Sovereign Risk and Financial Risk

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\textit{The Economics of Sovereign Debt and Default}
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DISCLAIMER

The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System.
Introduction

Significant amount of research on the country-specific determinants of sovereign default risk.
(Arellano [2008]; Yue [2010]; Mendoza & Yue [2012])

We study the relationship between sovereign default risk, local economic conditions, and global financial risk.
(Borri & Verdelhan [2011]; Longstaff, Pan, Pedersen & Singleton [2011])
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Empirics:
- Construct an extensive micro-level dataset of sovereign bond spreads that is matched with country-specific economic and financial variables.
- Examine the extent to which movements in sovereign bond spreads are driven by local vs. global risk factors.

Theory:
- Build a multi-country equilibrium model of sovereign debt and endogenous default:
  - Risk-averse global financial intermediary (i.e., lender)
  - Endogenous debt dynamics
  - Optimal default
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**Data Sources & Methods**

- Use information on virtually all $-denominated and €–denominated sovereign bonds traded in the secondary market.
- Construct micro-level credit spreads using synthetic risk-free securities priced off zero-coupon U.S. Treasuries and German bunds:

\[
P_{it}^f[k] = \sum_{s=1}^{S} C(s)D(t_s); \quad D(t) = \exp[-r_t^f t]
\]

- Sovereign bond-level datasets:
  - **Coverage**: 51 countries ($-bonds); 38 countries (€-bonds)
  - **Credit quality**: investment- and speculative-grade bonds
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  - **Credit quality**: investment- and speculative-grade bonds
### SAMPLE CHARACTERISTICS

($-$denominated bonds)

<table>
<thead>
<tr>
<th>Bond Characteristic</th>
<th>Mean</th>
<th>StdDev</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of bonds per country/month</td>
<td>8.3</td>
<td>40.4</td>
<td>1</td>
<td>4</td>
<td>676</td>
</tr>
<tr>
<td>Maturity at issue (years)</td>
<td>13.8</td>
<td>8.8</td>
<td>1</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Term to maturity (years)</td>
<td>8.0</td>
<td>8.1</td>
<td>0.1</td>
<td>5.6</td>
<td>99.9</td>
</tr>
<tr>
<td>Duration (years)</td>
<td>5.4</td>
<td>3.6</td>
<td>0.1</td>
<td>4.8</td>
<td>18.8</td>
</tr>
<tr>
<td>Credit rating (Moody’s)</td>
<td>-</td>
<td>-</td>
<td>Ca</td>
<td>A1</td>
<td>Aaa</td>
</tr>
<tr>
<td>Coupon rate (pct.)</td>
<td>4.67</td>
<td>3.58</td>
<td>0.00</td>
<td>4.95</td>
<td>13.6</td>
</tr>
<tr>
<td>Nominal yield to maturity (pct.)</td>
<td>5.82</td>
<td>3.51</td>
<td>0.17</td>
<td>5.28</td>
<td>35.8</td>
</tr>
<tr>
<td>Credit spread (bps.)</td>
<td>216</td>
<td>282</td>
<td>-100</td>
<td>124</td>
<td>2,988</td>
</tr>
</tbody>
</table>
**Sample Characteristics**

(€-denominated bonds)

<table>
<thead>
<tr>
<th>Bond Characteristic</th>
<th>Mean</th>
<th>StdDev</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of bonds per country/month</td>
<td>10.2</td>
<td>18.6</td>
<td>1</td>
<td>4</td>
<td>105</td>
</tr>
<tr>
<td>Maturity at issue (years)</td>
<td>6.82</td>
<td>7.9</td>
<td>1</td>
<td>5</td>
<td>53</td>
</tr>
<tr>
<td>Term to maturity (years)</td>
<td>6.4</td>
<td>6.9</td>
<td>0.1</td>
<td>4.3</td>
<td>50</td>
</tr>
<tr>
<td>Duration (years)</td>
<td>5.2</td>
<td>5.1</td>
<td>0.1</td>
<td>3.9</td>
<td>32</td>
</tr>
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<td>Credit rating (Moody’s)</td>
<td>-</td>
<td>-</td>
<td>Ca</td>
<td>Aa2</td>
<td>Aaa</td>
</tr>
<tr>
<td>Coupon rate (pct.)</td>
<td>3.18</td>
<td>3.33</td>
<td>0.00</td>
<td>3.25</td>
<td>12.6</td>
</tr>
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<td>Nominal yield to maturity (pct.)</td>
<td>4.55</td>
<td>2.58</td>
<td>0.12</td>
<td>4.18</td>
<td>31.8</td>
</tr>
<tr>
<td>Credit spread (bps.)</td>
<td>120</td>
<td>250</td>
<td>-99</td>
<td>286</td>
<td>2,920</td>
</tr>
</tbody>
</table>
SOVEREIGN BOND SPREADS
(By Type of Currency)

Credit spreads on dollar-denominated sovereign bonds

Credit spreads on euro-denominated sovereign bonds
Sovereign Bond Spreads
(By Credit Quality)

Average credit spread on investment-grade sovereign bonds

Average credit spread on speculative-grade sovereign bonds


**Global Financial Risk Factor**

- **Excess Bond Premium (EBP):** an indicator of financial distress based on U.S. nonfinancial corporate bond spreads: ([Gilchrist & Zakrajšek [2012]](#))
  
  - EBP contains substantial predictive power for many indicators of economic activity at business cycle frequencies
  - EBP comoves closely with balance sheet conditions of key financial intermediaries
  - EBP provides a useful summary measure of the effective risk-bearing capacity of the financial sector

- Global market return ($R^*$).

- As an alternative to the EBP, also consider the VXO (i.e., the option-implied volatility of the S&P 100).
**GLOBAL FINANCIAL RISK FACTOR**

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EXCESS BOND PREMIUM

Percentage points

Monthly


-2 -1 0 1 2 3 4

Percentage points

Monthly


-2 -1 0 1 2 3 4

Percentage points
SOVEREIGN BOND SPREADS AND THE EBP

(Investment-Grade Countries)
SOVEREIGN BOND SPREADS AND THE EBP

(Speculative-Grade Countries)
STOCK RETURNS
(By Credit Quality)
Panel Data Analysis

Regression specification:

\[ \Delta s_{it} = \beta_1' F_t + \beta_2' Z_{it} + \eta_i + \epsilon_{it} \]

- \( \Delta s_{it} \) = change in the sovereign spread for country \( i \)
- \( F_t \) = global risk factors (i.e., \( \Delta EBP \), \( R^* \), \( \Delta VXO \))
- \( Z_{it} \) = country-specific economic fundamentals (stock returns, equity volatility, \( \Delta FX \), \( \Delta FX\)-volatility, credit ratings, FX-regime)
- Allow coefficient vector \( \beta_1 \) to differ between investment- and speculative-grade countries.
# Sovereign Bond Spreads and Financial Risk

(Global Risk Factors: $\Delta EBP$ and $R^*$)

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Coef.</th>
<th>SE</th>
<th>Coef.</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta EBP_t \times \text{SG}$</td>
<td>0.2689</td>
<td>0.1103</td>
<td>0.2132</td>
<td>0.1057</td>
</tr>
<tr>
<td>$\Delta EBP_t \times \text{IG}$</td>
<td>0.1323</td>
<td>0.0546</td>
<td>0.0562</td>
<td>0.0584</td>
</tr>
<tr>
<td>$R^*_t \times \text{SG}$</td>
<td>-0.0034</td>
<td>0.0008</td>
<td>-0.0046</td>
<td>0.0012</td>
</tr>
<tr>
<td>$R^*_t \times \text{IG}$</td>
<td>0.0000</td>
<td>0.0004</td>
<td>-0.0009</td>
<td>0.0007</td>
</tr>
<tr>
<td>$R_{it}$</td>
<td>-0.0008</td>
<td>0.0002</td>
<td>-0.0007</td>
<td>0.0003</td>
</tr>
<tr>
<td>$VOL_{it}$</td>
<td>0.0031</td>
<td>0.0012</td>
<td>0.0025</td>
<td>0.0018</td>
</tr>
<tr>
<td>$R^2$ (within)</td>
<td>0.1783</td>
<td></td>
<td>0.2321</td>
<td></td>
</tr>
</tbody>
</table>
Construct **investment-grade** (IG) and **speculative-grade** (SG) portfolios of monthly sovereign credit spreads and stock returns.

9-variable VAR(6):

- **Fast-moving block**: IG sovereign bond spread; SG sovereign bond spread; IG equity return; SG equity return; 10-year (real) Treasury yield; oil return; EBP
- **Slow-moving block**: global IP growth; global CPI inflation
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GLOBAL IMPLICATIONS OF A FINANCIAL SHOCK

Excess bond premium
Percentage points

Credit spread - IG
Percentage points

Credit spread - SG
Percentage points

Cumulative stock return - IG
Percent

Cumulative stock return - SG
Percent

10-year real Treasury yield
Percentage points

Oil price
Percent

Global industrial production
Percent

Global CPI
Percent
Equilibrium Model of Sovereign Default

- Endowment economy model with a continuum of sovereign countries.
- Risk-averse financial intermediary (i.e., “deep-pocket” foreign lender) with an exogenous pricing kernel.  
  (Zhang [2005])
- Optimal default entails a permanent loss of output and financial exclusion (autarky).  
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Each country’s income follows a random walk process:

\[ Y' = Y \exp(\varepsilon') \]

where

\[ \varepsilon' = \nu' + \eta' \]

- \( \nu \) is an i.i.d. country-specific shock with
  \[ \nu' \sim N\left(-\frac{1}{2} \sigma_{\nu}^2, \sigma_{\nu}^2\right) \]

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Optimization Problem

Countries

- Maximize utility of consumption ($\ln C'$)
- Borrow from the global financial intermediary by issuing one-period discount bonds.

$q(Y, \eta, B') = \text{price of one unit of debt}$

Bellman equation:

$$V^{nd}(Y, \eta, B) = \max_{B', C} \ln C + \beta E \max \left\{ V^{nd}(Y', \eta', B'), V^d(Y', \eta') \right\}$$

subject to

$$C = Y + q(Y, \eta, B')B' - B$$
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VALUE OF DEFAULTING

- After defaulting, country is in autarky:

\[
V^d(Y, \eta) = \ln C + \beta E \left\{ V^d(Y', \eta') \right\}
\]

subject to

\[
C = (1 - \phi) Y
\]

where \(0 < \phi < 1\) is the fraction of output lost due to default.

- Given the income process:

\[
V^d(Y, \eta) = \alpha_j + \left[ \frac{1}{1 - \beta} \right] \ln Y
\]

where the state-specific \(\alpha_j\) solves the linear equation:

\[
\alpha_j = \ln[1 - \phi] + \beta E \left[ \alpha' + \frac{1}{1 - \beta} \eta' \right] - \frac{1}{2 (1 - \beta)} \sigma^2
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\]
**Value Function**

- **Conjecture:** \( q \) is independent of \( Y \) and normalize by income

\[
\begin{aligned}
    c &= \frac{C}{Y}; &
    d &= \frac{B'}{Y}; &
    b' &= \frac{d}{\exp(\nu' + \eta')} = \frac{B'}{Y'}
\end{aligned}
\]

- Value function satisfies

\[
V^{nd}(Y, \eta, b) = \frac{1}{1 - \beta} \ln(Y) + f(\eta, b)
\]

where

\[
\begin{aligned}
f(\eta, b) &= \max_d \ln \left[ 1 + q(\eta, d)d - b \right] \\
&\quad + \beta E \max \left\{ f \left( \eta', \frac{d}{\exp(\nu' + \eta')} \right), \alpha' \right\} \\
&\quad + \frac{\beta}{1 - \beta} \left( E(\eta') - \frac{\sigma^2_\nu}{2} \right)
\end{aligned}
\]
**Value Function**

- **Conjecture:** $q$ is independent of $Y$ and normalize by income

\[
\begin{align*}
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\[
+ \frac{\beta}{1 - \beta} \left( E(\eta') - \frac{\sigma^2_{\nu}}{2} \right)
\]
Value-matching condition implies a state-specific cutoff $\bar{\nu}$:

$$\alpha = f \left( \eta, \frac{d}{\exp(\bar{\nu} + \eta)} \right)$$

Equivalently, we can define the normalized cutoff:

$$\bar{\nu}(d) = \frac{\ln(d) - \eta - \ln f^{-1}(\eta, \alpha_d) - 0.5\sigma^2_{\nu}}{\sigma_{\nu}}$$

where $f^{-1}$ is the inverse value function.
**Default Choice**

- Value-matching condition implies a state-specific cutoff $\bar{\nu}$:

  $$\alpha = f \left( \eta, \frac{d}{\exp(\bar{\nu} + \eta)} \right)$$

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NORMALIZED VALUE FUNCTION

- Using the default cutoff:

\[ f(\eta, b) = \max_d \ln [1 + q(\eta, d)d - b] \]

\[ + \beta E \left[ \Phi(\bar{\nu}) \alpha' + \int_{\nu' > \bar{\nu}} f \left( \eta', \frac{d}{\exp(\nu' + \eta')} \right) d\Phi(\nu') \right] \]

\[ + \frac{\beta}{1 - \beta} \left[ E(\eta') - \frac{\sigma^2_\nu}{2} \right] \]

- Straightforward to evaluate integral with the log-normal process.
EXOGENOUS RISK-AVERSE LENDER

- Let $m = \ln M$ denote the SDF of the financial intermediary:
  
  $$m_{t+1} = \ln \lambda - \gamma_t [e_{t+1} - \kappa (e_{t+1})]$$

- The bond price $q$ satisfies:
  
  $$q (\eta, d) = E [m [1 - \Phi (\bar{\nu} (d))]]$$

  so that

  $$\frac{\partial q}{\partial d} = -E \left[ m \phi (\bar{\nu} (d)) \frac{1}{\sigma_{\nu} d} \right] < 0$$
**Exogenous Risk-Averse Lender**

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  \[ q(\eta, d) = E \left[ m \left[ 1 - \Phi(\bar{\nu}(d)) \right] \right] \]

so that
\[ \frac{\partial q}{\partial d} = -E \left[ m\phi(\bar{\nu}(d)) \frac{1}{\sigma_{\nu}d} \right] < 0 \]
Global Economy: 1,000 countries; 1,200 years

Annual model with $\beta = 0.9$

Global income growth process:

$$\eta_t = \rho \eta_{t-1} + e_t$$

with $\rho = 0.2$ and $\sigma_e = 0.01$ (discretized).

Country-specific i.i.d. income process with $\sigma_\nu = 0.05$

Conumption loss in default $\phi = 0.02$
Exogenous Pricing Kernel

- Pricing kernel process:

\[
\ln m_{t+1} = \ln \lambda - \gamma_t e_{t+1} + \frac{1}{2} \gamma_t^2 \sigma_e^2
\]

- Risk Neutral: \( \gamma_t = 0 \)
- Risk Averse: \( \gamma_t = \gamma_0 \)
- Time-Varying Risk Aversion: \( \gamma_t = \gamma_0 \exp (\theta_t) \)
  - \( \theta_t \) = “risk” shock
- Calibration:
  - \( \lambda = 0.95 \)
  - \( \gamma_0 = 50 \)
  - \( \theta_t = \ln 0.8 \) with \( \Pr = 0.8 \) and \( \ln 1.6 \) with \( \Pr = 0.2 \)
**Exogenous Pricing Kernel**

- **Pricing kernel process:**
  
  \[
  \ln m_{t+1} = \ln \lambda - \gamma_t \epsilon_{t+1} + \frac{1}{2} \gamma_t^2 \sigma_e^2
  \]

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  - \( \lambda = 0.95 \)
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**EXOGENOUS PRICING KERNEL**

- Pricing kernel process:
  \[
  \ln m_{t+1} = \ln \lambda - \gamma_t e_{t+1} + \frac{1}{2} \gamma_t^2 \sigma_e^2
  \]

- **Risk Neutral**: \( \gamma_t = 0 \)
- **Risk Averse**: \( \gamma_t = \gamma_0 \)
- **Time-Varying Risk Aversion**: \( \gamma_t = \gamma_0 \exp (\theta_t) \)
  - \( \theta_t \) = “risk” shock

- **Calibration**:
  - \( \lambda = 0.95 \)
  - \( \gamma_0 = 50 \)
  - \( \theta_t = \ln 0.8 \) with \( \Pr = 0.8 \) and \( \ln 1.6 \) with \( \Pr = 0.2 \)
### Baseline Model Simulations

(Micro-Level Moments)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Risk Neutral</th>
<th>Risk Averse</th>
<th>Risk Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$std(c_i)$</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>$mean(s_i)$</td>
<td>0.037</td>
<td>0.081</td>
<td>0.082</td>
</tr>
<tr>
<td>$std(s_i)$</td>
<td>0.042</td>
<td>0.063</td>
<td>0.070</td>
</tr>
<tr>
<td>$corr(s_i, c_i)$</td>
<td>-0.235</td>
<td>-0.271</td>
<td>-0.265</td>
</tr>
<tr>
<td>$corr(s_i, q)$</td>
<td>-0.975</td>
<td>-0.988</td>
<td>-0.976</td>
</tr>
<tr>
<td>$corr(s_i, \eta)$</td>
<td>-0.816</td>
<td>-0.900</td>
<td>-0.837</td>
</tr>
<tr>
<td>$corr(s_i, \gamma)$</td>
<td>-</td>
<td>-</td>
<td>0.388</td>
</tr>
<tr>
<td>$corr(s_i, \varepsilon_i)$</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>$corr(c_i, \varepsilon_i)$</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>
## Baseline Model Simulations

*(Aggregate Moments)*

<table>
<thead>
<tr>
<th>Moment</th>
<th>Risk Neutral</th>
<th>Risk Averse</th>
<th>Risk Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{mean}(b)$</td>
<td>0.285</td>
<td>0.284</td>
<td>0.284</td>
</tr>
<tr>
<td>default freq. (%)</td>
<td>0.028</td>
<td>0.028</td>
<td>0.041</td>
</tr>
<tr>
<td>$\text{corr}(s, \eta)$</td>
<td>-0.837</td>
<td>-0.911</td>
<td>-0.857</td>
</tr>
<tr>
<td>$\text{corr}(s, \gamma)$</td>
<td>-</td>
<td>-</td>
<td>0.397</td>
</tr>
<tr>
<td>$\text{corr}(s, c)$</td>
<td>-0.832</td>
<td>-0.820</td>
<td>-0.761</td>
</tr>
<tr>
<td>$\text{corr}(s, b)$</td>
<td>0.833</td>
<td>0.757</td>
<td>0.689</td>
</tr>
<tr>
<td>$\text{corr}(s, d)$</td>
<td>-0.592</td>
<td>-0.565</td>
<td>-0.553</td>
</tr>
<tr>
<td>$\text{mean}(s)$ (%)</td>
<td>0.037</td>
<td>0.080</td>
<td>0.082</td>
</tr>
<tr>
<td>$\text{std}(s)$</td>
<td>0.041</td>
<td>0.063</td>
<td>0.069</td>
</tr>
<tr>
<td>$\text{mean}(rb)$ (%)</td>
<td>5.27</td>
<td>5.31</td>
<td>5.31</td>
</tr>
<tr>
<td>$\text{std}(rb)$</td>
<td>0.136</td>
<td>0.122</td>
<td>0.159</td>
</tr>
</tbody>
</table>

**NOTE:** The aggregate bond spreads are computed using the sovereign bond portfolio weighted by $d_i$; $rb$ denotes the holding-period return on such a sovereign bond portfolio.
**Baseline Model Simulations**

(Risk-Neutral Lender)
**Baseline Model Simulations**

(Risk-Averse Lender)
**Baseline Model Simulations**

(Uncorrelated Risk Shocks)
**Asymmetric Costs**

(Micro-Level Moments)

<table>
<thead>
<tr>
<th>Moment</th>
<th>Risk Neutral</th>
<th>Risk Averse</th>
<th>Risk Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$std(c_i)$</td>
<td>0.032</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>$mean(s_i)$ (%)</td>
<td>0.110</td>
<td>0.145</td>
<td>0.106</td>
</tr>
<tr>
<td>$std(s_i)$</td>
<td>0.098</td>
<td>0.086</td>
<td>0.087</td>
</tr>
<tr>
<td>$corr(s_i, c_i)$</td>
<td>-0.360</td>
<td>-0.528</td>
<td>-0.437</td>
</tr>
<tr>
<td>$corr(s_i, q)$</td>
<td>-0.657</td>
<td>-0.739</td>
<td>-0.879</td>
</tr>
<tr>
<td>$corr(s_i, \eta)$</td>
<td>-0.070</td>
<td>-0.490</td>
<td>-0.694</td>
</tr>
<tr>
<td>$corr(s_i, \gamma)$</td>
<td>-</td>
<td>-</td>
<td>0.199</td>
</tr>
<tr>
<td>$corr(s_i, \varepsilon_i)$</td>
<td>-0.0006</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$corr(c_i, \varepsilon_i)$</td>
<td>-0.0004</td>
<td>-0.003</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

**NOTE:** Asymmetric costs: $\phi = \delta_0 + \delta_1 e_t$, $\delta_0, \delta_1 > 0$. 
### Asymmetric Costs

*(Aggregate Moments)*

<table>
<thead>
<tr>
<th>Moment</th>
<th>Risk Neutral</th>
<th>Risk Averse</th>
<th>Risk Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean(b)</td>
<td>0.547</td>
<td>0.537</td>
<td>0.530</td>
</tr>
<tr>
<td>default freq. (%)</td>
<td>0.120</td>
<td>0.048</td>
<td>0.042</td>
</tr>
<tr>
<td>corr(s, η)</td>
<td>-0.115</td>
<td>-0.662</td>
<td>-0.789</td>
</tr>
<tr>
<td>corr(s, γ)</td>
<td>-</td>
<td>-</td>
<td>0.225</td>
</tr>
<tr>
<td>corr(s, c)</td>
<td>-0.112</td>
<td>-0.564</td>
<td>-0.622</td>
</tr>
<tr>
<td>corr(s, b)</td>
<td>0.323</td>
<td>0.389</td>
<td>0.459</td>
</tr>
<tr>
<td>corr(s, d)</td>
<td>0.102</td>
<td>-0.401</td>
<td>-0.546</td>
</tr>
<tr>
<td>mean(s) (%)</td>
<td>0.111</td>
<td>0.146</td>
<td>0.107</td>
</tr>
<tr>
<td>std(s)</td>
<td>0.065</td>
<td>0.063</td>
<td>0.077</td>
</tr>
<tr>
<td>mean(rb) (%)</td>
<td>0.98</td>
<td>1.10</td>
<td>1.07</td>
</tr>
<tr>
<td>std(rb)</td>
<td>0.576</td>
<td>0.249</td>
<td>0.208</td>
</tr>
</tbody>
</table>

**NOTE:** The aggregate bond spreads are computed using the sovereign bond portfolio weighted by $d_i$; $rb$ denotes the holding-period return on such a sovereign bond portfolio.
ASYMMETRIC COSTS
(Uncorrelated Asymmetric Risk Shocks)
Credit spreads on sovereign bonds are heavily influenced by a global risk factor as measured by the excess bond premium.

Standard default model with risk-averse lender:

- Qualitatively captures the effect of global financial risk factor on sovereign bond credit spreads
- Has difficulty matching the average size of sovereign bond credit spreads (i.e., default probabilities are too low)
- Cannot account for time-variation in sovereign bond spreads

Possible extensions:

- Introduce multi-period debt
- Allow the pricing kernel to depend on sovereign bond returns to capture contagion effect of sovereign default
Summary

Credit spreads on sovereign bonds are heavily influenced by a global risk factor as measured by the excess bond premium.

Standard default model with risk-averse lender:
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- Has difficulty matching the average size of sovereign bond credit spreads (i.e., default probabilities are too low)
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Possible extensions:
- Introduce multi-period debt
- Allow the pricing kernel to depend on sovereign bond returns to capture contagion effect of sovereign default
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Possible extensions:

- Introduce multi-period debt
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