Welfare Implications of Heterogeneous Labor Markets in a Currency Area*

Céline Poilly\textsuperscript{1} \hspace{1cm} Jean-Guillaume Sahuc

\textit{Banque de France} \hspace{1cm} \textit{Banque de France}

\textit{and THEMA-University of Cergy Pontoise} \hspace{1cm} \textit{and Audencia School of Management}

October 3, 2007

Abstract

This paper investigates the role of labor markets heterogeneity in a monetary union and especially what are the welfare gains/costs of reforms on the labor market for each members of the area. To this end, we develop a medium-scale two-country model representing a currency union characterized by price and wage stickiness, real rigidities and labor market frictions. Several results emerge. First, the choice of the instruments to direct a reform (aiming at reducing the home unemployment rate) is not neutral on the amount of the social welfare gain in the union. Second, the way the monetary authorities conduct their policy slightly influences the amount of the welfare gain. Third, countries should have the same degree of wage flexibility on their labor market to fully profit from the effects of structural reforms, especially when this degree is weak.

\textbf{Keywords:} DSGE model, currency union, heterogeneity, matching frictions, welfare

\textbf{JEL Classification:} C3, C5

\textsuperscript{*}First and incomplete draft – comments welcome. This paper has been prepared for the workshop “Wage Bargaining, Employment and Monetary and Economic Policies” hosted by the Banque de France and the DARES, Paris, October 9-10, 2007. We would like to thank Vladimir Borgy, Fabrice Collard, Patrick Fève and Julien Matheron for helpful conversations. The views expressed in this paper are the responsibility of the authors and do not necessarily reflect those of the Banque de France. The usual caveat applies.

\textsuperscript{1}Corresponding author: Banque de France, Economic and Financial Research Division, 31 rue Croix des Petits Champs, F-75049 Paris, France. E-Mail: celine.poilly@banque-france.fr.
“It was the roaring economic growth of the European Economic Community, above all else, that made it such a success in its early days... Conversely, it was gloom about the economy, and particularly over persistent high unemployment, that played the biggest part in the rejection of the constitution and in the spread of Euroscepticism across the continent. If the European Union is to flourish far beyond its 50th birthday, it is its economy that most needs attention”.

The Economist, March 15, 2007

1 Introduction

It is common view that the euro area economy as a whole is characterized by nominal and real rigidities, inhibiting a better labor market performance in Europe. Unemployment is persistently high (between 8% and 9% since 1999), the level of participation is low (around 69% since 1999) and real wages seem to be rather inflexible. The bad performance of the euro area labor market influences the well being of both the economy and society. A perfect flexibility of the labor market would imply that following any change in the economic environment, the labor force would be immediately redeployed to its most efficient use, with unemployment remaining at its structural level. However, there are many costs and impediments to such instantaneous adjustment, such as institutional features. It means that it takes more time for relative prices and quantities to fully reflect the new economic environment in the euro area than in other large areas, like the U.S. for example.

Such frictions cannot be neglected and the understanding of labor market matching processes is of considerable importance for the conduct of monetary policy. Although recent fully-fledged models, like Christiano et al. (2005) and Smets and Wouters (2003, 2005), are successful at explaining a number of phenomena, their lack of implications about the labor market have lead researchers to depart from the competitive labor market hypothesis. For example, following the seminal contributions by Merz (1995) and Andolfatto (1996), some papers have attempted to incorporate labor market search and matching frictions in such New-Keynesian models (Moyen and Sahuc, 2005, Trigari, 2005, and Walsh, 2005). They show that introducing these features improves the empirical performance of the closed-economy sticky-price model in several directions: (i) the existence of involuntary unemployment in equilibrium allows to reproduce the labor market stylized facts characterized by the

Beveridge and Phillips curves; (ii) labor market frictions act as a necessary complement to nominal rigidities; (iii) monetary policy shocks can explain important features of labor market fluctuations.

However, in a single currency union, an additional difficulty due to the heterogeneity of the members appears. First, with a common currency, a low level of labor market flexibility is more costly within the area since neither independent monetary policy nor the exchange rate could be adjustment mechanisms in the face of asymmetric or symmetric economic shocks. For example, adjustment to a country-specific or asymmetric shock may require a change in the real exchange rate and relative wages between countries to keep the adverse impact on unemployment and output to a minimum. Outside a currency union, this can be achieved either through an adjustment of the nominal exchange rate or an adjustment of factor and goods prices complemented by an appropriate monetary policy. Without the nominal exchange rate within a single currency, this would imply a less flexible economy. Second, as the monetary authorities take their decisions on the basis of aggregate developments, national idiosyncrasies are left to the care of national governments. Although the monetary authorities are interested on the aggregate variables, there are clearly disparities among the members. For example, in 2006, the unemployment rate of the Netherlands or Ireland is low (resp. 3.9% and 4.4%) and rather high in France or Greece (resp. 9.5% and 8.9%). What does it imply? The former group of countries spends a shorter period than the later group out of equilibrium or, in terms of the output gap, actual output remains closer to trend or potential. It would appear, therefore, that the more flexible is the labor market the better.

Then, the key issue is not labor market flexibility per se since the common monetary policy can react to average responses in the union, but rather asymmetries across member countries. A body of evidence has discussed the effects of having disparities between the labor market institutions of different regions in a monetary union (Guichard and La芳argue, 2000, Hughes Hallett and Viegi, 2003, or Dellas and Tavlas, 2005). They show that a shock that initially has a symmetric effect across the monetary union will evolve into an asymmetric shock if the labor market of one region is more flexible than another and then it adjusts more rapidly. Compolmi and Faia (2007) also show that a calibrated model of a monetary union with country-specific labor market institutions is able to replicate the data evidence. To assess the need for structural labor market reforms, as stressed recurrently by the European Central Bank and recommended by the Lisbon Strategy (also known as the Lisbon Agenda)², it is necessary to investigate how labor markets will perform and interact into a monetary union. The issue is not only to know how the overall performance of the monetary union

²The Lisbon strategy sets up the ambitious target for a Europe that would be "the most competitive and dynamic knowledge-based economy in the world, capable of sustaining economic growth with more and better jobs and greater social cohesion".
is affected by a modification of the frictions on the labor market, but also how the country-specific environment is modified and what are the implications in terms of welfare.\(^3\)

The purpose of this paper is to study the role of labor markets heterogeneity in a monetary union and especially to know what are the welfare gains/costs of reforms on the labor market for each members of the area. We investigate which modification on the labor market structure (\textit{i.e.} the labor market parameters) could provide the weaker loss in order to achieve a (low) unemployment rate target. Therefore, we make various scenarios and elaborate which scenarios are preferable from a welfare perspective. To do so, we develop a model of a two-country monetary union which resorts to the “New Open Economy Macroeconomics” literature. By incorporating significant frictions in the form of nominal and real rigidities, such structural models have been shown to provide a sufficiently rich dynamics to fit the actual data fairly well. Cross-country differences in the structural parameters and home bias in preferences are incorporated in the model. Finally, the matching model is chosen as it may provide a simple and elegant representation of European labor market characteristics in capturing the salient features of the theory of involuntary unemployment.

The paper is organized as follows. In a second section, we describe the medium-scale monetary union model. A third section is devoted to the model’s dynamics. A fourth section analysis the welfare implications of heterogeneous labour markets in the monetary union. A last section offers some concluding remarks.

2 A model of a currency union

The world is composed of two countries, Home and Foreign (also denoted by \(\mathcal{H}\) and \(\mathcal{F}\) hereafter). The total population is ordered on a continuum of measure one. The population of country \(\mathcal{H}\) belongs to \([0, n)\), while the foreign population belongs to \([n, 1]\). Therefore, \(n\) is the relative measure of the home country size into the union. An agent in the home country is indexed by \(H \in [0, n)\), while a foreign agent is indexed by \(F \in [n, 1]\). Variables in the home country are denoted \(X_t\) while foreign variables are denoted \(X_t^*\). The home economy produces a continuum of differentiated goods indexed by \(h \in [0, n)\). Foreign goods (or, equivalently, goods produced in the rest of the area) are indexed by \(f \in [n, 1]\).

The two countries are part of a currency union so that monetary policy is chosen for the whole area. Financial markets are assumed to be complete both at the national and international level. The labor market specification is based on the economics of search. Wages and hours worked are set

\(^3\)Jondeau and Sahuc (2005) have shown that forgetting structural heterogeneity of the members implies large and significant welfare losses in the euro area.
by Nash-bargaining between households and wholesalers. In addition, we introduce wage rigidity by assuming that the inertia of wages is due to a social norm (Hall, 2005). Production of final goods takes place in two stages. Perfectly competitive wholesalers manage the production of the same homogeneous input good and make hiring decisions. Monopolistic retailers buy the input good to produce differentiated final goods sold by the households and set prices to the discrete time version of Calvo’s (1983) model. Finally, households are assumed to have a taste bias towards home-produced goods. Since preferences differ across countries, the price of consumption bundles will differ when expressed in a common currency. The real exchange rate thus deviates from purchasing power parity (PPP).

2.1 Households

The home economy is populated by a large number of infinitely-living identical households, consuming Dixit-Stiglitz aggregates of domestic and imported goods. A home household $H$ owns a firm producing goods $h$ and receives dividends from it. We assume that households in a given country have the same preferences and endowments, defined over a composite consumption good ($C_t$), the employment’s rate ($N_t$) and hours worked ($H_t$). Although there may be idiosyncratic shocks among households, we assume that they have access to complete markets for state-contingent claims, so that there is no heterogeneity among agents in a given country. Consequently, all households in the same country behave in the same manner and then we consider the optimization problem of a representative household. The representative household in country $H$ maximizes the following expected sequence of present and future utility flows given by

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{\sigma_c}{\sigma_c - 1} (C_t - b C_t) \frac{\sigma_c - 1}{\sigma_c} - v N_t \left( \frac{\sigma_h}{\sigma_h + 1} \frac{\sigma_h + 1}{\sigma_h} \right) \right],$$

subject to a series of real period budget constraints

$$C_t + \frac{B_t + 1}{(1 + R_t)P_t} + T_t \leq N_t W_t H_t + (1 - N_t) \Theta \Theta + \frac{B_t}{P_t} + \Pi_t,$$

where $E_t$ denotes the expectation operator conditional on the information set at time $t$, $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$, $v > 0$ is a scale parameter, $\sigma_c$ is the intertemporal elasticity of substitution of consumption, and $\sigma_h$ is the elasticity of labor disutility with respect to hours worked. Preferences display external habit formation where the habit stock is supposed to equal the level of aggregate consumption in the previous period ($C_t = C_{t-1}$), and $b$ represents the habit

---

\[4\] The perfect insurance system makes the representative household a weighted average of employed and unemployed households, where the weight is the employment rate. That is why the employment rate enters the utility function and the budget constraint. By simplification we suppose that there is no disutility to search a job.
persistence parameter, measuring the effect of past consumption on current utility \((0 \leq b < 1)\). In addition, \(W_t\) is the hourly real wage and \(\Theta\) is the unemployment benefits. Let \(P_t\) denote consumption price index (CPI), \(R_t\) is the nominal interest rate, \(T_t\) is the real lump sum tax and \(\Pi_t\) is the sum of the dividends derived from retailers (\(\Pi_t^r\)) and wholesalers (\(\Pi_t^w\)). Finally, we assume complete markets for state-contingent claims. Consequently, households can transfer wealth to the next period by holding \(B_{t+1}\) unit of the one-period nominal bond denominated in the domestic currency.

The maximization problem of the home household consists in maximizing equation (1) subject to constraint (2), yielding the optimal profile of consumption, holdings of domestic bond. The first-order conditions imply
\[
U_{C,t} = (C_t - bc_t)^{\frac{1}{\alpha}},
\]
\[
(1 + R_t)^{-1} = \beta\mathbb{E}_t \left[ \frac{U_{C,t+1}P_t}{U_{C,t}P_{t+1}} \right],
\]
where \(U_{C,t}\) denotes the derivative of utility \(U\) with respect to variable \(C\) at the period \(t\). Equation (3) defines the marginal utility of consumption. Equation (4) is the usual Euler equation for inter-temporal consumption flows. It establishes that the ratio of marginal utility of future and current consumption is equal to the inverse of the real interest rate.

### 2.1.1 Composite consumption index

The aggregate consumption index for home households and the corresponding consumption index for foreign households are defined by
\[
C_t = \left( C_{H,t}^{\omega} (C_{F,t})^{1-\omega} \right)^{\frac{1}{\omega}}\text{ and } C_t^* = \left( (C_{H,t}^*)^{\omega} (C_{F,t}^*)^{1-\omega} \right)^{\frac{1}{\omega}},
\]
where \(\omega\) and \(\omega^*\) denote the share of home goods in the consumption of home and foreign households respectively. \(C_{H,t}\) (resp. \(C_{F,t}\)) is the sub-index of consumption of imperfectly substitutable, home (resp. foreign) goods, which is in turn given by the following CES aggregators
\[
C_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\varphi}} \int_0^n C_t(h) \frac{\partial c_t}{\partial h} dh \right]^{\frac{1}{\varphi+1}}\text{ and } C_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\varphi}} \int_1^{1} C_t(f) \frac{\partial c_t}{\partial f} df \right]^{\frac{1}{\varphi+1}},
\]
where \(C_t(h)\) (resp. \(C_t(f)\)) is consumption of the generic good \(h\) (resp. \(f\)) produced in country \(H\) (resp. \(F\)). Parameter \(\varphi\) denotes the elasticity of substitution across goods produced within a given country. The corresponding consumption price indexes are given by
\[
P_t = (P_{H,t}^{\omega} (P_{F,t}^*)^{1-\omega})\text{ and } P_t^* = (P_{H,t}^*)^{\omega} (P_{F,t})^{1-\omega}.\]

We abstract here from the optimal intra-temporal allocations between domestic and foreign goods.
Here, $P_{H,t}$ (resp. $P_{F,t}$) is the price sub-index for home- (resp. foreign-) produced goods expressed in the home currency, defined as

$$P_{H,t} = \left[ \frac{1}{n} \int_{0}^{\infty} P_{H,t}(h)^{1-\varepsilon_{H}} dh \right]^{\frac{1}{1-\varepsilon_{H}}}$$

and

$$P_{F,t} = \left[ \frac{1}{n} \int_{0}^{\infty} P_{F,t}(f)^{1-\varepsilon_{F}} df \right]^{\frac{1}{1-\varepsilon_{F}}} ,$$

where $P_{H,t}(h)$ (resp. $P_{F,t}(f)$) is the price of a generic good $h$ (resp. $f$) produced in country $H$ (resp. $F$).

We also assume that prices are set in the producer’s currency and that the law of one price holds. We then have $P_{H,t}(h) = P_{H,t}(h) S_{t}$ and $P_{F,t}(f) = P_{F,t}(f) S_{t}$, where $S_{t}$ is the nominal exchange rate expressed as units of domestic currency needed for one unit of foreign currency. Under the currency union assumption the nominal exchange rate is equal to one ($S_{t} = 1$). Since we assume the same elasticity of substitution among goods in a given country, we also have $P_{H,t} = P_{H,t}$ and $P_{F,t} = P_{F,t}$. Yet, from the definition of the CPI, we obtain that

$$\frac{P_{t}}{P_{t}^{*}} = \left( \frac{P_{H,t}}{P_{F,t}} \right)^{\omega-\omega^{*}} \quad .$$

Therefore, if we assume that there exists a home bias in preferences ($\omega \neq \omega^{*}$), PPP does not necessarily hold, i.e. $P_{t} \neq P_{t}^{*}$. We expect $\omega > \omega^{*}$, so that home households put a higher weight on home goods than foreign households.

### 2.1.2 International risk sharing

Under the assumption of complete markets, domestic and foreign households trade in state-contingent claims denominated in the home currency. This implies the following perfect risk-sharing condition (Chari et al., 2002)

$$Q_{t} = \kappa \frac{U_{C,t}^{*}}{U_{C,t}} , \quad (8)$$

where the real exchange rate, defined as $Q_{t} \equiv S_{t} P_{t}^{*} / P_{t}$, is proportional to the ratio of the marginal utility of consumption between the two countries.\(^6\)

Since the real exchange rate deviates from PPP because of home bias in preferences, we also have

$$Q_{t} = \left( \frac{P_{H,t}^{*}}{P_{H,t}} \right)^{\omega^{*}} \left( \frac{P_{F,t}^{*}}{P_{F,t}} \right)^{1-\omega^{*}} \left( \frac{P_{F,t}}{P_{H,t}} \right)^{\omega-\omega^{*}} = (T_{t})^{\omega-\omega^{*}} \quad (9)$$

where $T_{t}$ is the home terms of trade, i.e. the relative price between foreign and home bundles of goods as perceived by the home resident. It is defined as\(^7\)

$$T_{t} = \frac{P_{F,t}}{P_{H,t}} = \frac{P_{F,t}^{*}}{P_{H,t}^{*}} , \quad (10)$$

\(^6\kappa = [S_{0} P_{0}^{*} U_{C,0}]/[P_{0} dU_{C,0}]\) is a constant that depicts initial condition.

\(^7\)The foreign terms of trade are simply given by $T_{t}^{*} = P_{H,t}^{*} / P_{F,t}^{*} = 1 / T_{t}$, because the law of one price holds.
From equalities (7), we easily deduce that
\[
\pi_t = \pi_{H,t} \left[ \frac{T_t}{T_{t-1}} \right]^{1-\omega} \quad \text{and} \quad \pi_t^* = \pi_{F,t} \left[ \frac{T_{t-1}}{T_t} \right]^{\omega^*},
\]
where \( \pi_t \) and \( \pi_t^* \) are the CPI inflation rate in the home and the foreign country, respectively. In addition, \( \pi_{H,t} \) and \( \pi_{F,t} \) are domestic inflation rate in the home and the foreign country, respectively, defined as the inflation of the index of domestic goods prices. Using equations (10), (3), (8) and (9), we obtain
\[
(T_t)^{\omega^*} = \kappa \frac{(C_t - bC_{t-1})^{\sigma}}{(C_t^* - bC_t^*)^{\sigma^*}}.
\]
Equation (11) provides a rather elegant way to escape the exchange rate non-stationarity and model indeterminacy issues. Note that, when there is no home bias in preferences \( (\omega = \omega^*) \), the perfect risk sharing assumption does not allow to determine the terms of trade anymore.

### 2.2 Labor market matching of the home country

At the macroeconomic level, the law of motion of aggregate employment \( (N_t) \) is
\[
N_{t+1} = (1 - s_t)N_t + M_t,
\]
where \( s_t \) is a stochastic process for the job separation rate evolving belong 0 and 1.\(^8\) Therefore, proportion \( s_t \) of all filled jobs disappears at each instant, and \( M_t \) is the mass of recruiting at period \( t \).\(^9\) Thus, matching which take place at the period \( t \) are only productive at the following period.

The matching function is a very convenient hypothetical concept whose basic idea is that the recruiting effort of employers and the search effort of workers serve as inputs in a market matching function that generates new hires.\(^10\) The job vacancies \( (V_t) \) and unemployed workers \( (U_t) \) are randomly matched with each other. The aggregate flow of job matches are deterministic and given by the following matching technology
\[
M_t = M(U_t, V_t) = \bar{m} \epsilon_t^m U_t^\vartheta V_t^{1-\vartheta},
\]
where \( \vartheta \in (0,1) \), \( \bar{m} > 0 \) is a scale parameter and \( \epsilon_t^m \) denotes a country-specific matching shock.\(^11\) This shock allows to measure the implications of a change in the number of successful matches.

---

\(^8\)In order to measure the effects of a change in the labor market institutions, we assume that the job destruction rate can be exogenously hit. It follows an AR(1) process: \( \ln(s_t) = (1 - \rho_s) \ln(\delta) + \rho_s \ln(s_{t-1}) + \eta_{s,t} \).

\(^9\)To simplify the analysis, we ignore any endogenous separation. Hall (2005) has argued that the separation rate varies little over the business cycle, although part of the literature disputes this position.

\(^10\)Firms have jobs that are filled or vacant and workers have a job or are unemployed but only the vacant jobs are offered and unemployed people are engaged in search. This assumption implies that the two activities of production of goods and trade in labour market are strictly separate activities.

\(^11\)It follows an AR(1) process: \( \ln(\epsilon_{m,t}) = (1 - \rho_m) \ln(\bar{\epsilon}_m) + \rho_m \ln(\epsilon_{m,t-1}) + \eta_{m,t} \).
(implying a change in the labor market tightness) on the model’s dynamics. The matching technology exhibits constant returns to scale. We choose a Cobb-Douglas form for its simplicity. The labor force being normalized to one, the number of unemployed workers at the beginning of any given period is \( U_t = 1 - N_t \).

The job vacancies and unemployed workers that are matched together in period \( t \) are randomly selected from the sets \( V_t \) and \( U_t \). Hence, the stochastic process governing the state of vacant jobs during an interval of time is Poisson with rate

\[
\tau_t = \frac{M_t}{V_t}.
\]

In other words, \( \tau_t \) can be interpreted as the instantaneous probability of a vacancy being filled. Also, the average steady-state duration of a job vacancy is \( 1/\tau \).

Similarly, the instantaneous probability that an unemployed worker finds a vacant position is given by

\[
\theta_t = \frac{M_t}{U_t},
\]

which means that the average steady-state duration of unemployment is \( 1/\theta \).

### 2.3 The production sector in the domestic and the foreign country

Production of final goods in each country takes place in two stages. First, perfectly competitive wholesalers manage the production of the same homogenous input good and make investment and hiring decisions. Second, there is a continuum of infinitely living and monopolistic retailers indexed by \( h \) on the interval \([0, n)\) for the home country and by \( f \) on the interval \([n, 1]\) for the foreign country. They buy the input good to produce differentiated goods which are bundled into homogeneous home and foreign goods by a constant returns to scale of the Dixit-Stiglitz form

\[
Y_t = \left[ \frac{1}{n} \right]^{1/p} \int_0^n Y_t(h)^{(p-1)/p} \, dh \quad \text{and} \quad Y_t^* = \left[ \frac{1}{1 - n} \right]^{1/p} \int_n^1 Y_t^*(f)^{(p-1)/p} \, df \right]^{1/p}.
\]

In addition, they set prices according to the discrete time version of Calvo’s (1983) model. Since, the maximization problems which characterize the production sector are symmetric across the two economies, we present only the ones for the home region.

#### 2.3.1 Wholesalers

We consider a representative firm in each country which acts on a perfect competition market and makes investment and hiring decisions. Each period, this firm uses physical capital \( (K_t) \) and labor (total hours, \( N_t H_t \)) as inputs in order to produce a homogeneous wholesale good \( (Y_t^{wh}) \) which cannot
be consumed and will be sold to retailers at relative price \( MC_t = P_t^w / P_t \) to produce a differentiated final good. The production technology is given by

\[
Y_t^w = \epsilon_{a,t} (z_t K_t) \phi (N_t H_t)^{1-\phi},
\]

where \( \phi \in (0, 1) \) is the elasticity of value added with respect to capital, \( z_t \) is the capital utilization rate, and \( \epsilon_{a,t} \) denotes an exogenous technology shock.\(^{12}\) For computational convenience, we assume constant return to scale.

The modelling of investment can be linked to Tobin’s Q-model, which couples investment decisions to forward-looking stock market valuations of the firm. This model can be derived from the theory if it is assumed that investment is subject to adjustment costs, which are a convex function of the rate of change of the firm’s capital stock. A necessary condition is convexity which implies that these installation costs increase at an increasing rate and too fast an accumulation of capital is more costly.

The firm’s stock of physical capital evolves according to

\[
K_{t+1} = (1 - \delta (z_t)) K_t + I_t,
\]

where \( I_t \) denotes time \( t \) purchases of investment goods and \( \delta (z_t) \) a positive, increasing and convex function of the utilization rate defined by

\[
\delta (z_t) = \delta \frac{z_t^d}{d},
\]

that reflects the fact that a higher utilization rate raises the depreciation rate of capital (with \( d > 1 \)). \( \delta > 0 \) is a scale parameter. The functional form chosen here for the adjustment costs is given by

\[
A_t = A(I_t, K_t, z_t) = \frac{\varpi}{2} \left( \frac{I_t}{K_t} - \delta (z_t) \right)^2 K_t,
\]

with \( \varpi > 0 \).

The representative firm chooses sequences of vacancies, investment, and utilization rate in order to maximize the expected sum of discounted profits, taking as given a per vacancy cost (\( \eta \)).\(^{13}\)

\[
\mathbb{E}_t \sum_{t=0}^{\infty} T_{t,t+j} \left[ MC_t \epsilon_{a,t} (z_t K_t) \phi (N_t H_t)^{1-\phi} - W_t N_t H_t - \eta V_t - (I_t + A_t) \right],
\]

\(^{12}\)It follows an AR(1) process: \( \ln(\epsilon_{a,t}) = (1 - \rho_a) \ln(\epsilon_{a,t-1}) + \epsilon_{a,t-1} \).

\(^{13}\)We assume that the firm bears a fixed cost of posting a vacancy. This assumption proposed by Mortensen and Pissarides (1999) is usual in the literature. However, as shown by Fujita and Ramey (2007) and Bodart et al. (2006), this assumption may not be sufficient to offset the too rapid adjustment of vacancies after a shock obtained in this kind of model. Alternative specifications, as a quadratic cost of adjusting labor or a sunk cost, can help to solve this issue. This is an interesting extension of our paper.
subject to the following constraints

\[ N_{t+1} = (1 - s_t) N_t + \tau_t V_t, \]  
\[ K_{t+1} = (1 - \delta(z_t)) K_t + I_t. \]  

and with \( \Upsilon_{t,t+j} = \beta^j U_{C_t}(C_{t+j}) / U_{C_t}(C_t) \) is the discount factor between time \( t \) and \( t + j \).

The first-order conditions of this program are given by\(^{14}\)

\[ \frac{\eta}{\tau_t} = \mathbb{E}_t \left[ \Upsilon_{t,t+1} \left( (1 - \phi) MC_{t+1} \frac{Y_w}{N_{t+1}} - W_{t+1} H_{t+1} + (1 - s_t) \frac{\eta}{\tau_{t+1}} \right) \right], \]  
\[ 1 + A_t = \mathbb{E}_t \left[ \Upsilon_{t,t+1} \left( \phi MC_{t+1} \frac{Y_w}{K_{t+1}} - A_{t+1} + (1 - \delta(z_{t+1}))(1 + A_{t+1}) \right) \right], \]

\[ \phi MC_t \frac{Y_w}{z_t} = K_t \delta z_t. \]

### 2.3.2 Wage and hours determination

As previously, we present only the labor decisions for the home country since sectors are symmetric across the two economies. In each country, wage and hours worked are determined by the generalized Nash-bargaining solution. Indeed, the matching between an unemployed person and a firm who coordinate each other gives rise to a surplus which must be shared between the meeting pair. This sharing takes place at the match level through a bilateral and decentralized wage/hours negotiation.

Knowing that there are a representative household and a representative firm, we are located directly at the symmetric equilibrium solution of the model.

Formally, the surplus generated by a successful match between an unemployed worker and a vacant job is the marginal value of employment. One can show that hourly real wage is given by

\[ W_t^* = \xi_t \left( (1 - \phi) MC_t \frac{Y_w}{H_t N_t} + (1 - s_t) \frac{\eta}{H_t \tau_t} \right) + (1 - \xi_t) \left[ \frac{\sigma_h v H_t}{1 + \sigma_h UC_t} + \Theta \right] \]

\[ - (1 - \theta_t - s_t) \mathbb{E}_t \left\{ \frac{(1 - \xi_t) \xi_{t+1}}{1 - \xi_{t+1}} \right\} \eta \frac{H_t \tau_t}{H_t \tau_t}, \]

where \( \xi_t \) is a stochastic process for the relative bargaining power of households evolving belong 0 and 1.\(^{15}\)

\(^{14}\)Let \( A_{\xi} \) denote the first derivative of \( A_t \) with respect to \( x_t \), where \( x_t = \{ I_t, K_t, z_t \} \) and \( \delta_{z_t} \) denotes the first derivative of \( \delta_t \) with respect to \( z_t \).

\(^{15}\)An increase in the labor market tightness \( (\theta_t / \tau_t) \) may result from a variation of the worker’s bargaining strength, all things being equal. By assuming that the household’s bargaining strength can be hit by an exogenous perturbation, we implicitly measure the effect of a change in the labor market tightness resulting from a change in the bargaining structure. We assume it follows an AR(1) process: \( \ln(\xi_t) = (1 - \rho_t) \ln(\bar{\xi}) + \rho_t \ln(\xi_{t-1}) + \eta_{\xi,t} \).
In addition, in order to avoid a too large procyclicity of wages, we introduce wage rigidities into the model in the form of a backward looking social norm (Hall, 2005). Precisely, we assume that the individual real wage is a weighted average of the one obtained through the Nash-bargaining process and a wage norm which is set independently of idiosyncratic conditions. Consequently, the real wage paid in job is defined by

$$W_t = (W_t^*)^{(1-\alpha_w)} \bar{W}^{\alpha_w},$$

where $\bar{W}$ is the wage norm, $\alpha_w \in [0, 1]$ is the adjustment rate. Following Hall (2005), we adopt the adaptive wage specification such that $\bar{W} = W_{t-1}$.

Finally, hours worked are determined by maximizing the joint surplus and are given by

$$H_t = \frac{(1 - \phi)^2 MC_t Y^w_t}{N_t} = \frac{vH_{t-1}^*}{U_{C,t}}.$$

### 2.3.3 Retailers

There is a continuum of monopolistically competitive retailers indexed by $h$ on the interval $[0, n)$. Each of them is infinitively lived and produces a differentiated final good $Y_t(h)$ with a technology that transforms one unit of wholesale goods into one unit of retail goods, so that $Y_t(h) = Y^w_t(h)$. Firms on the retail sector purchase output from wholesale producers at the price $MC_t$ (which becomes the firm’s real marginal cost) and directly sell to households. For the sake of simplicity, we assume that the government and the wholesaler have the same optimal intratemporal allocations for each differentiated goods as the household.

Retailers price setting decision is modelled through a modified version of the Calvo’s (1983) staggering mechanism. In addition to the baseline mechanism, we allow for the possibility that firms that do not optimally set their prices may nonetheless adjust it to keep up with the previous period increase in the general price level. In each period, a firm faces a constant probability, $1 - \alpha_p$, of being able to re-optimize its price and chooses the new price $P^*_t(h)$ that maximizes the expected discounted sum of profits

$$E \sum_{j=0}^{\infty} \alpha^j Y_{t,t+j} \left[ \frac{P^*_t(h) \Psi^H_{t,t+j}(1 + \text{tax})}{P_{H,t+j}} - MC_{t+j} \right] Y_{t+j}(h)$$

16In the standard matching frictions model, real wages are too procyclical since they are directly driven by the dynamics of output. This implies a too less volatility of employment.

17Rather than assuming that hours worked and real wages are determined simultaneously, we could assume that firms choose hours worked, by taking the bargained real wage as given. This “right to manage” assumption (see Trigari, 2006) introduces an additional channel from the real wages to the real marginal cost and inflation.

18For the sake of simplicity, we assume that the government and the wholesaler have the same optimal intratemporal allocations for each differentiated goods as the household.
subject to the sequence of demand equations

$$Y_{t+j}(h) = \left( \frac{P^*_H(h) \Psi^H_{t,t+j}}{P^H_{H,t+j}} \right)^{-\varepsilon_p} Y_{t+j}$$

(27)

with

$$\Psi^H_{t,t+j} = \begin{cases} \prod_{\nu=0}^{j-1} \bar{\pi}_H^{1-\gamma_p} \pi_{H,t+j} & j > 0 \\ 1 & j = 0 \end{cases}$$

(28)

where $\bar{\pi}_H$ is the domestic trend inflation, the coefficient $\gamma_p \in [0,1]$ indicates the degree of indexation to past inflation during the periods in which firm is not allowed to re-optimize. Finally, $\Psi^H_{t,t+j}$ is a correcting term that accounts for the fact that, if firm $h$ does not re-optimize its price, it updates it according to the rule

$$P_H(t,h) = (\bar{\pi}_H)^{1-\gamma_p} (\pi_{H,t-1})^{\gamma_p} P_{H,t-1}(h).$$

(29)

Consequently, the first-order condition associated to the profit maximization implies that firms set their price equal to the discounted stream of expected future real marginal costs

$$p^*_H(t,h) = \mu_\varepsilon \frac{E_t \sum_{j=0}^{\infty} \alpha_j^p Y_{t+j} \left( \frac{\Psi_{t,t+j}^H P_{H,t+j}}{P^H_{H,t+j}} \right)^{-\varepsilon_p}}{E_t \sum_{j=0}^{\infty} \alpha_j^p Y_{t+j} \left( (1 + \text{tax}) \left( \frac{\Psi_{t,t+j}^H P_{H,t+j}}{P^H_{H,t+j}} \right)^{1-\varepsilon_p} \right)^{-\varepsilon_p}} Y_{t+j}$$

(30)

where $p^*_H(t,h)$ is the relative price of domestic goods and $\mu_\varepsilon \equiv \varepsilon_\varepsilon / (\varepsilon_\varepsilon - 1)$ is the optimal markup in a flexible-price economy. In order to eliminate monopoly distortions, we assume that producers of intermediate goods are subsidized at rate tax such that $(1 + \text{tax}) = \mu_\varepsilon$. As there are no firm-specific shocks in this economy, all firms that are allowed to re-optimize their price at date $t$ select the same optimal price $p^*_H(t,h) = p^*_H(t,h)$.

Staggered price setting under partial indexation implies the following expression for the evolution of the domestic price index

$$P_H(t) = \alpha_p \left( (\bar{\pi}_H)^{1-\gamma_p} (\pi_{H,t-1})^{\gamma_p} P_{H,t-1} \right)^{1-\varepsilon_p} + (1 - \alpha_p) \left( P^*_H(t) \right)^{1-\varepsilon_p}. \left( \frac{1}{\varepsilon_p} \right)^{1-\varepsilon_p}.$$

(31)

The price setting problem solved by firms in the foreign country is similar and leads to an optimal rule analogous to equation (31).

### 2.4 Market clearing conditions

Domestic and foreign outputs may be either transformed into a single type of consumption good, invested, consumed by the government, used up in vacancy posting costs or capital adjustment cost.
The allocation of demand across each of the goods produced within a given country for consumers \( H, F \) are then given by

\[
X_t(h) = \frac{1}{n} \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\varepsilon_p} X_{H,t} \quad \text{and} \quad X_t^*(h) = \frac{1}{n} \left( \frac{P_{H,t}^*(h)}{P_{H,t}} \right)^{-\varepsilon_p} X_{H,t}^*,
\]

\[
X_t(f) = \frac{1}{1-n} \left( \frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\varepsilon_p} X_{F,t} \quad \text{and} \quad X_t^*(f) = \frac{1}{1-n} \left( \frac{P_{F,t}^*(f)}{P_{F,t}} \right)^{-\varepsilon_p} X_{F,t}^*,
\]

where \( X_t = \{C_t, I_t\} \) and \( X_t^* = \{C_t^*, I_t^*\} \).

The aggregator (5) implies that home and foreign demands for composite home and foreign are given by

\[
X_{H,t} = \omega \left( \frac{P_t}{P_{H,t}} \right) X_t \quad \text{and} \quad X_{H,t}^* = \omega^* \left( \frac{P_t^*}{P_{H,t}} \right) X_t^*,
\]

\[
X_{F,t} = (1-\omega) \left( \frac{P_t}{P_{F,t}} \right) X_t \quad \text{and} \quad X_{F,t}^* = (1-\omega^*) \left( \frac{P_t^*}{P_{F,t}} \right) X_t^*.
\]

Then, aggregate outputs in home and foreign goods are

\[
Y_t = \omega (T_t)^{1-\omega} (C_t + I_t) + \frac{1-n}{n} \omega^* (T_t)^{1-\omega^*} (C_t^* + I_t^*) + G_t + A_t + \eta V_t \tag{32}
\]

\[
Y_t^* = (1-\omega) (T_t)^{-\omega} \frac{n}{1-n} (C_t + I_t) + (1-\omega^*) (T_t)^{-\omega^*} (C_t^* + I_t^*) + G_t^* + A_t^* + \eta^* V_t^*. \tag{33}
\]

### 2.5 Fiscal and monetary policy

We close the model by specifying the governments and monetary authorities’ behaviors. Governments’ spending (\( G_t \) and \( G_t^* \)) are exogenous. Governments generate revenue from lump sum taxation and bond creation and they pay unemployment benefits. Since we do not consider distortions taxes, the two governments face the following budget constraints

\[
G_t = T_t + \frac{B_{t+1} \left( 1 + R_t \right) - B_t}{P_t} - (1-N_t) \Theta,
\]

\[
G_t^* = T_t^* + \frac{B_{t+1}^* \left( 1 + R_t \right) - B_t^*}{P_t^*} - (1-N_t^*) \Theta^*.
\]

In addition, we assume that the common monetary authorities follow a standard Taylor rule of the form

\[
(1 + R_t)^{\psi_H} \left( \tilde{\pi}_t \right)^{\psi_n} (1 - \psi_n) \left( \tilde{Y}_t \right)^{\psi_\eta} (1 - \psi_\eta).
\]

where \( \tilde{\pi}_t = n \pi_t + (1-n) \pi_t^* \) denotes the union-wide inflation (i.e. the weighted deviations of home and foreign CPI inflations from their steady-state values, normalized to one) and \( \tilde{Y}_t = n (Y_t/Y_t^n) + (1-n) (Y_t^*/Y_t^{*n}) \) is the union’s output gap (i.e. the weighted deviations of home and foreign outputs to their natural level, defined as the level of output that occurred when prices are flexible).
3 The dynamics of the currency union

In this section, we describe the dynamic behavior of the model in response to three shocks for several scenarios concerning the degree of flexibility of the labor market. As we are especially interested by the effects of shocks on the labor market, we consider (i) a domestic matching shock, meaning that the matches exogenously increases, (ii) a domestic job destruction rate shock, meaning that unemployment duration is increased, and (iii) a domestic bargaining shock, meaning that the wage bargaining power of firms is exogenously hit.\footnote{Shocks on technology, government spending, and monetary policy have been done and are available upon request.} For each shock, the impulse response functions (IRFs) of production, consumption, investment, aggregate employment, hours worked, vacancies, real wages and CPI inflation in the home and the foreign country are displayed.

3.1 Calibration

The model is calibrated to quarterly data. At this stage, we assume equal size between the two countries ($n = 0.5$) and a symmetric calibration for the two countries.

Preferences. We set the discount factor $\beta = 0.99$, which gives an annual steady state real interest rate equal to 4%. We assign values for the intertemporal elasticity of substitution ($\sigma_c = 0.8$), the elasticity of labor disutility ($\sigma_h = 0.45$), and the consumption habit parameter ($b = 0.7$) similar to those estimated by Sahuc and Smets (2006). The elasticity of substitution between differentiated goods $\varepsilon_p$ is set equal to 6, corresponding to a markup $\mu_p = 1.2$. Finally, the home bias parameter $\omega$, representing the share of home goods on total consumption is set to 0.8 (so $\omega^* = 0.2$).

Production. The share of government spending in the GDP is equal to 16% and the share of consumption in GDP is 56%. The rate of capital depreciation ($\delta$) is set to 2.5%. The capital share parameter ($\phi$) is set to 0.35. The capital adjustment cost parameter $\xi$ is set to 10.

The degree of price rigidity $\alpha_p$ is set equal to 0.7 implying an average duration of price contracts of less than one year (see Dhyne et al., 2006). In addition, the price indexation parameter $\gamma_p$ is set equal to 0.5, a conventional value in the euro area DSGE literature.

Exogenous shocks and monetary policy. The reaction function of the monetary authority is assumed to be an inertial Taylor rule with the usual parameter values (see Clarida et al., 1998): $\psi_R = 0.85$, $\psi_x = 1.5$ and $\psi_y = 0.125$. We assume that all shocks are persistent with an autocorrelation of 0.85.

Labor market. Concerning the matching process and the labor market, we follow Petrongolo and Pissarides (2001) survey of empirical studies for the Cobb-Douglas matching function parameter
and set $\vartheta = 0.6$. Aggregate expenditures on search activity ($\eta \bar{V}/\bar{Y}$) are fixed at 2.5%.$^{20}$ The relative bargaining power of households ($\bar{\xi}$) is then determined by the model’s steady state and is equal to 0.6.$^{21}$

<table>
<thead>
<tr>
<th></th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>8.5</td>
<td>6.9</td>
<td>6.6</td>
<td>7.5</td>
<td>8.2</td>
<td>8.4</td>
<td>8.4</td>
<td>8.2</td>
</tr>
<tr>
<td>Germany</td>
<td>7.9</td>
<td>7.2</td>
<td>7.4</td>
<td>8.2</td>
<td>9.0</td>
<td>9.5</td>
<td>9.4</td>
<td>8.4</td>
</tr>
<tr>
<td>Greece</td>
<td>12.0</td>
<td>11.2</td>
<td>10.7</td>
<td>10.3</td>
<td>9.7</td>
<td>10.5</td>
<td>9.8</td>
<td>8.9</td>
</tr>
<tr>
<td>Spain</td>
<td>12.5</td>
<td>11.1</td>
<td>10.3</td>
<td>11.1</td>
<td>11.1</td>
<td>10.6</td>
<td>9.2</td>
<td>8.5</td>
</tr>
<tr>
<td>France</td>
<td>10.5</td>
<td>9.1</td>
<td>8.4</td>
<td>8.7</td>
<td>9.5</td>
<td>9.6</td>
<td>9.7</td>
<td>9.5</td>
</tr>
<tr>
<td>Ireland</td>
<td>5.7</td>
<td>4.2</td>
<td>4.0</td>
<td>4.5</td>
<td>4.7</td>
<td>4.5</td>
<td>4.3</td>
<td>4.4</td>
</tr>
<tr>
<td>Italy</td>
<td>10.9</td>
<td>10.1</td>
<td>9.1</td>
<td>8.6</td>
<td>8.4</td>
<td>8.0</td>
<td>7.7</td>
<td>6.8</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>2.4</td>
<td>2.3</td>
<td>2.0</td>
<td>2.7</td>
<td>3.7</td>
<td>5.1</td>
<td>4.5</td>
<td>4.7</td>
</tr>
<tr>
<td>Netherlands</td>
<td>3.2</td>
<td>2.8</td>
<td>2.2</td>
<td>2.8</td>
<td>3.7</td>
<td>4.6</td>
<td>4.7</td>
<td>3.9</td>
</tr>
<tr>
<td>Austria</td>
<td>3.9</td>
<td>3.6</td>
<td>3.6</td>
<td>4.2</td>
<td>4.3</td>
<td>4.8</td>
<td>5.2</td>
<td>4.7</td>
</tr>
<tr>
<td>Portugal</td>
<td>4.5</td>
<td>4.0</td>
<td>4.0</td>
<td>5.0</td>
<td>6.3</td>
<td>6.7</td>
<td>7.6</td>
<td>7.7</td>
</tr>
<tr>
<td>Finland</td>
<td>10.2</td>
<td>9.8</td>
<td>9.1</td>
<td>9.1</td>
<td>9.0</td>
<td>8.8</td>
<td>8.4</td>
<td>7.7</td>
</tr>
<tr>
<td><strong>Euro Area</strong></td>
<td><strong>9.1</strong></td>
<td><strong>8.2</strong></td>
<td><strong>7.8</strong></td>
<td><strong>8.2</strong></td>
<td><strong>8.7</strong></td>
<td><strong>8.8</strong></td>
<td><strong>8.6</strong></td>
<td><strong>7.9</strong></td>
</tr>
</tbody>
</table>

Sources: Eurostat.

As shown in Tables 1 and 2, there is a great heterogeneity among the euro area countries concerning the properties of the labor market. Indeed, we can consider three groups in function of the level of the unemployment rate: (i) the countries that have a low unemployment rate (Ireland, Luxembourg, Netherlands and Austria), (ii) the countries closer to the euro area rate (Belgium, Italy, Portugal and Finland), and (iii) those having a high unemployment rate (Germany, Greece, Spain and France). In addition, in 2000, short-term unemployment (defined as unemployment with a duration of less than six months) represented around one-third of total unemployment in the euro area (see Table 2). This level was particularly high in Luxembourg, the Netherlands, Austria, Finland and, to a lesser extent, in France and Ireland.

$^{20}$This value is a compromise between the value of 1% reported by Andolfatto (1996) and the 5% reported by Krause and Lubik (2007a).

$^{21}$This value implies that the equilibrium unemployment is socially-efficient (Hosios, 1990).
Table 2. Unemployment duration (2000)

<table>
<thead>
<tr>
<th>Share in %</th>
<th>Short-term (less than 6 months)</th>
<th>Between 6 months and 1 year</th>
<th>Long-term (more than 1 year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>28.2</td>
<td>15.5</td>
<td>56.3</td>
</tr>
<tr>
<td>Germany</td>
<td>32.4</td>
<td>16.1</td>
<td>51.5</td>
</tr>
<tr>
<td>Greece</td>
<td>26.5</td>
<td>17.1</td>
<td>56.4</td>
</tr>
<tr>
<td>Spain</td>
<td>37.8</td>
<td>19.7</td>
<td>42.4</td>
</tr>
<tr>
<td>France</td>
<td>43.6</td>
<td>16.8</td>
<td>39.6</td>
</tr>
<tr>
<td>Ireland</td>
<td>43.1</td>
<td>20.0</td>
<td>36.6</td>
</tr>
<tr>
<td>Italy</td>
<td>22.4</td>
<td>16.3</td>
<td>61.3</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>56.0</td>
<td>18.8</td>
<td>25.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>53.5</td>
<td>13.8</td>
<td>32.7</td>
</tr>
<tr>
<td>Austria</td>
<td>56.2</td>
<td>15.4</td>
<td>28.4</td>
</tr>
<tr>
<td>Portugal</td>
<td>40.0</td>
<td>17.1</td>
<td>42.9</td>
</tr>
<tr>
<td>Finland</td>
<td>58.9</td>
<td>16.5</td>
<td>24.6</td>
</tr>
<tr>
<td>Euro Area</td>
<td>35.3</td>
<td>17.0</td>
<td>47.7</td>
</tr>
</tbody>
</table>

Sources: ECB (2002).

This may reflect the fact that, in these countries, a large part of unemployment is frictional and not persistent. The long-term unemployment rate for the euro area was very high in 2000, representing 47.7% of total unemployment. Especially, the share of long-term unemployment is very high in Italy, Greece, Belgium and Germany.

Following these statistics and in order to investigate the impact of labor market rigidities on the transmission channels, we assume different specifications concerning the degree of flexibility on the labor market, summarized in Table 3. In the first specification, $\mathcal{R}\mathcal{K}$, we assume that the home and the foreign countries are symmetric and their labor market is rigid (this case, corresponding to the euro area as a whole, is our benchmark calibration). The degrees of wage rigidity ($\alpha_w$ and $\alpha^*_w$) are then set to 0.6, as proposed by Faia (2007). In addition, the mean durations of unemployment and vacancies are respectively 12 and 6 months ($\bar{\tau} = \bar{\tau}^* = 0.50$ and $\bar{\varphi} = \bar{\varphi}^* = 0.25$). Finally, according to Table 1 (last line), the unemployment rate for the euro area as a whole is estimated at 7.9% in
2006 which implies a value for $\bar{N}$ and $\bar{N}^*$ of 0.921 ($\bar{s} = \bar{s}^* = 0.0214$). In the second calibration, $\mathcal{FR}$, the domestic labor market is more flexible than in the foreign one (it can correspond to the case of Ireland or Luxembourg vs. the rest of the euro area, for instance). In this context, the home degree of wage rigidity is set to zero ($\alpha_w = 0$) and the home mean durations of unemployment and vacancies are respectively 6 and 3 months ($\bar{\tau} = 0.95$ and $\bar{\varphi} = 0.50$). In addition, the steady-state level of unemployment is set to 5% ($\bar{s} = 0.0132$). The parameters of the foreign country remain at their benchmark calibration. Finally, in the last specification, $\mathcal{FF}$, the home and foreign labor markets are assumed to be flexible.

Table 3. Calibrations of the labor market rigidities and institutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mathcal{RR}$</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>0.6000</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>0.0235</td>
</tr>
<tr>
<td>$\bar{\tau}$</td>
<td>0.5000</td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>0.2500</td>
</tr>
</tbody>
</table>

3.2 Impulse responses

3.2.1 Home country matching shock

Figure 1 displays the model-based IRFs of the variables to a matching shock in the home country. The solid lines corresponds to the benchmark case (specification $\mathcal{RR}$). The lines with diamonds corresponds to a flexible home labor market and a rigid foreign one (specification $\mathcal{FR}$). Finally, the dashed lines corresponds to flexible home and foreign labor markets (specification $\mathcal{FF}$). This shock results in an increase in the number of successful matches in the home labor market and then a variation of the labor market tightness. At short term, it appears a rise in vacancies, since firms expect more successful matches, and total hours (along both the intensive and extensive margins). The response of posted vacancies becomes negative after 3 periods, probably due to the presence of congestion effects which make firms’ expectations about successful matches less optimistic.

The calibrated value of the job destruction rate is deduced from the equality $s = \bar{\varphi}U/(1 - U)$.

22
Figure 1. Impulse response functions to a positive matching shock in the home country
These congestion effects imply a fast drop of the home labor market tightness. However, this reduction is compensated by a drop of hours worked, implying that real wage rises. This improvement in the labor market structure results in an increase in consumption spending and production and a small decline of capital at short term (due to the substitution between labor and capital) supported with a raise of the capital utilization rate. Consequently, real marginal costs slightly rise during two periods. This implies that home CPI inflation increases in the short run before to being negative, probably because the dynamics of the real marginal cost is driven by the response of hours worked.

By assuming that the home labor market is more flexible (specifications $\mathcal{FR}$ and $\mathcal{FF}$), we obtain that the responses of employment, hours and vacancies return more rapidly to their steady state. The reason is that home economy benefits from successful matches without being constraint in the bargaining process by more restrictive labor institutions. Therefore, households and firms capture more rapidly the positive effects of the shock and the congestion effects happen earlier. This results in a more volatile response of output and home CPI inflation.

Although the home economy absorbs a large part of the matching shock, this latter spills over to the foreign country. The rise of home inflation and output spreads to foreign inflation and also implies an increase in foreign production at time 0. However, the responses of employment, vacancies and real wages depend on the degree of flexibility of the foreign labor market. Indeed, when the two labor markets are rigid (specification $\mathcal{RR}$), the recruiting efforts made by the firms ($V_t$) and the employment rate ($N_t$) are negatively affected by the expansive shock on the home labor market. This negative effect is eased in presence of higher flexibility of the two labor markets. This result means that the rigidity on all labor markets offsets the positive effect of a home matching shock on the foreign economy. We can also point out that the labor adjustment along the intensive margin is not strongly affected by the degree of rigidities on the foreign labor market since the dynamics of hours worked is closely related to the response of output.

3.2.2 Home country destruction rate shock

Figure 2 displays the model-based IRFs to a negative shock on the job destruction rate in the home country. This shock implies an increase in the employment duration resulting, for instance, from a stronger regulation of employment destructions. Since the job duration increases, the expected firms’ profitability of a job rises. However, this effect is compensated by the fact that the recruitments are more costly since flows of unemployed person in the home labor market decrease. Therefore, a reduction of the job destruction rate implies that the number of posted vacancies and the unemployment rate decrease.
Figure 2. Impulse response functions to a negative job destruction shock in the home country.
The labor market tightness rises since the reduction of unemployment is higher than the reduction of vacancies. The induced increase in real wages is magnified by the fall of hours worked. Consequently, consumption and output are positively affected by this shock. When the home labor market is more flexible (specifications $\mathcal{FR}$ and $\mathcal{FF}$), real wages are more volatile, as in the previous shock. This implies a stronger increase in the real marginal cost and then a negative effect on output in the short run.

The foreign country is slightly affected by a negative variation of the home job destruction rate. The results are similar to those of the previous shock. Indeed, foreign CPI inflation and output are positively affected in the short run. However, the responses of the foreign labor market variables are dependent on the degree of labor flexibility on home and foreign countries. Indeed, their positive responses are reinforced under flexible domestic and foreign labor markets. In addition, real wages volatility is also increased.

3.2.3 Home country bargaining shock

Figure 3 displays the model-based IRFs to a negative bargaining shock in the home country. This unexpected decrease in the household’s bargaining power makes a job less profitable. During the wage determination process, domestic households obtain a smaller share of the surplus and domestic firms profits more to the openness of new jobs. The rise in the profitability is naturally expected by firms which post immediately more jobs (vacancies increase). The negative impact on unemployment and the positive effect on the effort of employment’s creation leads the labor market tightness to increase. This indicator of the evolution of the difficulty for households to find a job encourage a wage moderation. Then real wages are under their long-run value. The effects are different from those obtained in the previous sub-section. Precisely, households supply more labor along the intensive and extensive margins, and consumption is negative in the short run. Consequently, total hours increase and households consume less when their relative bargaining power decreases. The rise of total labor implies a substitution effect between labor and capital. This substitution effect results in a decrease in the capital utilization rate and in investment spending. Then, after the initial positive effect on output, it slowly returns to its steady-state value. Finally, inflation has a negative hump-shape responses.

By assuming a flexible home labor market (specifications $\mathcal{FR}$ and $\mathcal{FF}$), we observe that the responses of total employment and real wages are less persistent, as in the previous shock, since firms can adjust more rapidly their labor inputs. Finally, a flexible home labor market implies a smoother responses of consumption and CPI inflation rate.
Figure 3. Impulse response functions to a negative bargaining shock in the home country.
This change in the home economy has spillover effects in the foreign country. The output growth of the home labor market implies that foreign output responds positively at the impact. This positive effect is related to the increase in hours worked. However, foreign firms’ expectations are pessimistic since they post less vacancies at short term and the labor and capital inputs decrease. Unlike the two previous shocks, the degree of flexibility of the two labor markets does not strongly change the sign of the responses of foreign labor market variables. Precisely, introducing flexibility in the home and/or foreign labor market implies that the responses of vacancies and employment are smoother but their response is rather negative. This is compensate by a more volatile response of real wages. This is not surprising since real wages are directly affected by the bargaining power of firms (\(\xi\)) and the duration of unemployment and vacancies (\(\hat{g}\) and \(\hat{v}\)).

4 The welfare consequences of heterogeneous labor markets

The first obstacle to the identification and implementation of European labor markets and redistribution policy reforms is the heterogenous status quo configuration across member countries of labor market and social policies which largely reflects country-specific history and characteristics and, therefore, calls for country-specific reform strategies. Welfare systems designed decades ago may well need to be redesigned in light of new demographic and technological trends, and changes in the structure of market and non-market economic interactions. But while the relevant changes are broadly similar in all industrialized economies, their impact and implications are quantitatively different across euro area member countries characterized by still very different economic and social structure, and by different policy configurations.

Our model is clearly not designed to discuss the labor markets’ institutional configuration neither the degree of regulation. However, the matching model allows to analyze three transmission mechanisms through which labor market reforms can act: (i) a change in the separation process (\(s\)), (ii) a change in the matching technology (\(\tilde{m}\)) and (iii) a change in the bargaining power (\(\tilde{\xi}\)).

We now study the welfare implications of labor market reforms on one side of the union (the domestic country) under different configurations. We make a permanent increase in these key parameters on the labor market at time 1 in order to reduce the unemployment rate from 7.9% to 5%.

4.1 Measuring welfare gains

As the model provides with a fully micro-founded utility criterion, we build on a welfare analysis. In each country \(j\), we define the welfare associated with the time-invariant stochastic allocation
conditional on the initial state \((\text{init})\) of the economy in period 0 as

\[
W_{j;0}^{\text{init}} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{\sigma_c}{\sigma_c - 1} \left( C_{j;t}^{\text{init}} - bC_{j;t-1}^{\text{init}} \right) \frac{\sigma_c - 1}{\sigma_c} - v \lambda_{j;0}^{\text{init}} \left( \frac{\sigma_h}{\sigma_h + 1} \left( H_{j;t}^{\text{init}} \right) \frac{\sigma_h - 1}{\sigma_h} \right) \right]
\]

where \(C_{j;t}^{\text{init}}, N_{j;t}^{\text{init}}\) and \(H_{j;t}^{\text{init}}\) denote contingent plans for consumption, employment and individual hours under the initial steady-state. Similarly, we define the conditional welfare associated with a new steady-state \((\text{final})\) as

\[
W_{j;0}^{\text{final}} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{\sigma_c}{\sigma_c - 1} \left( C_{j;t}^{\text{final}} - bC_{j;t-1}^{\text{final}} \right) \frac{\sigma_c - 1}{\sigma_c} - v \lambda_{j;0}^{\text{final}} \left( \frac{\sigma_h}{\sigma_h + 1} \left( H_{j;t}^{\text{final}} \right) \frac{\sigma_h - 1}{\sigma_h} \right) \right]
\]

Like Lucas (1987), in the case of the welfare costs of fluctuations, we express the welfare gains to a new steady-state in readily interpretable economic terms: The gain to the new steady state is then given by the fraction of consumption stream an individual should be given in order to compensate the fact that she has to switch from a initial steady state to a new one. In noting \(W_{j;0}^{\text{init}}, W_{j;0}^{\text{final}}\), we measure the welfare gain in percentage points, \(\text{Gain}_j = \lambda_j \times 100\), by solving for \(\lambda_j\) the following equation

\[
W_{j;0}^{\text{final}} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{\sigma_c}{\sigma_c - 1} \left( C_{j;t}^{\text{final}} - bC_{j;t-1}^{\text{final}} \right) \frac{\sigma_c - 1}{\sigma_c} (1 + \lambda_j) \frac{\sigma_c - 1}{\sigma_c} - v \lambda_{j;0}^{\text{init}} \left( \frac{\sigma_h}{\sigma_h + 1} \left( H_{j;t}^{\text{final}} \right) \frac{\sigma_h - 1}{\sigma_h} \right) \right] - W_{j;0}^{\text{init}},
\]

which gives

\[
\lambda_j = \frac{W_{j;0}^{\text{final}} + W_{j;0}^{\text{init}}}{W_{j;0}^{\text{init}} + W_{j;0}^{\text{final}}} \frac{\sigma_c - 1}{\sigma_c} - 1.
\]

### 4.2 Impacts of labor market reforms

We now clarify the incentives for unilateral structural reforms, leading changes in structural parameters in home country but leaving those in foreign country unchanged. Assuming that the model’s labor market parameters \((\bar{s}, \bar{m}, \bar{\xi})\) can be affected through various structural policies, we seek to determine the direction in which a country has an incentive to direct its reforms, when the social planner aims at maximizing the social welfare. In particular, we assume that the government in domestic country wants to reduce the unemployment rate from 7.9% to 5% whereas this rate is still unchanged in the foreign country.\(^{23}\)

#### 4.2.1 Benchmark case

In the first line of Table 4, we present the welfare gains of each country of changing one of the three labor market parameters \((\bar{s}, \bar{m}, \bar{\xi})\). In the first block-column, we assume that the job destruction rate

\(^{23}\)Technically, we assume a permanent shock on a structural parameter and we make deterministic simulations over 500 periods.
(s) decreases from 2.14% to 1.33% in order to reach a domestic unemployment rate of 5%, between time 0 and time 1. There are many reasons for this exogenous change in s. For instance, this could be a reduction of the share of time-limited contracts relatively to the number of permanent contracts or an increase in the firing cost. This higher rigidity in the home labor market implies a welfare improvement. Indeed, the aggregate welfare increases by 3.446% in the domestic country and 1.556% in the foreign one. To gauge these welfare results more concretely, we note that European personal consumption expenditures were about 15300€ per person in 2006; thus this would permanently increase welfare by about 527€ per person in the home country and 236€ in the foreign country.

Table 4. Welfare gains (in %) of labor market reforms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>̇s</th>
<th>̇m</th>
<th>ť</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Home</td>
<td>Foreign</td>
<td>Home</td>
</tr>
<tr>
<td>Benchmark</td>
<td>3.4457</td>
<td>1.5460</td>
<td>3.3931</td>
</tr>
<tr>
<td>Sensitivity to:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Monetary policy rules</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>3.4438</td>
<td>1.5439</td>
<td>3.3915</td>
</tr>
<tr>
<td>R2</td>
<td>3.4470</td>
<td>1.5475</td>
<td>3.3941</td>
</tr>
<tr>
<td>R3</td>
<td>3.4454</td>
<td>1.5457</td>
<td>3.3928</td>
</tr>
<tr>
<td>R4</td>
<td>3.4454</td>
<td>1.5457</td>
<td>3.3935</td>
</tr>
<tr>
<td>(ii) Wage rigidity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>̂αw = 0 and ̂α_w* = 0.6</td>
<td>3.4456</td>
<td>1.5458</td>
<td>3.3930</td>
</tr>
<tr>
<td>̂αw = ̂α_w* = 0</td>
<td>3.4459</td>
<td>1.5459</td>
<td>3.3933</td>
</tr>
</tbody>
</table>

In the second block-column, the decrease in the home unemployment rate from 7.9% to 5% is the result of an increase in the number of successful matches (given by ̇m). This means that the economy benefits to more efficient matches, which could be due to an improvement of the services offered by employment agencies. We obtain similar results than previously. Precisely, by increasing ̇m from 0.37 to 0.60, the welfare gain amounts to 3.393% in the home country and 1.530% in the
foreign one. Therefore, the permanent increase in welfare would be of 519€ per person in the home country and 234€ per person in the foreign country. Our model suggests that the welfare gain is slightly higher when we reach the new unemployment rate by decreasing the job separation rate rather than by increasing the efficiency of the labor match.

Finally, in the third block-column, we reach a decrease in the domestic unemployment rate to 5% by reducing the households’ bargaining power ($\xi$). This could result from a smaller weight of the trade unions in the bargaining process since $\xi$ is reducing from 60% to 40% at time 1. In this case, we obtain that the reduction of the home unemployment rate implies that home households bear a cost in terms of welfare. The reason may come from the fact that the Hosios’s condition stops to be verified. Since $\xi$ moves from the value that allows the equilibrium unemployment to be socially-efficient, trading costs now appear. The departure from the first-best allocation can then induce large welfare costs. Precisely, the home welfare cost reaches 0.371%, meaning that the permanent decrease in welfare would be of 57€ per person. The intuition of this result has been suggested in the previous section. Indeed, a large decrease in the household’s bargaining power implies that households have incentives to increase their labor supply but since real wages decrease, it also results in a fall of consumption.

Consequently, at time 1 the modification of $\xi$ implies that total hours increase and households consume less in comparison with their behavior at time 0. This means that a high change of the labor power struggle in negotiations can be sightly costly for the home economy. On the contrary, the foreign country profits from the change in the structure of the home economy since the welfare gains are equal to 0.308%, meaning that the permanent increase in welfare would be of 47€ per person.

4.2.2 Sensitivity exercises

Until now, we have considered that home and foreign countries are perfectly symmetric and their labor market is rigid. Here, we investigate whether a modification of the economic environment can modify our previous results.

We first check the robustness of our findings under different monetary policy rules. We consider four type of rules: (R1) an inflation targeting rule ($\psi_z = 1.5, \psi_R = \psi_y = 0$), (R2) a stronger reaction to inflation than in the benchmark rule ($\psi_R = 0.85, \psi_x = 2, \psi_y = 0.125$), (R3) a non interest rate smoothing rule ($\psi_R = 0, \psi_x = 1.5, \psi_y = 0.125$), and (R4) the benchmark rule but including also the unemployment rate (with the same weight as output gap). Although monetary policy is neutral in the long run, we are interested in its role during the transition period. As shown in the second block
of rows of Table 4, there are no significant differences between these specifications, meaning that the way the monetary authorities conduct their policy slightly influences the amount of the welfare gain. We can only say that rule \( R2 \) is the preferable rule since it gives the highest welfare gains for the two countries and the worst specification is the pure inflation targeting rule. Interestingly, we observe that introducing the unemployment rate in the rule does not ameliorate the welfare, except when \( \tilde{m} \) is modified.

Table 5. Sensitivity to the unemployment rate target

<table>
<thead>
<tr>
<th>( \bar{U}_{final} )</th>
<th>Parameter</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \bar{s} )</td>
<td>( \tilde{m} )</td>
<td>( \bar{\xi} )</td>
<td>( \Theta )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Home</td>
<td>Foreign</td>
<td>Home</td>
<td>Foreign</td>
<td>Home</td>
<td>Foreign</td>
<td>Home</td>
<td>Foreign</td>
</tr>
<tr>
<td>7.0%</td>
<td>1.0602</td>
<td>0.4819</td>
<td>1.0361</td>
<td>0.4746</td>
<td>0.0431</td>
<td>0.1474</td>
<td>0.0428</td>
<td>0.1475</td>
</tr>
<tr>
<td>6.0%</td>
<td>2.2306</td>
<td>1.0085</td>
<td>2.2035</td>
<td>1.0024</td>
<td>-0.0515</td>
<td>0.2642</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.0%</td>
<td>3.4457</td>
<td>1.5460</td>
<td>3.3931</td>
<td>1.5304</td>
<td>-0.3715</td>
<td>0.3081</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Second, we investigate whether our results in terms of welfare are dependant on the degree of wage rigidities in the home and foreign countries. Whereas in the benchmark calibration, the degrees of home and foreign wage rigidity are equal to 0.6, we consider now (i) the case of no real wages rigidity only in the home country (\( \alpha_w = 0 \) and \( \alpha_w^* = 0.6 \)) and (ii) the case of no real wages rigidity in all the union (\( \alpha_w = \alpha_w^* = 0 \)). We obtain that the degree of wage rigidities has negligible effect on the amount of welfare gain/cost. In addition, by focusing on a change of \( \bar{s} \) and \( \tilde{m} \), we show that more flexible labor markets in the whole monetary union increases the welfare gains of the home country. This increase is conditional to the degree of rigidity on the foreign labor market since this gain is slightly smaller than its benchmark counterpart if the labor market in the foreign country is not flexible. The foreign welfare gain is smaller than its benchmark counterpart as soon as we introduce flexibility in the home labor market. This means that countries should have the same degree of flexibility on the labor market to fully profit from the effects of structural reforms (by moving \( \bar{s} \) or \( \tilde{m} \)), especially when this degree is weak. Finally, when the households’ bargaining power (\( \bar{\xi} \)) is hit, the higher the flexibility on the labor markets the higher the countries’ welfare losses. Indeed, we may suspect that small values of \( \alpha_w \) and \( \alpha_w^* \) imply higher decreases in real wages, such that households’ consumption can be affected in a large extent.
Our previous results have been obtained by assuming that the home economy is confronted with a change in its labor institution resulting in a decrease in the unemployment rate from 7.9% to 5%. However, the choice of the value on the unemployment rate target could be arguable. Table 5 reports the welfare gains of modifying \( s, \tilde{m} \) or \( \bar{\xi} \) for two other configurations. In the first line, we assume that the unemployment rate moves from 7.9% to 7.0% between time 0 and time 1. In the second line, the unemployment rate moves from 7.9% to 6.0%. The last line reminds the benchmark results.

For \( s \) or \( \tilde{m} \), we show that the smaller the unemployment rate target the higher the welfare gain for both the home and the foreign country. This suggests that the welfare gains can be rather large when the decrease in the home unemployment rate is sufficient enough (around 500\( \varepsilon \) and 200\( \varepsilon \) for the domestic and the foreign country, respectively).

It is also interesting to note that the value of the welfare cost depends on the magnitude of the variation of the households’ bargaining power. Indeed, when the reforms aim at reducing the unemployment rate from 7.9% to 7%, \( \bar{\xi} \) moves from 60% to 55% and the permanent increase in home welfare amounts to 7\( \varepsilon \) per person. Therefore, this suggests that reaching a smaller unemployment rate by increasing the households’ bargaining strength have mitigate and negligible effects on the social welfare in the home country.

In order to widen the scope of our discussion, we also assume that the domestic government decides to modify the level of the unemployment benefits (fourth block-column). Precisely, we redo the previous exercise by assuming that the ratio \( \Theta/W \) decreases from 0.5 to 0.1. This means that the home unemployment rate target reaches 7%. In this case, the home welfare gain amounts to 0.025\% and the foreign one amounts to 0.147\%. This suggests that reducing the home unemployment benefits has insignificant effect in the home country (4\( \varepsilon \) per person) whereas the foreign country profits from a welfare gain of 22\( \varepsilon \) per person. The interpretation of this result is close to the one done for \( \bar{\xi} \), since \( \Theta \) plays a similar role on the dynamics of wages and employment. Domestic total hours increase and households consume less, due to the fall in real wages.

We have finally checked that the previous findings do not depend on the degree of rigidity on the home final goods markets. We assume that there is no price indexation (\( \gamma_p = 0 \)) and that the average duration of price contracts is less than three months. Similar results have been highlighted, meaning that the home and foreign welfare gains are higher for a variation of \( s \) rather than a variation of \( \tilde{m} \). In addition, the home country bears a welfare cost when the decrease in the unemployment rate is implied by a variation of \( \bar{\xi} \).


5 Conclusion

The bad performance of the euro area labor market influences the well being of both the economy and society. The literature has attempted to incorporate labor market search and matching frictions in closed-economy representations of the area, however, in a single currency union, an additional difficulty due to the heterogeneity of the members appears. In this paper, we studied the role of labor markets heterogeneity in a monetary union and more particularly we investigated what are the welfare gains/costs of reforms on the labor market for each members of the area. We developed a medium-scale two-country model representing a currency union characterized by price and wage stickiness, real rigidities and labor market frictions.

Assuming that the model’s labor market parameters can be affected through various structural policies, we seek to determine the direction in which a country has an incentive to direct its reforms, when the social planner aims at maximizing the social welfare. Precisely, we focus on three parameters related to the labor market structure: the job destruction rate, the scaling parameter in the matching function, and the households’ bargaining power. We assume a permanent shock on these structural parameters in order to achieve a lower home unemployment rate and we compute the social welfare gains/costs.

Several results emerge. First, the choice of the instruments to direct a reform (aiming at reducing the home unemployment rate) is not neutral on the amount of the social welfare gain in the union. Indeed, the welfare gain is slightly higher by decreasing the job separation rate rather than by increasing the efficiency of the labor match. Nevertheless, using these two instruments would permanently increase the welfare by about 500€ per person in the home country and 200€ in the foreign country. On the contrary, an decrease in the households’ bargaining power leads to mitigate and negligible effects on the social welfare. Targeting a too low unemployment rate can provide welfare deteriorations in the home country. This results from higher total hours and smaller consumption. In the same time, the foreign country profits from this reform. Second, the way the monetary authorities conduct their policy slightly influences the amount of the welfare gain. For instance, using an interest rate rule with a strong reaction to inflation allows to improve the welfare in both home and foreign country. Third, countries should have the same degree of wage flexibility on their labor market to fully profit from the effects of structural reforms, especially when this degree is weak.
References


