Capital Regulation in a Macroeconomic Model with Three Layers of Default*

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Abstract

We develop a model which aims at providing a framework for the positive and normative analysis of macroprudential policies. The basic model incorporates optimizing financial intermediaries ("bankers") who allocate their scarce wealth ("inside equity") together with funds raised from saving households across two lending activities, mortgage lending and corporate lending. External financing for all borrowers (including banks) takes the form of external debt which is subject to default risk. The model shows the interplay of three interconnected net worth channels as well as distortions due to deposit insurance, and can be extended to analyse the implications of securitization and liquidity risk. The setup allows an explicit welfare analysis of macroprudential policies.

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Non-technical summary

We develop a theoretical model which aims at providing a framework for the positive and normative analysis of macroprudential policies. The basic model incorporates optimizing financial intermediaries ("bankers") who allocate their scarce wealth ("inside equity") together with funds raised from saving households across two lending activities, mortgage lending to borrowing households and corporate lending to borrowing firms.

The external financing for all borrowers (including banks) takes the form of not contingent debt which is subject to default risk due to borrowers' exposure to both idiosyncratic and aggregate risk factors. Defaults are assumed to cause deadweight losses as in the costly state verification setup of Gale and Hellwig (1985). Households' and firms' leverage is an endogenous multiple of their net worth. In contrast, banks, which are assumed to obtain their outside funding in the form of government-guaranteed deposits, have their leverage limited by a regulatory capital requirement. Importantly, in spite of the presence of deposit insurance, we assume depositors to suffer some transaction costs if their banks fail. This generates a risk premium that acts as an important source of amplification when bank solvency is weak.

The model exhibits the operation of three interconnected net worth channels, which create the potential for amplification and propagation noted in various strands of the existing literature (including the one operating through the price of housing, which is the collateral used by the borrowing households), as well as distortions due to deposit insurance. While limited net worth typically leads to under-investment, the risk subsidization linked to deposit insurance creates the potential for an excessive supply of bank credit. The basic model is then suitable for a non-trivial welfare analysis of various forms of leverage regulation (maximum loan-to-value or debt-to-income ratios for mortgages, capital requirements imposed on banks' lending activities, countercyclical capital charges, et cetera), that are likely to be the core of macroprudential policy.

We use the basic model developed in this paper to analyze the effects of capital requirements on steady state and on the transmission of various types of shocks. But the basic model can be extended to allow for the possibility of securitization and liquidity risk (e.g. in the form of interim funding shocks suffered by banks). These extensions may allow expanding the analysis to the regulatory treatment of securitization, liquidity regulation, and the assessment of lending of last resort policies. While the basic model belongs to the class of non-monetary models, introducing nominal rigidities and a meaningful role for monetary policy constitutes a natural third possible extension that would allow us to assess the interactions between macroprudential policy and monetary policy.

1 Introduction

The financial crisis has clearly illustrated that prudential regulation uniquely based on the soundness of individual institutions is not sufficient to ensure financial stability. As a consequence, it has been proposed to develop a new approach which factors in the macroeconomic perspective and takes into account the connections between the financial and the non-financial sectors, and the central role of financial intermediaries. So-called macro-prudential policies should therefore be designed to address the contribution of financial stability, systemic risk and the pro-cyclicality of the financial system to overall economic performance.

However, little is known about the general equilibrium effects of the several macro-prudential instruments proposed so far. Policy makers demand analytical frameworks with which to assess the implications and effectiveness of such instruments, and address the optimal design of macro-prudential policy. Beyond incorporating financial frictions and distortions, a good model for policy analysis in this area should put the banking system and financial intermediation more generally at the centre of the stage.

A number of recent papers has focused on introducing bank frictions into otherwise mainstream macroeconomic models. Some of these papers (e.g. Gertler and Kiyotaki 2010, Meh and Moran 2010) describe a bank net worth channel that operates essentially along the same lines as the conventional entrepreneurial net worth channel (notably in Bernanke, Gertler and Gilchrist 1999, henceforth BGG) but causing fluctuations in the availability of bank credit rather than directly on entrepreneurial investment. Most papers either focus on one main bank friction (like the bank net worth channel) or otherwise capture several frictions in a reduced form manner, without explicitly modeling the optimizing behavior of financial intermediaries and without explicitly addressing the welfare analysis of macroprudential policies.[1]

The purpose of this paper is to encompass the most relevant aspects of macroprudential

[1] See the report of the Mars research network for a survey. See http://www.ecb.europa.eu/home/html/researcher_mars.en.html.

concern in a single model with enough microfoundations to allow us to perform a welfare analysis of macroprudential policy. The model is built with an eclectic perspective, trying to provide a synthesis of the most relevant interlinkages between the real and the financial sectors identified in the literature. Putting together some of the mechanisms previously analyzed on a stand alone basis or from a more partial equilibrium perspective may identify relevant sources of feedback and clarify the relative quantitative relevance of each channel. This multidimensional approach is essential to provide coherent advice to the new macroprudential authorities. While this paper is focused on bank capital regulation, the key microprudential policy tool and arguably one of the main tools for macroprudential policy as well, the model presented here is rich enough to allow for the analysis of other candidate macroprudential instruments, such as loan to value ratios.

The second main goal of the paper is to provide a model where default plays a central and material role. Up to the global financial crisis and even beyond it, the role of default has been largely overlooked in macroeconomics. A number of papers, following Kiyotaki and Moore (1997), allow for the *possibility* of default, but rule it out in equilibrium through appropriately chosen financial contracts. Other papers, following the seminal BGG model, do allow for default in equilibrium, but consider state contingent debt that prevents default to unexpectedly fluctuate due to aggregate shocks. Therefore, both approaches abstract from some of the consequences of default for financial stability and subsequently for the real economy. In our work, default impinges on the balance sheet of the lenders, influencing their optimal behaviour and thereby macroeconomic outcomes. Moreover, overall default risks can arise from both aggregate and idiosyncratic risk. For a discussion on the importance to introduce default in macro-models, see Geanakoplos (2011) and Goodhart, Tsomocos and Shubik (2013).

Our model belongs to the class of dynamic stochastic general equilibrium (DSGE) models, but several aspects of its construction make it quite distant from the typical DSGE model in use by central banks prior to 2007, which some observers blame for having ignored the relevance of financial intermediation and financial stability. By the same token, our construction is far away from the classical framework of microprudential supervision, which was very limited in its analysis of the impact of macroeconomic performance on financial intermediation and lacked the mission and the analytical tools to properly consider the impact of prudential policies on general macroeconomic performance. Hence, we try to bridge the gap between the micro and macro literature and build a framework which allows for the welfare analysis of the relevant policy instruments.

More in detail, the model developed in this paper tries to coherently put together the following ingredients: (i) non-trivial lending and borrowing decisions in the household sector, with some households demanding bank loans for the purchase of housing, (ii) non-trivial borrowing decisions in the corporate sector, with firms demanding bank loans for the funding of their capital accumulation, (iii) non-trivial default risk in all classes of borrowing, including the borrowing that financial intermediaries obtain from lending households in the form of bank deposits, (iv) a net worth channel operating at the level of each levered sector, indebted households, indebted entrepreneurs, and banks, and (v) a bank funding fragility channel which operates through the premium requested by risk averse depositors who suffer some deadweight losses if banks default.

The rationale for macro-prudential policies in our model arises from two distortions. First, external financing frictions resulting from the possibility of default, which in the form of bankruptcy costs (or costly state verification, CSV), restrict access to credit and may result in too little borrowing compared with a first best world without these frictions (*CSV distortion*). Second, we assume that the government guarantees the principal and interest of bank deposits in full (though not the transaction costs suffered by the depositors of a failed bank). This pushes banks to take on risk at the expense of the deposit insurance agency (DIA), which may result in more lending and borrowing in equilibrium than what a social planner would find optimal when internalizing the full costs of bank default (*limited liability distortion*). The interaction of these two distortions points in opposite directions and has a net result that may imply steady state levels of credit to the various sectors above or below the ones that a social planner would optimally choose. Indeed, we show that steady state household welfare in our model is a hump-shaped function of credit availability as determined by bank capital regulation.[2]

We use the model to analyze the effects of capital requirements on steady state and on the transmission of various types of shocks. On top of time-invariant capital requirements, possibly differentiated across classes of loans according to their risk, we also consider counter-cyclical adjustments to the capital ratios (adjustments that, in practice, might be implemented in the form of a countercyclical capital buffer, as in Basel III).

Three main results stand out in our analysis. First, in the context of our model, there is generally an optimal level of capital requirements. In effect, capital requirements reduce bank leverage, bank failure risk and the implicit subsidies associated with deposit insurance. Simultaneously, they force the banks to make a greater use of bankers' limited wealth. This second aspect makes capital requirements have a potential impact on the cost of equity funding (due to its scarcity in the short run) and on the pattern of accumulation of wealth by bankers (in the medium to long run). Lower leverage and, in the short run, a larger cost of equity funding lead banks to extend less credit and to be less fragile. However, too high levels of capital requirements may unduly restrict credit availability.

Second, we find that when bank leverage is high (because capital requirements are low), the economy is more responsive to shocks. This shows that limited liability and the implicit subsidies associated with deposit insurance, which allow banks to meet the required rate of return on equity with lower lending rates, constitute a potentially powerful channel of financial amplification.

Third, the counter-cyclical adjustment of capital ratios may significantly improve the benefits of high capital requirements, but once again only up to a certain level. Otherwise, the effects due to the increase in bank fragility will backfire. This is, in fact, what happens when the reference capital requirements are ex ante too low: their further relaxation in

[2] This hump-shaped welfare function is conceptually similar to Benigno et al. (2013), where it is not clear a priori whether there is too much or too little credit and whether regulation should restrict or subsidize credit. However, their model is substantially different from ours.

response to a negative shock can be harmful. On one hand, a countercyclical reduction in the requirements may allow the bank to charge lower loan rates on a larger amount of loans. On the other hand, if bank fragility gets further deteriorated, banks' funding costs will increase. This increase may then well off-set the intended impact of the countercyclical adjustment, causing long-lasting detrimental effects on credit supply and GDP.

The rest of the paper is organized as follows. In Section 2, we first discuss how the paper fits in the existing literature. In Section 3 we introduce the key elements of the core model, describing the equilibrium equations that directly emanate from agents' optimization. In Section 4, we complete the set of equilibrium equations with those referred to market clearing. Section 5 describes the calibration of the model. Section 6 contains the main positive and normative results for the model. Section 7 concludes. The Appendix summarizes the main model equations and provides a formal definition of equilibrium.

2 Relation to previous literature

Our model builds on a large literature which includes financial frictions in general equilibrium models, including among others Carlstrom and Fuerst (1997), BGG, and Kiyotaki and Moore (1997). Financial frictions are typically found to increase the persistence of shocks and to amplify their impact, though this is not necessarily the case in all models. Brunnermeier et al. (2012) provide a survey of the literature on the macroeconomic modelling of financial frictions. Brzoza-Brzezina et al. (2013) compare the properties of models with collateral constraints like in Kiyotaki and Moore with models with an external finance premium as in BGG, concluding that the business cycle properties of the external finance premium framework are more in line with empirical evidence.

A bunch of papers have incorporated banking in DSGE models; these include Curdia and Woodford (2008), Goodfriend and McCallum (2007), Gerali et al. (2010), Meh and Moran (2010), and Christiano, Motto and Rostagno (2010). As in our paper, this literature mostly focuses on direct lending by banks, excluding securitization and investment banking activities. In most of this work the emphasis is on the role of bank lending in the propagation of shocks (typically monetary policy shocks) or in the optimal conduct of monetary policy, not on the inefficiencies associated with financial frictions that create a role for macro-prudential policies. Moreover, in these papers default normally does not feature prominently or at all. An exception is Angeloni and Faia (2013), who focus on capital regulation in a model where banks are fragile and subject to runs, and the main distortion arises from the fact that the projects funded by banks may be subject to costly early liquidation.

Our modeling of default costs throughout the paper resembles the well-known setup of BGG. However, similar to Benes and Kumhof (2011) but in contrast to BGG we do not allow debt repayments to be contingent on the realization of aggregate shocks. This assumption implies a restriction in the contracting space: contracts are incomplete in that they cannot be made fully contingent on aggregate variables (perhaps due to verifiability problems, publication lags, potential manipulability if contractually relevant, etc.).

The emphasis on default is similar to several models that have analyzed macroprudential issues outside the DSGE tradition, such as Goodhart, Sunirand and Tsomocos (2006) and Goodhart et al. (2012). Van den Heuvel (2008) and Repullo and Suarez (2013) analyze optimal bank capital regulation in simple finite horizon models with financial frictions. In a way, our paper provides a bridge between this literature and the more traditional DSGE models with financial frictions.

Recent work by Mark Gertler and coauthors (Gertler and Karadi 2011; Gertler and Kiyotaki 2010; Gertler, Kiyotaki and Queralto 2011) has also emphasized the presence of financing constraints for banks as a key factor in the propagation of shocks. In their work, the financing constraint arises from the fact that bankers can choose to divert a fraction of available funds away from the funded projects and in favour of the household of which they are a member. To prevent this, bankers need to have some positive net worth in the project. In our work, the friction comes from the possibility of overinvestment (or excessive risk taking, from the society's point of view) due to limited liability, not from the possibility of diversion (as also suggested by Cole 2011).[3] In addition, we model explicitly the intermediation chain that links saving households that invest in bank deposits with household and corporate borrowers that obtain bank loans. This allows us to keep track of the transmission of default risk across sectors.

Hirakata et al. (2013) also consider the full intermediation chain and allow for borrowers' and banks' default. Our treatment differs from theirs in at least two important aspects. First, they consider a monopolist banker set up in which the contracts linking the bank with depositors and borrowers, respectively, are solved for jointly in order to maximize the utility of the banker. Secondly, Hirakata et al. (as Angeloni and Faia, 2012) consider uninsured deposits while we assume deposit insurance, and think of its effects on banks' risk taking as one of the reasons for the need for macroprudential policy. Collard et al. (2012) also look at the socially excessive risk taking by banks due to limited liability and deposit insurance and analyze the interplay between prudential and monetary policy instruments. In a similar vein, Martinez-Miera and Suarez (2012) focus on the effect of capital requirements on banks' incentive to extend loans with potentially highly correlated defaults in case a so-called systemic (rare but potentially devastating) shock occurs.

Our model belongs to the class of DSGE models and we solve it with standard methods, i.e. perturbation methods. This departs from Brunnermeier and Sannikov (2014), who use a continuous time methodology to solve for the full dynamics of a model with financial frictions. Their model is very stylized and features productive experts and less productive households. Owing to financial frictions the wealth of experts is important for the experts' ability to buy physical capital and use it productively. Brunnermeier and Sannikov find that the system's reaction is highly nonlinear and the financial system is inherently unstable; most of the effects are asymmetric and only arise in a downturn. The fact that the analysis that we perform does not allow to asses the importance of non-linearities in our setup should be taken as an important caveat.

[3] In Iacoviello (2013) bankers are more impatient and are subject to a capital constraint so as to prevent them from borrowing in an unlimited way.

Our paper shares the goal of finding a rationale for macroprudential policies with papers that have recently put the emphasis on pecuniary externalities, including Jeanne and Korinek (2010), Bianchi (2011) and Bianchi and Mendoza (2010), Gersbach and Rochet (2012), and Christiano and Ikeda (2013). The pecuniary externalities arise due to the fact that agents do not internalize the effect of their actions (e.g. borrowing and investment decisions) on the prices of housing and physical capital, which in turn affect agents' collateral constraints. In our model the endogenous leverage of households and firms is also affected by these prices, so pecuniary externalities might also play a role, but it is hard to predict in which direction (and to which extent) they distort the allocation of credit in our economy.

In models with pecuniary externalities, the bidirectional relationship between asset prices and financial constraints may produce feedback loops that result in excessive volatility of these prices and of macroeconomic variables more generally, calling for policies that promote financial stability. Similar mechanisms may operate in our model but assessing formally the welfare cost of such added volatility is challenging (since requires assessing welfare out of steady state) and is beyond the scope of this first exploration of our model. In a less normative vein, we analyze the potential stabilization role of capital regulation when the economy is hit by several types of shocks.

3 Model set-up

We consider an economy populated by households, entrepreneurs and bankers, whose main characteristics are as follows:

Households. Households are risk-averse and infinitely lived and derive utility from a storable *consumption* good and from a durable good, *housing*, which provides housing services to its owners. The consumption good acts as a numeraire in our economy and its price is normalized to one. The price of housing is denoted q_t^H. Similar to Iacoviello (2005) and subsequent literature there are two types of households that differ in their discount factor (patient and impatient). Moreover, they are grouped in two distinct representative dynasties which provide risk-sharing to their members: the saving dynasty and the borrowing dynasty. In equilibrium, the saving households consume out of their current income and save the rest for future use, while the borrowing households take mortgage debt from banks against their holdings of housing. Mortgage debt is provided to the individual members of the dynasty against their individual housing units on a limited-liability non-recourse basis. This implies the possibility of defaulting on mortgage debt at an individual level with the only implication for the borrower of losing the housing good with which the mortgage is secured. Thus, in contrast to Iacoviello (2005), mortgage loans feature default risk and, in case of default, the repossession of collateral by the banks involves verification costs similar to those considered in BGG. Both types of households supply labour in a competitive market.

Entrepreneurs. *Entrepreneurs* are risk neutral agents specialized in owning and maintaining the stock of physical capital, which they rent in each period to the firms involved in the production of the consumption good.[4] They live across two consecutive periods and derive utility from transferring a part of their final wealth to the saving dynasty (to whom they may be interpreted to serve as an altruistic agent) and from leaving another part as a bequest to the next generation of entrepreneurs (who can be interpreted as their heirs). Each of these parts can be realistically interpreted as "dividends" and "retained earnings," respectively. With this OLG formulation, which will also be postulated for the bankers, we will be able to generate the same sort of dynamics for entrepreneurial and banking net worth as in BGG. Its main virtue is to allow us to keep the number of agents to care about in the welfare calculations to a minimum (the patient and the impatient households) without neglecting any of the consumption capacity generated in the economy. Entrepreneurs finance their initial purchases of capital partly with the inherited net worth and partly with corporate loans provided by banks. Similar to household loans, corporate loans are subject to limited

[4] A possible interpretation is that physical capital would suffer prohibitively high depreciation rates if owned and maintained by any other class of agents.

liability and default risk, and recovering residual returns from bankrupt entrepreneurs leads banks to incur verification costs.

Bankers. *Bankers* are the providers of inside equity to perfectly competitive financial intermediaries that we call banks. Like entrepreneurs, they are risk neutral agents who live across two consecutive periods, and derive utility from making transfers to the saving dynasty and leaving bequests to the next cohort of bankers.[5] The active bankers use the wealth inherited from the previous generation of bankers to provide equity funding to banks. At the end of each period, the gross return on such equity is fully distributed to the bankers, who in turn distribute it to the patient households ("dividends") and the next cohort of bankers ("retained earnings").

To complete the model overview, we need to refer to banks, consumption good producing firms, and capital good producing firms:

Banks. Banks' outside funding is made up of fully insured deposits raised among the saving households. Banks operate under limited liability and may default due to *both* idiosyncratic and aggregate shocks to the performance of their loan portfolios. Deposit insurance is funded within each period by levying lump-sum taxes on patient households, if needed. From the standpoint of savers, however, we assume that recovering the fully insured principal and interest of their deposits in case of bank failure is costly in terms of time and effort, so that deposits may still pay a risk premium which will co-move with bank's default risk. Bankers' inside equity contributions are necessary for the banks to comply with the exogenous regulatory capital requirement. Banks are of two types, who lend to impatient households (banks H) and to entrepreneurs (banks F) respectively.

[5] The induced dynamics of bankers' net worth is similar to that in Gertler and Karadi (2011), where a fraction of households become bankers at random in every period and remain as bankers in subsequent periods with some probability.

Production of the consumption good. There is a perfectly competitive consumption good producing sector made up of firms owned by the patient households. These firms combine capital rented from entrepreneurs with household and entrepreneurial labour inputs in order to produce the consumption good. This sector is not directly affected by financial frictions.

Production of the capital good and housing. There is a perfectly competitive capital good and housing producing sector made up of firms owned by patient households. Like in Gertler, Kiyotaki and Queralto (2011), these firms optimize intertemporally in response to changes in the price of capital and are subject to technological illiquidity in the form of investment adjustment costs. Also this sector is not directly affected by financial frictions.

We now turn to describe the model in detail. For the sake of brevity, the full set of model equations, including the first order conditions, is in the Appendix.

3.1 Households

The economy is populated by two representative dynasties made up of a measure-one continuum of ex ante identical households each. Households are risk averse and maximize some time-separable expected utility functions. One dynasty, identified by the superscript s, is made of relatively patient households with a discount factor β^s. The other dynasty, identified by the superscript m, is made of more impatient households with a discount factor $\beta^m \leq \beta^s$. Thus, in equilibrium, the patient households save and the impatient households borrow. Dynasties provide consumption risk sharing to their members and are in charge of taking most household decisions.[6]

[6]This latter feature is convenient for the solution of the model with standard techniques (i.e. avoiding kinks).

3.1.1 Saving households

The dynasty of patient households maximizes

$$E_t\left[\sum_{i=0}^{\infty}(\beta^s)^{t+i}\left[\log\left(c^s_{t+i}\right)+v^s_{t+i}\log\left(h^s_{t+i}\right)-\frac{\varphi^s_{t+i}}{1+\eta}\left(l^s_{t+i}\right)^{1+\eta}\right]\right] \tag{1}$$

where c^s_t denotes the consumption of non-durable goods and h^s_t denotes the total stock of housing held by the various members of the dynasty. l^s_t denotes hours worked in the consumption good producing sector, with η the inverse of the Frisch elasticity of labor supply. v^s_t and φ^s_t are preference parameters which can vary over time (potentially causing fluctuations in, e.g., the equilibrium price of housing). Specifically, the housing preference parameter v^s_t follows the stationary process

$$\ln v^s_t = (1-\rho^v)\ln\bar{v}^s + \rho^v\ln v^s_{t-1} + \varepsilon^v_t, \tag{2}$$

where $\bar{v}^s > 0$ is the constant long-term average of the process, ρ^v is the persistence parameter and ε^v_t is an i.i.d. white noise shock with variance σ^2_v (housing demand shock).

The disutility of labor parameter φ^s_t follows the stationary process $\ln\varphi^s_t = (1-\rho^\varphi)\ln\bar{\varphi}^s + \rho^\varphi\ln\varphi^s_{t-1} + \varepsilon^\varphi_t$,where $\bar{\varphi}^s > 0$ is the constant long-term average of the process, ρ^φ is the persistence parameter and ε^φ_t is an i.i.d. white noise process with variance σ^2_φ (demand shock).

The patient households' dynamic budget constraints read as follows

$$c^s_t + q^H_t h^s_t + d_t \leq w_t l^s_t + q^H_t(1-\delta^H_t)h^s_{t-1} + \widetilde{R}^D_{t-1}d_{t-1} - T_t + \Pi^s_t \tag{3}$$

where q^H_t is the price of housing, δ^H_t is the (possibly time-varying) rate at which housing units depreciate, w_t is the wage rate, and

$$\widetilde{R}^D_{t-1} = R^D_{t-1}(1-\gamma PD^b_t) \tag{4}$$

where R^D_{t-1} is the fixed (gross) interest rate received at t on the savings deposited at banks at $t-1$ in the previous period. The principal and interest of bank deposits are fully guaranteed by a deposit insurance agency (DIA) that, for simplicity, is assumed to ex post balance its budget by imposing a lump sum tax T_t on patient households. However, we assume that households incur a linear transaction cost γ whenever they have to recover the funds deposited in a failed bank. So PD_t^b stands for the fraction of deposits in banks that fail in period t (which can be computed as the average deposit-weighted bank default rate realized in period t).[7] This transaction cost introduces a link between the bank probability of default and bank funding costs and a wedge between the rate of return on deposits and the risk free rate. At the same time, we model it in a way that does not affect the incentives for any individual bank and, thus, preserves the usual distortions associated with limited liability and deposit insurance (i.e. banks' incentives to take excessive risks at the expense of the DIA). Finally, Π_t^s includes the profits accruing to the saving households from the ownership of the capital good and housing producing firms as well as the dividend transfers received from entrepreneurs and bankers.

The housing depreciation rate is time-varying and subject to AR(1) shocks:

$$\delta_t^H = \delta^H + \varepsilon_t^{\delta^H} \tag{5}$$

$$\varepsilon_t^{\delta^H} = \rho^{\delta^H} \varepsilon_t^{\delta^H} + v_t^{\delta^H} \tag{6}$$

where $v_t^{\delta^H}$ is an i.i.d. shock.

3.1.2 Borrowing households

The objective function of the representative dynasty of impatient households has the same form and parameters as (1), except for the discount factor which for them is $\beta^m < \beta^s$ and will induce this dynasty to borrow rather than save in equilibrium. This explains the differences in their dynamic budget constraints, that read as follows:

$$c_t^m + q_t^H h_t^m - b_t^m \leq w_t l_t^m + \int_0^\infty \max\left\{\omega_t^m q_t^H (1-\delta_t^H) h_{t-1}^m - R_{t-1}^m b_{t-1}^m, 0\right\} dF^m(\omega_t^m) - T_t^m, \tag{7}$$

[7] PD_t^b is specified further in Section 3.4. For evidence that bank failure is costly to depositors even in the presence of deposit insurance, see Brown et al. (2013).

where b_t^m is the dynasty's aggregate borrowing from the banking system and R_{t-1}^m is the contractual gross interest rate on the housing loan of size b_{t-1}^m agreed with a bank in the previous period. The term in the integral reflect the fact that the housing good and the debt secured against it are assumed to be distributed across the individual households that constitute the dynasty. Each impatient household experiences at the beginning of each period t an idiosyncratic shock ω_t^m to efficiency units of housing owned from the previous period and have the option to (strategically) default on the non-recourse housing loans associated with those units.[8]

The shock ω_t^m is assumed to be independently and identically distributed across the impatient households, and to follow a log-normal distribution with density and cumulative distributions functions denoted by $f^m(\cdot)$ and $F^m(\cdot)$, respectively. The shock makes the effective resale value of the housing units acquired in the previous period be $\tilde{q}_t^H = \omega^m q_t^H(1 - \delta_t^H)$ and, given that default is costless for households, makes default on the underlying loan ex post optimal for the household whenever $\omega_t^m q_t^H(1-\delta_t^H)h_{t-1}^m < R_{t-1}^m b_{t-1}^m$.[9] This explains the presence of the max operator in the integral in (7).

Housing loans In each period, after the realization of the idiosyncratic shock ω_t^m, each individual household decides whether to default on the individual loans attached to the housing held from the previous period and the residual net worth is passed on to the dynasty, which is not liable for any unpaid debt. The dynasty then makes the decisions on consumption, housing, labor supply and debt for period t and allocates them evenly across its members.

Fluctuations in the net worth of the dynasty (as captured by the last term in the right hand side of (7)) are driven by the changes in the net worth of the loan-repaying households as well as the realization of zero net worth from all housing units owned by members that

[8]This shock is intended to capture idiosyncratic fluctuations in the value of houses and can be interpreted as a reduced form representation of a sudden improvement or worsening in the neighborhood, in the social equipment available nearby or in the resource cost of maintaining the property. See also Forlati and Lambertini (2011) who use a similar formulation.

[9]See Geanakoplos (2003) for a discussion of the ex post optimality of this type of behavior by the borrower, and Goodhart et al. (2012) for an extension to the analysis of mortgage contracts backed by housing collateral.

default on their housing loans. Default in period t will occur for

$$\omega_t^m \leq \overline{\omega}_t^m = \frac{x_{t-1}^m}{R_t^H},$$

where

$$R_t^H \equiv \frac{q_t^H(1-\delta_t^H)}{q_{t-1}^H}$$

is the ex post average realized gross return on housing, and

$$x_t^m \equiv \frac{R_t^m b_t^m}{q_t^H h_t^m}$$

is a measure of a household leverage. The fraction of defaulted mortgages at period t can then be expressed as $F^m(\overline{\omega}_t^m)$ and the net worth accruing to the dynasty out of its aggregate housing investment in the previous period can be written as

$$\Phi_t^m \equiv \left(\int_{\overline{\omega}_t^m}^{\infty}(\omega_t^m - \overline{\omega}_t^m)dF(\omega_t^m)\right) R_t^H q_{t-1}^H h_{t-1}^m. \tag{8}$$

Now, using the same intermediate notation as in BGG, we can more compactly write

$$\Phi_t^m = (1-\Gamma^m(\overline{\omega}_t^m))R_t^H q_{t-1}^H h_{t-1}^m, \tag{9}$$

where

$$\Gamma^m\left(\overline{\omega}_t^m\right) = \int_0^{\overline{\omega}_t^m} \omega_t^m f\left(\omega_t^m\right) d\omega_t^m + \overline{\omega}_t^m \int_{\overline{\omega}_t^m}^{\infty} f\left(\omega_t^m\right) d\omega_t^m. \tag{10}$$

The variable Φ_t^m can be interpreted as net housing equity after accounting for repossessions of defaulting households.

Since the borrowing households will default on the loans taken at period t according to a similar pattern of behavior, the terms of such loans must satisfy the following participation constraint for the lending bank:[10]

$$(1 - \Gamma^H(\overline{\omega}^H_{t+1}))(\Gamma^m\left(\overline{\omega}^m_{t+1}\right) - \mu^m G^m\left(\overline{\omega}^m_{t+1}\right))R^H_{t+1}q^H_t h^m_t \geq \rho_t \phi^H_t b^m_t. \tag{11}$$

Intuitively, this constraint says that the bankers who contribute equity $\phi^H_t b^m_t$ to the lending bank (where ϕ^H_t is the capital requirement on housing loans) should expect a gross expected return on their contribution at least as high as some market-determined required rate of return ρ_t which is exogenous for any individual bank although endogenous in aggregate (this will be determined later). Therefore, $\rho_t \phi^H_t b^m_t$ measures total gross equity returns for a given bank.

The expression in the left hand side of the inequality accounts for the total equity returns associated with a portfolio of housing loans to the various members of the impatient dynasty. The term $\mu^m G\left(\overline{\omega}^m_{t+1}\right)$ reflects the proportional verification costs μ^H incurred in the repossession of the fraction $G^m\left(\overline{\omega}^m_{t+1}\right)$ of housing units which defaulting loans were borrowing against, where $G^m\left(\overline{\omega}^m_{t+1}\right) = \int_0^{\overline{\omega}^m_{t+1}} \omega^m_{t+1} f^m\left(\omega^m_{t+1}\right) d\omega^m_{t+1}$.

The factor $(1 - \Gamma^H(\overline{\omega}^H_{t+1}))$ plays a similar role to the factor $(1 - \Gamma^m(\overline{\omega}^m_t))$ in (9) and accounts for bank leverage and the possibility that an individual bank that lend to households (identified by the superscript H) fails due to sufficiently adverse idiosyncratic or aggregate shocks to the performance of its portfolio of housing loans. The full details of the threshold $\overline{\omega}^H_{t+1}$ of the idiosyncratic shock below which the bank fails are presented in subsection 3.4.

Note that the assumption of limited liability and the fact that bank liabilities (deposits) are insured, a bank can meet the required return on equity with a lower lending rate. This suggests that the limited liability distortion acts in the direction of expanding credit availability for entrepreneurs and impatient households.

It should also be emphasized that the probability of a default event hinges on R^H_t, an aggregate variable which may be influenced by aggregate shocks (say, productivity shocks).

[10] In principle, the borrowing rate R^m_t is part of the housing loan contract and, hence, can be treated as part of the decision variables of the impatient dynasty in period t. However, treating the intermediate variable x^m_t as part of the contract variables (together with b^m_t and h^m_t) allows us to write the entire contract problem without explicit reference to R^m_t.

Therefore, default is a function of both idiosyncratic and aggregate shocks, unlike in BGG and most of the literature on modelling financial frictions.

Borrowing households' optimization problem The decision problem of the borrowing households can be compactly written as a contracting problem between the corresponding representative dynasty and its bank:

$$\max_{\{c^m_{t+i},h^m_{t+i},l^m_{t+i},x^m_{t+i},b^m_{t+i}\}_{i=0}^{\infty}} E_t\left[\sum_{i=0}^{\infty}(\beta^m)^{t+i}\left[\log\left(c^m_{t+i}\right)+v^m_{t+i}\log\left(h^m_{t+i}\right)-\frac{\varphi^m_{t+i}}{1+\eta}\left(l^m_{t+i}\right)^{1+\eta}\right]\right] \tag{12}$$

subject to the budget constraint of the dynasty,

$$c^m_t+q^H_t h^m_t-b^m_t\leq w_t l^m_t+\left(1-\Gamma^m\left(\frac{x^m_t}{R^H_{t+1}}\right)\right)R^H_{t+1}q^H_t h^m_t-T^m_t, \tag{13}$$

and the participation constraint of the bank,

$$E_t\left[(1-\Gamma^H(\overline{\omega}^H_{t+1}))\left(\Gamma^m\left(\frac{x^m_t}{R^H_{t+1}}\right)-\mu^m G^m\left(\frac{x^m_t}{R^H_{t+1}}\right)\right)R^H_{t+1}\right]q^H_t h^m_t=\rho_t\phi^H_t b^m_t, \tag{14}$$

which we impose with equality without loss of generality.

3.2 Entrepreneurs

We consider a sequence of overlapping generations of two-period lived risk-neutral entrepreneurs.[11] Each generation of entrepreneurs inherits wealth in the form of bequests n^e_t from the previous generation of entrepreneurs. Entrepreneurs are the only agents who can own and maintain the capital stock. They purchase new capital from capital goods producers and depreciated capital from the previous generation of entrepreneurs, and then rent it to the contemporaneous producers of the consumption good. Entrepreneurs finance their capital holdings with their own initial net worth n^e_t and with loans b^e_t received from the banks specialized in corporate loans.

An entrepreneur born at time t values the transfers made to the patient dynasty at time

[11] This assumption makes entrepreneurs risk-neutral in their decision-making as assumed in BGG.

$t+1$ ("dividends"), c^e_{t+1}, and the bequests left to the next cohort of entrepreneurs ("retained earnings"), n^e_{t+1}, according to the utility function $(c^e_{t+1})^{\chi^e}(n^e_{t+1})^{1-\chi^e}$. Thus, once in period $t+1$, the entrepreneur will solve

$$\max_{c^e_{t+1},n^e_{t+1}} (c^e_{t+1})^{\chi^e}(n^e_{t+1})^{1-\chi^e} \tag{15}$$

subject to:

$$c^e_{t+1} + n^e_{t+1} \leq W^e_{t+1}.$$

Optimizing behavior then yields the "dividend" rule

$$c^e_{t+1} = \chi^e W^e_{t+1} \tag{16}$$

and the "earnings retention" rule

$$n^e_{t+1} = (1-\chi^e)W^e_{t+1}. \tag{17}$$

These rules guarantee easy aggregation and generate the same type of net worth dynamics as in Bernanke, Gertler and Gilchrist (1999). Assuming that entrepreneurs derive utility from transferring consumption capacity to the saving households rather than from their own consumption will allow us to focus our welfare analysis on households' expected lifetime utility but without neglecting the consumption capacity associated with the profits of entrepreneurial firms. We return to this issue when describing bankers and in section 6.1.

The decision problem of the entrepreneur who starts up at t can then be written as:

$$\max_{k_t,b^e_t,R^F_t} E_t(W^e_{t+1}) \tag{18}$$

subject to the period t resource constraint

$$q^K_t k_t - b^e_t = n^e_t, \tag{19}$$

the definition

$$W_{t+1}^e = \max\left[\omega_{t+1}^e \left(r_{t+1}^k + (1-\delta_{t+1})\, q_{t+1}^K\right) k_t - R_t^F b_t^e - T_{t+1}^e, 0\right], \tag{20}$$

and a bank participation constraint which will be fully specified in the next subsection. In these expressions, q_t^K is the price of capital at period t, k_t is the capital held by the entrepreneur in period t, b_t^e is the amount borrowed from the bank in period t, δ_t is the time-varying depreciation rate of each efficiency unit of capital, r_t^K is the rental rate per efficiency unit of capital, and R_t^F is the contractual gross interest rate of the loan taken from the bank in period t. Finally, T_t^e is the lump sum tax possibly imposed on the entrepreneurs to finance the deposit insurance.

Note that the depreciation rate δ_t is time-varying because we also consider a depreciation shock ε_t^δ, whereby

$$\delta_t = \delta + \varepsilon_t^\delta \tag{21}$$

$$\varepsilon_t^\delta = \rho_\delta \varepsilon_t^\delta + v_t^\delta \tag{22}$$

and v is an i.i.d. shock.

The factor ω_{t+1}^e that multiplies the return from capital holdings is an idiosyncratic shock to the entrepreneur's efficiency units of capital. This shock realizes after the period t loan with the bank is agreed and prior to renting the available capital to consumption good producers in that date. With a role similar to the shock ω_{t+1}^m suffered by the housing held by borrowing households, the shock ω_{t+1}^e is a simple way to rationalize the existence of idiosyncratic shocks to the entrepreneurs' performance and to generate a non-trivial default rate on entrepreneurial loans. The shock is independently and identically distributed across entrepreneurs and follows a log-normal distribution with an expected value of one, and density and cumulative distribution functions denoted $f^e(\cdot)$ and $F^e(\cdot)$, respectively.

Similar to all other borrowers in our economy, an entrepreneur cannot be held liable for any contracted repayments due to banks (which amount $R_t^F b_t^e$ in period $t+1$) over and above the gross returns that she obtains on the capital investment undertaken in the previous period, $\left(r_{t+1}^K + (1-\delta_{t+1})\, q_{t+1}^K\right) \omega_{t+1}^e k_t$. This is why we have the max operator in (20): it takes into account limited liability and the possibility that entrepreneurs default on their bank loans.

3.2.1 Entrepreneurial loans

Let

$$R_{t+1}^K = \frac{r_{t+1}^K + (1-\delta_{t+1})\, q_{t+1}^K}{q_t^K}$$

denote the gross return per efficiency unit of capital obtained in period $t+1$ out of capital owned in period t. Then the entrepreneur will repay her loan at $t+1$ whenever her idiosyncratic shock ω_{t+1}^e exceeds the following threshold:

$$\overline{\omega}_{t+1}^e \equiv \frac{R_t^F b_t^e}{q_t^K k_t} \frac{1}{R_{t+1}^K} \equiv \frac{x_t^e}{R_{t+1}^K} \tag{23}$$

where $x_t^e \equiv \frac{R_t^F b_t^e}{q_t^K k_t}$ denotes entrepreneurial *leverage* as measured by the ratio of contractual debt repayment obligations at $t+1$, $R_t^F b_t^e$, to the value of the capital purchased at t, $q_t^K k_t$. Notice that (23) implies (differently from BGG where the contractual debt repayments are made contingent on R_{t+1}^K) that fluctuations in R_{t+1}^K will (realistically) produce fluctuations in entrepreneurial default rates.

When an entrepreneur defaults on her loan, the bank only recovers a fraction $1-\mu^e$ of the gross return of the capital available to the defaulted entrepreneur, where μ^e stands for verification costs incurred by the bank when taking possession of the returns and selling the underlying capital to other entrepreneurs. Hence a bank recovers $R_t^F b_t^e$ from performing loans and $(1-\mu^e)\, R_{t+1}^K q_t^K \omega^e k_t$ from non-performing loans. Ex ante, lenders recognize that under certain realizations of the idiosyncratic and the aggregate shocks, entrepreneurs will go bankrupt, especially when their ex ante leverage x_t^e is high.

The division between entrepreneurs and their bank of the total gross returns on a well-diversified portfolio of entrepreneurial investments at period t can be compactly expressed using notation similar to the one already introduced for borrowing households:

$$\Gamma^e\left(\overline{\omega}^e_{t+1}\right) = \int_0^{\overline{\omega}^e_{t+1}} \omega^e_{t+1} f^e\left(\omega^e_{t+1}\right) d\omega^e_{t+1} + \overline{\omega}^e_{t+1} \int_{\overline{\omega}^e_{t+1}}^{\infty} f^e\left(\omega^e_{t+1}\right) d\omega^e_{t+1}, \tag{24}$$

which gives the share of the gross returns (gross of verification costs) that will accrue to the bank, and

$$G^e\left(\overline{\omega}^e_{t+1}\right) = \int_0^{\overline{\omega}^e_{t+1}} \omega^e_{t+1} f^e\left(\omega^e_{t+1}\right) d\omega^e_{t+1}, \tag{25}$$

which denotes the part of those returns that comes from defaulted loans. Then the verification costs incurred by the bank on its portfolio of loans to entrepreneurs will be $\mu^e G^e\left(\omega^e_{t+1}\right)$, and the net share of the total gross returns of the portfolio that the bank appropriates can be expressed as $\Gamma^e\left(\overline{\omega}^e_{t+1}\right) - \mu^e G^e\left(\omega^e_{t+1}\right)$. We will use this expression below when introducing the bank's participation constraint into the entrepreneur's optimization problem.

3.2.2 Entrepreneurs' optimization problem

The contracting problem between the entrepreneur and her bank in period t can be written as one of maximizing the entrepreneur's expected wealth at $t+1$

$$\max_{x^e_t, k_t} E_t\left[\left(1 - \Gamma^e\left(\frac{x^e_t}{R^K_{t+1}}\right)\right) R^K_{t+1} q^K_t k_t\right]$$

subject to the participation constraint of the bank:

$$E_t\left[(1 - \Gamma^F(\overline{\omega}^F_{t+1}))\left(\Gamma^e\left(\frac{x^e_t}{R^K_{t+1}}\right) - \mu^e G^e\left(\frac{x^e_t}{R^K_{t+1}}\right)\right)\right] R^K_{t+1} q^K_t k_t = \rho_t \phi^F_t (q^K_t k_t - n^e_t), \tag{26}$$

which we can write with equality without loss of generality. Just like in the case of the bank extending loans to impatient households in (10), equation (26) states that the expected payoffs appropriated by the equity holders of a bank which holds a portfolio of loans to entrepreneurs must be sufficient to guarantee the expected rate of return ρ_t that the bankers require on the wealth that they contribute to bank. Bankers' equity contribution, $\phi^F_t (q^K_t k_t - n^e_t)$, is determined by the need to comply with a capital requirement ϕ^F_t on each unit of lending.

The factor $(1-\Gamma^F(\overline{\omega}_{t+1}^F))$ that multiplies the left hand side of (26) accounts for bank leverage and the possibility that an individual bank specialized in corporate loans (identified by the superscript F) fails due to sufficiently adverse idiosyncratic or aggregate shocks to the performance of its portfolio of entrepreneurial loans. The full details of the threshold $\overline{\omega}_{t+1}^F$ of the idiosyncratic shock below which the bank fails are presented below in subsection 3.4.

The final wealth of the entrepreneurs that start up in period t can be written as:

$$W_{t+1}^e = \frac{\left(1-\Gamma^e\left(\overline{\omega}_{t+1}^e\right)\right) R_{t+1}^K}{1-E_t\left\{\left(1-\Gamma^F\left(\overline{\omega}_{t+1}^F\right)\right)\left(\Gamma^e\left(\overline{\omega}_{t+1}^e\right)-\mu^e G^e\left(\overline{\omega}_{t+1}^e\right)\right)\frac{R_{t+1}^K}{\rho_t \phi_t^F}\right\}} n_t^e$$

and, since, a fraction $(1-\chi^e)$ of such wealth is left as a bequest to next generation of entrepreneurs, the law of motion of entrepreneurs' aggregate initial net worth can be written as

$$n_{t+1}^e = (1-\chi^e)\frac{\left(1-\Gamma\left(\overline{\omega}_{t+1}^e\right)\right) R_{t+1}^K}{1-E_t\left\{\left(1-\Gamma^F\left(\overline{\omega}_{t+1}^F\right)\right)\left(\Gamma^e\left(\overline{\omega}_{t+1}^e\right)-\mu^e G^e\left(\overline{\omega}_{t+1}^e\right)\right)\frac{R_{t+1}^K}{\rho_t \phi_t^F}\right\}} n_t^e.$$

3.3 Bankers

We model bankers in a very similar way to entrepreneurs: there are overlapping generations of risk-neutral two-period lived bankers. Bankers have exclusive access to the opportunity of investing their wealth as banks' inside equity capital. Each generation of bankers inherits its wealth in the form of bequests n_t^b from the previous generation of bankers and leaves bequests n_{t+1}^b to subsequent one. Aggregate banker net worth determines, for given capital requirement, the equilibrium required rate of return on bank equity and hence the lending rates.

A banker born at time t values the transfers to the patient dynasty at $t+1$, c_{t+1}^b, and the bequests left to the next cohort of bankers, n_{t+1}^b, according to the utility function $\left(c_{t+1}^b\right)^{\chi^b}\left(n_{t+1}^b\right)^{1-\chi^b}$. The banker who starts up at period t receives a bequest from the previous generation of bankers and decides how to allocate this wealth as inside capital across the two classes of existing banks: the banks specialized in housing loans (the H banks) and the banks specialized in entrepreneurial loans (the F banks). There is a continuum of ex ante identical perfectly competitive banks of each class. The ex post gross return at $t+1$ on the inside equity invested in H and F banks at t is denoted $\widetilde{\rho}^H_{t+1}$ and $\widetilde{\rho}^F_{t+1}$, respectively.

If a banker starting up with wealth n^b_t invests an amount e^F_t in inside equity of one or several F banks, and the rest in one or several of the H banks, his net worth after one period will be:

$$W^b_{t+1} = \widetilde{\rho}^F_{t+1} e^F_t + \widetilde{\rho}^H_{t+1} \left(n^b_t - e^F_t \right), \tag{27}$$

which the banker will devote to transfers c^b_{t+1} and bequests n^b_{t+1} by solving

$$\max_{c^b_{t+1}, n^b_{t+1}} \left(c^b_{t+1} \right)^{\chi^b} \left(n^b_{t+1} \right)^{1-\chi^b} \tag{28}$$

subject to:

$$c^b_{t+1} + n^b_{t+1} \leq W^b_{t+1}.$$

Optimizing behavior yields, the "dividend" rule

$$c^b_{t+1} = \chi^b W^b_{t+1} \tag{29}$$

and the "earnings retention" rule

$$n^b_{t+1} = (1-\chi^b) W^b_{t+1}. \tag{30}$$

Then the portfolio problem of the banker who starts up at t can be written as follows:

$$\max_{e^F_t} E_t(W^b_{t+1}) = E_t(\widetilde{\rho}^F_{t+1} e^F_t + \widetilde{\rho}^H_{t+1} \left(n^b_t - e^F_t \right)). \tag{31}$$

So, interior equilibria in which both classes of banks receive strictly positive inside equity from bankers will require the following equality to hold

$$E_t \widetilde{\rho}^F_{t+1} = E_t \widetilde{\rho}^H_{t+1} = \rho_t, \tag{32}$$

where ρ_t denotes bankers' required expected gross rate of return on equity investments undertaken at time t. This expected return is endogenously determined in equilibrium but both individual banks and bankers take it as given in their decisions. Specifically, ρ_t plays an essential role in writing of the bank participation constraints, (11) and (26), that appear in the problems that determine the terms of the debt contracts established between each class of banks and their borrowers.

Finally, the law of motion of the initial wealth available to each generation of bankers when they start up is:

$$n_{t+1}^b = \left(1-\chi^b\right)[\widetilde{\rho}_{t+1}^F e_t^F + \widetilde{\rho}_{t+1}^H \left(n_t^b - e_t^F\right)]. \tag{33}$$

This OLG set up makes bankers' risk neutrality operate as in a one-period model but makes bank capital an important variable for aggregate dynamics (such as e.g. in Gertler and Kiyotaki, 2011). As in the case of entrepreneurs, the transfer of c_{t+1}^b to the savings households will allow us to focus the welfare analysis on households' lifetime utility without neglecting the consumption capacity associated with bank profits.

3.4 Banks

The banks which issue the equity bought by bankers are institutions specialized in extending either mortgages or corporate loans. A bank lasts for one period only: it is an investment project created at t and liquidated at $t+1$. We assume a continuum of banking institutions of each class $j = H, F$. The equity payoffs π_{t+1}^j generated by a representative of class j after its period of operation is given by the positive part of the difference between the returns from its loans and the repayments due to its deposits:

$$\pi_{t+1}^j = \max\left[\omega_{t+1}^j \tilde{R}_{t+1}^j b_t^j - R_t^D d_t^j, 0\right], \tag{34}$$

where b_t^j and d_t^j are the loans extended and the deposits taken by the bank at t, respectively, R_t^D is the gross interest rate paid on the deposits taken at t (which is uniform across all banks given the presence of deposit insurance), and $\tilde{R}_{t+1}^j$ denotes the realized return on a well diversified portfolio of loans of class j. The max operator reflects the fact that the shareholders of the bank enjoy limited liability, so their payoffs cannot be negative.

The bank's idiosyncratic failure risk comes from the existence of an idiosyncratic portfolio return shock ω_{t+1}^j which is i.i.d. across the banks of class j and is assumed to follow a log-normal distribution with a mean of 1 and a distribution function $F^j(\omega_{t+1}^j)$. Bank default will be driven by fluctuations in the aggregate loan return $\tilde{R}_{t+1}^j$ (itself driven by firms' or households' default rates) and the bank-idiosyncratic shock ω_{t+1}^j which can be interpreted as a shock to each individual bank's ability to extract payoffs from its loans. A possible interpretation for this shock is a productivity shock in the production of banking services.

When a bank fails, its equity is written down to zero and its deposits are taken over by the deposit insurance fund which pays out all deposits in full. The deposit insurance fund recoups this by taking over the failed bank's loan portfolio minus resolution costs which are assumed to be a μ^j fraction of total bank assets.

The bank also faces a regulatory capital constraint:

$$e_t^j \geq \phi_t^j b_t^j, \tag{35}$$

where ϕ_t^j is the potentially time-varying capital-to-asset ratio of banks of class j. Thus, the bank is restricted by regulation to hold equity to at least a fraction ϕ_t^j of the assets it holds on its balance sheet. It is possible to show that in equilibrium the capital requirement holds with equality, so that the bank's loans can be written as $b_t^j = \frac{e_t^j}{\phi_t^j}$ and its deposits as $d_t^j = \frac{(1-\phi_t^j)e_t^j}{\phi_t^j}$. Allowing the capital requirement ϕ_t^j to vary across different classes of banks is consistent with thinking of them as risk-based (like under Basel III) or as sectoral requirements serving as tools of macroprudential policy.

Note that the role of the capital requirement is the mirror image of the effect of the limited liability that was described earlier. A higher capital requirement reduces leverage, forcing banks to get funded with a larger share of equity, which, due to its scarcity, is more expensive. Moreover, it reduces the probability of bank default and hence the subsidy implied by the deposit insurance, leading to a higher lending rate and more restricted access to credit for households and entrepreneurs. Indeed, it can be shown analytically that the implicit subsidy made possible by the existence of deposit insurance decreases with the capital requirement ϕ and increases with the probability of bank default.

Let $\overline{\omega}_{t+1}^{j}$ denote the threshold realization of ω_{t+1}^{j} below which the bank fails because the realized return on its loan portfolio is lower than its deposit repayment obligations, i.e.

$$\overline{\omega}_{t+1}^{j}\tilde{R}_{t+1}^{j}b_{t}^{ij} \equiv R_{t}^{D}d_{t}^{ij}. \tag{36}$$

Using our previous expressions for b_t^j and d_t^j, we can write the threshold as

$$\overline{\omega}_{t+1}^{j} = (1-\phi_{t}^{j})\frac{R_{t}^{D}}{\tilde{R}_{t+1}^{j}}, \tag{37}$$

that is, the product of the leverage ratio $1-\phi_t^j$ and the spread between the realized gross loan return and the gross deposit rate, $\tilde{R}_{t+1}^{j}/R_{t}^{D}$.

The equity payoffs in (34) can then be rewritten as

$$\begin{aligned}\pi_{t+1}^{j} &= \max\left[\omega_{t+1}^{j}-\overline{\omega}_{t+1}^{j},0\right]\frac{\tilde{R}_{t+1}^{j}}{\phi_{t}^{j}}e_{t}^{j} \\ &= \left[\int_{\overline{\omega}_{t+1}^{j}}^{\infty}\omega_{t+1}^{j}f^{j}\left(\omega_{t+1}^{j}\right)d\omega_{t+1}^{j}-\overline{\omega}_{t+1}^{j}\int_{\overline{\omega}_{t+1}^{j}}^{\infty}f^{j}\left(\omega_{t+1}^{j}\right)d\omega_{t+1}^{j}\right]\frac{\tilde{R}_{t+1}^{j}}{\phi_{t}^{j}}e_{t}^{j}\end{aligned} \tag{38}$$

where $f^{j}\left(\omega_{t+1}^{j}\right)$ denotes the density distribution of ω_{t+1}^{j} conditional on the information available when the loans are originated at time t. Hence $F^{j}(\overline{\omega}_{t+1}^{j})$ is the probability of default of a bank of class j (conditional upon a given realization of the aggregate loan return $\tilde{R}_{t+1}^{j}$).

Following BGG, it is useful to define

$$\Gamma^{j}(\overline{\omega}_{t+1}^{j}) = \int_{0}^{\overline{\omega}_{t+1}^{j}}\omega_{t+1}^{j}f^{j}\left(\omega_{t+1}^{j}\right)d\omega_{t+1}^{j}+\overline{\omega}_{t+1}^{j}\int_{\overline{\omega}_{t+1}^{F}}^{\infty}f^{j}\left(\omega_{t+1}^{j}\right)d\omega_{t+1}^{j} \tag{39}$$

and

$$G^{j}\left(\overline{\omega}_{t+1}^{j}\right) = \int_{0}^{\overline{\omega}_{t+1}^{j}}\omega_{t+1}^{j}f^{j}\left(\omega_{t+1}^{j}\right)d\omega_{t+1}^{j}, \tag{40}$$

which denotes the share of total bank assets which belong to banks which end up in default. Thus $\mu^j G^j\left(\overline{\omega}_{t+1}^j\right)$ is the total cost of bank default expressed as a fraction of total bank assets.

Using this notation we can write

$$\pi_{t+1}^j = \frac{\left[1 - \Gamma^j(\overline{\omega}_{t+1}^j)\right] \tilde{R}_{t+1}^j}{\phi_t^j} e_t^j \tag{41}$$

and define the ex post gross rate of return on equity invested in a bank of type j as:

$$\widetilde{\rho}_{t+1}^j = \frac{\left[1 - \Gamma^j(\overline{\omega}_{t+1}^j)\right] \tilde{R}_{t+1}^j}{\phi_t^j}. \tag{42}$$

For completeness, notice that derivations in prior sections in imply the following expressions for $\tilde{R}_{t+1}^j$, $j = H, F$:

$$\tilde{R}_{t+1}^H = \left(\Gamma^m\left(\frac{x_t^m}{R_{t+1}^H}\right) - \mu^m G^m\left(\frac{x_t^m}{R_{t+1}^H}\right)\right)\frac{R_{t+1}^H q_t^H h_t^m}{b_t^m},$$

$$\tilde{R}_{t+1}^F = \left(\Gamma^e\left(\frac{x_t^e}{R_{t+1}^K}\right) - \mu^e G^e\left(\frac{x_t^e}{R_{t+1}^K}\right)\right)\frac{R_{t+1}^K q_t^K k_t}{q_t^K k_t - n_t^e}.$$

The aggregate default rate for the banking system PD_t^b, which also enters the saving households' budget constraint, is given by

$$PD_t^b = \frac{E_{t-1}^F PD_t^F + ((1-\chi^b)N_{t-1}^b - E_{t-1}^F)PD_t^H}{(1-\chi^b)N_{t-1}^b} \tag{43}$$

3.5 Consumption good production

The consumption good is produced instantaneously by perfectly competitive firms which combine capital rented from entrepreneurs, k_{t-1}, and labor supplied by patient and impatient households, l_t, using a standard Cobb-Douglas production function:

$$y_t = A_t k_{t-1}^{\alpha} l_t^{1-\alpha},$$

where A_t is total factor productivity, and α and η are elasticity parameters. Optimality in the use of the capital input requires:

$$r_t^K = \alpha \frac{y_t}{k_{t-1}}.$$

Finally, optimality in the use of the household labour input requires:

$$w_t = (1-\alpha)\frac{y_t}{l_t}.$$

3.6 Capital good production

Capital producing firms obtain new units of capital from the consumption good and sell them to entrepreneurs at the price q_t^K. These firms are owned by the patient households and the technology they use is subject to adjustment costs. In order to produce $I_t = k_t - (1-\delta_t)\,k_{t-1}$ of new capital goods, the representative capital producing firm needs to spend resources of

$$\left[1 + g\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $g\left(\frac{I_t}{I_{t-1}}\right)$ is the investment adjustment cost function (see the Appendix for details). Since the firm is owned by the patient households, its objective is to choose investment I_t in order to maximize

$$E_t \sum_{i=0}^{\infty} (\beta^s)^i \frac{c_t^s}{c_{t+i}^s} \left\{ q_{t+i}^K I_{t+i} - \left[1 + g\left(\frac{I_{t+i}}{I_{t+i-1}}\right)\right] I_{t+i} \right\}$$

4 Market clearing

Housing market

The stock of housing $h_t = h_t^s + h_t^m$ is made of the depreciated previous stock plus the additions coming from residential investment I_t^H:

$$h_t = (1-\delta_t^H)h_{t-1} + I_t^H \tag{44}$$

In the calibration we will set $\delta_t^H < \delta_t$ on account of the fact that housing depreciates less quickly than physical capital. Housing producing firms obtain new units of capital from the consumption good and sell them to households at the price q_t^H. Also construction firms are owned by the patient households and the technology they use is subject to adjustment costs. In order to produce I_t^H of new housing goods, the representative construction firm needs to spend resources amounting to

$$\left[1 + g^H\left(\frac{I_t^H}{I_{t-1}^H}\right)\right] I_t^H,$$

where $g^H\left(\frac{I_t^H}{I_{t-1}^H}\right)$ is the housing investment adjustment cost function (see the Appendix for details).[12]

Banks' inside equity market

The total equity provided by bankers ($n_t^b = (1-\chi^b)W_t^b$) must equal the sum of the demand for bank equity from the banks making loans to households, $\phi_t^H\left(\frac{q_t^H h_t^m x_t^e}{R_t^m}\right)$, and from the banks making loans to entrepreneurs, $\phi_t^F\left[q_t^K k_t - (1-\chi^e)W_t^e\right]$:

$$(1-\chi^b)W_t^b = \phi_t^F\left[q_t^K k_t - (1-\chi^e)W_t^e\right] + \phi_t^H\left(\frac{q_t^H h_t^m x_t^m}{R_t^m}\right).$$

Deposit market

The deposits held by the saving households (d_t) must equal the sum of the demand for deposit funding from the banks making loans to households, $(1-\phi_t^H)\left(\frac{q_t^H h_t^m x_t^e}{R_t^m}\right)$, and from the banks making loans to entrepreneurs, $(1-\phi_t^F)\left[q_t^K k_t - (1-\chi^e)W_t^e\right]$:

$$d_t = (1-\phi_t^F)\left[q_t^K k_t - (1-\chi^e)W_t^e\right] + (1-\phi_t^H)\left(\frac{q_t^H h_t^m x_t^m}{R_t^m}\right).$$

Consumption good market

In the goods market, total output y_t should equal the total consumption demands of the savers c_t^s, the borrowers c_t^m, plus the resources absorbed in the production of the new capital I_t and the new housing I_t^H, plus the resources lost in the recovery by lenders of the proceeds

[12] Apart from the different rate of depreciation, housing production is modelled in the same way as capital production and we therefore do not develop it here in greater detail.

associated with defaulted bank loans, in transaction costs by depositors at failed banks, or by the deposit insurance agency in the recovery of bank assets:

$$\begin{aligned}
y_t &= c_t^s + c_t^m \\
&+ \left[1 + g\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + \left[1 + g^H\left(\frac{I_t^H}{I_{t-1}^H}\right)\right] I_t^H \\
&+ \mu^e G^e\left(\overline{\omega}_t^e\right) R_t^K q_{t-1}^K k_{t-1} + \mu^m G^m\left(\frac{x_{t-1}^m}{R_t^H}\right) R_t^H q_{t-1}^H h_{t-1}^m + \gamma P D_t^b R_{t-1}^D d_{t-1} \\
&+ \mu^B \left[G^H\left(\overline{\omega}_t^H\right) \widetilde{R}_t^H \left(\frac{q_{t-1}^H h_{t-1}^m x_{t-1}^m}{R_{t-1}^m}\right) + G^F\left(\overline{\omega}_t^F\right) \widetilde{R}_t^F \left[q_{t-1}^K k_{t-1} - \left(1 - \chi^e\right) W_{t-1}^e\right]\right].
\end{aligned}$$

We will also consider a measure of net output, $\widetilde{y}_t$, which is net of the expenditure associated to default:

$$\widetilde{y}_t = c_t^s + c_t^m + \left[1 + g\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + \left[1 + g^H\left(\frac{I_t^H}{I_{t-1}^H}\right)\right] I_t^H \tag{45}$$

This output measure is arguably more important when analyzing welfare, since costs associated with default do not increase household utility.

Labor market

Total demand for households' labor by the consumption good producing firms, $(1-\alpha)\frac{y_t}{w_t}$, must equal to the labour supply of the two types of households:

$$(1-\alpha)\frac{y_t}{w_t} = l_t^s + l_t^m.$$

Deposit insurance agency

Finally, the total costs to the deposit insurance agency T_t due to losses in H and F banks, and hence the total lump sum tax imposed on patient households, are given by

$$T_t^H = \left[\overline{\omega}_t^H - \Gamma^H\left(\overline{\omega}_t^H\right) + \mu^H G^H\left(\overline{\omega}_t^H\right)\right] \widetilde{R}_t^H \left(\frac{q_{t-1}^H h_{t-1}^m x_{t-1}^m}{R_{t-1}^m}\right)$$

and

$$T_t^F = \left[\overline{\omega}_t^F - \Gamma^F\left(\overline{\omega}_t^F\right) + \mu^F G^F\left(\overline{\omega}_t^F\right)\right] \widetilde{R}_t^F \left[q_{t-1}^K k_{t-1} - \left(1 - \chi^e\right) W_{t-1}^e\right].$$

Bank capital regulation

The regulatory capital requirement ϕ_t^j is generally specified as follows:

$$\phi_t^j = \bar{\phi}_0^j + \bar{\phi}_1^j \left[\log\left(b_{t-1}^H + b_{t-1}^F\right) - \log\left(\bar{b}^H + \bar{b}^F\right)\right] + \bar{\phi}_2^j \left[\log(GDP_{t-1}) - \log(\overline{GDP})\right], \quad (46)$$

where $\bar{\phi}_0^j$ is the structural capital requirement and the additional terms capture the risk-sensitive part of the requirement and/or the existence of a *countercyclical capital buffer* that depends on the state of the economy.[13] The above specification encompasses the possibility that the cyclical component of the requirement increases or decreases with the cyclical deviations of total bank credit, $b_{t-1}^H + b_{t-1}^F$, aggregate output, GDP_{t-1}, or the total bank-credit-to-gdp ratio, $\left(b_{t-1}^H + b_{t-1}^F\right)/GDP_{t-1}$, from their steady state levels, $\bar{b}^H + \bar{b}^F$, $\overline{GDP}$, and $\left(\bar{b}^H + \bar{b}^F\right)/\overline{GDP}$, respectively.

For computational convenience, our countercyclical adjustment of the capital requirements is not explicitly formulated as the type of countercyclical buffer (CCB) introduced by Basel III. In Basel III, the CCB is an add-on to the capital requirements (Core Tier 1 + the conservation buffer), that is to say a positive number. In our symmetric formulation above, the total capital charge may both increase or decrease relative to its time-invariant benchmark. In other words, in our set up, capital requirements can be relaxed even at low level of capital. In the context of Basel III, a similar outcome could arguably be obtained by adjusting the risk weights or through the exercise of some other form of supervisory forbearance in bad times.

5 Baseline parametrization

The baseline parametrization of the model is largely based on values that are standard in the literature. In particular, it relies mostly on Gerali et al. (2010) and Darracq-Pariès et al. (2011), who both develop DSGE models of the euro area, for most parameters of the house-

[13] To save on notation, when analyzing time-invariant capial requirements below ($\bar{\phi}_1^j = \bar{\phi}_2^j = 0$), we will refer to $\bar{\phi}_0^j$ by simply ϕ^j.

holds and entrepreneurs sectors as well as those governing exogenous shocks (persistence and volatility). Capital requirements are set at a benchmark level of 8% for corporate loans (compatible with the full weight level of Basel I and the treatment of not rated corporate loans in Basel II and III) and 4% for mortgage loans (compatible with their 50% risk weight in Basel I). *Table 1* reports all the parameter values. One period in the model corresponds to one quarter in calendar time.

Table 1. Baseline parametrization of the model

Description	Par.	Value	Description	Par.	Value
Patient household discount factor	β^s	0.995	Capital requirement for mortgage loans	ϕ^H	0.04
Impatient household discount factor	β	0.98	Capital requirement for corporate loans	ϕ^F	0.08
Patient household utility weight of housing	υ^s	0.25	Mortgage bank bankruptcy cost	μ^H	0.3
Impatient household utility weight of housing	υ^s	0.25	Corporate bank bankruptcy cost	μ^F	0.3
Patient household marginal disutility of labor	φ^s	1	Capital share in production	α	0.3
Impatient household marginal disutility of labor	φ^m	1	Capital depreciation rate	δ^K	0.025
Inverse of Frisch elasticity of labor	η	1	Capital adjustment cost parameter	ψ^K	2
Depositor cost of bank default	γ	0.10	Housing depreciation rate	δ^H	0.01
Variance of household idiosyncratic shocks	σ^2_m	0.08	Housing adjustment cost parameter	ψ^H	2
Household bankruptcy cost	μ^m	0.3	TFP shock persistence	ρ_A	0.9
Dividend payout of entrepreneurs	χ^e	0.05	Capital depreciation shock persistence	ρ^δ	0.9
Variance of entrepreneurial risk shock	σ^2_e	0.12	Housing depreciation shock persistence	ρ^{δ^H}	0.9
Entrepreneur bankruptcy cost	μ^e	0.3	Risk shock(s) persistence	ρ^σ	0.9
Dividend payout of bankers	χ^b	0.05	Housing demand shock(s) persistence	ρ^υ	0.9
Variance of mortgage bank risk shock	σ^2_H	0.0119	Labor disutility shock(s) persistence	ρ^φ	0.9
Variance of corporate bank risk shock	σ^2_F	0.0238			

6 Results

First, we analyze the long-run implications of different levels of capital requirements. Second, we analyze the effects of shocks to aggregate productivity, capital depreciation and bank risk in the dynamics around the steady state. We compare the transmission of shocks under the baseline capital requirements ($\phi^F = 0.08$; $\phi^H = 0.04$) and under higher capital requirements.

6.1 Steady state effects of capital requirements

In the following, we investigate the relationship between different levels of capital requirements, ϕ^F and ϕ^H, and welfare in steady state.[14] The welfare function for each agent is given by the conditional expectation of the corresponding lifetime utility as of a reference period t. Due to the presence of several classes of agents in the model, we consider a (utilitarian) social welfare measure that aggregates the individual welfare of the representative agents of each class. We will focus on households only.

Specifically, we compute the welfare gains associated with any particular policy change as a weighted average of the welfare gains of each household dynasties, the patient ($j = s$) and the impatient ($j = m$), measured in consumption-equivalent terms, i.e. the percentage increase in steady state consumption, Δ^j, that would make the welfare of such dynasty under the baseline policy ($\phi^F = 0.08$; $\phi^H = 0.04$) equal to the welfare under alternative values of ϕ^F and ϕ^H. And we weight each individual Δ^j with the share of dynasty j in aggregate consumption under the baseline policy. So the reported social welfare gains are given by

$$\Delta W \equiv \frac{c_0^s}{c_0^s + c_0^m}\Delta^s + \frac{c_0^m}{c_0^s + c_0^m}\Delta^m, \tag{47}$$

where c_0^j denotes the steady state consumption of dynasty j under the baseline policy.

Importantly, although entrepreneurs and bankers do not enter into our social welfare criterion on their own right, the contribution of entrepreneurial and bank profits to aggregate consumption capacity is taken into account through the (lump sum) transfers that these agents have been assumed to make to the patient dynasty.[15]

We start by providing a first key result of our paper, namely the steady state relationship between the capital requirement ratio and social welfare gains. *Figure 1* displays the steady state social welfare gains ΔW associated with capital requirements higher than the baseline

[14] The Appendix reports the key equations of interest for the deterministic steady state solution of the model.

[15] We have also considered a version of the model in which these transfers are split between the two dynasties and the results are qualitatively and quantitatively very similar.

value. The hump-shaped relationship between higher capital requirements and social welfare gains reflect the presence of a trade-off. Higher capital requirements reduce the implicit subsidy to banks associated with limited liability and deposit insurance. Thus, in comparison with the baseline policy, an increase in capital requirements implies both a reduction in the supply of loans (which are provided at higher interest rates) and a lower average default rate of banks (see *Figure 2a*. The implied reduction in the social cost of banks' default has a positive effects on economic activity, notably consumption and investment (*Figure 2b*). This effect dominates at first. In contrast, the negative effects on economic activity coming from the reduction in the supply of credit to the economy dominate when capital requirements are high enough (actually, at levels in which banks' default rate is virtually zero). Note that the initial increase in credit displayed in *Figure 2c* is due to the reduction in the cost of deposit funding. Indeed, banks are less fragile and depositors require a lower premium in compensation for their anticipated costs of bank default.

(Figures 1 and 2a-2b here)

Under the calibration reported in Section 5, we find that the optimal capital requirement should be around 10.5 per cent for business loans and 5.25 per cent for mortgages (since we are assuming a 50% risk weight of mortgages). This is consistent with BIS (2010) and Miles et al. (2013). However, our model would not support higher capital ratios, such as the value of 25% recently suggested by Admati and Hellwig (2013). In our model, too high capital requirements would excessively restricts credit availability while reducing default rates only marginally, resulting in a net welfare loss.

Overall, our setup provides a clear rationale for capital regulation, which arises as a welfare improving response to the excessive risk taking by banks. Importantly, banks' equity funding in the model is limited by the wealth endogenously accumulated by the bankers who own and manage the banks. So capital requirements reduce bank leverage, bank failure risk and the implicit subsidies associated with deposit insurance, and, simultaneously, they also force the banks to make a greater use of bankers' limited wealth. In the short run, this second aspect makes capital requirements have a potential impact on the cost of equity funding (due to the scarcity of bankers' wealth). However, over time, bankers accumulate additional wealth and the cost of equity funding in the new steady state is the same as under lower requirements. So the steady state results are entirely due to banks' lower leverage and their possibly higher weighted average cost of funds.

6.2 Capital requirements and shock propagation

The second set of results concerns the model responses to structural shocks, in a first order approximation around the deterministic steady state. *Figures 3a*reports the response of GDP to a 1 per cent decline in aggregate productivity. It compares the response of GDP under alternative parameterizations of the model. We find that higher capital requirements: (i) mitigate the effects of a reduction in aggregate productivity (*panel A*); (ii) mimic the dynamics of a no bank default economy (*panel B*). Comparing the benchmark economy with an economy with higher financial distress (higher level of bank risk σ^F and σ^H) in the banking sector, we also find that high financial distress greatly exacerbates the negative effect of productivity shocks (*panel C*).

Figures 3b and 3c report on the effect of a negative productivity shock on the key variables in the model. Each graph with the impulse response functions contains four lines. We report the responses of the variables in the benchmark economy, i.e. $\phi^F = 0.08$ and $\phi^H = 0.04$ (starred line) and in the economy with capital requirements closer to the welfare maximizing ones, i.e. $\phi^F = 0.105$ and $\phi^H = 0.0525$, (dashed line). Further, we also consider a parametrization with no bank default, i.e. $\sigma^F = \sigma^H \approx 0$, (solid line) and with high financial distress, i.e. $\sigma^F = 0.0238$ and $\sigma^H = 0.0119$ (dotted line). This set of results allows us to understand the role of capital regulation for the propagation of shocks.

(Figures 3a-3c)

An exogenous reduction in aggregate productivity implies a reduction in spending and production. Thus, the relative price of housing and physical capital decline leading to an increase in the default by households and entrepreneurs (*Figures 3b*). Higher borrowers' default reduces bank capital and, thus, the supply of loans (*bank capital channel*). At the same time, bank default increases leading to an increase in the cost of deposit funding, which further increases the bank lending rates that banks have to charge in order to satisfy bankers' participation constraints (*bank funding channel*). Both channels further contribute to the reduction in the price of houses and physical capital leading to higher default rates among borrowers (*Figures 3c).*

Figures 4a-4b replicate the same analysis for a depreciation shock, namely a negative shock to the value of the stocks of housing and physical capital (the shock is assumed to hit both stocks in the same proportion at the same time). Also here, even more so than for TFP shocks, the presence of bank default leads to very strong amplification, notably of the effects on GDP. In the model with a high capital requirement or no bank default we find a mild and short contraction of output, but under high bank risk the implied recession is much deeper and long lasting. The difference can be largely explained by the different effect on bank capital and bank defaults. Bank capital declines (Figure 4a) and this restricts the supply of loans in a very persistent way, specially under the high bank risk calibration. In addition, bank defaults increase leading to a rise in the cost of deposit funding, which further depresses economic activity and amplifies the decline in bank capital. Our model features a powerful interaction between bank capital and the bank cost of funding channels of crisis transmission. The result is a deep and persistent decline in economic activity in the economy with low capital requirements (i.e. the benchmark economy).

(Figures 4a-4b)

Figures 5a and 5b report on the dynamic effects of shocks to the standard deviation of the idiosyncratic shocks to banks' performance, which we interpret as a shock to "bank risk". Similar to the results for the depreciation shock, the effects of the shock are very mild in the economy with high capital requirements or an initially low level of bank risk ("no bank risk" economy). In the benchmark economy, a high starting value for bank risk, coupled with low capital requirements, has the opposite effect of greatly amplifying the transmission of the shock. Again, the difference is largely explained by the diverging paths for bankers' net worth and the cost of deposit funding.

(Figures 5a-5b)

Figure 6 provides an overview of the key results. Overall, these results suggest that, first, an economy with "high capital requirements" (set close to the welfare maximizing ones) behaves very similarly to an economy with no bank default. Thus, high capital requirements insulate the economy from the bank net worth channel and prevent excessive volatility due to banks' excessive lending and excessive failure risk. Additionally, the figures show that when bank leverage is high (because capital requirements are low), the economy is more responsive to shocks. This evidences that the limited liability and deposit insurance subsidies, which allow banks to meet the required rate of return on equity with lower lending rates, constitute a potentially powerful channel of financial amplification and contagion.

(Figure 6)

6.3 Countercyclical capital adjustments

Figures 7a-7b summarize the results of running the same exercises as in prior figures but comparing economies with cyclically-flat capital requirements like in the previous analysis with economies in which the capital requirements are cyclically adjusted. In particular, in terms of equation (47) we set $\bar{\phi}_1^j = 0.3$ *and* $\bar{\phi}_2^j = 0$ so that the capital requirements vary according to the percentage deviation of total credit from its steady state level, in a symmetric fashion.

The results suggest that introducing a countercyclical adjustment mitigates the reduction in the supply of credit to the economy, but does so at the cost of an increase in bank default and, thus, a higher overall cost of funds for banks. It turns out that the countercyclical adjustment adds stability when associated with a high level of capital requirements (i.e. when bank default risk is already very low).

In contrast, when the countercyclical adjustment is added to the economy with low capital requirements, we find that for most shocks and variables the result is more rather than less amplification. The countercyclical adjustment of the capital requirements actually helps moderate the negative output effects of the shocks in the short run. However, the effects are negative over the medium/long run. Overall, the lesson from this exercise is that relaxing capital standards only works well when the starting capital requirement position is strong.

7 Conclusions

In this paper we have proposed a DSGE model with multiple financial frictions affecting households, entrepreneurs and banks. One distinctive feature of our model is that it contains three layers of default and that, unlike in previous literature, a default has material consequences for the balance sheet of the lender. In this way, we allow both idiosyncratic and aggregate shocks to matter for the default frequency of the various classes of borrowers and we can assess the macroeconomic consequences of default. The model also allows us to study the impact of household and corporate defaults on banks' net worth and default, the feedback coming from the importance of the latter in determining the availability and the cost of bank loans.

From a policy perspective, our main focus has been on bank capital regulation. In our model, bank capital regulation finds a rationale in the presence of two distortions that may push credit provision away from the first best solution (the solution that a social planner would select). On the one hand, costly state verification makes lending costly and this reduces credit compared with the socially optimal level in an ideal economy without these costs. On the other hand, banks have limited liability and their deposits enjoy government guarantees, which encourages them to potentially extend excessive lending. Bank capital regulation needs to find a compromise between these two objectives. In our baseline calibration, we find that a reasonable compromise can be found at levels of the capital ratio around 10.5%, which is above the Basel III levels of capital but below more radical proposals such as those of Admati and Hellwig (2013). In terms of the dynamics of the model, we find that shock propagation and amplification are large when idiosyncratic bank risk is high and the bank capital requirements are low. Higher capital requirements largely eliminate the amplification due to the financial factors in the present model.

Needless to say, our results are highly model dependent and their robustness should be checked by considering richer models that relax some of the simplifying assumptions of this first step (such as banks' inability to raise outside equity or the binding nature of bank capital requirements). Additionally, the model could be extended to introduce liquidity risk (and its regulation) and to allow for securitization (and its regulation). Finally, our model is entirely real and considers no nominal rigidities and, hence, has no room for (conventional) monetary policy. However, it would be relatively straightforward to add nominal rigidities in order to study the interplay between macro-prudential policy (capital regulation) and monetary policy. Several of these extensions appear to be interesting avenues for further research.

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Appendix: Model equations

Patient households

The dynasty of saving households is assumed to maximize its present value of utility subject to the budget constraint

$$d_t + c_t^s + q_t^H \left(h_t^s - h_{t-1}^s \left(1 - \delta_t^H\right)\right) = l_t^s w_t + d_{t-1}\widetilde{R}_t^D - T_t^s + \Pi_t + \Pi_t^H \tag{48}$$

The first-order conditions for this program, with respect to c_t^s, l_t^s, d_t and h_t, are

$$\lambda_t^s = \frac{1}{c_t^s}, \tag{49}$$

$$\varphi^s (l_t^s)^\eta = w_t \lambda_t^s, \tag{50}$$

$$\lambda_t^s = \beta^s E_t \left(\lambda_{t+1}^s \widetilde{R}_{t+1}^D \right), \tag{51}$$

$$\lambda_t^s q_t^H = \beta^s E_t \left(\frac{1}{h_{t+1}^s} + \lambda_{t+1}^s \left(1 - \delta_{t+1}^H\right) q_{t+1}^H \right), \tag{52}$$

with $\widetilde{R}_t^D = R_{t-1}^D \left(1 - \gamma P D_t^b\right)$ and λ_t^s the multiplier associated to the budget constraint.

Impatient households

The dynasty of borrowing households also maximizes its present value of utility under the budget constraint

$$c_t^m + q_t^H h_t^m - (1 - \Gamma_t^m)\left(1 - \delta_t^H\right) R_t^H q_{t-1}^H h_{t-1}^m = b_t^m + l_t^m w_t - T_t^m \tag{53}$$

The first-order conditions for this program, with respect to c_t^m, l_t^m, x_t^m, h_t^m and b_t^m are

$$\lambda_t^m = \frac{1}{c_t^m}, \tag{54}$$

$$\varphi^m (l_t^m)^\eta = w_t \lambda_t^m, \tag{55}$$

$$E_t \left(\beta^m \lambda_{t+1}^m \Gamma_{t+1}^{m\prime} - \xi_t^m \left(1 - \Gamma_{t+1}^H\right) \left(\Gamma_{t+1}^{m\prime} - \mu^m G_{t+1}^{m\prime} \right)\right) = 0, \tag{56}$$

$$\beta^m E_t\left(\frac{1}{h^m_{t+1}}\right) - q^H_t\left(\lambda^m_t + \beta^m E_t\left(\lambda^m_{t+1}\left(1-\Gamma^m_{t+1}\right)R^H_{t+1}\right) + \xi^m_t E_t\left(R^H_{t+1}\left(1-\Gamma^H_{t+1}\right)\left(\Gamma^m_{t+1} - \mu^m G^m_{t+1}\right)\right)\right) = 0, \tag{57}$$

$$\lambda^m_t - \xi^m_t E_t\left(\widetilde{\rho}^H_{t+1}\right)\phi^H_t = 0 \tag{58}$$

with

$$\bar{\omega}^m_t = \frac{x^m_{t-1}}{R^H_t}, \tag{59}$$

$$x^m_t = \frac{R^m_t b^m_t}{q^H_t h^m_t}, \tag{60}$$

$$R^H_t = \frac{\left(1-\delta^H_t\right)q^H_t}{q^H_{t-1}}, \tag{61}$$

λ^m_t the multiplier associated to the budget constraint and ξ^m_t the multiplier associated to the participation constraint.

Entrepreneurs

The entrepreneurs' wealth follows the law of motion

$$W^e_t = k_{t-1} q^K_{t-1} R^K_t \left(1-\Gamma^e_t\right) - T^e_t. \tag{62}$$

As a result of their optimizing behavior, entrepreneurs' aggregate net worth obeys

$$n^e_t = \left(1-\chi^e\right) W^e_t. \tag{63}$$

The representative entrepreneur maximizes his expected wealth subject to the participation constraint of the bank. The first-order conditions for this program, with respect to x^e_t and k_t, are

$$E_t\left(\Gamma^{e'}_{t+1} - \xi^e_t\left(1-\Gamma^F_{t+1}\right)\left(\Gamma^{e'}_{t+1} - \mu^e G^{e'}_{t+1}\right)\right) = 0 \tag{64}$$

and

$$E_t\left(\left(1-\Gamma^e_{t+1}\right)R^K_{t+1} + \xi^e_t\left(R^K_{t+1}\left(1-\Gamma^F_{t+1}\right)\left(\Gamma^e_{t+1} - \mu^e G^e_{t+1}\right) - \widetilde{\rho}^F_{t+1}\phi^F_t\right)\right) = 0 \tag{65}$$

with ξ^e_t the multiplier associated to the participation constraint.

The rate of return on capital R_t^K, $\overline{\omega}_t^e$ and x_t^e are defined as

$$\overline{\omega}_t^e = \frac{x_{t-1}^e}{R_t^K}, \tag{66}$$

$$x_t^e = \frac{R_t^F \left(q_t^K k_t - n_t^e\right)}{q_t^K k_t} \tag{67}$$

and

$$R_t^K = \frac{r_t^K + (1 - \delta_t)\, q_t^K}{q_{t-1}^K}. \tag{68}$$

Bankers

The law of motion of bankers' aggregate net worth is

$$W_t^b = \left(\widetilde{\rho}_t^F e_{t-1}^F + \widetilde{\rho}_t^H \left(n_{t-1}^b - e_{t-1}^F\right)\right), \tag{69}$$

where inside equity follows

$$e_t^F = \left(q_t^K k_t - n_t^e\right) \phi_t^F. \tag{70}$$

As a result of his optimizing behavior, banker's wealth obeys

$$n_t^b = \left(1 - \chi^b\right) W_t^b. \tag{71}$$

Finally, the first order condition for the bankers' portfolio choice problem implies

$$E_t \widetilde{\rho}_{t+1}^F = E_t \widetilde{\rho}_{t+1}^H. \tag{72}$$

Banks

Banks' default thresholds are given by

$$\overline{\omega}_t^H = \frac{R_{t-1}^D \left(1 - \phi_{t-1}^H\right)}{\widetilde{R}_t^H} \tag{73}$$

and

$$\overline{\omega}_t^F = \frac{R_{t-1}^D \left(1 - \phi_{t-1}^F\right)}{\widetilde{R}_t^F}. \tag{74}$$

The ex post gross rates of return on equity are

$$\widetilde{\rho}_t^H = \frac{\widetilde{R}_t^H \left(1 - \Gamma_t^H\right)}{\phi_{t-1}^H} \tag{75}$$

and

$$\widetilde{\rho}_t^F = \frac{\widetilde{R}_t^F \left(1 - \Gamma_t^F\right)}{\phi_{t-1}^F}. \tag{76}$$

The realized gross loan return rates can be expressed as

$$\widetilde{R}_t^H = \frac{h_{t-1}^m q_{t-1}^H R_t^H \left(\Gamma_t^m - \mu^m G_t^m\right)}{b_{t-1}^m} \tag{77}$$

and

$$\widetilde{R}_t^F = \frac{k_{t-1} q_{t-1}^K R_t^K \left(\Gamma_t^e - \mu^e G_t^e\right)}{q_{t-1}^K k_{t-1} - n_{t-1}^e}. \tag{78}$$

Consumption good production

The aggregate production function is

$$y_t = A_t k_{t-1}^{\alpha} l_t^{1-\alpha}. \tag{79}$$

The rental rate of capital and the competitive real wage are

$$r_t^K = \frac{\alpha y_t}{k_{t-1}}, \tag{80}$$

and

$$w_t = \frac{(1-\alpha)\, y_t}{l_t}. \tag{81}$$

Capital good production

Capital producing firms' profits are given by

$$\Pi_t = q_t^K I_t - I_t \left(1 + g_t\right)$$

where $g_t \equiv g(\frac{I_t}{I_{t-1}}) = \frac{\psi^K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$. The evolution of capital stock is

$$k_t = k_{t-1}\left(1 - \delta_t\right) + I_t \left(1 - g_t\right). \tag{82}$$

Intertemporal profits maximization with respect to I_t yields

$$q_t^K = 1 + g_t + \frac{I_t}{I_{t-1}} g_t' - \beta^s E_t \left(\frac{\lambda_{t+1}^s}{\lambda_t^s} \left(\frac{I_{t+1}}{I_t} \right)^2 g_{t+1}' \right). \tag{83}$$

Housing supply

Residential good producing firms' profit function is given by

$$\Pi_t^h = q_t^H I_t^H - I_t^H \left(1 + g_t^H\right).$$

where $g_t^H \equiv g^H(\frac{I_t^H}{I_{t-1}^H}) = \frac{\psi^H}{2} \left(\frac{I_t^H}{I_{t-1}^H} - 1 \right)^2$. The evolution of housing stock is

$$h_t = \left(1 - \delta_t^H\right) h_{t-1} + I_t^H \left(1 - g_t^H\right). \tag{84}$$

Intertemporal profits maximization with respect to I_t^h implies

$$q_t^H = 1 + g_t^H + \frac{I_t^H}{I_{t-1}^H} g_t^{H\prime} - \beta^s E_t \left(\frac{\lambda_{t+1}^s}{\lambda_t^s} \left(\frac{I_{t+1}^H}{I_t^H} \right)^2 g_{t+1}^{H\prime} \right). \tag{85}$$

Bank inside equity and deposits markets

The banks' inside equity and the deposits market clear when

$$W_t^b \left(1 - \chi^b\right) = \phi_t^F \left(q_t^K k_t - W_t^e \left(1 - \chi^e\right)\right) + b_t^m \phi_t^H, \tag{86}$$

and

$$n_t^b + d_t = b_t^m + q_t^K k_t - n_t^e. \tag{87}$$

Labour and housing markets

Aggregate labour is given by total savers and borrowers labour supply:

$$l_t = l_t^s + l_t^m. \tag{88}$$

Total housing is

$$h_t = h_t^s + h_t^m. \tag{89}$$

Deposit insurance agency

The total costs to the deposit insurance agency are given by

$$T_t = T_t^F + T_t^H = T_t^s + T_t^m + T_t^e \tag{90}$$

where

$$T_t^F = \widetilde{R}_t^F \left(\bar{\omega}_t^F - \Gamma_t^F + \mu^F G_t^F\right) \left(q_{t-1}^K k_{t-1} - (1-\chi^e) W_{t-1}^e\right) \tag{91}$$

and

$$T_t^H = \frac{\widetilde{R}_t^H \left(\bar{\omega}_t^H - \Gamma_t^H + \mu^H G_t^H\right) x_{t-1}^m h_{t-1}^m . q_{t-1}^H}{R_{t-1}^m}. \tag{92}$$

Cyclically-adjusted capital requirements

Finally, the cyclically-adjusted capital requirements are:

$$\phi_t^F = \bar{\phi}_0^F + \bar{\phi}_1^F \left[\log\left(b_{t-1}^H + b_{t-1}^F\right) - \log\left(\bar{b}^H + \bar{b}^F\right)\right] + \bar{\phi}_2^F \left[\log(GDP_{t-1}) - \log(\overline{GDP})\right] \tag{93}$$

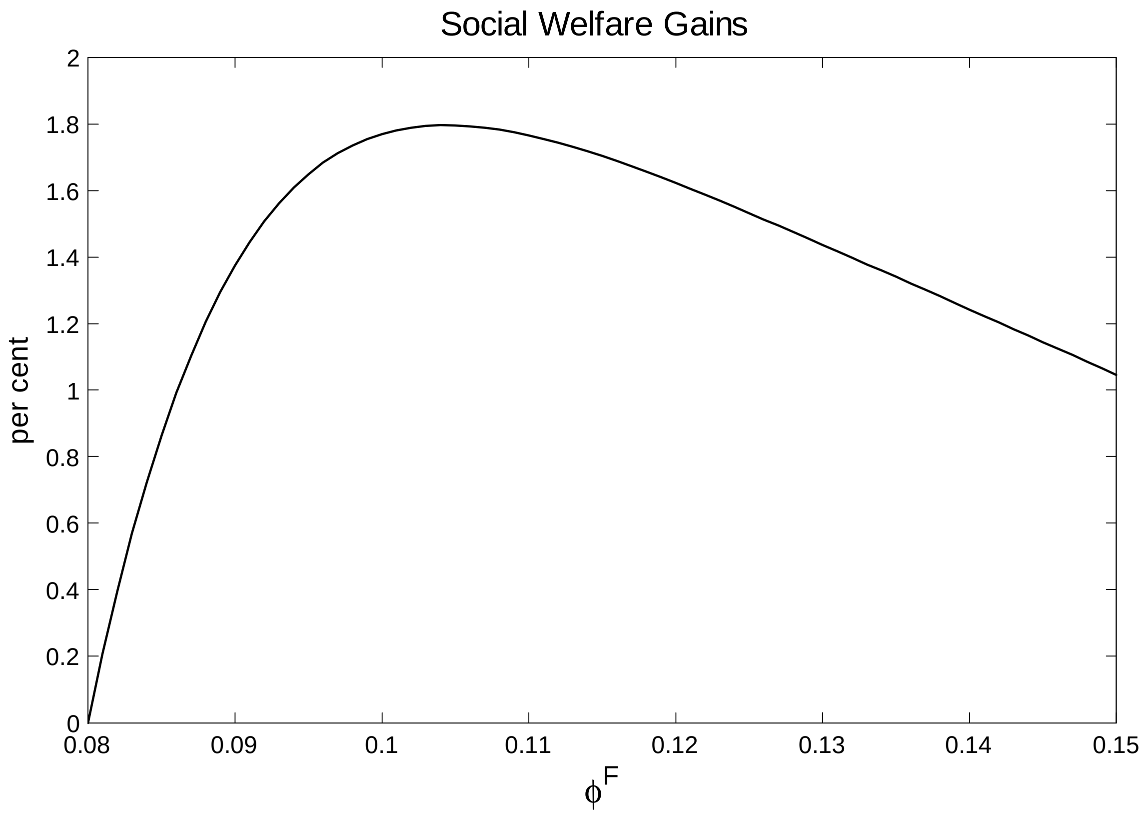
and

$$\phi_t^H = \bar{\phi}_0^H + \bar{\phi}_1^H \left[\log\left(b_{t-1}^H + b_{t-1}^F\right) - \log\left(\bar{b}^H + \bar{b}^F\right)\right] + \bar{\phi}_2^H \left[\log(GDP_{t-1}) - \log(\overline{GDP})\right]. \tag{94}$$

Competitive equilibrium

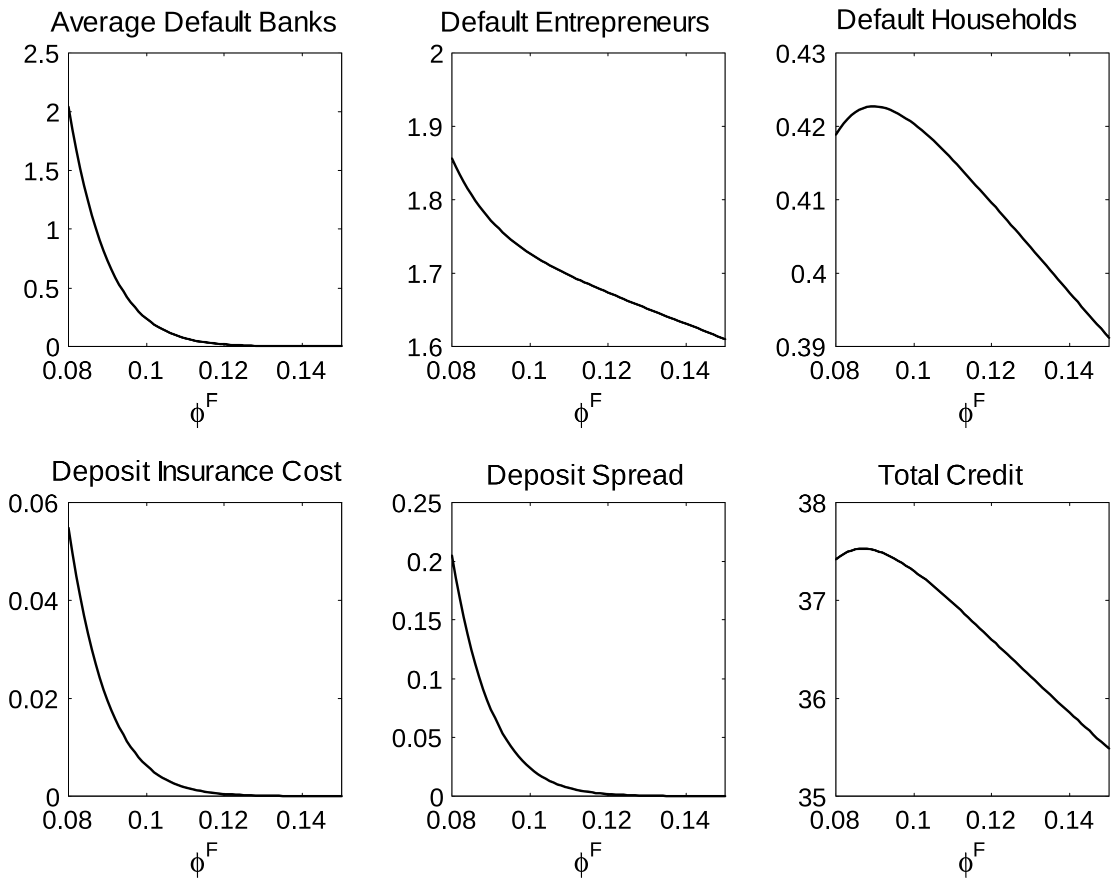
The competitive equilibrium of our model is a set of stationary processes b_t^m, c_t^m, c_t^s, d_t, e_t^F, h_t, h_t^m, h_t^s, I_t, I_t^H, k_t, l_t, l_t^m, l_t^s, λ_t^m, λ_t^s, n_t^b, n_t^e, $\bar{\omega}_t^e$, $\bar{\omega}_t^F$, $\bar{\omega}_t^H$, $\bar{\omega}_t^m$, ϕ_t^F, ϕ_t^H, q_t^H, q_t^K, r_t^K, R_t^D, R_t^F, $\widetilde{R}_t^F$, R_t^H, $\widetilde{R}_t^H$, R_t^K, R_t^m, $\widetilde{\rho}_t^F$, $\widetilde{\rho}_t^H$, T_t, T_t^F, T_t^H, w_t, W_t^b, W_t^e, x_t^e, x_t^m, ξ_t^e, ξ_t^m and y_t, satisfying the relations (48) to (94), given the exogenous stochastic processes A_t, δ_t, δ_t^H, v_t^m, v_t^s, φ_t^m, φ_t^s and the initial conditions b_{-1}^m, d_{-1}, e_{-1}^F, h_{-1}^m, h_{-1}^s, I_{-1}, I_{-1}^H, k_{-1}, n_{-1}^b, n_{-1}^e, ϕ_{-1}^F, ϕ_{-1}^H, q_{-1}^H, q_{-1}^K, R_{-1}^D, x_{-1}^e, x_{-1}^m.

Figure 1. Steady state.



Note: Social Welfare gains are the weighted average of the steady-state gains (losses) experienced by the representative agent of each type of houselhods (patient and impatient) measured in certainty-equivalent consumption terms. The weights are given by the consumption shares of each class of households under the initial reference policy (ϕ^F=0.08,ϕ^H=0.04). Alternative policies involve the value of ϕ^F described in the horizontal axes and ϕ^H= ϕ^F-0.04.

Figure 2a. Steady state values depending on the capital ratio (I).



Note: Alternative policies involve the value of ϕ^F of described in the horizontal axes and ϕ^H equal to $\phi^F - 0.04$.

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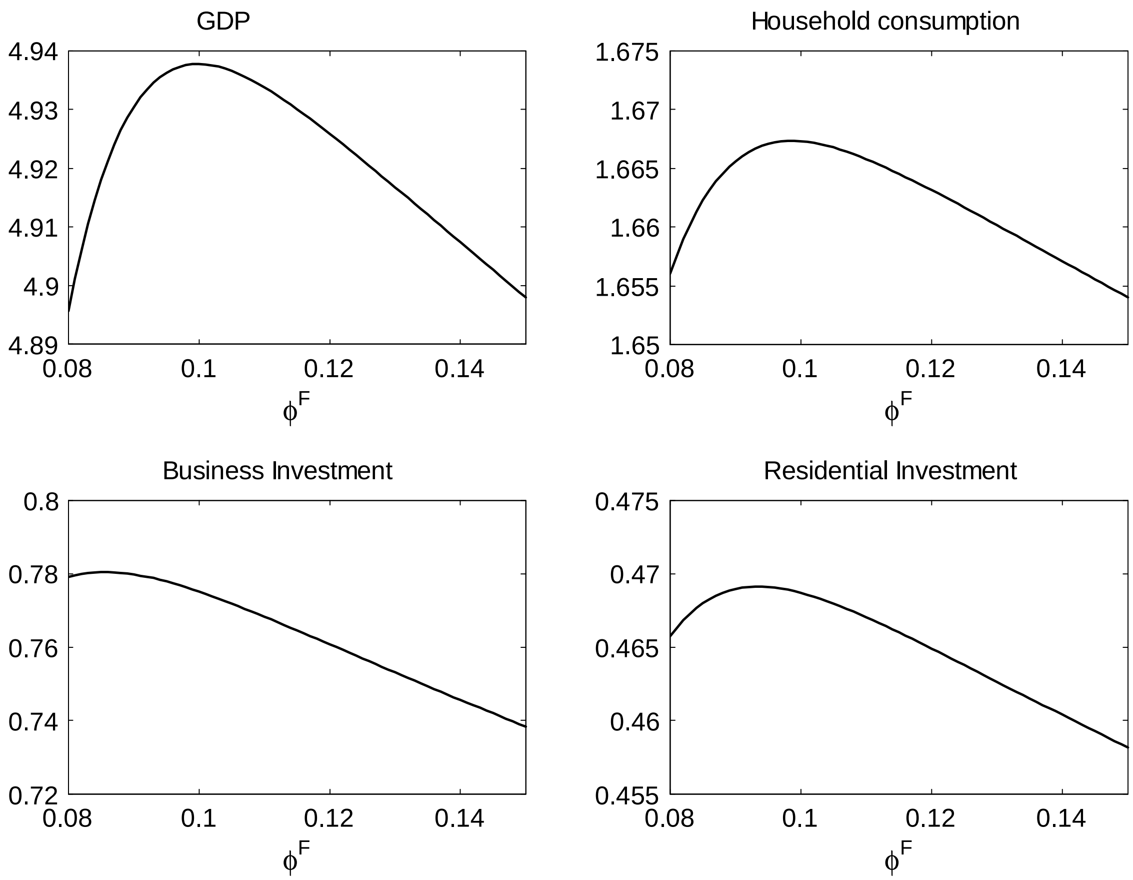
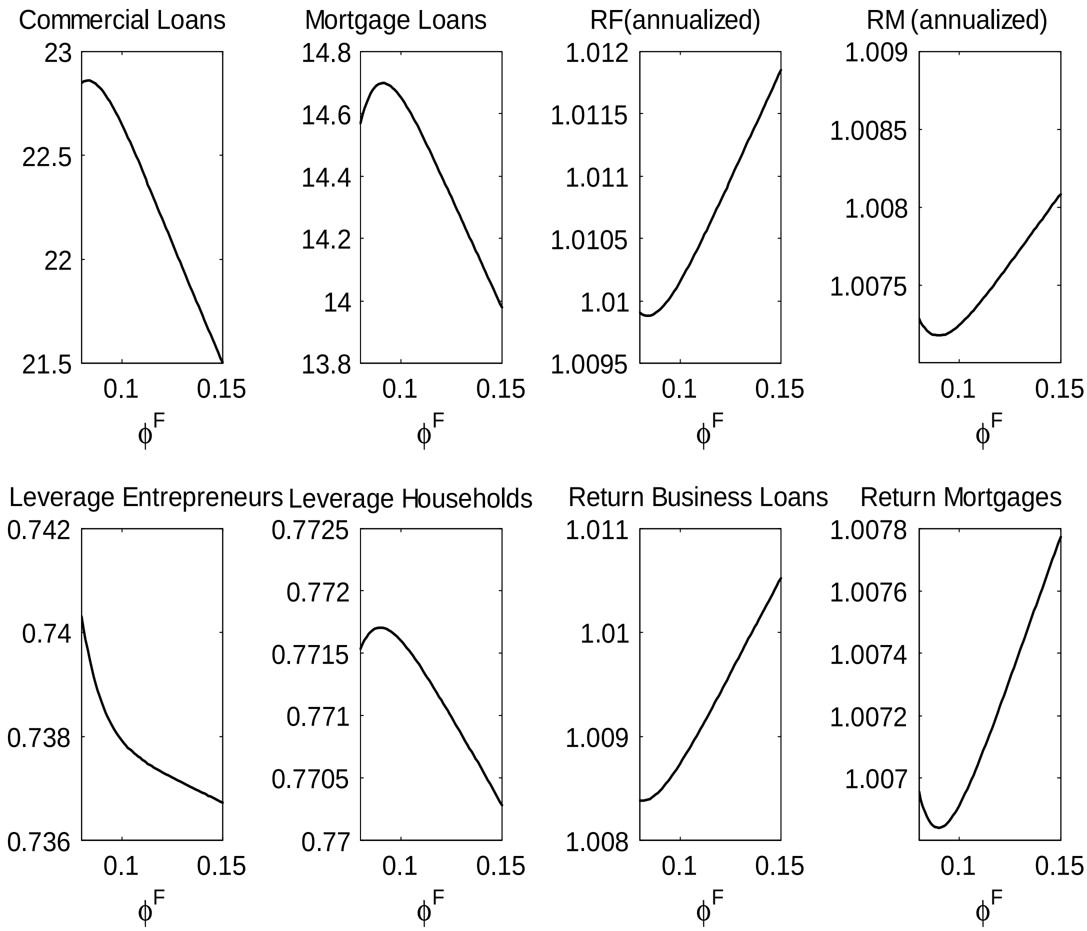
Figure 2b. Steady state values depending on the capital ratio (II).



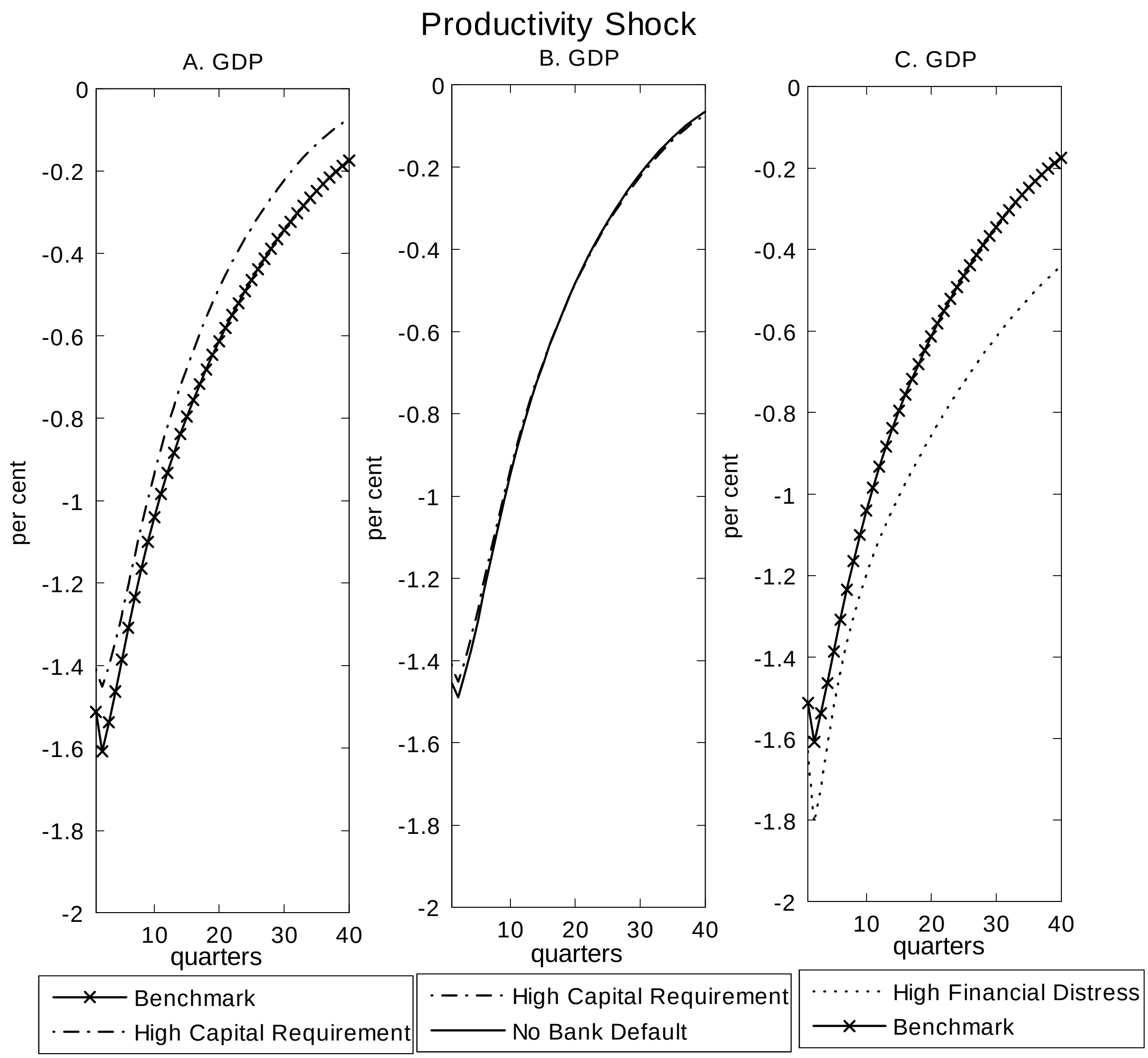
Note: Alternative policies involve the value of ϕ^F of described in the horizontal axes and ϕ^H equal to $\phi^F - 0.04$.

Figure 2c. Steady state values depending on the capital ratio (III).



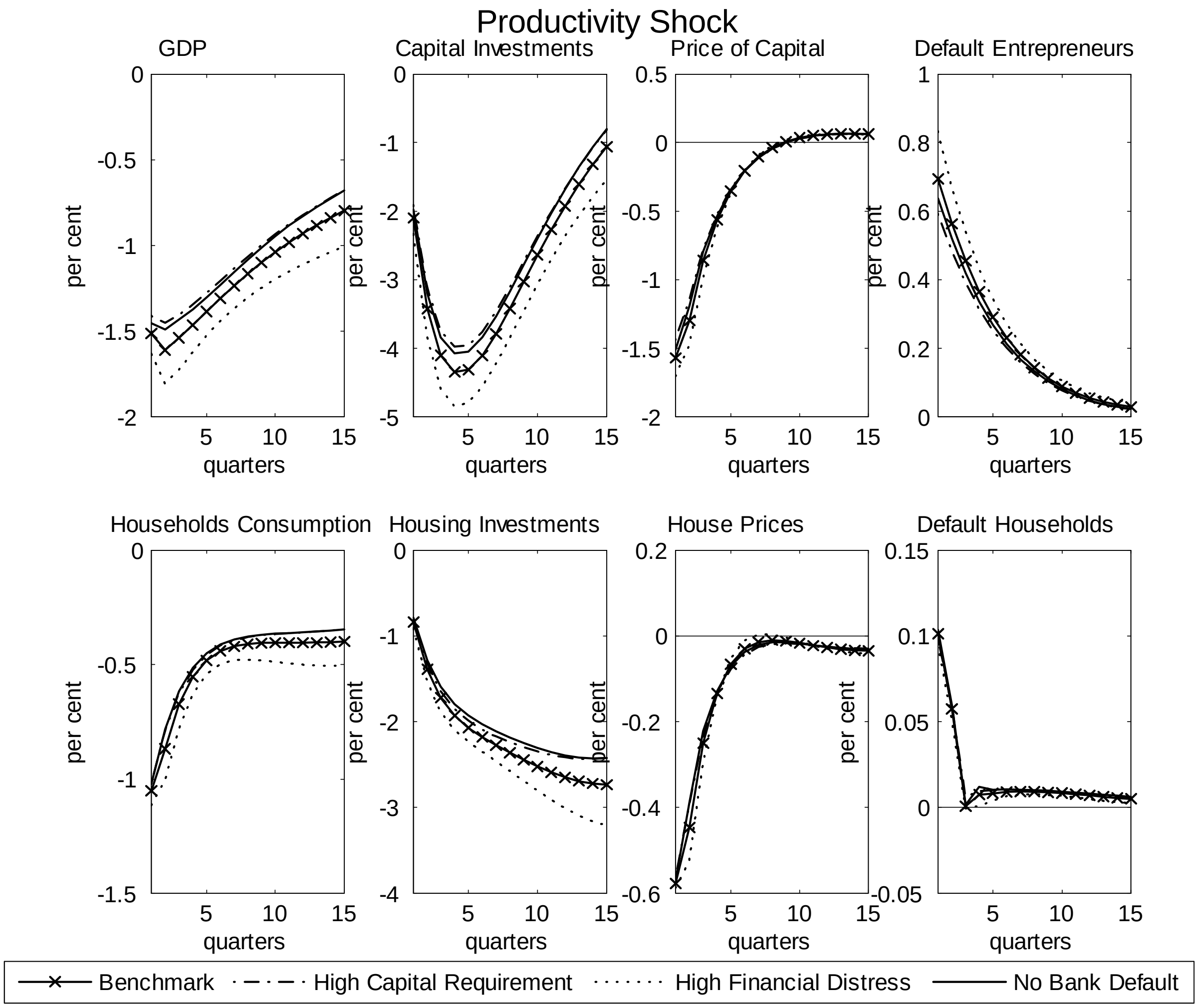
Alternative policies involve the value of ϕ^F of described in the horizontal axes and ϕ^H equal to $\phi^F - 0.04$.

Figure 3a. Impulse responses after a negative TFP shock: The effect of GDP under different assumptions on the bank capital ratio and bank risk



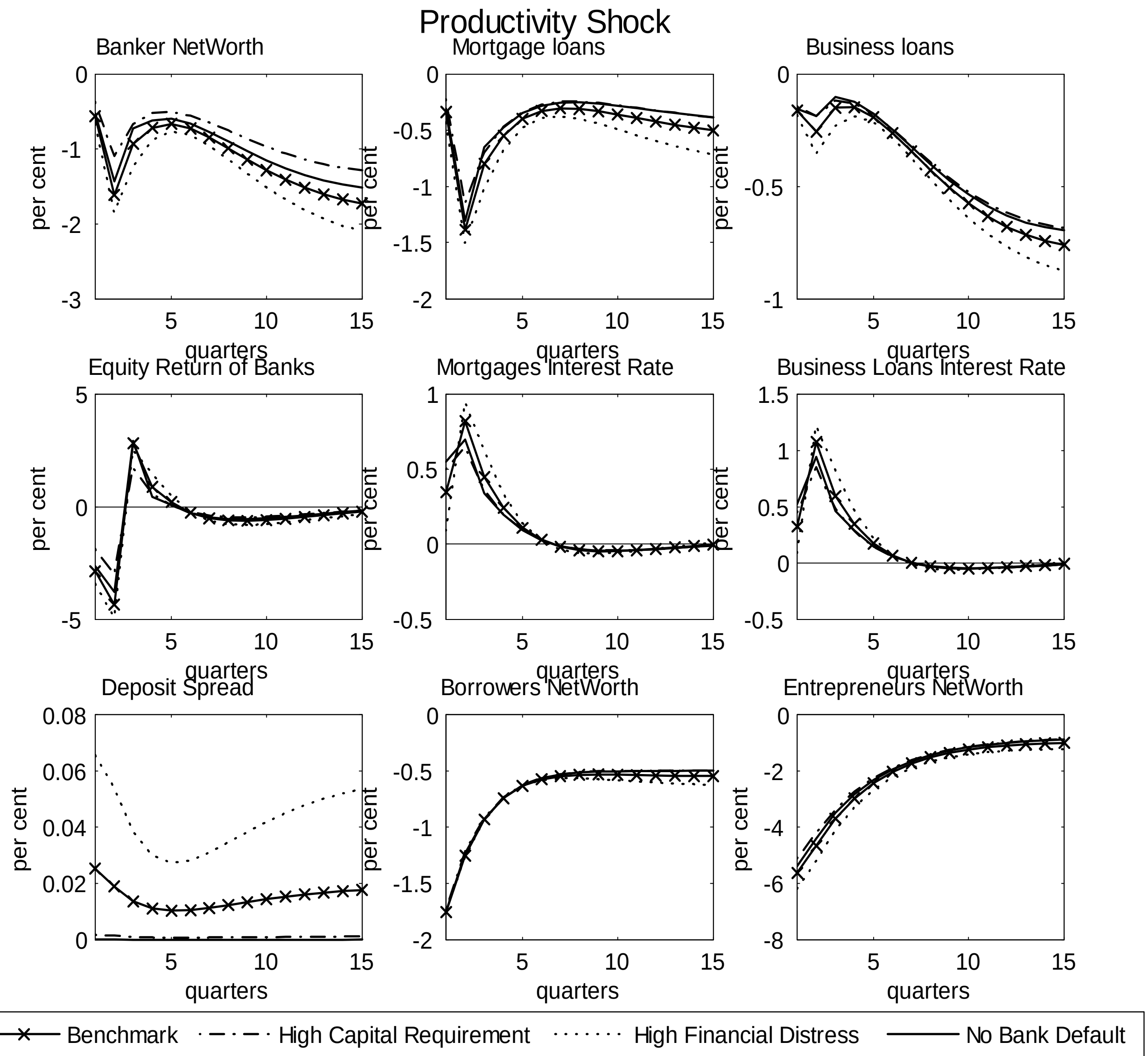
Note: "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output"

Figure 3b. Impulse responses after a negative TFP shock (II)



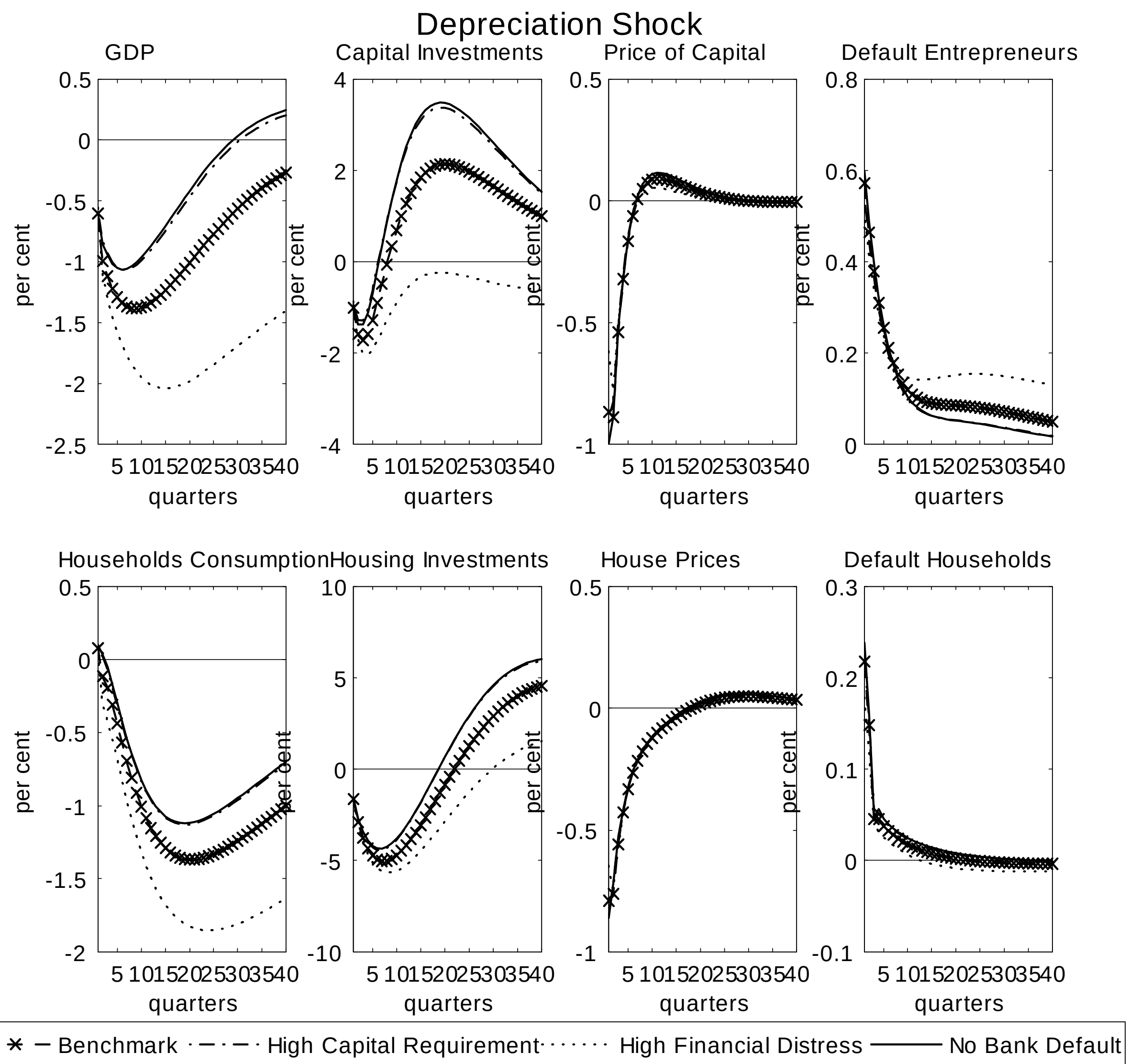
Note: "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 3c.. Impulse responses after a negative TFP shock (III)



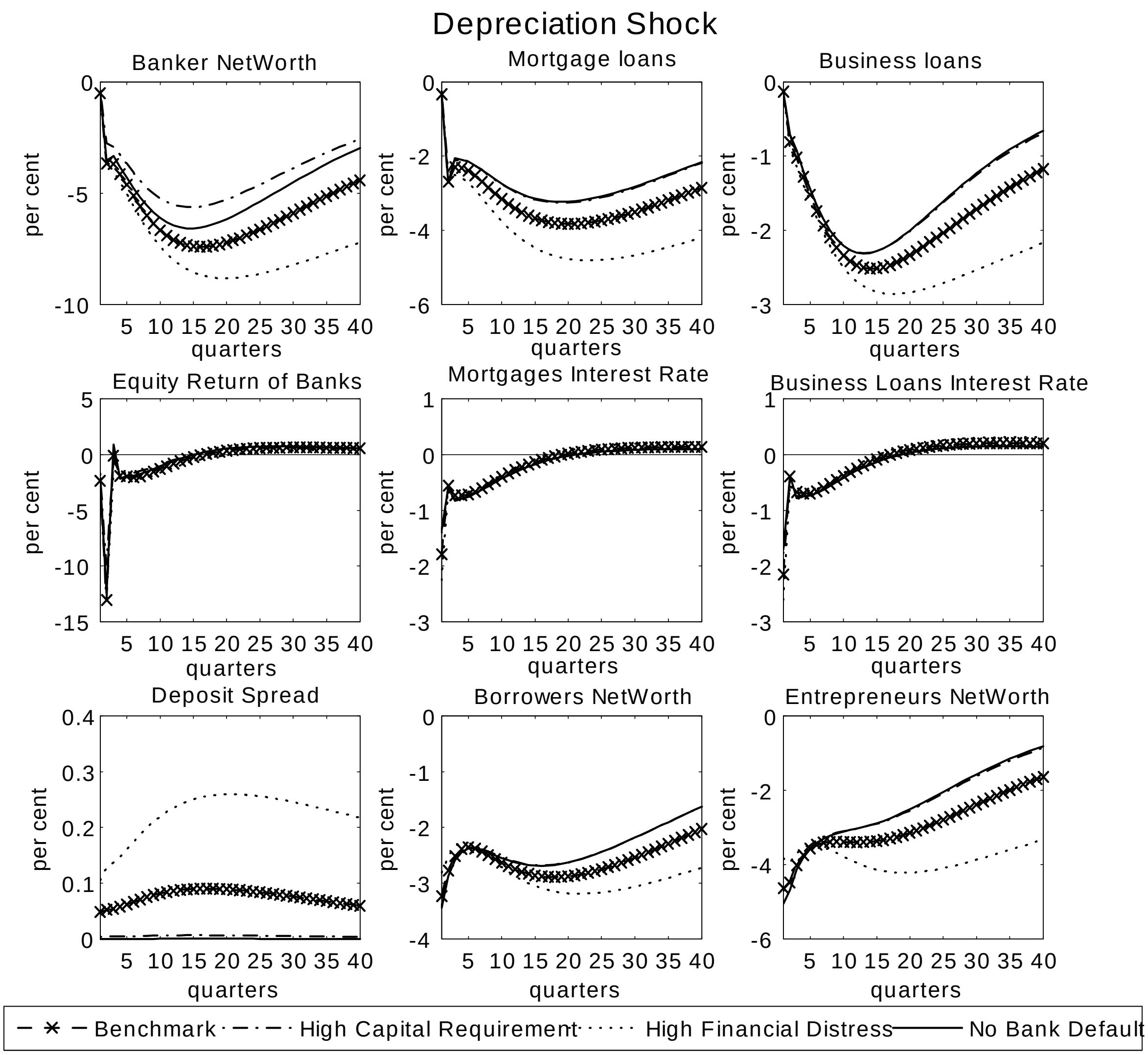
Note: "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 4a. Impulse responses after a shock to the housing and capital depreciation (I)



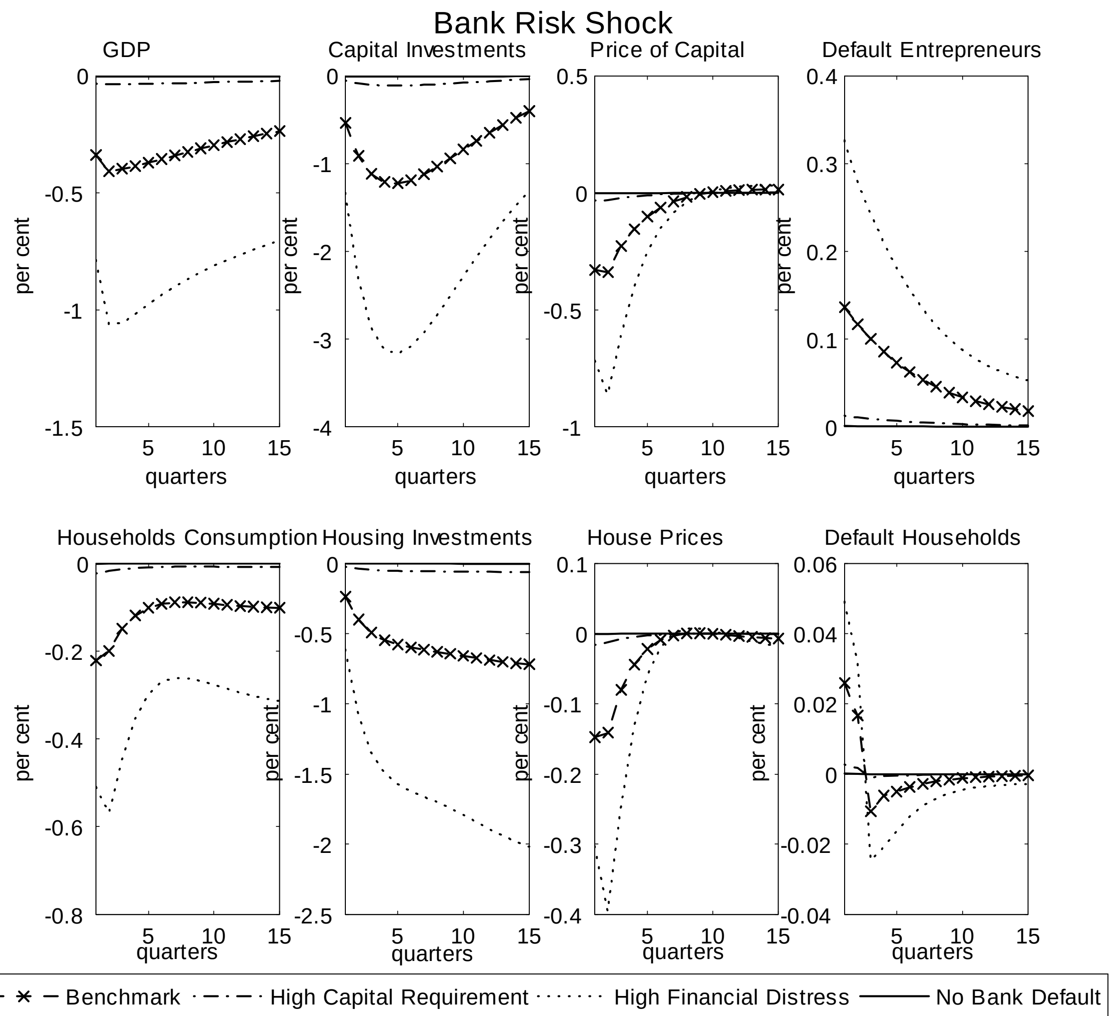
Note: "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 4b. Impulse responses after a shock to the housing and capital depreciation (II)



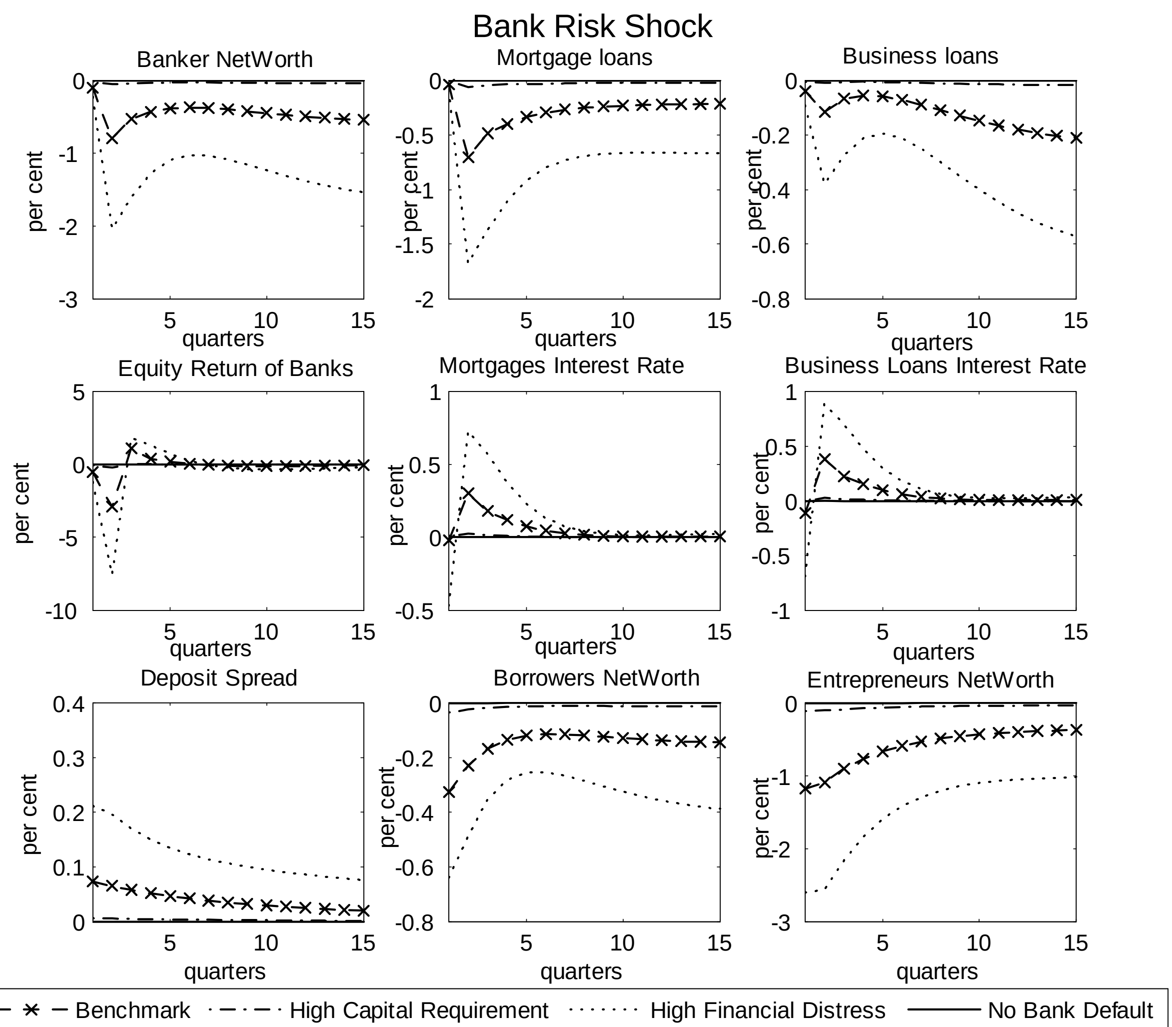
Note: "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 5a. Impulse responses after a shock to bank risk (I)



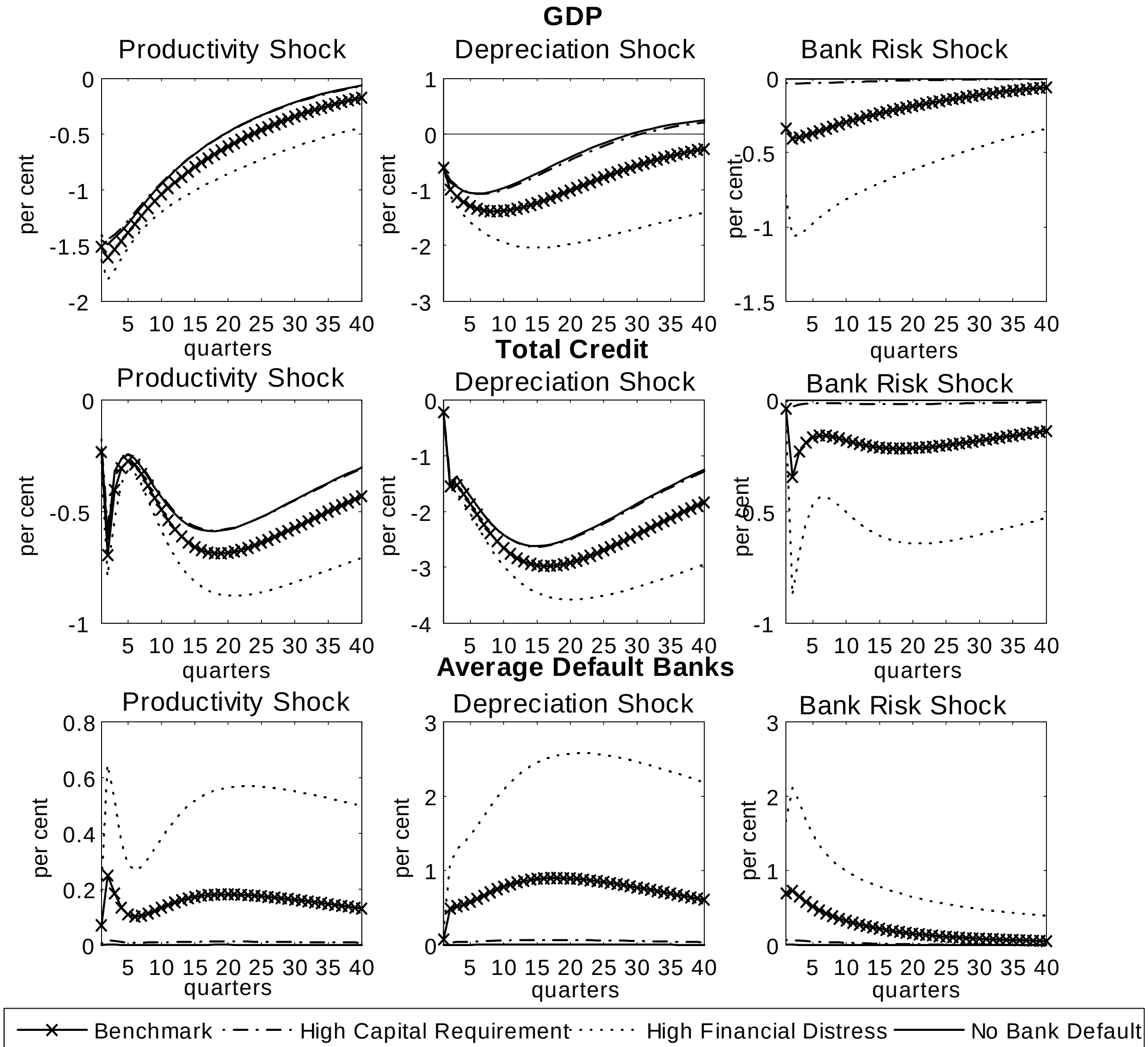
Note: A bank risk shock is an idiosyncratic shock to each bank's ability to extract payoffs from its loans. "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 5b. Impulse responses after a shock to bank risk (II)



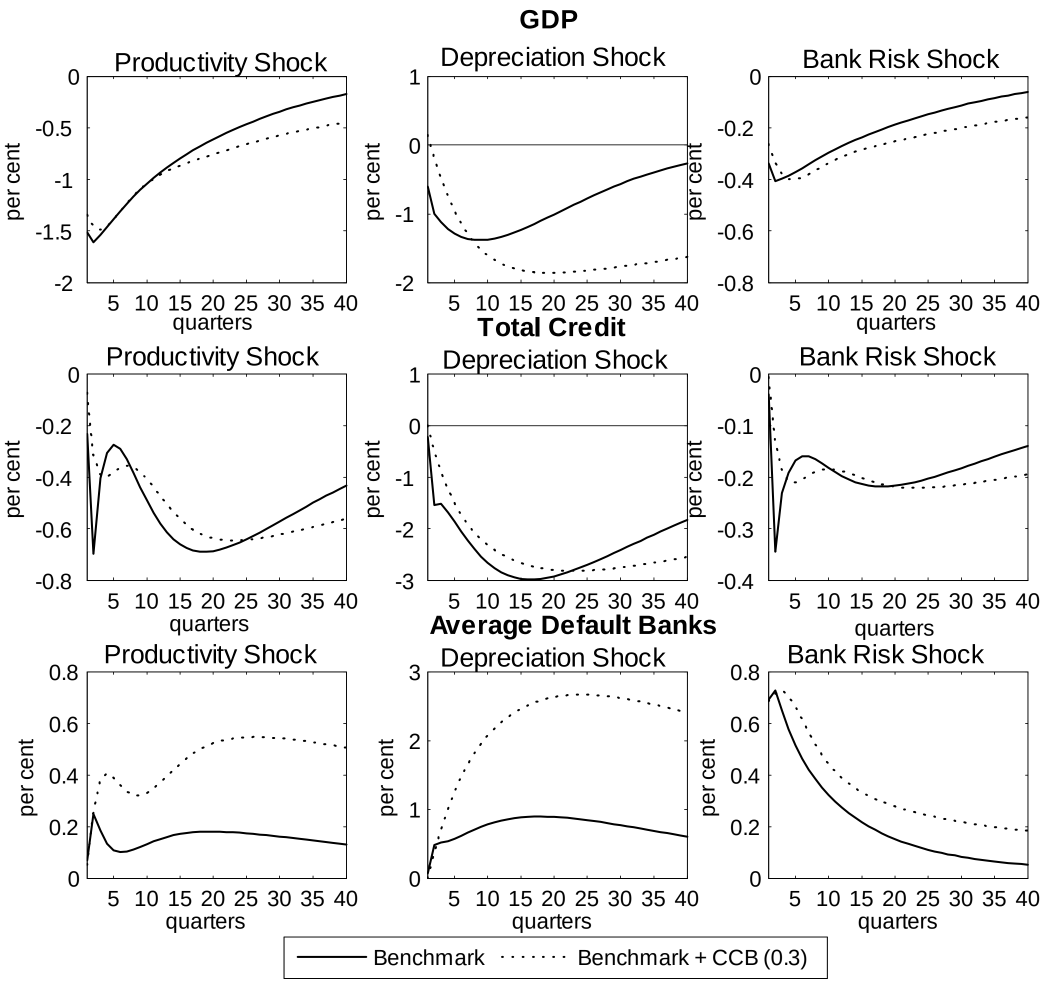
Note: A bank risk shock is an idiosyncratic shock to each bank's ability to extract payoffs from its loans. "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 6 Overview of key impulse responses after a shock to(i) productivity, (ii) depreciation and (iii) bank risk



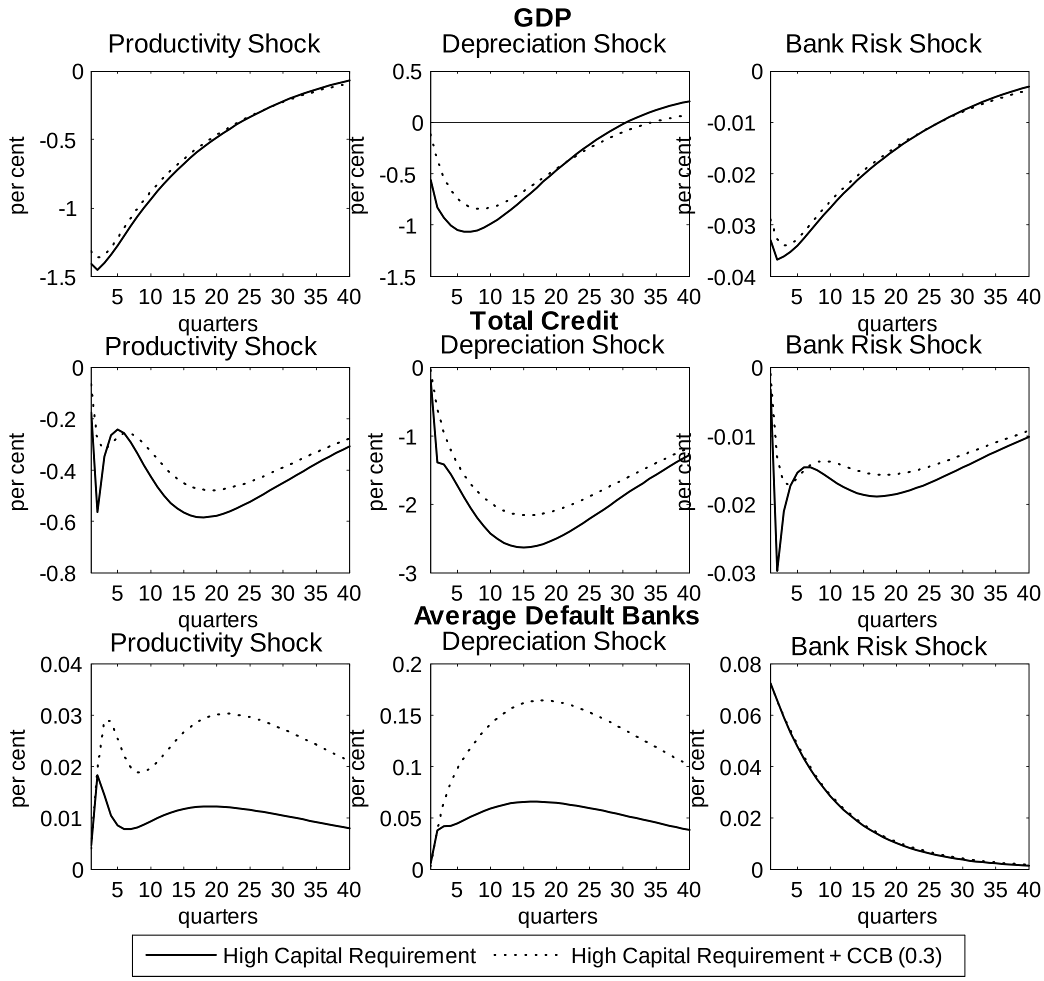
Note: A depreciation shock is a shock to the depreciation rates of capital and housing. A bank risk shock is an idiosyncratic shock to each bank's ability to extract payoffs from its loans. "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "High capital requirement" describes the economy with ϕ^F =10.5 and ϕ^H =6.5. "High financial distress" describes an economy with a variance of the idiosyncratic shock to banks 'performance higher than in the baseline parameterization. "No bank default" describes an economy in which the variance of the idiosyncratic shock to bank performance is zero. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 7a Overview of key impulse responses after a shock to(i) productivity, (ii) depreciation and (iii) bank risk: Benchmark capital requirements.



Note: A depreciation shock is a shock to the depreciation rates of capital and housing. A bank risk shock is an idiosyncratic shock to each bank's ability to extract payoffs from its loans. "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "Benchmark + CCB(0.3)" describes an economy in which the capital ratio reacts to the percentage deviation of total loans (corporate and mortgage) from their steady state values, with coefficient of 0.3. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".

Figure 7b Overview of key impulse responses after a shock to(i) productivity, (ii) depreciation and (iii) bank risk: Highcapital requirements.



Note: A depreciation shock is a shock to the depreciation rates of capital and housing. A bank risk shock is an idiosyncratic shock to each bank's ability to extract payoffs from its loans. "Benchmark" describes the economy with ϕ^F =0.08 and ϕ^H =0.04. "Benchmark + CCB(0.3)" describes an economy in which the capital ratio reacts to the percentage deviation of total loans (corporate and mortgage) from their steady state values, with coefficient of 0.3. GDP is defined as net of bankruptcy costs due to default. It is therefore a measure of "net output".