Money and Capital
in a persistent Liquidity Trap

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Disclaimer: the views expressed in this presentation are those of the speaker and do not necessarily reflect the views of the Banque de France.
Persistent liquidity traps
Increased real cash holdings in persistent liquidity traps

US

overnight interest rate

Japan

M1/CPI
Investment slowdown in persistent liquidity traps

**US**

**Japan**

**Investment (% of GDP)**
Investment slowdown in persistent liquidity traps

negative impact on potential output
Can increased real money holdings crowd out physical capital?
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Standard models

- no
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- money holdings have no real effects
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- usually kept out of the model
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**This paper**
- outside ZLB: no
Can increased real money holdings crowd out physical capital?

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A model of scarce assets with money

- Credit-constrained investors hold assets to finance investment.
- Deleveraging: borrowing constraint reduces supply of assets.
- Without money, arbitrarily low equilibrium interest rate (shadow rate).
- Sets Zero Lower Bound (ZLB) and creates gap between effective and shadow rate.
- Outside ZLB: Money only provides transaction services.
- At ZLB: Money used as saving instrument.
- First study flexible price steady states (after prices have adjusted).
- Supply-side view (≠ usual demand-side analyses).
- Also look at transition dynamics with short-run nominal rigidities.
A model of scarce assets with money

- model of scarce assets

  - credit-constrained investors hold assets to finance investment
  - deleveraging: borrowing constraint reduces supply of assets
  - w/o money: arbitrarily low equilibrium interest rate
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Model of scarce assets

- Credit-constrained investors hold assets to finance investment
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Introduce money explicitly

- Sets ZLB and creates gap between effective and shadow rate
- Outside ZLB: only provides transaction services
- At ZLB: used as saving instrument
## A model of scarce assets with money

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Main results

Consider a deleveraging shock that reduces net supply of assets
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Consider a deleveraging shock that reduces net supply of assets outside ZLB:

- interest rate decline: stimulates the supply of assets
- deleveraging shock need not affect capital and output

...other implications...

- interest rate gap widens & investors increase money holdings
- medium-term decline of capital and output
- why? low return of money & real balance effect
- exit the trap: decrease effective rate or increase shadow rate
- higher Gov't debt helps exiting ZLB but can lead to lower output
- QE widens interest rate gap and can extend the liquidity trap
Main results

Consider a deleveraging shock that reduces net supply of assets outside ZLB:
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at ZLB:
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outside ZLB

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policy implications

- exit the trap: decrease effective rate or increase shadow rate
- higher Gov't debt helps exiting ZLB but can lead to lower output
- QE widens interest rate gap and can extend the liquidity trap
Relation to the literature

Persistent liquidity traps in standard NK models: insufficient demand

- persistently negative output gap ⇔ persistent nominal rigidities

Supply-side effects at the ZLB


Money and liquidity

- fiat money as a saving instrument: OLG model of Samuelson 1958, turnpike model of Townends 1980

Real balance effect

- the Pigou effect: Pigou 1943, Patinkin 1956
Outline

1. A model with scarce assets and money
2. The effect of deleveraging
3. Policies in a liquidity trap
4. Extensions
1. A model with scarce assets and money
Main assumptions

One-good economy with nominal bonds and money
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Two types of agents: investors and workers
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One-good economy with nominal bonds and money

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Investors have a demand for assets
  - they save, waiting for investment opportunities
    (as in Woodford, 1990)
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Workers need money for transactions
Main assumptions

One-good economy with nominal bonds and money

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  - they save, waiting for investment opportunities (as in Woodford, 1990)
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  - Bonds dominate money as a saving vehicle, except at ZLB

Workers need money for transactions

Baseline model: perfect foresight & flexible prices
Investors

Maximize \( U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s}) \)
Maximize $U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$

Alternate between investing and saving phase
Maximize $U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$

Alternate between investing and saving phase

Investing phase in $t$: $c_t^I + k_{t+1} = a_t + \frac{M_t^S}{P_t} + \frac{b_{t+1}}{r_{t+1}}$
Maximize $U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$

Alternate between investing and saving phase

Investing phase in $t$: $c_t^I + k_{t+1} = a_t + \frac{M_t^S}{P_t} + \frac{b_{t+1}}{r_{t+1}}$

gross real interest rate $= \frac{i_{t+1}P_t}{P_{t+1}}$
Maximize $U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$

Alternate between investing and saving phase

Investing phase in $t$: $c_t^I + k_{t+1} = a_t + \frac{M_t^S}{P_t} + \frac{b_{t+1}}{r_{t+1}}$

Saving phase in $t$: $c_t^S + \frac{a_{t+1}}{r_{t+1}} + \frac{M_t^{S^1}}{P_t} = \rho_t k_t - b_t$

Gross real interest rate:

$\frac{i_{t+1} P_t}{P_{t+1}}$
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Borrowing constraint (relevant for investing phase)

$$b_{t+1} \leq \phi_t \rho_{t+1} k_{t+1}$$

gross real interest rate

$$= \frac{i_{t+1} P_t}{P_{t+1}}$$
Investors

Maximize  $U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$

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Borrowing constraint (relevant for investing phase)

$\phi_t \rho_{t+1} k_{t+1}$

gross real interest rate  
$= \frac{i_{t+1} P_t}{P_{t+1}}$
deleveraging shock:  $\phi \downarrow$
Maximize $U_t = \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$

Alternate between investing and saving phase

Investing phase in $t$: $c_t^I + k_{t+1} = a_t + \frac{M_t^S}{P_t} + \frac{b_{t+1}}{r_{t+1}}$

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Borrowing constraint (relevant for investing phase)

$$b_{t+1} \leq \phi_t \rho_{t+1} k_{t+1}$$

Capital rented to firms with production function $y_t = k_t^\alpha h_t^{1-\alpha}$

$\rho_t k_t = \alpha y_t$ (full depreciation in benchmark model)
Supply of assets by other agents

**Workers**

Cash-in-advance constraint:

\[ M_{t+1}^w = \text{wage bill} = (1 - \alpha)P_t y_t \]

Exogenous real debt limit \( l^w \)
## Supply of assets by other agents

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<td><strong>Budget constraint:</strong></td>
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<td>( M^w_{t+1} = \text{wage bill} = (1 - \alpha)P_t y_t )</td>
<td>( \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} + \frac{l^g_{t+1}}{r_{t+1}} = \frac{T^w}{P_t} + l^g_t )</td>
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<th>Exogenous real debt limit ( l^w )</th>
<th>Fiscal policy sets real debt ( l^g )</th>
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| Monetary policy: |  |
| \( M_{t+1}/M_t = \theta \geq 1 \) | (pins down long-term inflation) |
Solving the model

Analytical results
in benchmark model

- perfect foresight
- permanent shocks on $\phi$
  (proxy very persistent shocks)
- steady state equilibria
  (give asymptotic response)
- flexible prices and wages
  (+ full K depreciation)

Simulate transition dynamics in extended model

uncertainty
leverage
$\phi \in (\phi_H, \phi_L)$
deleveraging: $\phi$ drops from $\phi_H$ to $\phi_L$ with prob $\pi$ of switching back to $\phi_H$
downward wage rigidity
$W_t = \max\{\gamma W_{t-1}, W^*_t\}$
$W^*$ is market-clearing wage
(+ partial K depreciation)
Solving the model

Analytical results in benchmark model

- perfect foresight
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Equilibrium
Shortage of assets

Equilibrium on the bond market

\[ b_{t+1} + l^w_{t+1} + l^g_{t+1} = a_{t+1} \]
Shortage of assets

Equilibrium on the bond market

\[ b_{t+1} + l_{t+1}^w + l_{t+1}^g = a_{t+1} \]

\[ \equiv l_{t+1} \]

net supply of bonds to investors

Asset-scarce equilibrium if \( \phi \) and \( l \) low, borrowing constraints are binding, \( r < 1/\beta \) in the steady state

Assume "autarkic" investors, case \( l = 0 \) is actually realistic, implies \( b = a \)
Shortage of assets

Equilibrium on the bond market

\[ b_{t+1} + l^w_{t+1} + l^g_{t+1} = a_{t+1} \]

\[ \leq \phi_t \alpha y_{t+1} \]

\[ \equiv l_{t+1} \]

net supply of bonds to investors
Shortage of assets

Equilibrium on the bond market

\[ b_{t+1} + lw_{t+1} + lg_{t+1} = at+1 \]

\[ \leq \phi_t \alpha y_{t+1} \equiv lt+1 \]

net supply of bonds to investors

Asset-scarce equilibrium

if \( \phi \) and \( l \) low

borrowing constraints are binding

\( r < 1/\beta \) in the steady state
Shortage of assets

Equilibrium on the bond market

\[ b_{t+1} + I^w_{t+1} + I^g_{t+1} = a_{t+1} \]

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net supply of bonds to investors

Asset-scarce equilibrium

if \( \phi \) and \( l \) low

borrowing constraints are binding

\( r < 1/\beta \) in the steady state

Assume “autarkic” investors

\( l \) is net position of investors

case \( l = 0 \) is actually realistic

implies \( b = a \)
Euler equation of savers

\[ \beta(1 - \phi_{t-1}) \alpha y_t = \frac{1}{r_{t+1}} \left( \phi_t \alpha y_{t+1} + \frac{M_{t+1}^S}{P_{t+1}} \right) \]

(1)
Euler equation of savers

\[ \beta(1 - \phi_{t-1})\alpha y_t = \frac{1}{r_{t+1}} \left( \phi_t \alpha y_{t+1} + \frac{M^S_{t+1}}{P_{t+1}} \right) \]  \hspace{1cm} (1)

Demand for saving instruments

Supply of saving instruments

Normal equilibrium

\[ i_{t+1} > 1 \]

then \( M^S = 0 \)
Euler equation of savers

\[ \beta(1 - \phi_{t-1})\alpha y_t = \frac{1}{r_{t+1}} \left( \phi_t \alpha y_{t+1} + \frac{M_{t+1}^S}{P_{t+1}} \right) \]  

1. **Demand for saving instruments**
2. **Supply of saving instruments**

Normal equilibrium

\[ i_{t+1} > 1 \]

then \( M^S = 0 \)

- \( r \) adjusts
Euler equation of savers

\[ \beta(1 - \phi_{t-1})\alpha y_t = \frac{1}{r_{t+1}} \left( \phi_t \alpha y_{t+1} + \frac{M^S_{t+1}}{P_{t+1}} \right) \]  

(1)

Normal equilibrium

\[ i_{t+1} > 1 \]

then \( M^S = 0 \)

▶ \( r \) adjusts

Liquidity trap (ZLB)

\[ i_{t+1} = 1 \]

then \( r_{t+1} = \frac{P_t}{P_{t+1}} \)
Euler equation of savers

\[ \beta (1 - \phi_{t-1}) \alpha y_t = \frac{1}{r_{t+1}} \left( \phi_t \alpha y_{t+1} + \frac{M^S_{t+1}}{P_{t+1}} \right) \]  

(1)

Demand for saving instruments

Supply of saving instruments

Normal equilibrium

\[ i_{t+1} > 1 \]

then \( M^S = 0 \)

\( \triangleright r \) adjusts

Liquidity trap (ZLB)

\[ i_{t+1} = 1 \]

then \( r_{t+1} = P_t / P_{t+1} \)

\( \triangleright M^S / P > 0 \) adjusts
Euler equation of savers

\[ \beta(1 - \phi_{t-1})\alpha y_t = \frac{1}{r_{t+1}} \left( \phi_{t} \alpha y_{t+1} + \frac{M_{t+1}^S}{P_{t+1}} \right) \]  \hspace{1cm} (1)

**Normal equilibrium**

\[ i_{t+1} > 1 \]

then \( M^S = 0 \)

▶ \( r \) adjusts

**Liquidity trap (ZLB)**

\[ i_{t+1} = 1 \]

then \( r_{t+1} = P_t/P_{t+1} \)

▶ \( M^S/P > 0 \) adjusts

define shadow rate \( r^S \) that would obtain without ZLB
Aggregate budget constraint of investors

\[ k_{t+1} = \beta \alpha y_t - \frac{P_{t+1}}{P_t} \frac{M_t^{S}}{P_{t+1}} + \beta \frac{M_t^{S}}{P_t} \]  

(2)
Aggregate budget constraint of investors

\[ k_{t+1} = \beta \alpha y_t - \frac{P_{t+1}}{P_t} \frac{M_{t+1}^S}{P_{t+1}} + \beta \frac{M_t^S}{P_t} \]  

(2)

crowding-out effect

liquidity effect
Aggregate budget constraint of investors

\[ k_{t+1} = \beta \alpha y_t - \frac{P_{t+1}}{P_t} \frac{M^S_{t+1}}{P_{t+1}} + \beta \frac{M^S_t}{P_t} \]  

price of liquidity  
crowding-out effect  
liquidity effect
Equilibrium on money market

\[ M_{t+1} = (1 - \alpha)P_t y_t + M^S_{t+1} \]

transaction
(workers)
saving
(investors)
Equilibrium on money market

\[ M_{t+1} = (1 - \alpha) P_t y_t + M^S_{t+1} \]

(price \( P \) adjusts to accommodate demand for money)
Equilibrium on money market

\[ M_{t+1} = (1 - \alpha)P_t y_t + M^S_{t+1} \]

price \( P \) adjusts to accommodate demand for money
(with nominal rigidities: \( y \) temporarily adjusts downward)
2. The effect of deleveraging
Asymptotic response to long-lasting deleveraging (steady state analysis)
Deleveraging shock: $\phi \downarrow$

Euler equation

$$\beta(1 - \phi)\alpha y = \frac{1}{r}(\phi \alpha y + m^S)$$

with $m^S_t = M^S_t / P_t$

Aggregate budget constraint

$$k = \beta \alpha y - (\theta - \beta)m^S$$
Deleveraging shock: \( \phi \downarrow \)

**Euler equation**

\[
\beta(1 - \phi)\alpha y = \frac{1}{r}(\phi\alpha y + m^S)
\]

**Aggregate budget constraint**

\[
k = \beta\alpha y - (\theta - \beta)m^S
\]

with \( m^S = M_t^S/P_t \)

- \( m^S = 0 \)
- \( r = r^S = \frac{\phi}{\beta(1 - \phi)} \downarrow \)

normal equil. low return (\( \theta > \beta \)) takes away resources from investment liquidity trap
Deleveraging shock: \( \phi \downarrow \)

Euler equation

\[
\beta (1 - \phi) \alpha y = \frac{1}{r} (\phi \alpha y + m^S)
\]

Aggregate budget constraint

\[
k = \beta \alpha y - (\theta - \beta)m^S
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with \( m^S = M_t^S / P_t \)

normal equil.

\[
\begin{align*}
& \quad m^S = 0 \\
& \quad r = r^S = \frac{\phi}{\beta (1 - \phi)}
\end{align*}
\]

\( k = \beta \alpha y \) doesn't depend on \( \phi \)
Deleveraging shock: \( \phi \downarrow \)

**Euler equation**

\[
\beta (1 - \phi) \alpha y = \frac{1}{r} (\phi \alpha y + m^S)
\]

with \( m^S = M_t^S / P_t \)

**Aggregate budget constraint**

\[
k = \beta \alpha y - (\theta - \beta) m^S
\]

- \( m^s = 0 \)
- \( r = r^S = \frac{\phi}{\beta (1 - \phi)} \)
- \( k = \beta \alpha y \) doesn't depend on \( \phi \)

**normal equil.**

**liquidity trap**

- \( r - r^S = \frac{1}{\theta} - \frac{\phi}{\beta (1 - \phi)} \)
- \( m^S = \alpha \left[ (1 - \phi) \frac{\beta}{\theta} - \phi \right] y \)
- \( \alpha (r - r^S) \)
Deleveraging shock: $\phi \downarrow$

**Euler equation**

$$\beta(1 - \phi)\alpha y = \frac{1}{r}(\phi\alpha y + m^S)$$

with $m_t^S = M_t^S/P_t$

**Aggregate budget constraint**

$$k = \beta\alpha y - (\theta - \beta)m^S$$

- $m^S = 0$
- $r = r^S = \frac{\phi}{\beta(1 - \phi)}$
- $k = \beta\alpha y$ doesn't depend on $\phi$
- $r - r^S = \frac{1}{\theta} - \frac{\phi}{\beta(1 - \phi)}$
- $k = \beta\alpha y - (\theta - \beta)m^S$
- low return ($\theta > \beta$) takes away resources from investment

- $m^S = \alpha \left[ (1 - \phi)\frac{\beta}{\theta} - \phi \right] y$
- $\alpha(r - r^S)$

**normal equil.**

**liquidity trap**
Investors’ deleveraging

Dashed line = shadow variables
Transition dynamics during transitory deleveraging
## Calibration: US economy pre-crisis

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
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<tbody>
<tr>
<td><strong>Time period</strong></td>
<td>1 year</td>
<td></td>
</tr>
<tr>
<td><strong>Balance sheet parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$l^g$</td>
<td>0</td>
<td>Gov't supply of assets, net of RoW demand</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Flow of Funds 2006)</td>
</tr>
<tr>
<td>$l^w$</td>
<td>0</td>
<td>Autarkic investors</td>
</tr>
<tr>
<td><strong>Rates of return</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>4% real return on capital</td>
</tr>
<tr>
<td>$\phi^H$</td>
<td>0.495</td>
<td>2% real interest rate</td>
</tr>
<tr>
<td><strong>Deleveraging parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi^L / \phi^H - 1$</td>
<td>-3.9%</td>
<td>20% peak-to-trough non-resid. investment</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.005</td>
<td>5.5 pp increase civilian unemployment</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.10</td>
<td>10% probability of exit each year</td>
</tr>
<tr>
<td><strong>Conventional parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.02</td>
<td></td>
</tr>
</tbody>
</table>
Response to a 10 year deleveraging shock

- **interest rate** $i_{t+1}$
- **borrowing** $b_{t+1}/r_{t+1}$
- **money holding** $M_{t+1}^S/M_{t+1}$

- **capital** $K_{t+1}$
- **labor** $h_t$
- **output** $y_t$

- **no ZLB**
- **downwardly rigid wages**
- **flexible wages**

Strong Keynesian demand-side effects in the short run, supply-side effects remain after wages have adjusted.
Response to a 10 year deleveraging shock

- interest rate $i_{t+1}$
- borrowing $b_{t+1}/r_{t+1}$
- money holding $M^S_{t+1}/M_{t+1}$
- capital $K_{t+1}$
- labor $h_t$
- output $y_t$

- ▶ strong keynesian demand-side effects in short run
- ▶ supply-side effects remain after wages have adjusted
3. Policies in a liquidity trap
Addressing short-run keynesian unemployment
Helicopter money can mimic flexible wages

In the following, focus on flexible wages.
Helicopter money can mimic flexible wages

In the following, focus on flexible wages
Exiting the liquidity trap
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi + (l^w + l^g)/(\alpha y)}{\beta(1 - \phi)} \]
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi + (l^w + l^g)/(\alpha y)}{\beta (1 - \phi)} \]

Decrease effective rate

- higher inflation \( \theta \)

Scarce-asset setting: low rates are inefficient (impair consumption smoothing and in some cases lead to capital overaccumulation)
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi + (l^w + l^g)/(\alpha y)}{\beta (1 - \phi)} \]

Decrease effective rate

- higher inflation \( \theta \)
- negative nominal rate \( i \)
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi + (l^w + l^g)/(\alpha y)}{\beta(1 - \phi)} \]

**Decrease effective rate**
- higher inflation \( \theta \)
- negative nominal rate \( i \)

**Increase shadow rate**
- increase public debt \( l^g = \) public supply of liquidity
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi + (l^w + l^g)/(\alpha y)}{\beta(1 - \phi)} \]

**Decrease effective rate**
- higher inflation \( \theta \)
- negative nominal rate \( i \)

**Increase shadow rate**
- increase public debt \( l^g = \)
  - public supply of liquidity
- QE = decrease shadow rate and deepens liquidity trap
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi +(l^W + l^g)/(\alpha y)}{\beta(1 - \phi)} \]

- Decrease effective rate
  - higher inflation \( \theta \)
  - negative nominal rate \( i \)

- Increase shadow rate
  - increase public debt \( l^g = \) public supply of liquidity
  - QE = decrease shadow rate and deepens liquidity trap

What is the effect on capital and output?
Exiting the liquidity trap

Requires closing the interest rate gap

\[ r - r^S = \frac{i}{\theta} - \frac{\phi + (l^w + l^g)/(\alpha y)}{\beta(1 - \phi)} \]

Decrease effective rate
- higher inflation $\theta$
- negative nominal rate $i$

Increase shadow rate
- increase public debt $l^g = $ public supply of liquidity
- QE = decrease shadow rate and deepens liquidity trap

What is the effect on capital and output?

Scarce-asset setting: low rates are inefficient
(impair consumption smoothing and in some cases lead to capital overaccumulation)
Decrease effective rate

Large enough decrease: exit ZLB

- higher capital and output
Decrease effective rate

Large enough decrease: exit ZLB
  ➤ higher capital and output

But timid decrease has ambiguous impact on capital and output
  • low real rate decreases the demand for money
    (b/c relaxes borrowing constraint)
  • but also decreases real return on money
Negative interest rate

Baseline deleveraging shock (4%, with $\pi = 1/10$)

- interest rate $i_{t+1}$
- capital $K_{t+1}$
- output $y_t$
Negative interest rate

Baseline deleveraging shock (4%, with $\pi = 1/10$)

Stronger deleveraging shock (9%, with $\pi = 1/20$)
Increase shadow rate

Large enough increase of public debt: exit ZLB
   • small increase offset by ↓ $m^S$
Increase shadow rate

Large enough increase of public debt: exit ZLB
  ▶ small increase offset by ↓ $m^S$

When exiting the liquidity trap
  • possible negative impact on capital and output for small increase in $l^g$
  • positive impact if large enough increase in $l^g$
Increase Government debt

Debt increase by 5% of GDP in 2 years
baseline deleveraging shock (4%, with $\pi = 1/10$)
Increase Government debt

Debt increase by 5% of GDP in 2 years
baseline deleveraging shock (4%, with $\pi = 1/10$)

Debt increase by 18% of GDP in 2 years
stronger deleveraging shock (8%, with $\pi = 1/20$)
QE with late exit can extend the liquidity trap

bonds $l_{t+1}^g / y_{t+1}$

money holding $M_{t+1}^S / M_{t+1}$

gap $\Delta_{t+1}$

interest rate $i_{t+1}$

labor $K_{t+1}$

output $y_t$
QE with late exit can extend the liquidity trap

- Early exit
- No QE

- Expected late exit sustains somewhat output during deleveraging

- Interest rate $i_{t+1}$
- Labor $K_{t+1}$
- Output $y_t$
A non-ZLB steady state with high enough public debt is Pareto-efficient.
First best policy

A non-ZLB steady state with high enough public debt is Pareto-efficient

but need to (i) make sure investment is not hurt by higher rates during transition
First best policy

A non-ZLB steady state with high enough public debt is Pareto-efficient

but need to (i) make sure investment is not hurt by higher rates during transition
  ▶ capital subsidy
First best policy

A non-ZLB steady state with high enough public debt is Pareto-efficient

but need to (i) make sure investment is not hurt by higher rates during transition
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(ii) help investors smooth consumption during transition
A non-ZLB steady state with high enough public debt is Pareto-efficient

but need to (i) make sure investment is not hurt by higher rates during transition
  ▶ capital subsidy

(ii) help investors smooth consumption during transition
  ▶ corporate tax
First best policy

A non-ZLB steady state with high enough public debt is Pareto-efficient

but need to (i) make sure investment is not hurt by higher rates during transition
  ➤ capital subsidy

(ii) help investors smooth consumption during transition
  ➤ corporate tax

and (iii) make sure no agent is worse off
First best policy

A non-ZLB steady state with high enough public debt is Pareto-efficient

but need to (i) make sure investment is not hurt by higher rates during transition
  ➤ capital subsidy

(ii) help investors smooth consumption during transition
  ➤ corporate tax

and (iii) make sure no agent is worse off
  ➤ consumption tax
4. Extensions
Extensions

- Tightening workers’ borrowing limit also decreases asset supply.
- Same effect on interest rate and money holdings.
- But positive effect on capital and output.

Workers’ deleveraging

- Bubble can appear when \( r \leq 1 \), both at/outside ZLB.
- Bubble sustains a higher interest rate.
- Ambiguous effect on capital.
- \( \uparrow \) in discount factor or \( \downarrow \) in productivity growth can lead to ZLB.
- But no negative medium-run impact on capital.
- Because saving increases.
- Financial intermediation, inefficient saving technology, idiosyncratic uncertainty.

Similar results.
Extensions

workers’ deleveraging
• tightening workers’ borrowing limit also decreases asset supply
• same effect on interest rate and money holdings
• but positive effect on capital and output

bubbles
• bubble can appear when $r \leq 1$, both at/outside ZLB
• bubble sustains a higher interest rate
• ambiguous effect on capital
**Extensions**

- **workers’ deleveraging**
  - tightening workers’ borrowing limit also decreases asset supply
  - same effect on interest rate and money holdings
  - but positive effect on capital and output

- **bubbles**
  - bubble can appear when \( r \leq 1 \), both at/outside ZLB
  - bubble sustains a higher interest rate
  - ambiguous effect on capital

- **preference and growth shocks**
  - \( \uparrow \) in discount factor or \( \downarrow \) in productivity growth can lead to ZLB
  - but no negative medium-run impact on capital
  - because saving increases
Extensions

- workers’ deleveraging
  - tightening workers’ borrowing limit also decreases asset supply
  - same effect on interest rate and money holdings
  - but positive effect on capital and output

- bubbles
  - bubble can appear when \( r \leq 1 \), both at/outside ZLB
  - bubble sustains a higher interest rate
  - ambiguous effect on capital

- preference and growth shocks
  - \( \uparrow \) in discount factor or \( \downarrow \) in productivity growth can lead to ZLB
  - but no negative medium-run impact on capital
  - because saving increases

- other
  - financial intermediation, inefficient saving technology, idiosyncratic uncertainty
  - similar results
Conclusion

Deleveraging of investors in a liquidity trap can explain both:
► cash hoarding
► persistent slowdown in investment

Persistent liquidity trap has supply-side policy implications
► focus on the supply of assets
► complementary to demand-side policies in the short term
Money and Capital in a persistent Liquidity Trap

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¹University of Lausanne
²CEPR
³Banque de France

“Investment in the new monetary and financial environment”
Sciences Po, Banque de France, Banca d’Italia, Paris, July 5–6, 2018
Appendix
Investors are in autarky in the US data

Balance sheet for Nonfinancial Corporate Business in Financial Accounts of the US

Simple definition of net position
- Net worth - Nonfinancial Assets
- between -2% of GDP in 2000 and 6% of GDP in 2006

More restricted definition
- Net position in interest bearing assets
- between -9% of GDP in 2000 and -2% of GDP in 2006
Financial Accounts of the US in 2006

Net position of Government (incl. monetary authority) in interest-bearing instruments $\approx -40\%$ of GDP

Net position of rest of world in interest-bearing instruments $\approx 40\%$ of GDP

$\star$ available supply of Government assets $\approx 0$
Investors’ deleveraging with \( l \neq 0 \)
Investors’ deleveraging with $l \neq 0$

- Shadow rate $r^S$ increases with $l$:  
  $$r = \frac{\phi + l/(\alpha y)}{\beta(1-\phi)}$$

- $k = \beta \alpha y - (\frac{1}{r} - \beta)l$ now depends on $r$ and $\phi$

normal equil.
Investors’ deleveraging with \( l \neq 0 \)

- **normal equil.**
  - shadow rate \( r^S \) increases with \( l \): \[ r = \frac{\phi + l}{\alpha y} \frac{1}{\beta(1-\phi)} \]
  - \( k = \beta \alpha y - (\frac{1}{r} - \beta)l \) now depends on \( r \) and \( \phi \)

- **liquidity trap**
  - total liquidity \( s = m^S + l = \alpha \left[ (1 - \phi) \frac{\beta}{\theta} - \phi \right] y \) \( \uparrow \) when \( \phi \downarrow \)
  - \( k = \beta \alpha y - (\theta - \beta)s \) \( \downarrow \) when \( \phi \downarrow \)
Workers’ deleveraging ($I^w \downarrow$)

**Outside ZLB**
- similar to investors’s deleveraging
  - asset shortage: $r \downarrow$
  - lower $r$ has a positive effect on capital

**Liquidity trap**
- no effect on $k$
  - does not affect investors’ asset demand, which is still
    \[ \alpha[(1 - \phi)\beta/\theta - \phi]y \]
  - effect on supply of assets to investors $m^S + l$ is fully offset by increase in $m^S$
Credit easing

- Government issues new debt $\Delta l^g$ to lend to constrained investors
- Does not affect net government debt
- Offsets deleveraging and allows to get out of liquidity trap
Bubbles

- bubble can appear when \( r \leq 1 \)
- equivalent to money when \( \theta = 1 \)

- intermediate (low) leverage: bubble crowds out (in) capital
Financial intermediation

- money mainly in bank deposits
- a model with banks is isomorphic to baseline model
- increase in cash holdings by investors at ZLB shows up as an increase in excess bank reserves at the Central Bank