requirements and LTV ratios. In the decentralized economy absent financial frictions of section D.2, the commercial bank credit market equilibrium is given by

$$\underbrace{\frac{K^C}{\nu^*}}_{\text{Credit supply}} = \underbrace{\beta_P m^E K}_{\text{Credit demand}} . \tag{D.35}$$

Solving for the efficient level of capital requirements yields

$$\nu^* = \frac{K^C}{\beta_P m^E K}. \tag{D.36}$$

However, in the distorted steady state of the economy featuring financial frictions, the commercial bank credit market equilibrium reads

$$\underbrace{\frac{K^C}{\nu^C}}_{\text{Credit supply}} = \underbrace{\frac{\beta_P}{1+\beta_P\mu} m^E K}_{\text{Credit demand}} \tag{D.37}$$

such that

$$\nu^C = \frac{K^C(1+\beta_P\mu)}{\beta_P m^E K} \tag{D.38}$$

implying

$$\begin{aligned} \nu^C &> \nu^* \text{ if } \mu > 0 \\ \nu^C &= \nu^* \text{ if } \mu = 0 \end{aligned} \tag{D.39}$$

where ν^C refers to the market-clearing level of capital requirements in the commercial bank credit market. Market power in the commercial banking sector therefore provides a rationale for regulators to raise capital requirements above the efficient level. Intuitively, the social cost induced from bank market power ceteris paribus provides an incentive to shift more intermediation towards the perfectly competitive non-bank financial sector. As higher capital charges on commercial banks induce credit leakage, raising regulatory costs for commercial banks increases the share of credit intermediation provided by NBFIs.

If NBFIs are assumed to be risk-free intermediaries, it would ultimately be welfare-improving to shift intermediation completely to these perfectly competitive intermediaries to minimize the welfare loss stemming from bank market power. However, as NBFIs are risky lenders (captured by the spread parameter τ^S), increasing the share of credit intermediated increases potential costs from NBFI default. The non-bank credit market equilibrium in the steady state of the decentralized economy without frictions is given by

$$\underbrace{\beta_P K - B^{E,C*}}_{\text{Credit supply}} = \underbrace{\beta_P(1-m^E)K}_{\text{Credit demand}} \tag{D.40}$$

$$\beta_P K - \frac{K^C}{\nu^*} = \beta_P(1-m^E)K. \tag{D.41}$$

Solving for ν^*, the steady-state level of commercial bank capital requirements that results in the clearing of the NBFI credit market, again implies

$$\nu^* = \frac{K^C}{\beta_P m^E K}. \tag{D.42}$$

In the distorted steady-state of the financial friction economy, the NBFI credit market equilibrium is given by

$$\underbrace{\beta_P K - \frac{K^C}{\nu^S}}_{\text{Credit supply}} = \underbrace{(1-\tau^S)\beta_P\left(1 - \frac{1}{1+\beta_P\mu}m^E\right)K}_{\text{Credit demand}}. \tag{D.43}$$

Solving for ν^S yields

$$\nu^S = K^C \left[\beta_P K - (1-\tau^S)\beta_P \left(1 - \frac{1}{1+\beta_P \mu} m^E \right) K \right]^{-1} \tag{D.44}$$

implying

$$\begin{aligned} \nu^S &> \nu^* \text{ if } \mu > 0, \tau^S = 0 \\ \nu^S &< \nu^* \text{ if } \mu = 0, \tau^S > 0 \\ \nu^S &= \nu^* \text{ if } \mu = 0, \tau^S = 0 \\ \nu^S &< \nu^* \text{ if } \mu > 0, \tau^S > 0. \end{aligned} \tag{D.45}$$

Comparing across markets in the distorted steady state, we observe from conditions D.39 and D.45 that

$$\nu^C > \nu^* > \nu^S \text{ if } \mu > 0, \tau^S > 0. \tag{D.46}$$

Proposition 6 (Implications on capital requirements)**.** *In the economy featuring financial frictions, the distorted steady state implies that the market-clearing level of commercial bank capital requirements is larger than zero. In the frictionless decentralized economy, there is a unique market-clearing level of capital requirements. In the economy featuring financial frictions, no single market-clearing level of commercial bank capital requirement can be determined. Time-invariant macroprudential policy faces a trade-off, as the level of requirements*

- *increases when the commercial bank loan markup increases*
- *decreases when NBFI risk premia increase*

D.5.2 Welfare-Optimal Levels of Macroprudential Regulation

Having established how market-clearing levels of steady-state capital requirements depend on the distortion parameters μ and τ^S, I discuss how time-invariant macroprudential policies can be employed to bring credit aggregates to efficient levels such that permanent steady-state distortions due to financial market inefficiencies disappear. In the analysis, I assume that regulators first set borrower-side LTV ratios such that the efficiency gap in the NBFI sector is closed and then, conditional on the resulting level of LTV ratios, the optimal level of steady-state capital requirements that additionally closes the efficiency gap in the commercial bank credit market.

NBFI credit Regulators set the borrower-oriented permanent LTV ratio such that NBFI credit is at its efficient level. To do so, one must find the optimal level of the steady-state LTV ratio $\widehat{m}^E$ that results in $\widehat{B}^{E,S} = B^{E,S} - B^{E,S*} = 0$. Letting $\widehat{m}^E$ determine the optimal LTV ratio closing the credit gap, we get from equation D.34

$$\begin{aligned} 0 &= \left[\left(1 - \frac{1-\tau^S}{1+\beta_P\mu}\right)\widehat{m}^E - \tau^S\right]K\beta_P \\ \Leftrightarrow \widehat{m}^E &= \tau^S \frac{1+\beta_P\mu}{\tau^S+\beta_P\mu} \end{aligned} \tag{D.47}$$

which implies

$$\begin{aligned} \widehat{m}^E &= 0 \text{ if } \mu > 0, \tau^S = 0 \\ \widehat{m}^E &= 1 \text{ if } \mu = 0, \tau^S > 0. \end{aligned} \tag{D.48}$$

Proposition 7 (Optimal level of LTV ratio)**.** *The optimal level of the LTV ratio, i.e. the level that brings steady-state NBFI credit to its efficient level in the distorted economy,*

- *is equal to zero whenever NBFI risk is zero and implies a complete shift of intermediation from welfare-costly commercial to welfare-costless NBFIs in this case.*
- *is equal to one whenever commercial banks are perfectly competitive and NBFIs are risky, and implies a complete shift of intermediation from welfare-costly shadow to welfare-costless commercial banks in this case.*

Furthermore, the optimal level of the LTV ratio

- *decreases when the commercial bank loan markup increases*
- *increases when NBFI risk premia increase*

Commercial bank credit From the analysis in section D.5.1, we know that due to market power in the commercial banking sector, $\nu^C > \nu^*$ if $\mu > 0$ and that $B^{E,C} = B^{E,C*}$ whenever $\nu^C = \frac{K^C(1+\beta_P\mu)}{\beta_P m^E K}$. We can now derive the efficient level of steady-state capital requirements $\widehat{\nu}$ that closes the commercial bank credit gap D.22 taking into account the efficient level of the LTV ratio $\widehat{m}^E$ that closes the NBFI credit gap D.34 which is given by

$$\begin{aligned} \widehat{\nu} &= \frac{K^C(1+\beta_P\mu)}{\beta_P\widehat{m}^E K} \\ \Leftrightarrow \widehat{\nu} &= \frac{K^C(1+\beta_P\mu)}{\beta_P K}\frac{\tau^S+\beta_P\mu}{\tau^S(1+\beta_P\mu)} \\ \Leftrightarrow \widehat{\nu} &= \frac{K^C(\tau^S+\beta_P\mu)}{\beta_P\tau^S K}. \end{aligned} \tag{D.49}$$

The efficient capital requirements ν^* is now given by

$$\nu^* = \frac{K^C}{\beta_P \widehat{m}^E K}$$
$$\Leftrightarrow \nu^* = \frac{K^C}{\beta_P K} \frac{\tau^S + \beta_P \mu}{\tau^S(1 + \beta_P \mu)} \tag{D.50}$$

such that

$$\widehat{\nu} > \nu^* \text{ if } \mu > 0, \tau^S \to 0$$
$$\widehat{\nu} = \nu^* \text{ if } \mu = 0, \tau^S > 0$$
$$\widehat{\nu} = \nu^* \text{ if } \mu = 0, \tau^S \to 0$$
$$\widehat{\nu} > \nu^* \text{ if } \mu > 0, \tau^S > 0. \tag{D.51}$$

Proposition 8 (Optimal level of capital requirements)**.** *The conditional optimal level of the commercial bank capital requirement, i.e. the level that brings steady-state commercial bank credit to its efficient level in the distorted economy taking the level of the LTV ratio that closes the NBFI credit gap into account,*

- *is larger than the level obtained in the decentralized economy without financial frictions whenever commercial banks act under monopolistic competition and NBFI credit is at the efficient level. In this case, ceteris paribus, welfare increases with the share of intermediation conducted by perfectly competitive NBFIs.*

- *is equal to the level obtained in the decentralized economy without financial frictions whenever commercial banks act under perfect competition and NBFI credit is at the efficient level. In this case, the efficient level of commercial bank credit of the decentralized economy absent financial frictions is reached whenever $\widehat{\nu} = \nu^*$, see section D.5.1.*

E Appendix: Utility-Based Welfare Functions

E.1 No Non-Bank FInance

The welfare function is derived following Benigno and Woodford (2012) from a second-order approximation of aggregate utility. Following Lambertini et al. (2013) and Rubio (2011), the social welfare measure is given by a weighted average of patient households' and impatient firms' welfare functions:

$$\mathscr{W}_{t_0} = (1 - \beta_P)\mathscr{W}^P_{t_0} + (1 - \beta_E)\mathscr{W}^E_{t_0}. \tag{E.1}$$

For patient household and firms, the respective welfare function is given by the conditional expectation of lifetime utility at date t_0,

$$\mathscr{W}_{t_0}^P \approx E_{t_0} \sum_{t=t_0}^{\infty} \beta_P^{t-t_0} [U(C_t^P, L_t^P)] \tag{E.2}$$

and

$$\mathscr{W}_{t_0}^E \approx E_{t_0} \sum_{t=t_0}^{\infty} \beta_E^{t-t_0} [U(C_t^E)] \tag{E.3}$$

E.1.1 Patient Household Welfare

As in Benigno and Woodford (2005), I assume patient households to derive utility from consuming a Dixit-Stiglitz aggregate consumption good given by

$$C_t^P \equiv \left[\int_0^1 c_t^P(i)^{\frac{\theta^P-1}{\theta^P}} di \right]^{\frac{\theta^P}{\theta^P-1}} \tag{E.4}$$

with $\theta^P > 1$. Each type of the differentiated goods is supplied by one monopolistic competitive entrepreneur. Entrepreneurs in industry j use a differentiated type of labor specific to the respective industry, whereas prices for each class of differentiated goods produced in sector j are identically set across firms in that sector. I assume that each household supplies all types of labor and consumes all types of goods. Therefore, the representative household's period utility is of the form

$$U_t^P(C_t^P, L_t^P) = \tilde{u}^P(C_t^P; \varepsilon_t) - \int_0^1 \tilde{\nu}^P(L_t(j); \varepsilon_t) dj \tag{E.5}$$

where

$$\tilde{u}^P(C_t^P; \varepsilon_t) \equiv \frac{C_t^{P\,1-\sigma}}{1-\sigma} \tag{E.6}$$

$$\tilde{\nu}^P(L_t^P; \varepsilon_t) \equiv \frac{L^{P\,1+\phi^P}}{1+\phi^P}. \tag{E.7}$$

Employment

The production technology is identical across sectors, even though each firm uses the industry-specific labor type as input:

$$y_t(i) = a_t K^\alpha L_t(i)^{1-\alpha}. \tag{E.8}$$

By inverting the production function, one can express the second term in equation E.5 as a function of equilibrium production. Furthermore, as in Benigno and Woodford (2005), the relative quantities of the differentiated goods demanded can be expressed as a function of the relative prices for these goods. We can thus express

$$\int_0^1 \tilde{\nu}^P(y_t(i);\varepsilon_t)dj = \frac{1}{1+\phi^P}\frac{Y_t^{1+\omega}}{a_t^{1+\omega}L_t^{\phi^P}}\Delta_t \equiv \nu^P(Y_t;\varepsilon_t)\Delta_t \equiv \mathbb{V} \tag{E.9}$$

with $\omega \equiv \frac{1}{1-\alpha}(1+\phi^P)-1$ and where Δ_t depicts the price dispersion term stemming from the use of the Calvo (1983) pricing framework.[50] The law of motion for price dispersion is given by

$$\Delta_t = h(\Delta_{t-1},\pi_t) \tag{E.10}$$

where

$$h(\Delta_t,\pi_t) = \theta^\pi \Delta \pi^{\theta^P(1+\omega)} + (1-\theta^\pi)\Big(\frac{1-\theta^\pi \pi^{\theta^P-1}}{1-\theta^\pi}\Big)^{\frac{\theta^P(1+\omega)}{\theta^P-1}}. \tag{E.11}$$

The Calvo parameter θ^π measures the fraction of prices that remain unchanged by entrepreneurs in a certain period.[51] The gross inflation rate is given by $\pi_t = P_t/P_{t-1}$ where P_t depicts the overall price level in period t.

Using the respective expressions in equation E.5, period utility is thus given by

$$U_t^P(C_t^P,L_t^P) = \frac{C_t^{P\,1-\sigma}}{1-\sigma} - \mathbb{V}. \tag{E.12}$$

Following again Benigno and Woodford (2005), one can derive a second-order approximation of $\mathbb{V}$ that yields

$$\widehat{\mathbb{V}} = (1-\Phi)YU^P_{C^P}\Big\{\frac{1}{2}\frac{\theta^\pi}{(1-\theta^\pi)(1-\theta^\pi\beta^P)}\theta^P(1+\omega\theta^P)\widehat{\pi}_t^2 + \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega\widehat{Y}_t q_t\Big\} + \\ + t.i.p. + O^3 \tag{E.13}$$

where

$\Phi \equiv 1-\left(\frac{\theta^P-1}{\theta^P}\right)\frac{1}{\mu}$

$q_t \equiv \frac{\phi^P L^P + \frac{1}{1-\alpha}(1+\phi^P)\widehat{a}_t}{\omega}$ and where a Taylor approximation of equation E.10 has been

[50] See Benigno and Woodford (2005, 2012) for a detailed derivation.

[51] Under pricing à la Calvo (1983), the entrepreneurs in each industry can fix monetary prices for their goods only in some periods, and the probability with which a certain firm can adjust its price in the next period is given exogenously. Thus, only a subset of firms adjusts prices in each period, and consequently the overall price level adjusts only gradually in response to exogenous disturbances.

used[52] and bars indicate steady-state values and hats log-deviations from steady-state.

The second-order approximation of equation E.12 around the steady-state therefore yields

$$U_t^P - U^P = U_{C^P}^P C^P (\frac{C_t^P - C^P}{C^P}) + \frac{1}{2}\Big[U_{C^P C^P}^P C^{P2} (\frac{C_t^P - C^P}{C^P})^2\Big] - \\ -(1-\Phi) Y U_{C^P}^P \Big\{\frac{1}{2}\frac{\theta^\pi}{(1-\theta^\pi)(1-\theta^\pi \beta^P)}\theta^P (1+\omega\theta^P)\widehat{\pi}_t^2 + \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega \widehat{Y}_t q_t\Big\} + t.i.p. + O^3 \tag{E.14}$$

or in terms of log-deviations

$$U_t^P - U^P = U_{C^P}^P C^P \Big[\widehat{C}_t^P + \frac{1}{2}(1-\psi)(\widehat{C}_t^P)^2\Big] - \\ -(1-\Phi) Y U_{C^P}^P \Big\{\frac{1}{2}\frac{\theta^\pi}{(1-\theta^\pi)(1-\theta^\pi \beta^P)}\theta^P (1+\omega\theta^P)\widehat{\pi}_t^2 + \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega \widehat{Y}_t q_t\Big\} + t.i.p. + O^3 \tag{E.15}$$

where $\psi \equiv -\frac{U_{C^P C^P}^P}{U_{C^P}^P} C^P$. Following Benigno and Woodford (2012), $t.i.p.$ refers to terms independent of policy and O^3 captures terms of higher-order terms.

Defining $\widehat{W}_t^P \equiv \frac{U_t^P - U^P}{U_{C^P}^P C^P}$ and plugging in expressions for the derivative terms delivers

$$\widehat{W}_t^P = \widehat{C}_t^P + (1-\sigma)\frac{1}{2}(\widehat{C}_t^P)^2 - (1-\Phi)\frac{Y}{C^P}\Big\{\frac{1}{2}\frac{\theta^\pi}{(1-\theta^\pi)(1-\theta^\pi\beta^P)}\theta^P(1+\omega\theta^P)\widehat{\pi}_t^2 + \\ + \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega\widehat{Y}_t q_t\Big\} + t.i.p. + O^3. \tag{E.16}$$

Collecting terms yields

$$\widehat{W}_t^P = \widehat{C}_t^P + (1-\sigma)\frac{1}{2}(\widehat{C}_t^P)^2 - \frac{1}{2}\psi_{(0)}^{\pi^2}\widehat{\pi}_t^2 - \psi_{(0)}^{Y}\widehat{Y}_t - \frac{1}{2}\psi_{(0)}^{Y^2}\widehat{Y}_t^2 + \psi^{YA}\widehat{a}_t\widehat{Y}_t + t.i.p. + O^3 \tag{E.17}$$

with

$\psi_{(0)}^{\pi^2} \equiv (1-\Phi)\frac{Y}{C^P}\frac{\theta^\pi}{(1-\theta^\pi)(1-\theta^\pi\beta^P)}\theta^P(1+\omega\theta^P)$
$\psi_{(0)}^{Y} = (1-\Phi)\frac{Y}{C^P}$
$\psi_{(0)}^{Y^2} = (1-\Phi)\frac{Y}{C^P}(1+\omega)$
$\psi^{YA} = (1-\Phi)\frac{Y}{C^P}(\phi^P L^P + \frac{1}{1-\alpha}(1+\phi^P))$

[52] See again Appendix B.3 of Benigno and Woodford (2005) for details.

Consumption

From the aggregate consumption condition B.40, we know that $\widehat{C}_t^P$ is given by

$$\widehat{C}_t^P = \frac{C}{C^P}\widehat{C}_t - \frac{C^E}{C^P}\widehat{C}_t^E. \tag{E.18}$$

Plugging in $\widehat{W}_t^P$ and rewriting yields

$$\widehat{W}_t^P = \frac{C}{C^P}\Big(\widehat{C}_t + (1-\sigma)\frac{1}{2}\frac{C}{C^P}\widehat{C}_t^2\Big) - \frac{C^E}{C^P}\widehat{C}_t^E - (1-\sigma)\frac{CC^E}{C^{P2}}\widehat{C}_t\widehat{C}_t^E + (1-\sigma)\Big(\frac{C^E}{C^P}\Big)^2\frac{1}{2}(\widehat{C}_t^E)^2 - \\ - \frac{1}{2}\psi_{(0)}^{\pi^2}\widehat{\pi}_t^2 - \psi_{(0)}^{Y}\widehat{Y}_t - \frac{1}{2}\psi_{(0)}^{Y^2}\widehat{Y}_t^2 + \psi^{YA}\widehat{a}_t\widehat{Y}_t + t.i.p. + O^3. \tag{E.19}$$

We now derive an expression for $\mathbb{C} \equiv \widehat{C}_t + (1-\sigma)\frac{1}{2}\frac{C}{C^P}\widehat{C}_t^2$. Using the second-order approximation of the aggregate resource constraint (equation B.38) we can get the expression

$$\begin{aligned}\mathbb{C} = &\Big[\frac{1}{2}\frac{Y}{C} - \sigma'\frac{1}{2}\big(\frac{Y}{C}\big)^2\Big]\widehat{Y}_t^2 - \Big[\frac{1}{2}\frac{\delta^C K^C}{\pi C} + \sigma'\frac{1}{2}\big(\frac{\delta^C K^C}{\pi C}\big)^2\Big]\big((\widehat{K}_{t-1}^C)^2 + \widehat{\pi}_t^2\big) + \\ &+ \frac{Y}{C}\widehat{Y}_t - \frac{\delta^C K^C}{\pi C}\big(\widehat{K}_{t-1}^C - \widehat{\pi}_t\big) + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.20}$$

where $covars$[53] contains covariance terms between the endogenous variables $\widehat{Y}_t$, $\widehat{K}_{t-1}^C$, and $\widehat{\pi}_t$, and $\sigma' = 1-(1-\sigma)\frac{C}{C^P}$.

We can now replace the log-deviations of lagged commercial bank capital from steady state with the second-order approximation of the law of motion of bank capital (equation B.25) to get:

$$\begin{aligned}\mathbb{C} = &\frac{1}{2}\frac{Y}{C}\Big[1 - \sigma'\frac{Y}{C}\Big]\widehat{Y}_t^2 + \frac{1}{2}\Big[\frac{\sigma'(\psi^{K^C})^2}{(1-\delta^C)^2} - \frac{1}{1-\delta^C}\psi^{K^C}\Big](\widehat{K}_t^C)^2 + \\ &+ \frac{1}{2}\Big[\frac{J}{(1-\delta^C)K^C}\psi^{K^C} + \sigma'(\psi^{K^C})^2\Big]\widehat{J}_t^2 - \frac{1}{2}\psi^{K^C}(1+\psi^{K^C})\widehat{\pi}_t^2 + \\ &+ \frac{Y}{C}\widehat{Y}_t - \frac{1}{1-\delta^C}\psi^{K^C}\widehat{K}_t^C + \frac{J}{(1-\delta^C)K^C}\psi^{K^C}\widehat{J}_t + \psi^{K^C}\widehat{\pi}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.21}$$

where $\psi^{K^C} \equiv \frac{\delta^C K^C}{\pi C}$.

[53] In the following derivations, the term $covars$ will be extended by the covariance terms of all the endogenous and exogenous variables introduced each step. Due to space limitations, not all these terms will be written out until the end of the derivations.

Using the second-order approximation of the commercial bank profit function (equation B.26), we can substitute out $\widehat{J}_t$ and $\widehat{J}_t^2$ to get

$$\begin{aligned}\mathbb{C} &= \tfrac{1}{2}\psi^{Y^2}\widehat{Y}_t^2 + \tfrac{1}{2}\psi^{K^{C^2}}(\widehat{K}_t^C)^2 - \tfrac{1}{2}\psi^{\pi^2}\widehat{\pi}_t^2 + \tfrac{1}{2}\psi^{r^{bC^2}}(\widehat{r}_t^{bC})^2 + \tfrac{1}{2}\psi^{B^2}\widehat{B}_t^2 + \tfrac{1}{2}\psi^{r^2}\widehat{r}_t^2 + \tfrac{1}{2}\psi^{D^2}\widehat{D}_t^2 - \tfrac{1}{2}\psi^{\nu^2}\widehat{\nu}_t^2 + \\ &+ \psi^Y\widehat{Y}_t - \psi^{K^C}\widehat{K}_t^C + \psi^\pi\widehat{\pi}_t + \psi^{r^{bC}}\widehat{r}_t^{bC} + \psi^B\widehat{B}_t - \psi^r\widehat{r}_t - \psi^D\widehat{D}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.22}$$

with

$$\begin{aligned}
\psi^{Y^2} &\equiv \tfrac{Y}{C}\left(1 - \sigma'\tfrac{Y}{C}\right) & \psi^Y &\equiv \tfrac{Y}{C}\\
\psi^{K^{C^2}} &\equiv \tfrac{\psi^{K^C}}{1-\delta^C}\left(\tfrac{\sigma'\psi^{K^C}}{1-\delta^C} - 1\right) - \tfrac{\theta\nu^2}{1-\delta^C}\psi^{K^C} & \psi^{K^C} &\equiv \tfrac{1}{1-\delta^C}\psi^{K^C}\\
\psi^{\pi^2} &\equiv \psi^{K^C}\left(1 + \psi^{K^C}\right) & \psi^\pi &\equiv \psi^{K^C}\\
\psi^{r^{bC^2}} &\equiv \sigma'\psi^{K^C}\left(\tfrac{r^{bC}B^C}{J^C}\right)^2 + \tfrac{r^{bC}B^C}{(1-\delta^C)K^C}\psi^{K^C} & \psi^{r^{bC}} &\equiv \tfrac{r^{bC}B^C}{(1-\delta^C)K^C}\psi^{K^C}\\
\psi^{B^2} &\equiv \psi^{r^{bC^2}} - \tfrac{\theta\nu^2}{1-\delta^C} & \psi^B &\equiv \tfrac{r^{bC}B^C}{(1-\delta^C)K^C}\\
\psi^{r^2} &\equiv \sigma'\psi^{K^C}\left(\tfrac{rD^C}{J^C}\right)^2 - \tfrac{\theta\nu^2}{(1-\delta^C)K^C}\psi^{K^C} & \psi^r &\equiv \tfrac{rD^C}{(1-\delta^C)K^C}\\
\psi^{D^2} &\equiv \sigma'\psi^{K^C}\left(\tfrac{rD^C}{J^C}\right)^2 - \tfrac{rD^C}{(1-\delta^C)K^C}\psi^{K^C} & \psi^D &\equiv \tfrac{rD^C}{(1-\delta^C)K^C}\psi^{K^C}.\\
\psi^{\nu^2} &\equiv \tfrac{\theta\nu^2}{1-\delta^C}\psi^{K^C}
\end{aligned}$$

Next, we can eliminate second-order terms related to $\widehat{D}_t$ in $\mathbb{C}$ by using the commercial bank balance sheet (equation B.28) which yields:

$$\begin{aligned}\mathbb{C} &= \tfrac{1}{2}\psi^{Y^2}\widehat{Y}_t^2 + \tfrac{1}{2}\psi_{(2)}^{K^{C^2}}(\widehat{K}_t^C)^2 - \tfrac{1}{2}\psi^{\pi^2}\widehat{\pi}_t^2 + \tfrac{1}{2}\psi^{r^{bC^2}}(\widehat{r}_t^{bC})^2 + \tfrac{1}{2}\psi_{(2)}^{B^2}\widehat{B}_t^2 + \tfrac{1}{2}\psi^{r^2}\widehat{r}_t^2 - \tfrac{1}{2}\psi^{\nu^2}\widehat{\nu}_t^2 + \\ &+ \psi^Y\widehat{Y}_t - \psi_{(2)}^{K^C}\widehat{K}_t^C + \psi^\pi\widehat{\pi}_t + \psi^{r^{bC}}\widehat{r}_t^{bC} + \psi_{(2)}^B\widehat{B}_t - \psi^r\widehat{r}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.23}$$

with

$$\begin{aligned}
\psi_{(2)}^{K^{C^2}} &\equiv \psi^{K^{C^2}} + \psi^D\tfrac{K^C}{D} - 2\psi^{K^CD}\tfrac{D}{K^C} + 2\psi^{D^2}\left(\tfrac{K^C}{D}\right)^2\\
\psi_{(2)}^{B^2} &\equiv \psi^{B^2} - \psi^D\tfrac{B^C}{D} - 2\psi^{B^CD}\tfrac{B^C}{K^C} + 2\psi^{D^2}\left(\tfrac{B^C}{D}\right)^2\\
\psi_{(2)}^{K^C} &\equiv \psi^D\tfrac{K^C}{D} - \psi^{K^CB^C}\\
\psi_{(2)}^B &\equiv \psi^B - \psi^D\tfrac{B^C}{D}\\
\psi^{K^CD} &\equiv \psi^{K^{C^2}}\tfrac{\sigma' rD}{J}\\
\psi^{BD} &\equiv \sigma'\tfrac{r^{bC}B^C rD}{J^2}\\
\psi^{K^CB} &\equiv \tfrac{\theta\nu^2}{1-\delta^C} - \psi^{K^{C^2}}\sigma'\tfrac{r^{bC}B^C}{J}.
\end{aligned}$$

We use the profit equation B.26 to replace commercial bank capital:

$$\begin{aligned}\mathbb{C} &= \tfrac{1}{2}\psi^{Y^2}\widehat{Y}_t^2 - \tfrac{1}{2}\psi^{\pi^2}\widehat{\pi}_t^2 + \tfrac{1}{2}\psi_{(2)}^{r^{bC^2}}(\widehat{r}_t^{bC})^2 + \tfrac{1}{2}\psi_{(3)}^{B^2}\widehat{B}_t^2 + \tfrac{1}{2}\psi_{(2)}^{r^2}\widehat{r}_t^2 + \tfrac{1}{2}\psi_{(2)}^{\nu^2}\widehat{\nu}_t^2 + \\ &+ \psi^Y\widehat{Y}_t + \psi^\pi\widehat{\pi}_t + \psi_{(2)}^{r^{bC}}\widehat{r}_t^{bC} + \psi_{(3)}^B\widehat{B}_t + \psi_{(2)}^r\widehat{r}_t + \psi_{(2)}^{K^C}\widehat{\nu}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.24}$$

with

$\psi^{r^{bC2}}_{(2)} \equiv \psi^{r^{bC2}} - \frac{r^{bC}}{\theta\nu^3}\psi^{K^C}_{(2)}(1+4\frac{r^{bC}}{\theta\nu^3})$
$\psi^{B^2}_{(3)} \equiv \psi^{B^2}_{(2)} + \psi^{K^CB}_{(2)} + \psi^{K^{C2}}_{(2)} - 5\psi^{K^C}_{(2)}$
$\psi^{r^2}_{(2)} \equiv \psi^{r^2} + \frac{r}{\theta\nu^3}(\psi^{K^C}_{(2)} + \psi^{K^Cr}_{(2)}) + (\frac{r}{\theta\nu^3})^2(\psi^{K^{C2}}_{(2)} - 5\psi^{K^C}_{(2)})$
$\psi^{\nu^2}_{(2)} \equiv \psi^{K^{C2}}_{(2)} - 8\psi^{K^C}_{(2)} - \psi^{\nu^2} - \psi^{K^C\nu}$
$\psi^{r^{bC}}_{(2)} \equiv \psi^{r^{bC}} - \frac{r^{bC}}{\theta\nu^3}\psi^{K^C}_{(2)}$
$\psi^{B}_{(3)} \equiv \psi^{B}_{(2)} + \psi^{K^C}_{(2)}$
$\psi^{r}_{(2)} \equiv \frac{r}{\theta\nu^3}\psi^{K^C}_{(2)} - \psi^r$
$\psi^{K^CB}_{(2)} \equiv \psi^{K^CB} + \frac{B^C}{D}\psi^{K^CD} + \frac{K^C}{D}\psi^{BD} - 2\frac{B^CK^C}{D^2}\psi^{D^2}$
$\psi^{K^Cr}_{(2)} \equiv \psi^{K^Cr} - \frac{K^C}{D}\psi^{Dr}$
$\psi^{K^C\nu} \equiv \frac{\theta\nu^2}{1-\delta^C}$
$\psi^{K^Cr} \equiv \psi^{K^{C2}}\frac{\sigma' rD}{J}$
$\psi^{Dr} \equiv \big(\sigma'(\frac{rD}{J})^2 - \frac{rD}{(1-\delta^C)K^C}\big)\psi^{K^C}.$

Use equations B.16 and B.22 to replace $\widehat{r}^{bC}_t$:

$$\begin{aligned}\mathbb{C} = &\tfrac{1}{2}\psi^{Y^2}\widehat{Y}^2_t - \tfrac{1}{2}\psi^{\pi^2}\widehat{\pi}^2_t + \tfrac{1}{2}\psi^{B^2}_{(4)}\widehat{B}^2_t + \tfrac{1}{2}\psi^{r^2}_{(2)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(2)}\widehat{\nu}^2_t + \\ &+ \psi^Y\widehat{Y}_t + \psi^\pi\widehat{\pi}_t + \psi^B_{(4)}\widehat{B}_t + \psi^r_{(2)}\widehat{r}_t + \psi^{K^C}_{(2)}\widehat{\nu}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.25}$$

with

$\psi^{B^2}_{(4)} \equiv \psi^{B^2}_{(3)} + \psi^{r^{bC2}}_{(2)} + \psi^{r^{bC}B}_{(3)}$
$\psi^{B}_{(4)} \equiv \psi^{B}_{(3)} + \psi^{r^{bC}}_{(2)}$
$\psi^{r^{bC}B}_{(3)} \equiv$
$\psi^{r^{bC}B}_{(2)} + \psi^{K^Cr^{bC}}_{(2)} - (\psi^{K^{C2}}_{(2)} - 5\psi^{K^C}_{(2)})\frac{r^{bC}r}{(\theta\nu^3)^2}$
$\psi^{r^{bC}B}_{(2)} \equiv \psi^{r^{bC}B} - \frac{B^C}{D}\psi^{r^{bC}D}$

$\psi^{K^Cr^{bC}}_{(2)} \equiv \frac{K^C}{D}\psi^{r^{bC}D} - \psi^{K^Cr^{bC}}$
$\psi^{r^{bC}B} \equiv \big(\frac{r^{bC}B^C}{(1-\delta^C)K^C} + \sigma'(\frac{r^{bC}B^C}{J})^2\big)\psi^{K^C}$
$\psi^{r^{bC}D} \equiv \sigma'\frac{r^{bC}B^CrD}{J^2}\psi^{K^C}$
$\psi^{K^Cr^{bC}} \equiv \frac{\sigma' r^{bC}B^C}{J}\psi^{K^{C2}}.$

We can then use the definition of the commercial bank credit-to-GDP ratio Z_t (equation B.42) to express lending in relation to GDP:

$$\begin{aligned}\mathbb{C} = &\tfrac{1}{2}\psi^{Y^2}_{(2)}\widehat{Y}^2_t - \tfrac{1}{2}\psi^{\pi^2}\widehat{\pi}^2_t + \tfrac{1}{2}\psi^{r^2}_{(2)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(2)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}\widehat{Z}^2_t + \\ &+ \psi^Y_{(2)}\widehat{Y}_t + \psi^\pi\widehat{\pi}_t + \psi^r_{(2)}\widehat{r}_t + \psi^{K^C}_{(2)}\widehat{\nu}_t + \psi^B_{(4)}\widehat{Z}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.26}$$

with

$\psi^{Y^2}_{(2)} \equiv 1 + \psi^{Y^2} + \psi^{YB}_{(4)} + \psi^{B^2}_{(4)}$
$\psi^{z,cb^2} \equiv \psi^{B^2}_{(4)} + \psi^{B}_{(4)}$
$\psi^{Y}_{(2)} \equiv \psi^{Y} + \psi^{B}_{(4)}$
$\psi^{YB}_{(4)} \equiv \psi^{YB}_{(3)} - \psi^{Yr^{bC}}$
$\psi^{YB}_{(3)} \equiv \psi^{YB}_{(2)} - \psi^{YK^C}$
$\psi^{Yr^{bC}} \equiv \frac{\sigma' Y r^{bC} B^C}{(1-\delta^C)K^C C}\psi^{K^C}$

$\psi^{YB}_{(2)} \equiv \frac{B}{D}\psi^{YD} - \psi^{YB}$
$\psi^{YB} \equiv \frac{\sigma' Y r^{bC} B^C}{(1-\delta^C)K^C C}\psi^{K^C}$
$\psi^{YD} \equiv \frac{\sigma' Y r D}{(1-\delta^C)K^C C}\psi^{K^C}$
$\psi^{YK^C}_{(2)} \equiv \psi^{YK^C} + \frac{K^C}{D}\psi^{YD}$
$\psi^{YK^C} \equiv \frac{\sigma' Y}{(1-\delta^C)C}\psi^{K^C}.$

Finally, I use the first-order approximation of the monetary policy rule B.37 to replace $\widehat{r}_t$[54]

$$\begin{aligned} \mathbb{C} = {} & \tfrac{1}{2}\psi^{Y^2}_{(3)}\widehat{Y}_t^2 + \tfrac{1}{2}\psi^{\pi^2}_{(2)}\widehat{\pi}_t^2 + \tfrac{1}{2}\psi^{r^2}_{(2)}\widehat{r}_t^2 + \tfrac{1}{2}\psi^{\nu^2}_{(2)}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi^{z,cb^2}\widehat{Z}_t^2 + \\ & + \psi^{Y}_{(3)}\widehat{Y}_t + \psi^{\pi}_{(2)}\widehat{\pi}_t + \psi^{K^C}_{(2)}\widehat{\nu}_t + \psi^{B}_{(4)}\widehat{Z}_t + \\ & + covars + t.i.p. + O^3 \end{aligned} \tag{E.27}$$

with

$\psi^{Y^2}_{(3)} \equiv \psi^{Y^2}_{(2)} + 2\frac{\phi^y}{1+r}\psi^{Yr}_{(3)}$
$\psi^{\pi^2}_{(2)} \equiv 2\frac{\phi^\pi \pi}{1+r}\psi^{\pi r}_{(2)} - \psi^{\pi}$
$\psi^{Y}_{(3)} \equiv \psi^{Y}_{(2)} + \frac{\phi^y}{1+r}\psi^{r}_{(2)}$
$\psi^{\pi}_{(2)} \equiv \psi^{\pi} + \frac{\phi^\pi \pi}{1+r}\psi^{r}_{(2)}$
$\psi^{Yr}_{(3)} \equiv \psi^{Yr}_{(2)} + \psi^{Br}_{(3)}$

$\psi^{Yr}_{(2)} \equiv \psi^{Yr} - \frac{r}{\theta\nu^3}\psi^{YK^C}_{(2)}$
$\psi^{Br}_{(3)} \equiv \psi^{Br}_{(2)} + \frac{r}{\theta\nu^3}\left(\psi^{K^C B}_{(2)} + 5\psi^{K^{C^2}}_{(2)}\right) + \psi^{K^C r}_{(2)}$
$\psi^{Yr} \equiv \psi^{YD}$
$\psi^{Br}_{(2)} \equiv \frac{B}{D}\psi^{Dr} - \psi^{Br}$
$\psi^{Br} \equiv \psi^{r^{bC}D}.$

In the next step, I substitute $\mathbb{C}$ in $\widehat{W}_t^P$. Rearranging terms yields:

$$\begin{aligned} \widehat{W}_t^P = {} & \tfrac{1}{2}\psi^{Y^2}_{(4)}\widehat{Y}_t^2 + \tfrac{1}{2}\psi^{\pi^2}_{(3)}\widehat{\pi}_t^2 + \tfrac{1}{2}\psi^{r^2}_{(3)}\widehat{r}_t^2 + \tfrac{1}{2}\psi^{\nu^2}_{(3)}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi^{z,cb^2}_{(2)}\widehat{Z}_t^2 + \tfrac{1}{2}\psi^{ce^2}(\widehat{C}_t^E)^2 + \\ & + \psi^{Y}_{(4)}\widehat{Y}_t + \psi^{\pi}_{(3)}\widehat{\pi}_t + \psi^{\nu}\widehat{\nu}_t + \psi^{z,cb}\widehat{Z}_t - \psi^{ce}\widehat{C}_t^E + \\ & + covars + t.i.p. + O^3 \end{aligned} \tag{E.28}$$

with

$\psi^{Y^2}_{(4)} \equiv \frac{C}{C^P}\psi^{Y^2}_{(3)} - \psi^{Y^2}_{(0)}$
$\psi^{\pi^2}_{(3)} \equiv \frac{C}{C^P}\psi^{\pi^2}_{(2)} - \psi^{\pi^2}_{(0)}$
$\psi^{r^2}_{(3)} \equiv \frac{C}{C^P}\psi^{r^2}_{(2)}$
$\psi^{\nu^2}_{(3)} \equiv \frac{C}{C^P}\psi^{\nu^2}_{(2)}$

$\psi^{z,cb^2}_{(2)} \equiv \frac{C}{C^P}\psi^{z,cb^2}$
$\psi^{ce^2} \equiv \frac{C}{C^P}(1-\sigma')$

[54] I use the first-order instead of second-order approximation of the monetary policy rule, as I assume the central bank not to evaluate the second moments of Y_t and π_t in its decision making.

$\psi^{Y}_{(4)} \equiv \frac{C}{C^P}\psi^{Y}_{(3)} - \psi^{Y}_{(0)}$
$\psi^{\pi}_{(3)} \equiv \frac{C}{C^P}\psi^{\pi}_{(2)}$
$\psi^{\nu} \equiv \frac{C}{C^P}\psi^{K^C}_{(2)}$
$\psi^{z,cb} \equiv \frac{C}{C^P}\psi^{B}_{(4)}$
$\psi^{ce} \equiv \frac{C}{C^P}.$

Entrepreneur consumption can then be substituted by combining equations B.16, B.18, and B.20:

$$\begin{aligned} \widehat{W}^P_t &= \tfrac{1}{2}\psi^{Y^2}_{(6)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{\pi^2}_{(3)}\widehat{\pi}^2_t + \tfrac{1}{2}\psi^{r^2}_{(3)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(3)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(4)}\widehat{Z}^2_t + \\ &+ \psi^{Y}_{(5)}\widehat{Y}_t + \psi^{\pi}_{(3)}\widehat{\pi}_t + \psi^{\nu}\widehat{\nu}_t + \psi^{z,cb}_{(2)}\widehat{Z}_t + \\ &+ covars + t.i.p. + O^3 \end{aligned} \tag{E.29}$$

with

$\psi^{Y^2}_{(6)} \equiv \psi^{Y^2}_{(5)} + \left(\frac{K\chi}{\beta_E NW^E}\right)^2\psi^{ce^2}_{(2)}$
$\psi^{Y^2}_{(5)} \equiv \psi^{Y^2}_{(4)} + 2\psi^{Yce}\frac{K\chi}{\beta_E NW^E}$
$\psi^{ce^2}_{(2)} \equiv \psi^{ce^2} + \psi^{ce}$
$\psi^{z,cb^2}_{(4)} \equiv \psi^{z,cb^2}_{(3)} + \left(\frac{K\chi}{\beta_E NW^E}\right)^2\psi^{ce^2}_{(2)}$
$\psi^{z,cb^2}_{(3)} \equiv \psi^{z,cb^2}_{(2)} + \frac{K\chi}{\beta_E NW^E}\psi^{ce} + 2\frac{K\chi}{\beta_E NW^E}\psi^{zce}$
$\psi^{Y}_{(5)} \equiv \psi^{Y}_{(4)} + \frac{K\chi}{\beta_E NW^E}\psi^{ce}$
$\psi^{z,cb}_{(2)} \equiv \psi^{z,cb} + \frac{K\chi}{\beta_E NW^E}\psi^{ce}$
$\psi^{Yce} \equiv (1-\sigma')\frac{CC^E}{C^P}\psi^{Y}_{(3)}$
$\psi^{zce} \equiv (1-\sigma')\frac{CC^E}{C^P}\psi^{B}_{(4)}.$

Again using the first-order approximation of policy rule B.37, one can replace the inflation variance term $\widehat{\pi}^2_t$ and furthermore get:

$$\begin{aligned} \widehat{W}^P_t &= \tfrac{1}{2}\psi^{Y^2}_{(8)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(4)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(3)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(4)}\widehat{Z}^2_t + \\ &+ \psi^{Y}_{(7)}\widehat{Y}_t + \psi^{\pi}_{(4)}\widehat{\pi}_t + \psi^{\nu}\widehat{\nu}_t + \psi^{z,cb}_{(2)}\widehat{Z}_t + \\ &+ covars + t.i.p. + O^3 \end{aligned} \tag{E.30}$$

with

$\psi^{Y^2}_{(8)} \equiv \psi^{Y^2}_{(7)} - \frac{\phi^y}{1+r}\psi^{Yr}_{(4)}$
$\psi^{Y^2}_{(7)} \equiv \psi^{Y^2}_{(6)} + \left(\frac{\phi^y}{\phi^\pi\pi}\right)^2\psi^{\pi^2}_{(3)}$
$\psi^{r^2}_{(4)} \equiv \psi^{r^2}_{(3)} + \left(\frac{1+r}{\phi^\pi\pi}\right)^2\psi^{\pi^2}_{(3)}$
$\psi^{Y}_{(7)} \equiv \psi^{Y}_{(6)} + \frac{\phi^y}{1+r}\psi^{r}_{(3)}$
$\psi^{Y}_{(6)} \equiv \psi^{Y}_{(5)} - \frac{(1+r)\phi^y}{(\phi^\pi\pi)^2}\psi^{\pi^2}_{(3)}$
$\psi^{\pi}_{(4)} \equiv \psi^{\pi}_{(3)} + \frac{\phi^\pi\pi}{1+r}\psi^{r}_{(3)}$
$\psi^{r}_{(3)} \equiv \left(\frac{1+r}{\phi^\pi\pi}\right)^2\psi^{\pi^2}_{(3)}$
$\psi^{Yr}_{(4)} \equiv \frac{(1+r)\phi^y}{(\phi^\pi\pi)^2}\psi^{\pi^2}_{(3)}.$

E.1.2 Impatient Entrepreneur Welfare

For the impatient firm, period utility is given by

$$U^E_t(C^E_t) = \frac{{C^E_t}^{1-\sigma}}{1-\sigma}. \tag{E.31}$$

We can thus derive a similar expression for period welfare as for households:

$$\widehat{W}_t^E = \widehat{C}_t^E + (1-\sigma)\frac{1}{2}(\widehat{C}_t^E)^2. \tag{E.32}$$

As above, we can combine equations B.16, B.18, and B.20 to get:

$$\widehat{C}_t^E = -\frac{K\chi}{\beta_E NW^E}(\widehat{Z}_t + \frac{1}{2}\widehat{Z}_t^2) - \frac{K\chi}{\beta_E NW^E}\widehat{Y}_t - \frac{K\chi}{\beta_E NW^E}\widehat{Y}_t\widehat{Z}_t - \frac{1}{2}(\widehat{C}_t^E)^2. \tag{E.33}$$

Plugging in $\widehat{W}_t^E$ yields

$$\begin{aligned}\widehat{W}_t^E = & -\frac{1}{2}\sigma\Big(\frac{K\chi}{\beta_E NW^E}\Big)^2\widehat{Y}_t^2 - \frac{1}{2}\Big[\frac{K\chi}{\beta_E NW^E} + \sigma\Big(\frac{K\chi}{\beta_E NW^E}\Big)^2\Big]\widehat{Z}_t^2 - \\ & - \frac{K\chi}{\beta_E NW^E}(\widehat{Y}_t + \widehat{Z}_t) - \Big[\frac{K\chi}{\beta_E NW^E} + \sigma\Big(\frac{K\chi}{\beta_E NW^E}\Big)^2\Big]\widehat{Y}_t\widehat{Z}_t.\end{aligned} \tag{E.34}$$

E.1.3 Joint Welfare

We can now derive period welfare along the lines of equation E.1. Period joint welfare is given by

$$W_t = (1-\beta_P)W_t^P + (1-\beta_E)W_t^E. \tag{E.35}$$

Approximating yields:

$$\widehat{W}_t = (1-\beta_P)\frac{W^P}{W}\widehat{W}_t^P + (1-\beta_E)\frac{W^E}{W}\widehat{W}_t^E. \tag{E.36}$$

We can now plug in expressions $\widehat{W}_t^P$ and $\widehat{W}_t^E$ to get

$$\begin{aligned}\widehat{W}_t = & \tfrac{1}{2}\psi_{(9)}^{Y^2}\widehat{Y}_t^2 + \tfrac{1}{2}\psi_{(5)}^{r^2}\widehat{r}_t^2 + \tfrac{1}{2}\psi_{(4)}^{\nu^2}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi_{(5)}^{z,cb^2}\widehat{Z}_t^2 + \\ & + \psi_{(8)}^{Y}\widehat{Y}_t + \psi_{(5)}^{\pi}\widehat{\pi}_t + \psi_{(2)}^{\nu}\widehat{\nu}_t + \psi_{(3)}^{z,cb}\widehat{Z}_t + \\ & + covars + t.i.p. + O^3\end{aligned} \tag{E.37}$$

with

$\psi_{(9)}^{Y^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(8)}^{Y^2} - (1-\beta_E)\frac{W^E}{W}\sigma'\big(\frac{K\chi}{\beta_E NW^E}\big)^2$

$\psi_{(5)}^{r^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(4)}^{r^2}$

$\psi_{(4)}^{\nu^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(3)}^{\nu^2}$

$\psi_{(5)}^{z,cb^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(4)}^{z,cb^2} - (1-\beta_E)\frac{W^E}{W}\Big[\frac{K\chi}{\beta_E NW^E} + \sigma'\big(\frac{K\chi}{\beta_E NW^E}\big)^2\Big]$

$\psi_{(8)}^{Y} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(7)}^{Y} - (1-\beta_E)\frac{W^E}{W}\frac{K\chi}{\beta_E NW^E}$

$\psi_{(5)}^{\pi} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(4)}^{\pi}$

$\psi_{(2)}^{\nu} \equiv (1-\beta_P)\frac{W^P}{W}\psi^{\nu}$

$\psi_{(3)}^{z,cb} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(2)}^{z,cb} - (1-\beta_E)\frac{W^E}{W}\frac{K\chi}{\beta_E NW^E}.$

We can remove the linear term $\widehat{\nu}_t$ by combining the first-order approximation of the credit supply condition B.27 with the first-order approximations of the commercial bank balance sheet condition (equation B.24), bank profits (equation B.26), the law of motion for bank capital (equation B.25), and the aggregate resource constraint (equation B.38) to express $\widehat{\nu}_t$ only in linear terms of $\widehat{Z}_t$, $\widehat{Y}_t$, and $\widehat{\pi}_t$ such that

$$\begin{aligned}\widehat{W}_t = &\tfrac{1}{2}\psi^{Y^2}_{(10)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(5)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\pi^2}_{(4)}\widehat{\pi}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(4)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(6)}\widehat{Z}^2_t+\\ &+\psi^{Y}_{(9)}\widehat{Y}_t + \psi^{\pi}_{(6)}\widehat{\pi}_t + \psi^{z,cb}_{(4)}\widehat{Z}_t+\\ &+covars + t.i.p. + O^3\end{aligned} \tag{E.38}$$

with

$$\psi^{Y^2}_{(10)} \equiv \psi^{Y^2}_{(9)} + \psi^{Y\nu}_{(5)}\Omega^{y}_{(5)} \qquad \psi^{Y}_{(9)} \equiv \psi^{Y}_{(8)} - \psi^{\nu}_{(2)}\Omega^{y}_{(5)}$$
$$\psi^{\pi^2}_{(4)} \equiv 2\psi^{\pi\nu}_{(3)}\Omega^{\pi}_{(4)} \qquad \psi^{\pi}_{(6)} \equiv \psi^{\pi}_{(5)} - \psi^{\nu}_{(2)}\Omega^{\pi}_{(4)}$$
$$\psi^{z,cb^2}_{(6)} \equiv \psi^{z,cb^2}_{(5)} - \psi^{\nu z}_{(3)}\Omega^{B}_{(4)} \qquad \psi^{z,cb}_{(4)} \equiv \psi^{z,cb}_{(4)} - \psi^{\nu}_{(2)}\Omega^{B}_{(4)}$$

where the auxiliary parameters $\Omega^{y}_{(5)}$, $\Omega^{\pi}_{(4)}$, and $\Omega^{B}_{(4)}$ were derived during the side step of replacing $\widehat{\nu}_t$. Due to space limitations, their derivation is not discussed in detail here and results are available upon request. Using the approximation of the Taylor rule to replace $\widehat{\pi}^2_t$ yields

$$\begin{aligned}\widehat{W}_t = &\tfrac{1}{2}\psi^{Y^2}_{(11)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(6)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(4)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(6)}\widehat{Z}^2_t+\\ &+\psi^{Y}_{(9)}\widehat{Y}_t + \psi^{\pi}_{(7)}\widehat{\pi}_t + \psi^{z,cb}_{(4)}\widehat{Z}_t+\\ &+covars + t.i.p. + O^3\end{aligned} \tag{E.39}$$

with

$$\psi^{Y^2}_{(11)} \equiv \psi^{Y^2}_{(10)} - \left(\tfrac{\phi^y}{\phi^\pi\pi}\right)^2\psi^{\pi^2}_{(4)}$$
$$\psi^{r^2}_{(6)} \equiv \psi^{r^2}_{(5)} + \left(\tfrac{1+r}{\phi^\pi\pi}\right)^2\psi^{\pi^2}_{(4)}$$
$$\psi^{\pi}_{(7)} \equiv \psi^{\pi}_{(6)} + \tfrac{1+r}{\phi^\pi\pi}\psi^{\pi^2}_{(4)}.$$

Finally, I follow the same strategy as in Benigno and Woodford (2005) and use an iterated expression of the second-order approximation of the aggregate-supply relationship to replace the linear output term $\widehat{Y}_t$ in the lifetime welfare criterion

$$\begin{aligned}\widehat{W}_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}[&\tfrac{1}{2}\psi^{Y^2}_{(11)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(6)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(4)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(6)}\widehat{Z}^2_t+\\ &+\psi^{Y}_{(9)}\widehat{Y}_t + \psi^{\pi}_{(7)}\widehat{\pi}_t + \psi^{z,cb}_{(4)}\widehat{Z}_t] + t.i.p. + O^3.\end{aligned} \tag{E.40}$$

In the process, I replace the linear inflation term $\widehat{\pi}_t$ in the infinite sum by iterating forward the first-order approximation of the New-Keynesian Phillips curve and collect the covariances of $\widehat{Y}_t$, $\widehat{r}_t$, and $\widehat{Z}_t$ by defining efficiency gaps for these variables in a similar fashion as in Benigno and Woodford (2005).[55] Following these steps, one can express discounted lifetime welfare as

$$\widehat{W}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta_P^{t-t_0} [\tfrac{1}{2}\psi_{(12)}^{Y^2}(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \tfrac{1}{2}\psi_{(7)}^{r^2}(\widehat{r}_t - \widehat{r}_t^*)^2 + \tfrac{1}{2}\psi_{(5)}^{\nu^2}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi_{(7)}^{z,cb^2}(\widehat{Z}_t - \widehat{Z}_t^*)^2 + \\ + \psi_{(5)}^{z,cb}\widehat{Z}_t] + t.i.p. + O^3 + T_0 \quad \text{(E.41)}$$

where T_0 depicts a transitory component similar to the expression derived in Benigno and Woodford (2005). The coefficients can then directly be mapped in the parameters of the period loss function given by equation 39.

E.2 Non-Bank Finance

E.2.1 Patient Household Welfare

In the model, the introduction of NBFIs affects both the saving decision of patient households and the borrowing decision of impatient entrepreneurs as both agents can intermediate funds now with both financial institutions. The introduction of non-bank finance alters the above derivation of the welfare loss function via the entrepreneur problem, as entrepreneur net worth now depends on borrowing from both commercial banks and NBFIs (equationB.19)[56]. As indicated by equation B.18, net worth in turn affects entrepreneur consumption, and therefore steady state levels NW^E and C^E are affected by the introduction of non-bank finance. Adding NBFIs to the model does therefore not affect the above derivation until equation E.28, but only enters in the following step when steady-state entrepreneur consumption C^E is replaced.

[55]Due to space limitations, the respective steps are not reported here as they strictly follow the procedure introduced by Benigno and Woodford (2005). Detailed derivations are available upon request.

[56]In the model without NBFIs, equation B.19 would be identical except for the last term related to non-bank finance not being in place.

Following the subsequent derivations analogously, the term

$$\begin{aligned}\widehat{W}_t^P &= \tfrac{1}{2}\psi_{(6)}^{Y^2}\widehat{Y}_t^2 + \tfrac{1}{2}\psi_{(3)}^{\pi^2}\widehat{\pi}_t^2 + \tfrac{1}{2}\psi_{(3)}^{r^2}\widehat{r}_t^2 + \tfrac{1}{2}\psi_{(3)}^{\nu^2}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi_{(4)}^{z,cb^2}\widehat{Z}_t^2 + +\tfrac{1}{2}\psi_{(2)}^{z,sb^2}(\widehat{Z}_t^{SB})^2 + \\ &+ \psi_{(5)}^{Y}\widehat{Y}_t + \psi_{(3)}^{\pi}\widehat{\pi}_t + \psi^{\nu}\widehat{\nu}_t + \psi_{(2)}^{z,cb}\widehat{Z}_t + +\psi^{z,sb}\widehat{Z}_t^{SB} + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.42}$$

with

$$\begin{aligned}
\psi_{(6)}^{Y^2} &\equiv \psi_{(5)}^{Y^2} + \left(\tfrac{K(\chi^C+\chi^S)}{\beta_E NW^E}\right)^2\psi_{(2)}^{ce^2} \\
\psi_{(5)}^{Y^2} &\equiv \psi_{(4)}^{Y^2} + 2\psi^{Yce}\tfrac{K(\chi^C+\chi^S)}{\beta_E NW^E} \\
\psi_{(2)}^{ce^2} &\equiv \psi^{ce^2} + \psi^{ce} \\
\psi_{(4)}^{z,cb^2} &\equiv \psi_{(3)}^{z,cb^2} + \left(\tfrac{K\chi^C}{\beta_E NW^E}\right)^2\psi_{(2)}^{ce^2} \\
\psi_{(3)}^{z,cb^2} &\equiv \psi_{(2)}^{z,cb^2} + \tfrac{K\chi^C}{\beta_E NW^E}\psi^{ce} + 2\tfrac{K\chi^C}{\beta_E NW^E}\psi^{zce} \\
\psi_{(2)}^{z,sb^2} &\equiv \psi^{z,sb^2} + \left(\tfrac{K\chi^S}{\beta_E NW^E}\right)^2\psi_{(2)}^{ce^2} \\
\psi^{z,sb^2} &\equiv \tfrac{K\chi^S}{\beta_E NW^E}\psi^{ce}
\end{aligned}$$

$$\begin{aligned}
\psi_{(5)}^{Y} &\equiv \psi_{(4)}^{Y} + \tfrac{K(\chi^C+\chi^S)}{\beta_E NW^E}\psi^{ce} \\
\psi_{(2)}^{z,cb} &\equiv \psi^{z,cb} + \tfrac{K\chi^C}{\beta_E NW^E}\psi^{ce} \\
\psi^{z,sb} &\equiv \tfrac{K\chi^S}{\beta_E NW^E}\psi^{ce} \\
\psi^{Yce} &\equiv (1-\sigma')\tfrac{CC^E}{C^P}\psi_{(3)}^{Y} \\
\psi^{zce} &\equiv (1-\sigma')\tfrac{CC^E}{C^P}\psi_{(4)}^{B}.
\end{aligned}$$

As above, the first-order approximation of the Taylor-type policy rule B.37 can be used to replace the inflation variance term $\widehat{\pi}_t^2$ to get

$$\begin{aligned}\widehat{W}_t^P &= \tfrac{1}{2}\psi_{(8)}^{Y^2}\widehat{Y}_t^2 + \tfrac{1}{2}\psi_{(4)}^{r^2}\widehat{r}_t^2 + \tfrac{1}{2}\psi_{(3)}^{\nu^2}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi_{(4)}^{z,cb^2}\widehat{Z}_t^2 + +\tfrac{1}{2}\psi_{(2)}^{z,sb^2}(\widehat{Z}_t^{SB})^2 + \\ &+ \psi_{(7)}^{Y}\widehat{Y}_t + \psi_{(4)}^{\pi}\widehat{\pi}_t + \psi^{\nu}\widehat{\nu}_t + \psi_{(2)}^{z,cb}\widehat{Z}_t + +\psi^{z,sb}\widehat{Z}_t^{SB} + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.43}$$

where the updated parameters on output and the interest rate are identical to the values derived for equation E.30.

E.2.2 Impatient Entrepreneur Welfare

The entrepreneur's period utility is again given by

$$U_t^E(C_t^E) = \frac{C_t^{E^{1-\sigma}}}{1-\sigma} \tag{E.44}$$

such that

$$\widehat{W}_t^E = \widehat{C}_t^E + (1-\sigma)\frac{1}{2}(\widehat{C}_t^E)^2 \tag{E.45}$$

follows. Combining equations B.16, B.18, and B.20 now yields:

$$\begin{aligned}\widehat{C}_t^E = &-\frac{K\chi^C}{\beta_E NW^E}(\widehat{Z}_t + \frac{1}{2}\widehat{Z}_t^2) - \frac{K(\chi^C+\chi^S)}{\beta_E NW^E}\widehat{Y}_t - \frac{K\chi^C}{\beta_E NW^E}\widehat{Y}_t\widehat{Z}_t - \\ &- \frac{K\chi^S}{\beta_E NW^E}(\widehat{Z}_t^{SB} + \frac{1}{2}(\widehat{Z}_t^{SB})^2) - \frac{K\chi^S}{\beta_E NW^E}\widehat{Y}_t\widehat{Z}_t^{SB} - \frac{1}{2}(\widehat{C}_t^E)^2.\end{aligned} \tag{E.46}$$

Plugging in $\widehat{W}_t^E$ now yields

$$\widehat{W}_t^E = -\frac{1}{2}\sigma\Big(\frac{K(\chi^C+\chi^S)}{\beta_E NW^E}\Big)^2\widehat{Y}_t^2 - \frac{1}{2}\Big[\frac{K\chi^C}{\beta_E NW^E} + \sigma\Big(\frac{K\chi^C}{\beta_E NW^E}\Big)^2\Big]\widehat{Z}_t^2 - $$
$$-\frac{1}{2}\Big[\frac{K\chi^S}{\beta_E NW^E} + \sigma\Big(\frac{K\chi^S}{\beta_E NW^E}\Big)^2\Big](\widehat{Z}_t^{SB})^2 - \frac{K(\chi^C+\chi^S)}{\beta_E NW^E}\widehat{Y}_t -$$
$$-\frac{K\chi^C}{\beta_E NW^E}\widehat{Z}_t - \frac{K\chi^S}{\beta_E NW^E}\widehat{Z}_t^{SB} - \Big[\frac{K\chi^C}{\beta_E NW^E} + \sigma\frac{K^2\chi^C(\chi^C+\chi^S)}{(\beta_E NW^E)^2}\Big]\widehat{Y}_t\widehat{Z}_t -$$
$$-\Big[\frac{K\chi^S}{\beta_E NW^E} + \sigma\frac{K^2\chi^S(\chi^C+\chi^S)}{(\beta_E NW^E)^2}\Big]\widehat{Y}_t\widehat{Z}_t^{SB} - \sigma\frac{K^2\chi^C\chi^S}{(\beta_E NW^E)^2}\widehat{Z}_t\widehat{Z}_t^{SB} \qquad \text{(E.47)}$$

as $\widehat{Z}_t^{SB}$ enters the derivations.

E.2.3 Joint Welfare

Again, following E.1, period joint welfare is given by

$$W_t = (1-\beta_P)W_t^P + (1-\beta_E)W_t^E \qquad \text{(E.48)}$$

with the same approximating as before where expressions $\widehat{W}_t^P$ and $\widehat{W}_t^E$ are again substituted to get

$$\widehat{W}_t = \tfrac{1}{2}\psi_{(9)}^{Y^2}\widehat{Y}_t^2 + \tfrac{1}{2}\psi_{(5)}^{r^2}\widehat{r}_t^2 + \tfrac{1}{2}\psi_{(4)}^{\nu^2}\widehat{\nu}_t^2 + \tfrac{1}{2}\psi_{(5)}^{z,cb^2}\widehat{Z}_t^2 + +\tfrac{1}{2}\psi_{(3)}^{z,sb^2}(\widehat{Z}_t^{SB})^2 +$$
$$+\psi_{(8)}^{Y}\widehat{Y}_t + \psi_{(5)}^{\pi}\widehat{\pi}_t + \psi_{(2)}^{\nu}\widehat{\nu}_t + \psi_{(3)}^{z,cb}\widehat{Z}_t + +\psi_{(2)}^{z,sb}\widehat{Z}_t^{SB} + \qquad \text{(E.49)}$$
$$+\, covars + t.i.p. + O^3$$

with

$\psi_{(9)}^{Y^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(8)}^{Y^2} - (1-\beta_E)\frac{W^E}{W}\sigma'\big(\frac{K(\chi^C+\chi^S)}{\beta_E NW^E}\big)^2$
$\psi_{(5)}^{r^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(4)}^{r^2}$
$\psi_{(4)}^{\nu^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(3)}^{\nu^2}$
$\psi_{(5)}^{z,cb^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(4)}^{z,cb^2} - (1-\beta_E)\frac{W^E}{W}\Big[\frac{K\chi^C}{\beta_E NW^E} + \sigma'\big(\frac{K\chi^C}{\beta_E NW^E}\big)^2\Big]$
$\psi_{(3)}^{z,sb^2} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(2)}^{z,sb^2} - (1-\beta_E)\frac{W^E}{W}\Big[\frac{K\chi^S}{\beta_E NW^E} + \sigma'\big(\frac{K\chi^S}{\beta_E NW^E}\big)^2\Big]$
$\psi_{(8)}^{Y} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(7)}^{Y} - (1-\beta_E)\frac{W^E}{W}\frac{K(\chi^C+\chi^S)}{\beta_E NW^E}$
$\psi_{(5)}^{\pi} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(4)}^{\pi}$
$\psi_{(2)}^{\nu} \equiv (1-\beta_P)\frac{W^P}{W}\psi^{\nu}$
$\psi_{(3)}^{z,cb} \equiv (1-\beta_P)\frac{W^P}{W}\psi_{(2)}^{z,cb} - (1-\beta_E)\frac{W^E}{W}\frac{K\chi^C}{\beta_E NW^E}$
$\psi_{(2)}^{z,sb} \equiv (1-\beta_P)\frac{W^P}{W}\psi^{z,sb} - (1-\beta_E)\frac{W^E}{W}\frac{K\chi^S}{\beta_E NW^E}$.

The linear term $\widehat{\nu}_t$ can be removed as stated above. As $\widehat{\nu}_t$ can be replaced with variables related to commercial bank credit only, the side steps outlined above are identical to the case without NBFIs and do not affect the parameters on NBFI credit-to-GDP. Thus, we get

$$\begin{aligned}\widehat{W}_t = &\tfrac{1}{2}\psi^{Y^2}_{(10)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(5)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\pi^2}_{(4)}\widehat{\pi}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(4)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(6)}\widehat{Z}^2_t + +\tfrac{1}{2}\psi^{z,sb^2}_{(3)}(\widehat{Z}^{SB}_t)^2 + \\ &+ \psi^{Y}_{(9)}\widehat{Y}_t + \psi^{\pi}_{(6)}\widehat{\pi}_t + \psi^{z,cb}_{(4)}\widehat{Z}_t + +\psi^{z,sb}_{(2)}\widehat{Z}^{SB}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.50}$$

with the same parameter values (except for terms including $\widehat{Z}^{SB}_t$) as derived for equation E.38. Using once again the approximation of the monetary policy rule to replace $\widehat{\pi}^2_t$ yields

$$\begin{aligned}\widehat{W}_t = &\tfrac{1}{2}\psi^{Y^2}_{(11)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(6)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(4)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(6)}\widehat{Z}^2_t + +\tfrac{1}{2}\psi^{z,sb^2}_{(3)}(\widehat{Z}^{SB}_t)^2 + \\ &+ \psi^{Y}_{(9)}\widehat{Y}_t + \psi^{\pi}_{(7)}\widehat{\pi}_t + \psi^{z,cb}_{(4)}\widehat{Z}_t + +\psi^{z,sb}_{(2)}\widehat{Z}^{SB}_t + \\ &+ covars + t.i.p. + O^3\end{aligned} \tag{E.51}$$

and updated parameter values are identical to the ones derived for equation E.39, as none of the added NBFI parameters is affected by the Taylor rule substitution. Following Benigno and Woodford (2005) again by using an iterated expression of the second-order approximation of the aggregate-supply relationship to replace the linear output term $\widehat{Y}_t$ in the lifetime welfare criterion

$$\begin{aligned}\widehat{W}_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}[&\tfrac{1}{2}\psi^{Y^2}_{(11)}\widehat{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(6)}\widehat{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(4)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(6)}\widehat{Z}^2_t + +\tfrac{1}{2}\psi^{z,sb^2}_{(3)}(\widehat{Z}^{SB}_t)^2 \\ &+ \psi^{Y}_{(9)}\widehat{Y}_t + \psi^{\pi}_{(7)}\widehat{\pi}_t + \psi^{z,cb}_{(4)}\widehat{Z}_t + \psi^{z,sb}_{(2)}\widehat{Z}^{SB}_t] + t.i.p. + O^3.\end{aligned} \tag{E.52}$$

Again, I replace the linear inflation term $\widehat{\pi}_t$ in the infinite sum by iterating forward the first-order approximation of the New-Keynesian Phillips curve and collect the covariances of $\widehat{Y}_t$, $\widehat{r}_t$, $\widehat{Z}_t$,and $\widehat{Z}^{SB}_t$ by defining efficiency gaps for these variables as in Benigno and Woodford (2005).[57] Discounted lifetime welfare with NBFIs is thus given by

$$\begin{aligned}\widehat{W}_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}[&\tfrac{1}{2}\psi^{Y^2}_{(12)}\widetilde{Y}^2_t + \tfrac{1}{2}\psi^{r^2}_{(7)}\widetilde{r}^2_t + \tfrac{1}{2}\psi^{\nu^2}_{(5)}\widehat{\nu}^2_t + \tfrac{1}{2}\psi^{z,cb^2}_{(7)}\widetilde{Z}^2_t + \tfrac{1}{2}\psi^{z,sb^2}_{(4)}(\widetilde{Z}^{SB}_t)^2 \\ &+ \psi^{z,cb}_{(5)}\widehat{Z}_t + \psi^{z,sb}_{(3)}\widehat{Z}^{SB}_t] + t.i.p. + O^3 + T_0\end{aligned} \tag{E.53}$$

where $\widetilde{Z}^{SB}_t = \widehat{Z}^{SB}_t - \widehat{Z}^{SB*}_t$ and coefficients can again directly be mapped in the parameters of the period loss function given by equation 40.

[57]Respective steps and the following updates of the parameters are again not reported here as they again strictly follow the procedure introduced by Benigno and Woodford (2005). Detailed derivations are again available upon request.

F Appendix: Conditional Welfare Costs in Consumption Equivalents

In this section, I derive the consumption equivalence expression of welfare applied in section 6. Assuming $\sigma \to 1$, lifetime welfare given by equation 33 is

$$\mathscr{W}^*_{t_0} = (1-\beta_P)E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}\left\{\ln[C_t^{P*}] - \frac{L_t^{P*1+\phi^P}}{1+\phi^P}\right\} + (1-\beta_E)E_{t_0}\sum_{t=t_0}^{\infty}\beta_E^{t-t_0}\ln[C_t^{E*}] \tag{F.1}$$

under the Ramsey policy in the decentralized economy of section D.2 absent nominal rigidities as well as real and financial frictions. Lifetime welfare under the economy featuring nominal rigidities as well as real and financial frictions and distortions are given by equation 33:

$$\mathscr{W}_{t_0} = (1-\beta_P)E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}\left\{\ln[C_t^{P}] - \frac{L_t^{P1+\phi^P}}{1+\phi^P}\right\} + (1-\beta_E)E_{t_0}\sum_{t=t_0}^{\infty}\beta_E^{t-t_0}\ln[C_t^{E}]. \tag{F.2}$$

Let ξ^P and ξ^E determine the welfare costs for patient households and impatient entrepreneurs, respectively. Thus, in the economy with frictions, the welfare costs in terms of consumption relative to the levels in the counterfactual frictionless economy can be expressed as

$$\mathscr{W}_{t_0} = (1-\beta_P)E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}\left\{\ln[(1-\xi^P)^{\frac{1}{1-\beta_P}}C_t^{P*}] - \frac{L_t^{P*1+\phi^P}}{1+\phi^P}\right\} + \\ + (1-\beta_E)E_{t_0}\sum_{t=t_0}^{\infty}\beta_E^{t-t_0}\ln[(1-\xi^E)^{\frac{1}{1-\beta_E}}C_t^{E*}] \tag{F.3}$$

where the welfare cost of each agent is assumed to be proportional to the welfare share in equation 33. Rewriting yields

$$\mathscr{W}_{t_0} = (1-\beta_P)E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}\ln[(1-\xi^P)^{\frac{1}{1-\beta_P}}] + (1-\beta_P)E_{t_0}\sum_{t=t_0}^{\infty}\beta_P^{t-t_0}\left\{\ln[C_t^{P*}] - \frac{L_t^{P*1+\phi^P}}{1+\phi^P}\right\} + \\ (1-\beta_E)E_{t_0}\sum_{t=t_0}^{\infty}\beta_E^{t-t_0}\ln[(1-\xi^E)^{\frac{1}{1-\beta_E}}] + (1-\beta_E)E_{t_0}\sum_{t=t_0}^{\infty}\beta_E^{t-t_0}\ln[C_t^{E*}] \tag{F.4}$$

which yields

$$\mathscr{W}_{t_0} = \frac{1}{1-\beta_P}\ln(1-\xi^P) + \frac{1}{1-\beta_E}\ln(1-\xi^E) + \mathscr{W}^*_{t_0}. \tag{F.5}$$

Rearranging yields

$$\frac{1}{1-\beta_P}\ln(1-\xi^P)+\frac{1}{1-\beta_E}\ln(1-\xi^E)=\mathscr{W}_{t_0}-\mathscr{W}^*_{t_0} \tag{F.6}$$

$$\ln(1-\xi^P)+\frac{1-\beta_P}{1-\beta_E}\ln(1-\xi^E)=(\mathscr{W}_{t_0}-\mathscr{W}^*_{t_0})(1-\beta_P) \tag{F.7}$$

$$(1-\xi^P)(1-\xi^E)^{\frac{1-\beta_P}{1-\beta_E}}=\exp[(\mathscr{W}_{t_0}-\mathscr{W}^*_{t_0})(1-\beta_P)] \tag{F.8}$$

$$1-\xi\equiv(1-\xi^P)^{1-\beta_E}(1-\xi^E)^{1-\beta_P}=\exp[(\mathscr{W}_{t_0}-\mathscr{W}^*_{t_0})(1-\beta_P)]^{1-\beta_E}. \tag{F.9}$$

G Appendix: Optimal Policy Rule with Non-Banks

Minimizing the loss function

$$\widehat{L}'_t=\tfrac{1}{2}\lambda^{y^2\prime}\widetilde{Y}_t^2+\tfrac{1}{2}\lambda^{r^2\prime}\widetilde{r}_t^2+\tfrac{1}{2}\lambda^{z,cb^2\prime}\widetilde{Z}_t^2+\tfrac{1}{2}\lambda^{z,sb^2\prime}(\widetilde{Z}_t^{SB})^2+\tfrac{1}{2}\lambda^{\nu^2\prime}\widehat{\nu}_t^2 \tag{F.1}$$

subject to the linearized structural equations given in appendix B yields the following set of first-order conditions

$$0 = \Xi_{1t} + \Xi_{17t} \tag{F.2}$$

$$0 = \Xi_{1t} + \Xi_{3t} \tag{F.3}$$

$$0 = \Xi_{3t} - \theta^{p}\Xi_{19t} \tag{F.4}$$

$$0 = \Xi_{4t} - \varphi_{1}\Xi_{24t} \tag{F.5}$$

$$0 = \Xi_{8t} - \Xi_{4t} \tag{F.6}$$

$$0 = \Xi_{5t} - \varphi_{2}\Xi_{10t} - \varphi_{3}\Xi_{9t} \tag{F.7}$$

$$0 = \Xi_{11t} + \varphi_{4}\Xi_{6t} \tag{F.8}$$

$$0 = \Xi_{9t} - \Xi_{25t} - \varphi_{5}\Xi_{13t} \tag{F.9}$$

$$0 = \Xi_{15t} - \varphi_{6}\Xi_{13t} - \varphi_{7}\Xi_{11t} \tag{F.10}$$

$$0 = \Xi_{25t} + \Xi_{12t} - \Xi_{49t} - \Xi_{15t} \tag{F.11}$$

$$0 = \Xi_{50t} + \Xi_{49t} - \phi^{y}\Xi_{28t} + \Xi_{22t} + \Xi_{18t} - \Xi_{1t} - \Xi_{30t} \tag{F.12}$$

$$0 = \Xi_{1t} - \phi^{P}\Xi_{17t} - \varphi_{8}\Xi_{18t} \tag{F.13}$$

$$0 = \Xi_{20t} - \alpha\beta_{P}\Xi_{18t+1} \tag{F.14}$$

$$0 = \Xi_{12t} - \varphi_{9}\Xi_{13t} \tag{F.15}$$

$$0 = \Xi_{15t} + \Xi_{14t} - \varphi_{10}\Xi_{14t+1} - \varphi_{11}\Xi_{22t+1} - \nu\Xi_{12t} \tag{F.16}$$

$$0 = \Xi_{13t} - \varphi_{12}\Xi_{14t+1} \tag{F.17}$$

$$0 = \varphi_{13}(\Xi_{41t} - \Xi_{42t}) - \varphi_{14}\Xi_{36t} - \Xi_{32t} + \Xi_{28t} + \Xi_{16t} - \varphi_{15}\Xi_{13t} - \Xi_{11t} + \varphi_{16}\Xi_{44t+1} \tag{F.18}$$

$$0 = \Xi_{24t} - \varphi_{17}\Xi_{22t} \tag{F.19}$$

$$0 = \Xi_{19t} - \phi^{\pi}\Xi_{28t} - \varphi_{18}(\Xi_{16t-1} + \beta_{P}\Xi_{19t-1}) + \frac{\varphi_{11}}{\beta_{P}}\Xi_{22t} \tag{F.20}$$

$$0 = \Xi_{16t} - \varphi_{19}\Xi_{24t} - \Xi_{17t} - \varphi_{18}\Xi_{16t-1} \tag{F.21}$$

$$0 = 2\lambda^{\nu^{2}\prime}\widehat{\nu}_{t} - \Xi_{15t} \tag{F.22}$$

$$0 = \Xi_{40t} + \Xi_{10t} - \Xi_{26t} \tag{F.23}$$

$$0 = \varphi_{20}\Xi_{44t} - \varphi_{21}\Xi_{7t} + \varphi_{22}\Xi_{41t} + \varphi_{23}\Xi_{38t+1} \tag{F.24}$$

$$0 = \varphi_{24}\Xi_{36t} - \varphi_{25}\Xi_{38t+1} \tag{F.25}$$

$$0 = \Xi_{50t} - \Xi_{26t} + \Xi_{39t} + \varphi_{26}\Xi_{38t+1} \tag{F.26}$$

$$0 = \varphi_{27}\Xi_{38t+1} - \Xi_{39t} - \Xi_{38t} \tag{F.27}$$

$$0 = \varphi_{2}\Xi_{10t} - \Xi_{7t} - \varphi_{28}\Xi_{8t} \tag{F.28}$$

$$0 = \Xi_{6t} + \varphi_{28}\Xi_{8t} - \varphi_{3}\Xi_{9t} \tag{F.29}$$

$$0 = \Xi_{50t} - \Xi_{35t} \tag{F.30}$$

$$0 = \Xi_{49t} - \Xi_{34t} \tag{F.31}$$

$$0 = \eta^{S}\,\Xi_{42t} - \Xi_{40t} - \varphi_{29}\Xi_{42t-1} \tag{F.32}$$

$$0 = \nu^{S}\,\Xi_{41t} + \varphi_{30}\Xi_{40t} - \varphi_{31}\Xi_{41t-1} \tag{F.33}$$

$$0 = \Xi_{39t} - \Xi_{43t} + \beta_{P}\,\Xi_{43t+1} - \varphi_{32}\Xi_{44t+1} \tag{F.34}$$

$$0 = \Xi_{43t} - \varphi_{37}\Xi_{41t-1} \tag{F.35}$$

$$0 = \Psi^{S}\,\Xi_{44t} - \Xi_{43t} - \beta_{P}{}^{(-1)}\eta^{S}\,\theta^{S}\,\beta_{S}\,\Psi^{S}\Xi_{42t-1} \tag{F.36}$$

$$0 = 2\lambda^{y^{2\prime}}\widetilde{Y}_{t} + \Xi_{30t} \tag{F.37}$$

$$0 = \Xi_{30t} + \Xi_{29t} \tag{F.38}$$

$$0 = \Xi_{32t} + \Xi_{31t} \tag{F.39}$$

$$0 = 2\lambda^{r^{2\prime}}\widetilde{r}_{t} + \Xi_{32t} \tag{F.40}$$

$$0 = \Xi_{34t} + \Xi_{33t} \tag{F.41}$$

$$0 = 2\lambda^{z,cb^{2\prime}}\widetilde{Z}_{t} + \Xi_{34t} \tag{F.42}$$

$$0 = 2\lambda^{z,sb^{2\prime}}\widetilde{Z}_{t}^{SB} + \Xi_{35t} \tag{F.43}$$

where the Lagrange multipliers are given by $\Xi_{m,t+n}, m \in \{1,\ldots,50\}; n \in \{-1,0,1\}$. The vector of initial conditions is given by

$$\Upsilon = \begin{bmatrix} \Xi_{16-1} \\ \Xi_{19-1} \\ \Xi_{41-1} \\ \Xi_{42-1} \end{bmatrix} \tag{F.44}$$

and the auxiliary parameters are composites of deep parameters and steady-state relations:

$\varphi_1 = \frac{C^E}{C}$
$\varphi_2 = \frac{\chi^S K}{B^{E,S}}$
$\varphi_3 = \frac{\chi^C K}{B^{E,C}}$
$\varphi_4 = \frac{1}{1+r^{bc}}$
$\varphi_5 = \frac{R+\Delta^C}{\Delta^C+R\nu}$
$\varphi_6 = \frac{\theta\,\nu^4}{\Delta^C+\nu R}$
$\varphi_7 = \theta\,\nu^3$
$\varphi_8 = 1-\alpha$
$\varphi_9 = \frac{R}{\Delta^C+\nu R}$
$\varphi_{10} = \beta_P(1-\delta^b)$
$\varphi_{11} = \beta_P \frac{\delta^b\,Kb}{Y}$
$\varphi_{12} = \beta_P\delta^b$
$\varphi_{13} = R(1-\theta^S)\beta_S$
$\varphi_{14} = \frac{1}{1+R}$
$\varphi_{15} = \frac{\nu}{\Delta^C+\nu R}$
$\varphi_{16} = \beta_P\, R(\phi^S-1)$
$\varphi_{17} = \frac{C}{Y}$
$\varphi_{18} = {\beta_P}^{(-1)}$
$\varphi_{19} = \frac{C^P}{C}$
$\varphi_{20} = r^{bS}\,\phi^S$
$\varphi_{21} = \frac{1}{1+r^{bS}}$
$\varphi_{22} = r^{bS}\,\left(1-\theta^S\right)\,\beta_S$
$\varphi_{23} = \beta_P \frac{q\,B^{E,S}}{K^S}\,\sigma^S$
$\varphi_{24} = \frac{1}{1+r^{dS}}$
$\varphi_{25} = \beta_P \sigma^S\,\left(1-\frac{q\,B^{E,S}}{K^S}\right)$
$\varphi_{26} = \beta_P \frac{q\,B^{E,S}}{K^S}\,\left(\sigma^S\Delta_t^S+\omega^S\right)$
$\varphi_{27} = \beta_P\left(1+r^{dS}\right)\,\sigma^S$
$\varphi_{28} = \frac{\frac{K}{\beta_E}\,\chi^S}{NW}$
$\varphi_{29} = {\beta_P}^{(-1)}\eta^S\,\theta^S\,\beta_S\,\Psi^S$
$\varphi_{30} = \frac{\nu^S}{\theta^S-\nu^S}$
$\varphi_{31} = {\beta_P}^{(-1)}\nu^S\,\theta^S\,\beta_S\,\Xi^S$
$\varphi_{32} = \beta_P\phi^S\,\left(r^{bS}-R\right)$
$\varphi_{33} = (1+\phi^P)\theta^p$
$\varphi_{34} = \alpha\beta_P$
$\varphi_{35} = \frac{\Psi^S}{\varphi_{32}}$
$\varphi_{36} = \frac{\varphi_{29}}{\eta^S}$
$\varphi_{37} = {\beta_P}^{(-1)}\nu^S\,\theta^S\,\beta_S\,\Xi^S.$

Treating initial conditions Υ as parameters, the system given by equations F.2 to F.43 can be simplified such that

$$0 = \varphi_{38}\Xi_{15t} - \varphi_{39} - \varphi_{40}\Xi_{22t} - \varphi_{41}\Xi_{38t} - \varphi_{42}\Xi_{38t+1} + \varphi_{43}\widetilde{Y}_t - \varphi_{44}\widetilde{Z}_t - \varphi_{45}\widetilde{Z}_t^{SB} - 2\varphi_7\lambda^{r^2}\widetilde{r}_t \tag{F.45}$$

$$0 = \Xi_{14t} + \varphi_{46}\Xi_{15t} - \varphi_{10}\Xi_{14t+1} - \varphi_{11}\Xi_{22t+1} - \varphi_{47} - \varphi_{48}\Xi_{22t} - \varphi_{49}\Xi_{38t} - \varphi_{50}\Xi_{38t+1} - \varphi_{51}\widetilde{Z}_t - \varphi_{52}\widetilde{Z}_t^{SB} \tag{F.46}$$

$$0 = \varphi_{53} + \varphi_{54}\Xi_{15t} + \varphi_{55}\Xi_{22t} + \varphi_{56}\Xi_{38t} + \varphi_{57}\Xi_{38t+1} + \varphi_{58}\widetilde{Z}_t + \varphi_{59}\widetilde{Z}_t^{SB} - \varphi_{12}\Xi_{14t+1} \tag{F.47}$$

$$0 = \varphi_{60}\Xi_{22t} + \varphi_{61}\Xi_{38t} + \varphi_{62}\Xi_{38t+1} + \varphi_{63}\widetilde{Z}_t^{SB} - \varphi_{64} \tag{F.48}$$

$$0 = 2\lambda^{\nu^{2\prime}}\widehat{\nu}_t - \Xi_{15t} \tag{F.49}$$

where φ_{38} to φ_{64} depict auxiliary parameters defined for simplification.[58] Treating the period-t values of Lagrange multipliers Ξ_{14t}, Ξ_{15t}, Ξ_{22t}, and Ξ_{38t} as endogenous variables, one can solve the system defined by equations F.45 to F.48. Combining the solution for Ξ_{15t} with equation F.49, one can derive

$$2\lambda^{\nu^{2\prime}}\widehat{\nu}_t = \varphi_{65} + \varphi_{66}\Xi_{14t+1} + \varphi_{67}\Xi_{38t+1} + \varphi_{68}\widetilde{r}_t + \varphi_{69}\widetilde{Y}_t + \varphi_{70}\widetilde{Z}_t + \varphi_{71}\widetilde{Z}_t^{SB} \tag{F.50}$$

with φ_{65} to φ_{71} again depicting auxiliary parameters. In addition to the capital requirement $\widehat{\nu}_t$ and potential target variables $\widetilde{r}_t$, $\widetilde{Y}_t$, $\widetilde{Z}_t$, and $\widetilde{Z}_t^{SB}$, equation F.50

[58] Due to space limitations, auxiliary parameters are not reported in the following but available upon request.

contains expected values of Lagrange multipliers, $E_t[\Xi_{14t+1}, \Xi_{38t+1}]$. To derive a direct rule in the definition of Giannoni and Woodford (2003a,b), I express these multipliers in terms of policy and target variables only. By extending the system of equations, one can iteratively include the expected values of the Lagrange multipliers as endogenous variables and find explicit solutions. Starting by lagging equation F.50 by one period, I extend the system of equations F.45 to F.49 to get

$$0 = \varphi_{38}\Xi_{15t} - \varphi_{39} - \varphi_{40}\Xi_{22t} - \varphi_{41}\Xi_{38t} - \varphi_{42}\Xi_{38t+1} + \varphi_{43}\widetilde{Y}_t - \varphi_{44}\widetilde{Z}_t - \varphi_{45}\widetilde{Z}_t^{SB} - 2\varphi_7\lambda^{r^2}\widetilde{r}_t \tag{F.51}$$

$$0 = \Xi_{14t} + \varphi_{46}\Xi_{15t} - \varphi_{10}\Xi_{14t+1} - \varphi_{11}\Xi_{22t+1} - \varphi_{47} - \varphi_{48}\Xi_{22t} - \varphi_{49}\Xi_{38t} - \varphi_{50}\Xi_{38t+1} - \varphi_{51}\widetilde{Z}_t - \varphi_{52}\widetilde{Z}_t^{SB} \tag{F.52}$$

$$0 = \varphi_{53} + \varphi_{54}\Xi_{15t} + \varphi_{55}\Xi_{22t} + \varphi_{56}\Xi_{38t} + \varphi_{57}\Xi_{38t+1} + \varphi_{58}\widetilde{Z}_t + \varphi_{59}\widetilde{Z}_t^{SB} - \varphi_{12}\Xi_{14t+1} \tag{F.53}$$

$$0 = \varphi_{60}\Xi_{22t} + \varphi_{61}\Xi_{38t} + \varphi_{62}\Xi_{38t+1} + \varphi_{63}\widetilde{Z}_t^{SB} - \varphi_{64} \tag{F.54}$$

$$0 = 2\lambda^{\nu^2\prime}\widehat{\nu}_{t-1} - \varphi_{65} - \varphi_{66}\Xi_{14t} - \varphi_{67}\Xi_{38t} - \varphi_{68}\widetilde{r}_{t-1} - \varphi_{69}\widetilde{Y}_{t-1} - \varphi_{70}\widetilde{Z}_{t-1} - \varphi_{71}\widetilde{Z}_{t-1}^{SB} \tag{F.55}$$

$$0 = 2\lambda^{\nu^2\prime}\widehat{\nu}_t - \Xi_{15t}. \tag{F.56}$$

Solving the system of equations F.51 to F.55, I derive a solution for Lagrange multipliers Ξ_{14t}, Ξ_{15t}, Ξ_{22t}, and Ξ_{38t} as well as for $E_t[\Xi_{38t+1}]$:

$$E_t[\Xi_{38t+1}] = \varphi_{72}E_t[\Xi_{14t+1}] + \varphi_{73}E_t[\Xi_{22t+1}] + \varphi_{74}\widetilde{r}_t + \varphi_{75}\widetilde{r}_{t-1} + \varphi_{76}\widetilde{Y}_t + \varphi_{77}\widetilde{Y}_{t-1} + \varphi_{78}\widetilde{Z}_t + \varphi_{79}\widetilde{Z}_{t-1} + \varphi_{80}\widetilde{Z}_t^{SB} + \varphi_{81}\widetilde{Z}_{t-1}^{SB} + \varphi_{82} + \varphi_{83}\widehat{\nu}_{t-1} \tag{F.57}$$

with auxiliary parameters φ_{72} to φ_{83}. The solution not only depends on contemporaneous and lagged values of the policy tool $\widehat{\nu}_t$ and the potential target variables, but also on $E_t[\Xi_{14t+1}]$ and $E_t[\Xi_{22t+1}]$. I extend the system and find explicit solutions for the latter terms. Adding the lag of equation F.57 to system F.51 to F.56 yields:

$$0 = \varphi_{38}\Xi_{15t} - \varphi_{39} - \varphi_{40}\Xi_{22t} - \varphi_{41}\Xi_{38t} - \varphi_{42}\Xi_{38t+1} + \varphi_{43}\widetilde{Y}_t - \varphi_{44}\widetilde{Z}_t - \varphi_{45}\widetilde{Z}_t^{SB} - 2\varphi_7\lambda^{r^2}\widetilde{r}_t \tag{F.58}$$

$$0 = \Xi_{14t} + \varphi_{46}\Xi_{15t} - \varphi_{10}\Xi_{14t+1} - \varphi_{11}\Xi_{22t+1} - \varphi_{47} - \varphi_{48}\Xi_{22t} - \varphi_{49}\Xi_{38t} - \varphi_{50}\Xi_{38t+1} - \varphi_{51}\widetilde{Z}_t - \varphi_{52}\widetilde{Z}_t^{SB} \tag{F.59}$$

$$0 = \varphi_{53} + \varphi_{54}\Xi_{15t} + \varphi_{55}\Xi_{22t} + \varphi_{56}\Xi_{38t} + \varphi_{57}\Xi_{38t+1} + \varphi_{58}\widetilde{Z}_t + \varphi_{59}\widetilde{Z}_t^{SB} - \varphi_{12}\Xi_{14t+1} \tag{F.60}$$

$$0 = \varphi_{60}\Xi_{22t} + \varphi_{61}\Xi_{38t} + \varphi_{62}\Xi_{38t+1} + \varphi_{63}\widetilde{Z}_t^{SB} - \varphi_{64} \tag{F.61}$$

$$0 = 2\lambda^{\nu^{2\prime}}\widehat{\nu}_{t-1} - \varphi_{65} - \varphi_{66}\Xi_{14t} - \varphi_{67}\Xi_{38t} - \varphi_{68}\widetilde{r}_{t-1} - \varphi_{69}\widetilde{Y}_{t-1} - \varphi_{70}\widetilde{Z}_{t-1} - \varphi_{71}\widetilde{Z}_{t-1}^{SB} \tag{F.62}$$

$$0 = \Xi_{38t} - \varphi_{72}\Xi_{14t} - \varphi_{73}\Xi_{22t} - \varphi_{74}\widetilde{r}_{t-1} - \varphi_{75}\widetilde{r}_{t-2} - \varphi_{76}\widetilde{Y}_{t-1} - \varphi_{77}\widetilde{Y}_{t-2} - \varphi_{78}\widetilde{Z}_{t-1} - \varphi_{79}\widetilde{Z}_{t-2} - \varphi_{80}\widetilde{Z}_{t-1}^{SB} - \varphi_{81}\widetilde{Z}_{t-2}^{SB} - \varphi_{82} - \varphi_{83}\widehat{\nu}_{t-2} \tag{F.63}$$

$$0 = 2\lambda^{\nu^{2\prime}}\widehat{\nu}_t - \Xi_{15t}. \tag{F.64}$$

Solving the system given by equations F.58 to F.63 again for Lagrange multipliers Ξ_{14t}, Ξ_{15t}, Ξ_{22t}, and Ξ_{38t}, $E_t[\Xi_{38t+1}]$ and additionally for $E_t[\Xi_{22t+1}]$, one can derive a solution for Ξ_{15t} which only depends on contemporaneous and lagged values of the policy tool and potential target variables, but still includes $E_t[\Xi_{14t+1}]$:

$$\begin{aligned}\Xi_{15t} = {} & \varphi_{84} + \varphi_{85}E_t[\Xi_{14t+1}] + \\ & + \varphi_{86}\widetilde{r}_t + \varphi_{87}\widetilde{r}_{t-1} + \varphi_{88}\widetilde{r}_{t-2} + \\ & + \varphi_{89}\widetilde{Y}_t + \varphi_{90}\widetilde{Y}_{t-1} + \varphi_{91}\widetilde{Y}_{t-2} + \\ & + \varphi_{92}\widetilde{Z}_t + \varphi_{93}\widetilde{Z}_{t-1} + \varphi_{94}\widetilde{Z}_{t-2} + \\ & + \varphi_{95}\widetilde{Z}_t^{SB} + \varphi_{96}\widetilde{Z}_{t-1}^{SB} + \varphi_{97}\widetilde{Z}_{t-2}^{SB} + \\ & + \varphi_{98}\widehat{\nu}_{t-1} + \varphi_{99}\widehat{\nu}_{t-2}\end{aligned} \tag{F.65}$$

with auxiliary parameters φ_{84} to φ_{99}. Finally, lagging equation F.65 by one period and adding to system F.58 to F.64, one can derive the system

$$0 = \varphi_{38}\Xi_{15t} - \varphi_{39} - \varphi_{40}\Xi_{22t} - \varphi_{41}\Xi_{38t} - \varphi_{42}\Xi_{38t+1} + \varphi_{43}\widetilde{Y}_t - \varphi_{44}\widetilde{Z}_t - \varphi_{45}\widetilde{Z}_t^{SB} - 2\varphi_7\lambda^{r^2}\widetilde{r}_t \tag{F.66}$$

$$0 = \Xi_{14t} + \varphi_{46}\Xi_{15t} - \varphi_{10}\Xi_{14t+1} - \varphi_{11}\Xi_{22t+1} - \varphi_{47} - \varphi_{48}\Xi_{22t} - \varphi_{49}\Xi_{38t} - \varphi_{50}\Xi_{38t+1} - \varphi_{51}\widetilde{Z}_t - \varphi_{52}\widetilde{Z}_t^{SB} \tag{F.67}$$

$$0 = \varphi_{53} + \varphi_{54}\Xi_{15t} + \varphi_{55}\Xi_{22t} + \varphi_{56}\Xi_{38t} + \varphi_{57}\Xi_{38t+1} + \varphi_{58}\widetilde{Z}_t + \varphi_{59}\widetilde{Z}_t^{SB} - \varphi_{12}\Xi_{14t+1} \tag{F.68}$$

$$0 = \varphi_{60}\Xi_{22t} + \varphi_{61}\Xi_{38t} + \varphi_{62}\Xi_{38t+1} + \varphi_{63}\widetilde{Z}_t^{SB} - \varphi_{64} \tag{F.69}$$

$$0 = 2\lambda^{\nu^{2\prime}}\widehat{\nu}_{t-1} - \varphi_{65} - \varphi_{66}\Xi_{14t} - \varphi_{67}\Xi_{38t} - \varphi_{68}\widetilde{r}_{t-1} - \varphi_{69}\widetilde{Y}_{t-1} - \varphi_{70}\widetilde{Z}_{t-1} - \varphi_{71}\widetilde{Z}_{t-1}^{SB} \tag{F.70}$$

$$0 = \Xi_{38t} - \varphi_{72}\Xi_{14t} - \varphi_{73}\Xi_{22t} - \varphi_{74}\widetilde{r}_{t-1} - \varphi_{75}\widetilde{r}_{t-2} - \varphi_{76}\widetilde{Y}_{t-1} - \varphi_{77}\widetilde{Y}_{t-2} - \varphi_{78}\widetilde{Z}_{t-1} - \varphi_{79}\widetilde{Z}_{t-2} - \varphi_{80}\widetilde{Z}_{t-1}^{SB} - \varphi_{81}\widetilde{Z}_{t-2}^{SB} - \varphi_{82} - \varphi_{83}\widehat{\nu}_{t-2} \tag{F.71}$$

$$0 = \Xi_{15t-1} - \varphi_{84} - \varphi_{85}\Xi_{14t} - \varphi_{98}\widehat{\nu}_{t-2} - \varphi_{99}\widehat{\nu}_{t-3} - \varphi_{86}\widetilde{r}_{t-1} - \varphi_{87}\widetilde{r}_{t-2} - \varphi_{88}\widetilde{r}_{t-3} - \varphi_{89}\widetilde{Y}_{t-1} - \varphi_{90}\widetilde{Y}_{t-2} - \varphi_{91}\widetilde{Y}_{t-3} - \varphi_{92}\widetilde{Z}_{t-1} - \varphi_{93}\widetilde{Z}_{t-2} - \varphi_{94}\widetilde{Z}_{t-3} - \varphi_{95}\widetilde{Z}_{t-1}^{SB} - \varphi_{96}\widetilde{Z}_{t-2}^{SB} - \varphi_{97}\widetilde{Z}_{t-3}^{SB} \tag{F.72}$$

$$0 = 2\lambda^{\nu^{2\prime}}\widehat{\nu}_t - \Xi_{15t}. \tag{F.73}$$

Solving equations F.66 to F.72 for Lagrange multipliers Ξ_{14t}, Ξ_{15t}, Ξ_{22t}, Ξ_{38t}, $E_t[\Xi_{38t+1}]$, $E_t[\Xi_{22t+1}]$, and $E_t[\Xi_{14t+1}]$, the solution for Ξ_{15t} is now given by

$$\begin{aligned} \Xi_{15t} = {} & \varphi_{100} + \varphi_{101}\widehat{\nu}_{t-1} + \varphi_{102}\widehat{\nu}_{t-2} + \varphi_{103}\widehat{\nu}_{t-3} + \\ & + \varphi_{104}\widetilde{r}_t + \varphi_{105}\widetilde{r}_{t-1} + \varphi_{106}\widetilde{r}_{t-2} + \varphi_{107}\widetilde{r}_{t-3} + \\ & + \varphi_{108}\widetilde{Y}_t + \varphi_{109}\widetilde{Y}_{t-1} + \varphi_{110}\widetilde{Y}_{t-2} + \varphi_{111}\widetilde{Y}_{t-3} + \\ & + \varphi_{112}\widetilde{Z}_t + \varphi_{113}\widetilde{Z}_{t-1} + \varphi_{114}\widetilde{Z}_{t-2} + \varphi_{115}\widetilde{Z}_{t-3} + \\ & + \varphi_{116}\widetilde{Z}_t^{SB} + \varphi_{117}\widetilde{Z}_{t-1}^{SB} + \varphi_{118}\widetilde{Z}_{t-2}^{SB} + \varphi_{119}\widetilde{Z}_{t-3}^{SB} \end{aligned} \tag{F.74}$$

with auxiliary parameters φ_{100} to φ_{119}. Combining equations F.49 and F.74, yields a solution for $\widehat{\nu}_t$ which only depends on lagged values of the policy tools and target variables which depicts the capital requirement rule 49 stated in section 7.1:

$$\begin{aligned} \widehat{\nu}_t = {} & \rho^\nu + \rho_1^\nu\widehat{\nu}_{t-1} + \rho_2^\nu\widehat{\nu}_{t-2} + \rho_3^\nu\widehat{\nu}_{t-3} + \\ & + \phi_1^r\widetilde{r}_t + \phi_2^r\widetilde{r}_{t-1} + \phi_3^r\widetilde{r}_{t-2} + \phi_4^r\widetilde{r}_{t-3} + \\ & + \phi_1^y\widetilde{Y}_t + \phi_2^y\widetilde{Y}_{t-1} + \phi_3^y\widetilde{Y}_{t-2} + \phi_4^y\widetilde{Y}_{t-3} + \\ & + \phi_1^{z,cb}\widetilde{Z}_t + \phi_2^{z,cb}\widetilde{Z}_{t-1} + \phi_3^{z,cb}\widetilde{Z}_{t-2} + \phi_4^{z,cb}\widetilde{Z}_{t-3} + \\ & + \phi_1^{z,sb}\widetilde{Z}_t^{SB} + \phi_2^{z,sb}\widetilde{Z}_{t-1}^{SB} + \phi_3^{z,sb}\widetilde{Z}_{t-2}^{SB} + \phi_4^{z,sb}\widetilde{Z}_{t-3}^{SB} \end{aligned} \tag{F.75}$$